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Mach-Zehnder interferometer and the teleporter

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We suggest a self-testing teleportation configuration for photon qubits based on a Mach-Zehnder interferometer. That is, Bob can tell how well the input state has been teleported without knowing what that input state was. One could imagine building a ''locked'' teleporter based on this configuration. The analysis is performed for continuous variable teleportation but the arrangement could be applied equally to discrete manipulations.

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Quantum teleportation [1–5] is a method via which quantum information can be passed through a classical channel and successfully retrieved at a distant location. The sharing of entanglement between the sender (Alice) and receiver (Bob) is essential for teleportation as it provides the ''quantum key'' needed to retrieve the quantum information [6]. In this way the quantum state of an object can be transferred in a ''disembodied'' way between Alice and Bob, without either knowing the state.

This lack of knowledge about the state being transferred presents a problem for verification of teleportation. It would appear that it is not possible for Alice and Bob to test if the teleporter is operating correctly. Thus verification protocols as proposed [7,8] and carried out [2,3,5] involve a ''third'' person, Victor (the verifier), who must examine the teleported state to determine if the machine is working. Victor prepares the original input state and (in principle) is the only person who knows its identity. For example, Victor may prepare photons in various polarization states and then send them to Alice. Alice teleports them to Bob who then sends them back to Victor. Victor then checks to what degree they are still in the same polarization state after teleportation [2,3]. For continuous variable experiments the signal and noise properties of the input and output can be compared in various ways [5,8]. Because of the imperfect nature of experiments Victor must be careful not to be tricked in deciding if some level of teleportation has occurred.

In this paper we show that it *is* possible for Alice and Bob to verify that their teleporter is operating correctly without knowing the input states. This represents a unique type of verification procedure whose versatility may find important applications.

Consider first the setup shown schematically in Fig. 1(a). Basically we place a teleporter in one arm of a Mach-Zehnder interferometer, inject a single photon state, in an arbitrary polarization superposition state into one port, then use the interference visibility at the output ports to characterize the efficacy of teleportation. The beauty of such a setup is the visibility does not depend on the input state, so we can assess how well the teleporter is working without knowing what is going into it. Let us see how this works.

The input for one port of the interferometer is in the arbitrary polarization superposition state

$$|\phi\rangle_a = \frac{1}{\sqrt{2}}(x|1,0\rangle + y|0,1\rangle), \tag{1}$$

where $|n_h, n_v\rangle \equiv |n_h\rangle_h \otimes |n_v\rangle_v$, n_h and n_v are the photon number in the horizontal and vertical polarizations, respectively, and $|x|^2+|y|^2=1$. The input of the other port is in the vacuum state $|\phi\rangle_b=|0,0\rangle$. The operators in the Heisenberg picture for the four input modes (two spatial times two polarization) are a_h and a_v (superposition), and b_h and b_v (vacuum). We propagate these operators through the Mach-Zehnder interferometer (including the teleporter). After the first beamsplitter we can write

$$c_{h,v} = \frac{1}{\sqrt{2}}(a_{h,v} + b_{h,v}),$$
$$d_{h,v} = \frac{1}{\sqrt{2}}(a_{h,v} - b_{h,v}). \tag{2}$$

One of the beams (c) is then teleported. The teleporter we consider in this paper uses continuous variable (squeezing) entanglement as a quantum resource, as discussed in Refs. [9,10]. This model is chosen for its simplicity. Similar results would be obtained with other types of teleportation devices. The individual polarization modes of c are separated using a polarizing beamsplitter. Each mode is then mixed on a 50:50 beamsplitter with a correspondingly polarized member of an entangled pair of beams. The entangled pairs may come from two separate two-mode squeezers [11] or, alternatively, a single polarization and number entangler could be used [9]. Amplitude and phase quadrature measurements are carried out, respectively, on the two output beams for each mode. This can be achieved through either homodyne detection [9] or parametric amplification [10]. A classical channel for each of the polarization modes is formed from these measurements which are passed to the reconstruction site where they are used to displace the corresponding entangled pair for each mode. The output c_T is formed by combining the two displaced polarization modes on a polarizing beamsplitter. Under conditions for which losses can be neglected the output from the teleporter is

$$c_{h,v,T} = \lambda c_{h,v} + (\lambda\sqrt{H} - \sqrt{H-1}) f^{\dagger}_{h,v,1} + (\sqrt{H} - \lambda\sqrt{H-1}) f_{h,v,2}, \tag{3}$$

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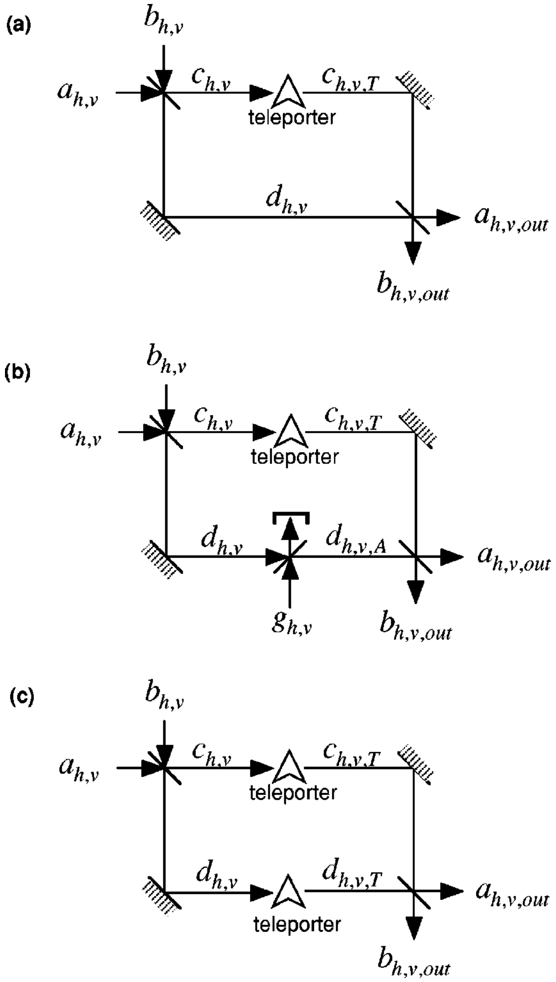

FIG. 1. Schematics of various Mach-Zehnder plus teleporter arrangements.

where λ is the feedforward gain in the teleporter, the $f_{h,v,i}$ are vacuum inputs to the two-mode squeezer providing the entanglement for the teleporter [see Fig. 2(a)], and H is the parametric gain of the squeezer. The fields are recombined in phase at the final beamsplitter giving the outputs

$$a_{h,v,out}=\frac{1}{\sqrt{2}}(c_{h,v,T}+d_{h,v}),$$
$$b_{h,v,out}=\frac{1}{\sqrt{2}}(c_{h,v,T}-d_{h,v}). \tag{4}$$

The expectation values for photon counting at the two outputs of the interferometer are

$$\begin{aligned}\langle a^\dagger_{out}a_{out}\rangle &= \langle\phi|_a\langle\phi|_b\langle\phi|_f(a^\dagger_{h,out}+a^\dagger_{v,out})(a_{h,out}+a_{v,out})\\ &\quad\times|\phi\rangle_a|\phi\rangle_b|\phi\rangle_f\\ &=0.25(1+\lambda)^2+(\lambda\sqrt{H}-\sqrt{H-1})^2,\end{aligned}$$

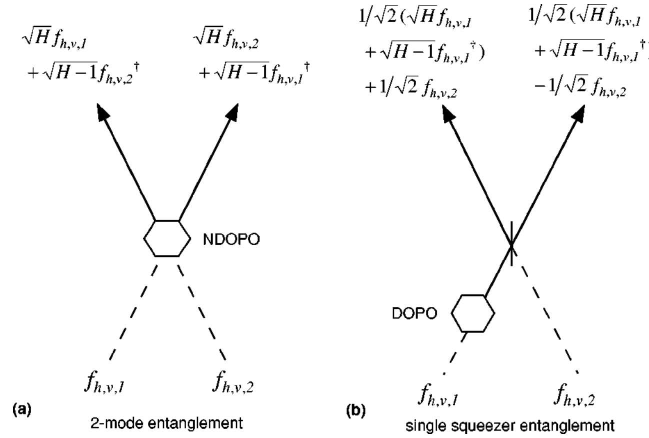

FIG. 2. Schematic of the two types of entanglement used for teleportation. NDOPO stands for nondegenerate optical parametric oscillator and DOPO stands for degenerate optical parametric oscillator. A separate pair of entangled beams is needed to teleport each of the two polarization modes. Alternatively, type-II polarization entanglement could be used [9].

$$\begin{aligned}\langle b^\dagger_{out}b_{out}\rangle &= \langle\phi|_a\langle\phi|_b\langle\phi|_f(b^\dagger_{h,out}+b^\dagger_{v,out})\\ &\quad\times(b_{h,out}+b_{v,out})|\phi\rangle_a|\phi\rangle_b|\phi\rangle_{f|\phi\rangle_b}\\ &=0.25(1-\lambda)^2+(\lambda\sqrt{H}-\sqrt{H-1})^2. \end{aligned}\tag{5}$$

In the limit of very strong entanglement squeezing ($\sqrt{H}-\sqrt{H-1}\rightarrow 0$) we find from Eq. (3) that $c_{h,v,T}\rightarrow c_{h,v}$ for unity gain ($\lambda=1$), i.e., perfect teleportation. For the same conditions (and only for these conditions) the visibility of the Mach-Zehnder outputs,

$$V=\frac{\langle a^\dagger_{out}a_{out}\rangle-\langle b^\dagger_{out}b_{out}\rangle}{\langle a^\dagger_{out}a_{out}\rangle+\langle b^\dagger_{out}b_{out}\rangle} \tag{6}$$

goes to one, indicating the state of the teleported arm exactly matches that of the unteleported arm. Notice that the expectation values [Eq. (5)], and thus the visibility, do not depend on the actual input state (no dependence on x and y). Hence we can demonstrate that the teleporter is operating ideally even if we do not know the state of the input, which can be assumed to be varying randomly. Classical limits can be set by examining the visibility obtained with no entanglement ($H=1$). In Fig. 3 we plot the visibility versus feedforward gain in the teleporter for the cases of no entanglement (0%), 50% entanglement squeezing and 90% entanglement squeezing. Maximum visibility in the classical case is $V_{max,c}=\sqrt{1/5}$. Increasing entanglement leads to increasing visibility.

It is known that using a single-mode squeezed beam, divided in half on a beamsplitter [see Fig. 2(b)], instead of a true two-mode squeezed source (which exhibits Einstein, Podolsky, Rosen correlations [12]), can still produce fidelities of teleportation higher than the classical limit for coherent state inputs. Loock and Braunstein [13] have recently contrasted various single-mode and two-mode squeezing schemes on the basis of their fidelity. It is educational to

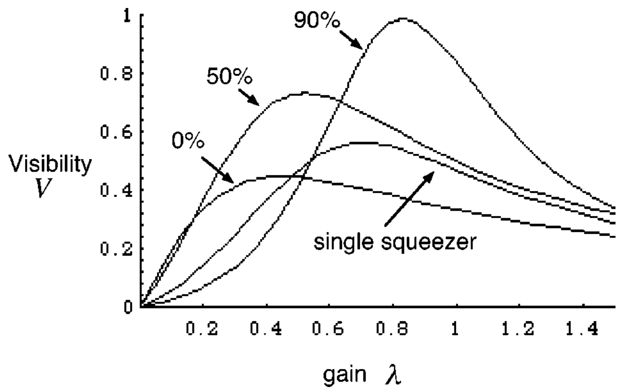

FIG. 3. Visibility versus gain for the setup shown in Fig. 1(a) and various levels of two-mode entanglement (0%, 50%, and 90%) and 87.5% single-mode squeezing (single squeezer).

examine how well the single squeezer teleporter performs in our single photon Mach-Zehnder. The input-output relation for a single squeezer teleporter is

$$c_{h,v,S}=\lambda c_{h,v}+\frac{1}{\sqrt{2}}[(\lambda\sqrt{H}-\sqrt{H-1})f^{\dagger}_{h,v,1}+(\sqrt{H}-\lambda\sqrt{H-1})f_{h,v,1}+\lambda f^{\dagger}_{h,v,2}+f_{h,v,2}]. \quad (7)$$

The expectation values for the outputs then become

$$\langle a^{\dagger}_{out}a_{out}\rangle=0.25(1+\lambda)^2+0.5(\lambda\sqrt{H}-\sqrt{H-1})^2+0.5\lambda^2,$$
$$\langle b^{\dagger}_{out}b_{out}\rangle=0.25(1-\lambda)^2+0.5(\lambda\sqrt{H}-\sqrt{H-1})^2+0.5\lambda^2. \quad (8)$$

On Fig. 3 we also present the visibility as a function of gain for the single squeezer case with squeezing of 87.5%. The squeezing is picked such that the average coherent state unity gain fidelity is the same as for the 50% squeezed two-mode entanglement (the criteria used in Ref. [13]). The performance of the single squeezer teleporter is clearly inferior. Although achieving a better visibility than the classical teleporter it never exceeds, or equals, for any gain, the performance of the 50% squeezed two-mode teleporter. The maximum visibility of the two-mode teleporter is 25% higher. We conclude that the entanglement of the single squeezer is not as useful for teleportation as might be suggested by the coherent state average fidelity measure.

In the experiments we have modeled so far the level of visibility has been determined not only by the ability of the teleporter to reproduce the input polarization states of the photons (the mode overlap) but also the efficiency with which input photons to the teleporter lead to correct output photons (the power balance). It is of interest to try to separate these effects. We can investigate just state reproduction if we allow attenuation to be applied to beam d, thus ''balancing'' the Mach-Zehnder by compensating for the loss introduced by the teleporter [see Fig. 1(b)]. The attenuated beam d becomes

$$d_{h,v,A}=\sqrt{\eta}d_{h,v}+\sqrt{1-\eta}g_{h,v}, \quad (9)$$

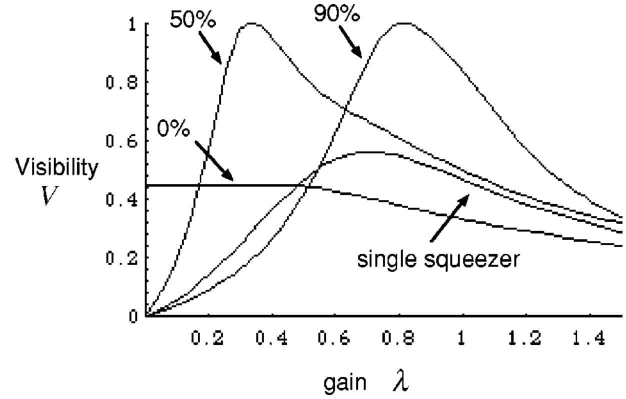

FIG. 4. Visibility versus gain with ''attenuation balancing'' [setup shown in Fig. 1(b)] for various levels of two-mode entanglement (0%, 50%, and 90%) and 87.5% single-mode squeezing (single squeezer).

where g is another vacuum field and η is the intensity transmission of the attenuator. The expectation values of the outputs (using two-mode entanglement) are now

$$\langle a^{\dagger}_{out}a_{out}\rangle=0.25(\sqrt{\eta}+\lambda)^2+(\lambda\sqrt{H}-\sqrt{H-1})^2,$$
$$\langle b^{\dagger}_{out}b_{out}\rangle=0.25(\sqrt{\eta}-\lambda)^2+(\lambda\sqrt{H}-\sqrt{H-1})^2. \quad (10)$$

In Fig. 4 we plot visibility versus gain, using the attenuation η to optimize the visibility ($\eta\leqslant 1$). Now we can always achieve unit visibility for any finite level of entanglement by operating at gain $\lambda_{opt}=(\sqrt{H-1}/\sqrt{H})$ and balancing the interferometer by setting $\eta=\lambda^2_{opt}$. The high visibility is achieved because at gain λ_{opt} the teleporter behaves like pure attenuation [9]. That is, the photon flux of the teleported field is reduced, but no ''spurious photons'' are added to the field. Thus, at this gain, all output photons from the teleporter are in the right state, but various input photons are ''lost.'' This effect does not occur for the single squeezer teleporter (also plotted in Fig. 4) whose performance is not improved by balancing the interferometer, further emphasizing its lack of useful entanglement.

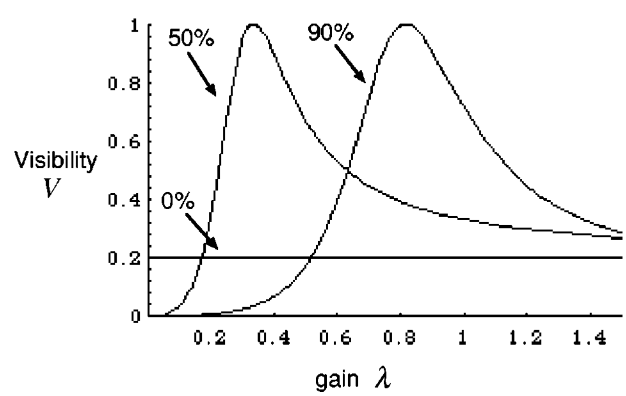

FIG. 5. Visibility versus gain for self-testing teleporter [setup shown in Fig. 1(c)] for various levels of two-mode entanglement (0%, 50%, and 90%).

This contrast between state reproduction and efficiency has been a topic of vigorous debate [14,15]. It is of note that our interferometric test can separate the two effects. It should also be noted that our test is sensitive not only to the relative phase of the polarization superposition, but also the overall phase of the teleported field. The overall phase is defined with respect to the field in the unteleported arm of the interferometer and is a constituent of the mode overlap. If the overall phase is randomized by the teleporter then very low visibility will result from our interferometric test.

So far we have considered test arrangements in which a teleported field is compared with one which is not teleported. However, the result of Eq. (10) suggests a self-testing arrangement for a teleporter. Suppose we place a teleporter in both arms of the interferometer as portrayed in Fig. 1(c). Writing an expression for the teleported beams d similar to Eq. (3) we find the expectation values of the outputs are now

$$\langle a_{out}^{\dagger} a_{out} \rangle = \lambda^2 + 2(\lambda \sqrt{H} - \sqrt{H-1})^2,$$
$$\langle b_{out}^{\dagger} b_{out} \rangle = 2(\lambda \sqrt{H} - \sqrt{H-1})^2, \qquad (11)$$

where we have assumed the gains of the two teleporters are the same. By monitoring the ''dark'' output port (b_{out}) it may be possible to keep the system ''locked'' to maximum visibility, without any knowledge of the input state or requiring the destruction of the output state (a_{out}). Once again, under low loss conditions, unit visibility is achieved for gain λ_{opt} as illustrated in Fig. 5. The added complexity of using two teleporters may be justified in practice by the greater versatility of this system.

In conclusion, we have examined a Mach-Zehnder arrangement for testing the efficacy of single photon qubit teleportation. The major advantage of this arrangement is it does not require the tester to know the input states of the photons. We have contrasted the results obtained with no entanglement, single-mode entanglement, and true two-mode entanglement using continuous variable teleportation. The highest visibilities are always achieved with two-mode entanglement. Indeed with low loss and power balancing it is always possible to realize unit visibility with two-mode entanglement. If the entanglement squeezing is high, good efficiency can also be obtained. We have only examined here the case where losses can be neglected. Losses reduce visibilities but the general trends discussed here remain the same.

It is not difficult to show that our test also works for mixed state inputs [16]. Thus the input to be tested could be one arm of a down-conversion source producing entangled pairs of photons. High visibility would indicate the teleported photons were still entangled with those in the other arm. A test of teleportation fidelity performed on only one arm of the down-converter cannot make such a determination. This is because fidelity is *not* sensitive to the overall phase of the field. This ability to check teleportation of entanglement locally may have applications in quantum information processing.

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