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M2-M5 systems in $\mathcal{N} = 6$ Chern-Simons theory

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Abstract: We study two aspects of M5-branes in $\mathcal{N} = 6$ $\mathrm{U}(N) \times \mathrm{U}(N)$ Chern-Simons gauge theory. We first examine multiple M2-branes ending on a M5-brane. We study Basu-Harvey type cubic equations, fuzzy funnel configurations, and derive the M5-brane tension from the $\mathcal{N} = 6$ theory. We also find a limit in which the above M2-M5 system reduces to a D2-D4 system and we recover the Nahm equation from the $\mathcal{N} = 6$ theory. We then examine domain wall configurations in mass-deformed $\mathcal{N} = 6$ theory with a manifest $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ global symmetry. We derive tensions of domain walls connecting between arbitrary M5-brane vacua of the deformed theory and observe their consistency with gravity dual expectations.

Keywords: Gauge-gravity correspondence, M-Theory, Brane Dynamics in Gauge Theories.

Contents

1. Introduction

Multiple M2-brane theory with a manifest SO(8) R-symmetry was shown [1, 2] to be consistent with a totally antisymmetric 3-algebraic description. The only finite dimensional Euclidean 3-algebra assuming total antisymmetry was based on the $so(4)$ 3-algebra with a quantized 4-index structure constant [11, 5]. The corresponding theory can be presented as a $\mathrm{SU}(2)_k \times \mathrm{SU}(2)_{-k}$ Chern-Simons gauge theory [7, 8] coupled to 8 scalars and 8 fermions in bi-fundamental representations [7]. The theory was shown to arise from two M2-branes moving in an orbifold of transverse R^8 space [9], and reduce to a maximally supersymmetric multiple D2-brane theory in a large k and large scalar vev limit [6, 9]. One-loop corrections to the couplings were considered in [41]. Generalizations to include an arbitrary higher rank non-abelian gauge symmetry lead to the Lorentzian 3-algebra [10, 20], but the corresponding theory contains ghost degrees of freedom due to the Lorentzian signature [10]. A ghost-removing procedure turns the theory into that of a dual description of the 3d maximally supersymmetric Yang-Mills theory [16 − 19]. Besides, infinite dimensional 3-algebras also exist [40, 21].

An alternative method to include a higher rank gauge symmetry was obtained very recently by considering the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ Chern-Simons gauge theory coupled to four $\mathcal{N} = 2$ superfields in bi-fundamental representations [12]. The Lagrangian of the theory exhibits a manifest SU(4) R-symmetry [13 − 15], see also [24 − 26, 38], and was proposed to arise from multiple M2-branes moving in a Z_k quotient of the transverse R^8 space [12]. The theory was also shown to be consistent with a 3-algebraic description with a less antisymmetric structure constant [14].

The present paper is motivated by trying to understand better the properties of this new $\mathcal{N}=6$ theory. A nice feature of the previous $\mathcal{N}=8$ theory is that it admits the Basu-Harvey equation [4, 42] with a SO(4) symmetry, and as a result, there are fuzzy funnel configurations describing multiple M2-branes gradually ending on a M5-brane wrapping a fuzzy 3-sphere. Another nice feature is that the $\mathcal{N}=8$ theory also admits a mass deformation keeping a $\mathrm{SO}(4)\times\mathrm{SO}(4)$ global symmetry [34−37, 22], which has multiple M5-brane vacua charaterized by M5-branes wrapping concentric fuzzy 3-spheres in two possible orthogonal R^4 spaces. In this paper, we will study these two aspects in the context of the $\mathcal{N}=6$ theory.

The organization of this paper is as follows. In section 2.1, we derive Basu-Harvey type equations by the method of forming perfect squares combining the kinetic terms with F-terms or D-terms. Related discussion but with slightly different methods was given in [24, 25]. In section 2.2, we analyze properties of the fuzzy funnel solutions and derive the M5-brane tension from the $\mathcal{N}=6$ theory. In section 2.3, we show a limit that the above Basu-Harvey equations reduce to Nahm equations describing D2-D4 systems, thus giving another consistency check. In section 3.1, we derive domain wall equations in the mass-deformed $\mathcal{N}=6$ theory keeping a $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ global symmetry [24]. In section 3.2, we analyze properties of the domain walls and compute their tensions, which are consistent with gravity dual descriptions in terms of M5-brane actions. In section 4, we briefly draw conclusions.

2. Basu-Harvey configurations and M2-M5 system

2.1 Bogomol'nyi completion

We begin by examining the bosonic potential in $\mathcal{N}=6$ $\mathrm{U}(N)\times\mathrm{U}(N)$ Chern-Simons theory and expressing it as a sum of several perfect squares. We basically follow the notation of [13], but use a different normalization condition for $\mathrm{U}(N)$ generators $\mathrm{tr}(T^aT^b)=(1/2)\delta^{ab}$. In this notation, the potential can be rewritten as

$$\begin{aligned} V_{\rm scalar} &= V_D + V_F \\ &= \frac{4\pi^2}{k^2}\mathrm{tr}\,\big(|Z^A Z^\dagger_A Z^B - Z^B Z^\dagger_A Z^A - W^{\dagger A} W_A Z^B + Z^B W_A W^{\dagger A}|^2 \\ &\qquad\qquad +|W^{\dagger A} W_A W^{\dagger B} - W^{\dagger B} W_A W^{\dagger A} - Z^A Z^\dagger_A W^{\dagger B} + W^{\dagger B} Z^\dagger_A Z^A|^2\big) \\ &\quad + \frac{16\pi^2}{k^2}\mathrm{tr}\left(|\epsilon_{AC}\epsilon^{BD} W_B Z^C W_D|^2 + |\epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D|^2\right), \end{aligned} \tag{2.1}$$

where $Z^A, W^{\dagger A},\ A=1,2$ are the lowest components of four $\mathcal{N}=2$ superfields respectively, and are all in the $(N,\overline{N})$ representations and have overall $\mathrm{U}(1)$ charges $+1$. The classical vacuum moduli space can be determined by demanding all the squares to be zero simultaneously. In this theory there is an additional residual Z_k symmetry which orbifolds the moduli space.

Next we want to consider Basu-Harvey type BPS equations, which have the dependence of only one of the spatial worldvolume coordinate, say $x^2=s$. The equations can be obtained by combining the kinetic terms and potential terms in the Hamiltonian and rewriting it as a sum of perfect squares plus some topological terms.

There are two ways to make combinations. If we combine the kinetic terms with F-term potentials, we obtain

$$\begin{aligned}
H &= \int dx^1 ds\ \mathrm{tr}(|\partial_s W^{\dagger A}|^2 + |\partial_s Z^A|^2 + V_{\mathrm{scalar}}) \\
&= \int dx^1 ds\ \mathrm{tr}\Big(|\partial_s W^{\dagger A} - \frac{4\pi}{k}\epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D|^2 + |\partial_s Z^A - \frac{4\pi}{k}\epsilon^{AC}\epsilon_{BD} W^{\dagger B} Z_C^\dagger W^{\dagger D}|^2 \\
&\qquad + \frac{4\pi^2}{k^2}|Z^A Z_A^\dagger Z^B - Z^B Z_A^\dagger Z^A - W^{\dagger A} W_A Z^B + Z^B W_A W^{\dagger A}|^2 \\
&\qquad + \frac{4\pi^2}{k^2}|W^{\dagger A} W_A W^{\dagger B} - W^{\dagger B} W_A W^{\dagger A} - Z^A Z_A^\dagger W^{\dagger B} + W^{\dagger B} Z_A^\dagger Z^A|^2 \Big) \\
&\quad + \frac{4\pi}{k}\epsilon_{AC}\epsilon^{BD}\int dx^1\ \mathrm{tr}(Z^A W_B Z^C W_D + W^{\dagger A} Z_B^\dagger W^{\dagger C} Z_D^\dagger)
\end{aligned} \tag{2.2}$$

or, if the kinetic terms are combined with D-term potentials, we get:

$$\begin{aligned}
H = \int dx^1 ds \mathrm{tr}\Big(& |\partial_s W^{\dagger A} + \frac{2\pi}{k}(W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^\dagger W^{\dagger A} + W^{\dagger A} Z_B^\dagger Z^B)|^2 \\
& + |\partial_s Z^A + \frac{2\pi}{k}(Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B})|^2 \\
& + \frac{16\pi^2}{k^2}|\epsilon_{AC}\epsilon^{BD} W_B Z^C W_D|^2 + \frac{16\pi^2}{k^2}|\epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D|^2 \Big) \\
+ \frac{\pi}{k}\int dx^1\ \mathrm{tr}(& W_A W^{\dagger A} W_B W^{\dagger B} - W^{\dagger A} W_A W^{\dagger B} W_B + 2 W^{\dagger A} W_A Z^B Z_B^\dagger \\
& - 2 W_A W^{\dagger A} Z_B^\dagger Z^B + Z_A^\dagger Z^A Z_B^\dagger Z^B - Z^A Z_A^\dagger Z^B Z_B^\dagger).
\end{aligned} \tag{2.3}$$

In each case, the last term is topological and doesn't affect the dynamics in the bulk. So we get a set of BPS equations, which minimizes the energy in a given topological sector:

$$\partial_s W^{\dagger A} - \frac{4\pi}{k}\epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D = 0 \tag{2.4}$$

$$\partial_s Z^A - \frac{4\pi}{k}\epsilon^{AC}\epsilon_{BD} W^{\dagger B} Z_C^\dagger W^{\dagger D} = 0 \tag{2.5}$$

$$Z^A Z_A^\dagger Z^B - Z^B Z_A^\dagger Z^A - W^{\dagger A} W_A Z^B + Z^B W_A W^{\dagger A} = 0 \tag{2.6}$$

$$W^{\dagger A} W_A W^{\dagger B} - W^{\dagger B} W_A W^{\dagger A} - Z^A Z_A^\dagger W^{\dagger B} + W^{\dagger B} Z_A^\dagger Z^A = 0 \tag{2.7}$$

for the F-term combination, and

$$\partial_s W^{\dagger A} + \frac{2\pi}{k}(W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^\dagger W^{\dagger A} + W^{\dagger A} Z_B^\dagger Z^B) = 0 \tag{2.8}$$

$$\partial_s Z^A + \frac{2\pi}{k}(Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B}) = 0 \tag{2.9}$$

$$\epsilon_{AC}\epsilon^{BD} W_B Z^C W_D = \epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D = 0 \tag{2.10}$$

for the D-term combination, respectively. The topological term gives the energy of the configuration when the BPS equations are satisfied.

JHEP09(2008)067

2.2 Fuzzy funnel solution and M5-brane tension

The new Basu-Harvey equation proposed in [25, 24] can be obtained by setting two complex scalars to be zero, and look at the non-trivial equations for the other two complex scalars. For example, we can set $W^{\dagger A} = 0,$ and $Z^A \neq 0$ in (2.9). The scalar part of the Hamiltonian is given as a square term plus a topological term:

$$H = \int dx^1 ds \ \mathrm{tr}\bigg(|\partial_s Z^A + \frac{2\pi}{k}(Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B)|^2\bigg) \\ + \frac{\pi}{k}\int dx^1 ds \ \partial_s \mathrm{tr}(Z_A^\dagger Z^A Z_B^\dagger Z^B - Z^A Z_A^\dagger Z^B Z_B^\dagger). \tag{2.11}$$

The first line gives a pair of BPS equations

$$\partial_s Z^A + \frac{2\pi}{k}(Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B) = 0, \tag{2.12}$$

where $A, B = 1, 2$. As opposed to the original Basu-Harvey equation in [4] which has a manifest SO(4) symmetry, the equation (2.12) has a manifest $\mathrm{SU}(2) \times \mathrm{U}(1)$ symmetry. As was argued in [25], this equation preserves half of the supersymmetries of the theory. For a configuration on which this equation is satisfied, the energy of the system is given by

$$E = \frac{\pi}{k}\int dx^1 \ \mathrm{tr}(Z_A^\dagger Z^A Z_B^\dagger Z^B - Z^A Z_A^\dagger Z^B Z_B^\dagger) \tag{2.13}$$

$$= 2\int ds dx^1 \mathrm{tr}(\partial_s Z_A^\dagger \partial_s Z^A). \tag{2.14}$$

We used the BPS equation (2.12) to obtain the second line.

To solve the BPS equation (2.12), we may separate the s-dependent and independent part:

$$Z^A = f(s) G^A, \quad f(s) = \sqrt{\frac{k}{4\pi s}}, \tag{2.15}$$

where G^As are $N \times \overline{N}$ matrices satisfying

$$G^A = G^B G_B^\dagger G^A - G^A G_B^\dagger G^B. \tag{2.16}$$

This equation is solved in [24] (see also [25]). One can diagonalize $G_1^\dagger$ using the $\mathrm{U}(N) \times \mathrm{U}(N)$ transformations and find that the other matrix $G_2^\dagger$ must be off-diagonal. The $G_A^\dagger$s have some nice properties: For a N dimensional irreducible solution,

$$(G_1^\dagger)_{m,n} = \sqrt{m-1}\delta_{m,n}, \quad (G_2^\dagger)_{m,n} = \sqrt{N-m}\delta_{m+1,n}, \tag{2.17}$$

$$G^1 G_1^\dagger = \mathrm{diag}\ (0, 1, 2, \ \ldots, N-1) = \ G_1^\dagger G^1 \tag{2.18}$$

$$G^2 G_2^\dagger = \mathrm{diag}\ (N-1, N-2, \ \ldots, 1, 0) \tag{2.19}$$

$$G_2^\dagger G^2 = \mathrm{diag}\ (0, N-1, N-2, \ \ldots, 1) \tag{2.20}$$

$$G^A G_A^\dagger = (N-1)\mathbf{1}_{N\times N}, \quad \mathrm{tr}(G^A G_A^\dagger) = N(N-1). \tag{2.21}$$

The eigenvalues of the matrices $G^1 G_1^\dagger$ and $G^2 G_2^\dagger$ may be interpreted as the squares of the radial positions of the points on a fuzzy 3-sphere projected onto 2 complex planes, respectively. Since there is a overall Z_k residual symmetry, the solution would describe a fuzzy S^3/Z_k.

The energy formula (2.14) is expressed in terms of fields Z^A, which is of mass dimension 1/2 and does not have the correct mass dimension -1 as a spatial coordinate. The correct normalization should reproduce the scalar kinetic term of the form,

$$S_{\text{kinetic}} = -T_2 \int d^3x \mathrm{tr}(\partial_\mu X_A^\dagger \partial^\mu X^A), \tag{2.22}$$

where T_2 is the M2-brane tension and X^A is the (complexified) spatial coordinate. This implies that we should relate X^A and Z^A by

$$X^A = \sqrt{\frac{1}{T_2}} Z^A. \tag{2.23}$$

Using this, we can define the radius averaged over each M2-brane as

$$\begin{aligned} R^2 &= \frac{2\mathrm{tr}(X_A^\dagger X^A)}{N} = \frac{2(N-1)}{T_2} f^2 & (2.24) \\ &= \frac{k(N-1)}{2\pi T_2} \cdot \frac{1}{s} & (2.25) \end{aligned}$$

The factor of two in the numerator comes from our normalization condition $\mathrm{tr}(T^a T^b) = (1/2)\delta^{ab}$. The radius vanishes for $N = 1$, and there are non-trivial fuzzy 3-spheres only for $N \geq 2$.

Combining all the above results, after some algebra, we obtain

$$\begin{aligned} E &= \frac{T_2^2}{2\pi} \frac{N}{N-1} \int dx^1 \left(\frac{2\pi^2}{k}\right) R^3 dR & (2.26) \\ &= \frac{T_2^2}{2\pi} \frac{N}{N-1} \int d^5x. & (2.27) \end{aligned}$$

The factor k in the denominator represents the fact that this M5-brane is divided by the Z_k orbifold action, and $\frac{2\pi^2}{k}$ is the volume of an S^3/Z_k with a unit radius. So the M5-brane wraps an S^3/Z_k. The M5-brane tension predicted from the $\mathcal{N} = 6$ theory is

$$T_5 = \frac{T_2^2}{2\pi} \frac{N}{N-1}. \tag{2.28}$$

The relation between M2-brane and M5-brane tension can also be derived in different ways, by matching the M-theory and type II string theory BPS spectrum [28], or by applying flux and Dirac quantization rules in eleven dimensions [29]:

$$T_5 = \frac{T_2^2}{2\pi}. \tag{2.29}$$

We see that for large N including the numerical coefficient, (2.28) exactly agrees with the known result (2.29). The $1/N$ deviation is due to the fuzziness of the 3-sphere in the finite N regime, and will disappear in the continuum limit for the fuzzy 3-sphere.

JHEP09(2008)067

2.3 Basu-Harvey equations and reduction to Nahm equations

In this section we take a limit in which M2-brane theory reduces to D2-brane theory [30, 31] and show that the Basu-Harvey equation (which describes D2-D4 system.

We take a diagonal expectation value in one of the direction, for example, the direction labelled by 3 and expand the fields around the vacuum:

$$Z^1 = (x^{10} + ix^{20})T^0 + X^1 + iX^2 \tag{2.30}$$

$$Z^2 = ((v + x^{30}) + ix^{40})T^0 + X^3 + iX^4 \tag{2.31}$$

Here, x's represent the $\mathrm{U}(1)$ part and $T^0 = \frac{1}{\sqrt{2N}}\mathbf{1}$ for normalization purpose, $\mathrm{tr}(T^0T^0) = 1/2$. X's take value on $\mathrm{SU}(N)$. We take N and v/k finite and fixed, and suppose v is large, and then we will neglect $o(1/v)$ terms in the calculation below.

By plugging (2.30) into the BPS equation (2.12), we see that

$$\partial_s Z^2 = \frac{2\pi}{k}(Z^2 Z_1^\dagger Z^1 - Z^1 Z_1^\dagger Z^2) \tag{2.32}$$

$$= \frac{2\pi v}{k\sqrt{2N}}[Z_1^\dagger, Z^1] \tag{2.33}$$

$$= \frac{4\pi v}{k\sqrt{2N}} i[X^1, X^2] \tag{2.34}$$

$\mathrm{U}(1)$ part decouples from the equations and we simply set them to zero. $\mathrm{SU}(N)$ part implies

$$\partial_s X^3 = \frac{4\pi v}{k\sqrt{2N}} i[X^1, X^2], \quad \partial_s X^4 = 0 \tag{2.35}$$

where we compared hermitian and anti-hermitian parts respectively.

In the same way, we can calculate the other component equation

$$\partial_s Z^1 = \frac{2\pi}{k}(Z^1 Z_2^\dagger Z^2 - Z^2 Z_2^\dagger Z^1) \tag{2.36}$$

$$= \frac{2\pi v}{k\sqrt{2N}}[Z^1, Z_2^\dagger + Z^2] \tag{2.37}$$

$$= \frac{4\pi v}{k\sqrt{2N}}([X^1, X^3] + i[X^2, X^3]) \tag{2.38}$$

So we get

$$\partial_s X^1 = \frac{4\pi v}{k\sqrt{2N}} i[X^2, X^3] \tag{2.39}$$

$$\partial_s X^2 = \frac{4\pi v}{k\sqrt{2N}} i[X^3, X^1] \tag{2.40}$$

Combining the above results, we get

$$\partial_s X^i = i\frac{1}{2} g_{\mathrm{YM}} \epsilon^{ijk}[X^j, X^k] \tag{2.41}$$

where $i, j, k = 1, 2, 3$ and ϵ^{ijk} is the totally antisymmetric tensor. By using $g_{\mathrm{YM}} = 4\pi v/k\sqrt{2N}$ as in the M2 to D2 reduction [30, 31] for the $\mathcal{N} = 6$ theory, we get the Nahm equation with the exact coefficient, in the large v and large k limit, with N and v/k fixed and finite. This describes multiple D2-branes ending on a D4-brane wrapping an S^2, and the reduction process makes an S^3/Z_k reducing to an S^2 that the D4-brane wraps.

JHEP09(2008)067

3. Domain wall configurations and M2-M5 system

3.1 Domain wall equations

In this section, we turn to the discussion of another aspect of the M5-branes in the $\mathcal{N}=6$ theory. For the $\mathcal{N}=8$ M2-brane theory on flat space, we can turn on four fermion mass terms, which preserve at least $\mathcal{N}=2$ supersymmetry. The most symmetric mass deformation is the one preserving a $\mathrm{SO}(4)\times\mathrm{SO}(4)$ symmetry [34–36] and a $\mathrm{SU}(2|2)\times\mathrm{SU}(2|2)$ superalgebra. In this case, M5-branes can wrap either of the two geometric S^3s in orthogonal R^4s.

In the case of $\mathcal{N}=6$ formulation, the most symmetric mass deformation turns out to preserve a manifest $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry [24] (see also related discussion [38, 39]) and we expect to have a $\mathrm{SU}(2|2)\times\mathrm{SU}(1|1)$ superalgebra. While, in this case, M5-branes can wrap either of two possible geometric (S^3/Z_k)s, where the Z_k action is due to the residual symmetry, which squash the 3-spheres along their Hopf fiber directions while maintaining a manifest $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry, as in (2.12).

We can turn on a D-term deformation corresponding to adding a FI term as found in [24]. In our notation, we have the deformed potential

$$\begin{aligned}V_{\rm scalar} &= V_D+V_F\\ &=\frac{4\pi^2}{k^2}\mathrm{tr}\bigg(|-\frac{k}{2\pi}\mu Z^B+Z^AZ^\dagger_AZ^B-Z^BZ^\dagger_AZ^A-W^{\dagger A}W_AZ^B+Z^BW_AW^{\dagger A}|^2\\ &\qquad +|-\frac{k}{2\pi}\mu W^{\dagger B}+W^{\dagger A}W_AW^{\dagger B}-W^{\dagger B}W_AW^{\dagger A}-Z^AZ^\dagger_AW^{\dagger B}+W^{\dagger B}Z^\dagger_AZ^A|^2\bigg)\\ &\quad+\frac{16\pi^2}{k^2}\mathrm{tr}\left(|\epsilon_{AC}\epsilon^{BD}W_BZ^CW_D|^2+|\epsilon^{AC}\epsilon_{BD}Z^BW_CZ^D|^2\right)\end{aligned}\tag{3.1}$$

where μ is a canonical mass parameter.

We perform the Bogomol'nyi completion combining the kinetic terms and D-terms similar to (2.3), and we get

$$\begin{aligned}H=&\int dx^1ds\mathrm{tr}\bigg(|\partial_sW^{\dagger A}-\mu W^{\dagger A}+\frac{2\pi}{k}(W^{\dagger B}W_BW^{\dagger A}-W^{\dagger A}W_BW^{\dagger B}\\ &\qquad\qquad -Z^BZ^\dagger_BW^{\dagger A}+W^{\dagger A}Z^\dagger_BZ^B)|^2\\ &\qquad +|\partial_sZ^A-\mu Z^A+\frac{2\pi}{k}(Z^BZ^\dagger_BZ^A-Z^AZ^\dagger_BZ^B-W^{\dagger B}W_BZ^A+Z^AW_BW^{\dagger B})|^2\\ &\qquad +\frac{16\pi^2}{k^2}|\epsilon_{AC}\epsilon^{BD}W_BZ^CW_D|^2+\frac{16\pi^2}{k^2}|\epsilon^{AC}\epsilon_{BD}Z^BW_CZ^D|^2\bigg)\\ &+\frac{\pi}{k}\int dx^1\ \mathrm{tr}(W_AW^{\dagger A}W_BW^{\dagger B}-W^{\dagger A}W_AW^{\dagger B}W_B+2W^{\dagger A}W_AZ^BZ^\dagger_B\\ &\qquad -2W_AW^{\dagger A}Z^\dagger_BZ^B+Z^\dagger_AZ^AZ^\dagger_BZ^B-Z^AZ^\dagger_AZ^BZ^\dagger_B)\\ &+\int dx^1\ \mathrm{tr}(\mu W^{\dagger A}W_A+\mu Z^AZ^\dagger_A)\end{aligned}\tag{3.2}$$

New boundary topological terms are produced at the same time when the BPS equations are modified.

The BPS domain wall equations are

$$\partial_s W^{\dagger A} - \mu W^{\dagger A} + \frac{2\pi}{k}(W^{\dagger B} W_B W^{\dagger A} - W^{\dagger A} W_B W^{\dagger B} - Z^B Z_B^\dagger W^{\dagger A} + W^{\dagger A} Z_B^\dagger Z^B) = 0 \quad (3.3)$$

$$\partial_s Z^A - \mu Z^A + \frac{2\pi}{k}(Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B - W^{\dagger B} W_B Z^A + Z^A W_B W^{\dagger B}) = 0 \quad (3.4)$$

$$\epsilon_{AC}\epsilon^{BD} W_B Z^C W_D = \epsilon^{AC}\epsilon_{BD} Z^B W_C Z^D = 0. \quad (3.5)$$

The equations are modified by just adding the linear terms.

3.2 Domain wall solutions and their tensions

In this section we discuss solutions of these domain wall configurations and derive their tensions. Setting $W^{\dagger A} = 0$ in equations (3.3)–(3.5), we need to solve

$$\partial_s Z^A - \mu Z^A + \frac{2\pi}{k}(Z^B Z_B^\dagger Z^A - Z^A Z_B^\dagger Z^B) = 0 \quad (3.6)$$

We assume the ansatz

$$Z^A = h(s) G^A, \qquad G^A = G^B G_B^\dagger G^A - G^A G_B^\dagger G^B \quad (3.7)$$

$$\partial_s h - \mu h + \frac{2\pi}{k} h^3 = 0 \quad (3.8)$$

We then obtain two solutions

$$h_1(s) = \sqrt{\frac{k\mu}{2\pi\,(1 - e^{-2\mu s})}} \quad (3.9)$$

$$h_2(s) = \sqrt{\frac{k\mu}{2\pi\,(1 + e^{-2\mu s})}} \quad (3.10)$$

The first solution h_1 describes a fuzzy funnel where $s \in (0, \infty)$, and in the $\mu \to 0$ limit reproduces (2.15). The second solution h_2 is a domain wall solution where $s \in (-\infty, \infty)$. We have

$$h_2(-\infty) = 0, \qquad h_2(+\infty) = \sqrt{\frac{k\mu}{2\pi}} \quad (3.11)$$

so this domain wall solution

$$Z^A = \sqrt{\frac{k\mu}{2\pi\,(1 + e^{-2\mu s})}} G^A \quad (3.12)$$

connects a trivial vacuum with a nontrivial fuzzy sphere vacuum $\sqrt{\frac{k\mu}{2\pi}} G^A$.

The non-vanishing boundary terms when $W^{\dagger A} = 0$ are

$$H = \int dx^1 ds \partial_s \mathrm{tr}(\mu Z^A Z_A^\dagger) + \frac{\pi}{k}\int dx^1 ds \partial_s \mathrm{tr}(Z_A^\dagger Z^A Z_B^\dagger Z^B - Z^A Z_A^\dagger Z^B Z_B^\dagger) \quad (3.13)$$

$$= \int dx^1 \ \mathrm{tr}\left(\frac{1}{2}\mu Z^A Z_A^\dagger\right)\Big|_{s=-\infty}^{s=\infty} = 2\int dx^1 ds \ \mathrm{tr}(\partial_s Z^A \partial_s Z_A^\dagger) \quad (3.14)$$

$$= \int dx^1 \left(\frac{k\mu^2}{4\pi}\right) \mathrm{tr}(G^A G_A^\dagger)|_{s=-\infty}^{s=\infty} \tag{3.15}$$

$$= \int dx^1 \frac{k}{4\pi}\mu^2 N(N-1) \tag{3.16}$$

where in deriving the second line in (3.14) we have used the equation of motion (3.6) to simplify

$$\frac{\pi}{k}(Z^A Z_B^\dagger Z^B Z_A^\dagger - Z^B Z_B^\dagger Z^A Z_A^\dagger) = -\frac{1}{2}\mu Z^A Z_A^\dagger + \frac{1}{2}(\partial_s Z^A) Z_A^\dagger \tag{3.17}$$

and used the fact that $\frac{1}{2}(\partial_s Z^A)Z_A^\dagger$ vanishes for both $s = -\infty$ and $s = \infty$.

Thereby the tension of this domain wall is

$$\tau = \frac{k}{4\pi}\mu^2 N(N-1) \tag{3.18}$$

It agrees with other results for slightly different theories as discussed in [1], and the second ref. in [22].

Since (3.17), (3.14) are the general results for general domain wall solutions, we see that the expression (3.15) should be a general result for the tension of a domain wall between two arbitrary vacua labelled by integers $\{N'_i|_{s=-\infty}, i = 1, \ldots, p'\}$, $\{N_i|_{s=\infty}, i = 1, \ldots, p\}$, in which the integers label the dimensions of irreducible solutions of the p' and p diagonal-block matrices in $G^A|_{s=-\infty}$ and $G^A|_{s=\infty}$ respectively. The tension of the domain wall between these two arbitrary vacua is therefore

$$\tau = \frac{k\mu^2}{4\pi}\sum_{i=1}^{p} N_i(N_i - 1)|_{s=\infty} - \frac{k\mu^2}{4\pi}\sum_{i=1}^{p'} N'_i(N'_i - 1)|_{s=-\infty} \tag{3.19}$$

The dependence of (3.18) on mass and N also agrees with the gravity dual analysis in [32] based on computing the action of a M5-brane filling a 4-ball bounded by the 3-sphere on which the M5-brane constructed from M2-branes wraps. The probe M5-brane is also along the $R^{1,1}$ part of the M2-brane worldvolume directions. This computation can also be performed by calculating the action of a M5-brane wrapping a S^3 as well as the x_2 line-segment across the fermion band at $y = 0$ in the gravity geometry in [36, 37]. In this gravity picture, it is suggestive that if the fermion band is narrow, the M5-brane action is expected to be small.

4. Conclusions and discussion

In this paper we have studied two problems of M5-branes in the $\mathcal{N} = 6$ theory. We analyzed the Basu-Harvey type equations and found evidence that the equations describe multiple M2-branes ending on a M5-brane, which wraps on a fuzzy 3-sphere. We derived the tension of M5-brane and it exactly agrees with the known result in large N limit. We also found that the 3-sphere is orbifolded by a Z_k action as the volume of the M5-brane is suppressed by $1/k$. This is also consistent with the $\mathrm{SU}(2)\times\mathrm{U}(1)$ symmetry of the equations. We also derived the Nahm equation describing D2-branes ending on a D4-brane wrapping an S^2 starting from the above Basu-Harvey type equations and taking a large k limit, providing further evidence for consistency.

We then turned to another situation where M5-branes wrapping on fuzzy 3-sphere emerge as the vacua of the mass-deformed $\mathcal{N} = 6$ theory. We find domain wall solutions and computed their tensions, in agreement with known gravity analysis, thereby adding another evidence for the existence of the M5-branes in the $\mathcal{N} = 6$ theory.

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JHEP09(2008)067