Macroeconomic Default Modeling and Stress Testing*

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This paper applies a macroeconomic-based model for estimating probabilities of default. The first part of the paper focuses on the relation between macroeconomic variables and the default behavior of Dutch firms. A convincing relationship with GDP growth and oil price and, to a lesser extent, the interest and exchange rate exists. The second part of the paper assesses the default behavior based on a stress scenario of two consecutive quarters of zero GDP growth as required by the Basel II framework. It can be concluded that a stress-test scenario covering two quarters of zero GDP growth does not influence the default rate significantly and thus does not seem to be very severe.

JEL Codes: C12, C13, C15, E32, E44, E47, G21, G28.

1. Introduction

Estimating probabilities of default is the first step in assessing the credit exposure and potential losses faced by financial institutions. Probabilities of default are also the basic inputs when evaluating systemic risk and stress testing financial systems. Therefore, predictors of credit risk are of natural interest to practitioners in the financial industry, as well as to regulators—especially under the new capital adequacy framework (Basel II), which encourages the active involvement of banks in measuring the likelihood of defaults.

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Financial literature has brought forth a variety of models that attempt to measure the probability of default, e.g., macroeconomic-variable-based models. Macroeconomic-based models are motivated by the observation that default rates in the financial, corporate, and household sectors increase during recessions. This observation has led to the implementation of econometric models that attempt to explain default indicators, such as probabilities of default or default rates, using macroeconomic variables.

In this paper we consider the default rate in relation to macroeconomic variables. Specifically, we will explore the relationship between the default rate and the macro economy by developing a logit model with macroeconomic parameters. This fairly simplistic model has three advantages: First, the model is relatively easy to understand. Second, it presents robust results. Third, the model takes the correlation of default rates amongst sectors into account. We call this special feature the "correlation" factor. Relatively little work has been done in previous research on estimating such a "correlation" factor, although it has often been confirmed in the literature that default rates are highly correlated amongst sectors.

A direct consequence of this model and its advantages is that the direction of causality is only one way—from macro variables to default—and that we do not allow for feedback from financial factors to the macro variables. We chose not to incorporate the banking sector, in order to develop a relatively simple, easy-to-implement model and focus solely on the relation between macroeconomic variables and the default rate.

By means of the logit model and selected variables, first we will assess which macroeconomic variables are related to the default behavior of firms and, second, examine the default behavior in 2007 on the basis of an unfavorable macroeconomic scenario of two quarters of zero GDP growth in the third and fourth quarters of 2006. We will compare it with a base scenario and the situation where we took the 2.5 percent worst cases of the base scenario.

The remainder of this paper is organized as follows. Section 2 discusses related studies. Section 3 describes the construction of the data set. Section 4 formulates the estimation model. Section

5 discusses the estimation results. Section 6 studies the stress-test scenarios. Section 7 concludes.

2. The Concept of Macroeconomic-Based Models

Estimating probabilities of default is a challenging subject. The types of models used to assess credit risk can be broadly classified either as market-based models or as fundamental-based models. Market-based models build on Merton's option-pricing theory and rely on security prices. Chan-Lau (2006) distinguishes four approaches within fundamental-based modeling to model probabilities of default: macroeconomic-based, accounting-based, rating-based, and hybrid models.

Macroeconomic models, as used in this research, explain changes in the default rate on the basis of macroeconomic conditions. These macroeconomic variables are cyclical indicators—for instance, GDP growth or interest rates—and financial market indicators—for instance, stock market prices and stock market volatility. Accounting-based models, on the other hand, generate probabilities of default for individual firms using accounting information. Furthermore, rating-based models can be used to infer probabilities of default when external ratings information is available. Hybrid models generate probabilities of default using a combination of economic variables, financial ratios, and ratings data.

Chan-Lau (2006) lists three advantages of macroeconomic models. First, an advantage is that this type of model is very suitable for designing stress scenarios. Second, because long data series are available for most countries, it is also possible to conduct cross-country comparative studies. Third, the default rate used to estimate the model is observed historically, so that one can avoid making assumptions.

On the other hand, a disadvantage of macroeconomic models is that the time span of the data needs to be longer than one business cycle; otherwise, the model would not capture the impact of the business cycle on probabilities of default. Furthermore, this type of model is subject to Lucas critique since the parameters or functional forms are unlikely to remain stable; i.e., it is virtually impossible to capture the complex interaction between the state of the economy and the default risk. Finally, aggregate economic data are usually reported with substantial lags. This makes it difficult to estimate or forecast macroeconomic models with up-to-date information.

Macroeconomic models can be classified into exogenous and endogenous models, depending on whether the model allows feedback between financial distress and the explanatory economic variables. The first category of macroeconomic-based models assumes that the economic variables are exogenous and not affected by financial distress. The general approach to modeling this category is described by the following equation:

$$pd_t = g(x_1, x_2, \ldots, x_n) + \varepsilon, \tag{1}$$

where pd is the probability of default over a given period t. A general aggregate model sets pd_t equal to a function g, $X = (x_1, x_2, \ldots, x_n)$, a function of a set of economic variables and a random variable ε.

A problem of the exogenous approach is that the relationship between macroeconomic variables and the default rate is assumed to be the same during periods of economic downturn and expansion. This seems intuitively to be implausible.

The second category of macroeconomic-based models assumes that the economic variables are endogenous and differ in times of financial distress. The typical econometric framework used in these models is the vector autoregressive (VAR) methodology. See, for example, Hoggarth, Logan, and Zicchino (2005). We can write the VAR in a more general form as

$$Z_{t+1} = \alpha_t + \sum_{j=1}^{p} \beta_j z_{t+1-j} + \varepsilon_{t+1}, \tag{2}$$

where α_t is a constant vector, β_j are (lagged) coefficients matrices, ε_{t+1} is a vector of residuals/shocks, and z is the vector of endogenous variables that includes both probabilities of default and the aggregate economic variables associated with the state of the business cycle. In principle, inference in VAR models is sensitive to the choice of lags. If a large number of lags is included, degrees of freedom are lost. If the lag length is too short, important lag dependencies may be omitted.

We end this section with a short overview of which macroeconomic variables are related to the default rate. Appendix 1 lists an extensive set of papers on macroeconomic default modeling, with a short description of each. It seems that the literature can be divided into that on quoted firms and that on unquoted firms. The papers mainly confirm the significance of GDP growth in relation to the default rate. Relations with stock market variables have been identified several times, but only for the literature on quoted firms. Furthermore, some papers show that the influence of the interest rate and the exchange rate is significant in certain sectors. Remarkably, none of the papers on our list examines the oil price as an explanatory variable.

3. The Model

3.1 Aggregate Default Modeling

Consider a general aggregate model that can be estimated by maximum likelihood. Let pd_t be the fraction (proportion) of firms that default in period t. We set pd_t equal to a function $g(.)$ of the relevant explanatory variables $\mathbf{z}_t$, a parameter vector $\boldsymbol{\theta}$, and a disturbance v_t. In addition, controlling the distribution of v_t controls the distribution of pd_t.

$$pd_t = g(\boldsymbol{\theta}, \mathbf{z}_t, v_t) \tag{3}$$

More specifically, $pd_{t,i}$ is the fraction of firms in sector i, $\forall i \in \{0, 1, \ldots, s\}$ that default in period t, with s the total number of sectors in the economy. The economy as a whole is denoted by $i = 0$. Furthermore, $\mathbf{z}_t$ is a vector of variables including intercept, relevant for the default rate at time t. $v_{t,i}$ is a disturbance and β_i is a vector of parameters.

$$pd_{t,i} = \frac{\exp\left(\mathbf{z}_t'\boldsymbol{\beta}_i + v_{t,i}\right)}{1 + \exp\left(\mathbf{z}_t'\boldsymbol{\beta}_i + v_{t,i}\right)} \tag{4}$$

Taking the logit[1] of both sides, we find

$$\tilde{pd}_{t,i} : \text{logit}(pd_{t,i}) = \mathbf{z}_t'\boldsymbol{\beta}_i + \upsilon_{t,i}. \tag{5}$$

From this general model, we obtain two separate models, an economy model ($i = 0$) and a sector model $\forall i \in \{1, 2, \ldots, s\}$. For the economy default rate ($i = 0$), we assume the disturbances $\upsilon_{t,0}$ are independent and identically distributed (i.i.d.). Letting $\sigma^2_{\psi,0} = \text{var}(\upsilon_{t,0})$, the economy model is

$$\tilde{pd}_{t,0} = \mathbf{z}_t'\boldsymbol{\beta}_0 + \upsilon_{t,0}$$
$$\text{where } \upsilon_{t,0} \overset{iid}{\sim} \left(0, \sigma^2_{\psi,0}\right). \tag{6}$$

For the sector default rates, $\forall i \in \{1, 2, \ldots, s\}$, the disturbances are divided into a latent systematic (ξ_t) and an idiosyncratic ($\psi_{t,i}$) part. The systematic part captures the correlation between the sector default rates. Estimation and inference of the parameters are based on maximizing the Gaussian quasi log-likelihood. Letting $\sigma_{\xi,i}$ and $\sigma_{\psi,i}$ be non-negative parameters, $\forall i \in \{1, \ldots, s\}$. The sector model is

$$\tilde{pd}_{t,i} = \mathbf{z}_t'\boldsymbol{\beta}_i + \upsilon_{t,i} \tag{7}$$

$$\text{where } \upsilon_{t,i} = \sigma_{\xi,i}\xi_t + \sigma_{\psi,i}\psi_{t,i}$$

$$\xi_t \overset{iid}{\sim} (0, 1), \psi_{t,i} \overset{iid}{\sim} (0, 1).$$

An advantage of taking the correlation of the sector default rates into account is that combining such a factor with macroeconomic indicators provides a natural test of the specification of the macro relationship. If the macroeconomic indicators are indeed informative, then the fluctuations explained by the factor will be relatively small.

[1]The logit transformation is given by $\text{logit}(x) = \ln(\frac{x}{1-x})$. Since $\frac{\exp(\text{logit}(x))}{1+\exp(\text{logit}(x))} = \frac{x/(1-x)}{1+x/(1-x)} = x$, the equation $x = \frac{\exp(y)}{1+\exp(y)}$ is solved for y by $y = \text{logit}(x)$.

3.2 Dynamic Effects of Shocks in the Aggregate Default Model

This section assesses the effect of shocks in $\mathbf{z}_t$ over the long and short term. Therefore, we differentiate (4) with respect to $\mathbf{z}_t$ to find

$$D_{\mathbf{z}_t} pd_{t,i} = \frac{pd_{t,i}\boldsymbol{\beta}_i}{\left(1 + \exp\left(\tilde{pd}_{t,i}\right)\right)^2}. \tag{8}$$

Given that $\tilde{pd}_{t,i}$ is generally low, the denominator can be ignored. Accordingly, for small $\Delta\mathbf{z}_t$, the elements of $\boldsymbol{\beta}_i$ are approximate semi-elasticities:

$$\Delta\% pd_{t,i} \approx \boldsymbol{\beta}_i' \Delta\mathbf{z}_t. \tag{9}$$

The next step is to capture persistence in the default rate. Therefore, we include the lagged default rate as an explanatory variable. Letting $\mathbf{z}_t^*$ denote explanatory variables other than the lagged default rate and the intercept, and $\boldsymbol{\beta}_i^*$ the corresponding parameter vector, we may write equation (5) as

$$\tilde{pd}_{t,i} = \boldsymbol{\beta}_{i,0} + \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i} + \boldsymbol{\beta}_i^{*\prime}\mathbf{z}_{t-1}^* + \upsilon_{t,i} \tag{10}$$

or, equivalently,

$$\tilde{pd}_{t,i} = \frac{\boldsymbol{\beta}_{i,0}}{1 - \boldsymbol{\beta}_{i,1}} + \boldsymbol{\beta}_i^{*\prime} \sum_{j=0i,1}^{\infty\boldsymbol{\beta}j} \mathbf{z}_{t-1-j}^* + \sum_{j=0i,1}^{\infty\boldsymbol{\beta}j} \upsilon_{t-j,i}. \tag{11}$$

Specifically, the inclusion of the lagged default rate makes the current default rate depend on all lags of the explanatory variables with coefficients declining at rate $\boldsymbol{\beta}_{i,1}$. In other words, a default depends not only on the previous period but also on the entire history, with more recent developments being more important.

Furthermore, to estimate the short- and long-term effects of a small shock $\Delta\mathbf{z}^*$ that occurs in period t_0 and persists indefinitely through time, equations (9) and (11) are combined. Applying equation (9) to (11) and considering $\sum_{j=0i,1}^{\infty\boldsymbol{\beta}j} \mathbf{z}_{t-1-j}^*$ as the explanatory vector, the estimated effects are as follows:

Short-term effect:

$$\Delta\%pd_{t_0+1,i} \approx \boldsymbol{\beta}_i^{*\prime}\Delta\left(\sum_{j=0i,1}^{\infty\boldsymbol{\beta}j}\mathbf{z}_{t_0-j}^{*}\right) = \boldsymbol{\beta}_i^{*\prime}\Delta\mathbf{z}^{*} \tag{12}$$

Long-term effect:

$$\lim_{t\to\infty}\Delta\%pd_{t+1,i} \approx \lim_{t\to\infty}\boldsymbol{\beta}_i^{*\prime}\Delta\left(\sum_{j=0i,1}^{\infty\boldsymbol{\beta}j}\mathbf{z}_{t-j}^{*}\right) = \frac{\boldsymbol{\beta}_i^{*\prime}}{1-\boldsymbol{\beta}_{i,1}}\Delta\mathbf{z}^{*} \tag{13}$$

It must be noted that, at time $t > t_0$, the effect on the default rate is

$$\Delta\%pd_{t,i} \approx \boldsymbol{\beta}_i^{*\prime}\Delta\left(\sum_{j=0i,1}^{\infty\boldsymbol{\beta}j}\mathbf{z}_{t-1-j}^{*}\right) = \left(1-\boldsymbol{\beta}_{i,1}^{t-t_0}\right)\frac{\boldsymbol{\beta}_i^{*\prime}}{1-\boldsymbol{\beta}_{i,1}}\Delta z^{*}. \tag{14}$$

Equation (14) shows that at time t, a fraction $(1-\boldsymbol{\beta}_{i,1}^{t-t_0})$ of the long-term effect is more or less realized. For this reason, $\boldsymbol{\beta}_{i,1}$ controls the speed at which the default rate responds to shocks. For shocks that do not persist indefinitely, the long-term effect can be interpreted as an upper bound to the maximum effect that will be attained.

4. Data and Definitions

4.1 Definitions

In this section we give a short description of both the default rate and the macroeconomic variables. The default rate is defined as the ratio of the number of firms in default to the total number of firms in quarter t. If $pd_{t,0}$ is the fraction of all firms that default in quarter t and $pd_{t,i}$ is the fraction of firms in sector i that default in quarter t, $pd_{t,0}$ and $pd_{t,i}$ are the economy and the sector i default rates, respectively.

$$\begin{aligned} pd_{t,0} &= \frac{\text{Number of defaults in all sectors in quarter } t}{\text{Average number of firms in all sectors in quarter } t} \\ pd_{t,i} &= \frac{\text{Number of defaults in sector } i \text{ in quarter } t}{\text{Average number of firms in sector } i \text{ in quarter } t} \end{aligned} \tag{15}$$

With regard to the macroeconomic variables, we chose macroeconomic variables that give rise to particular concerns about movements in unfavorable directions, so that the variables are useful for stress testing or scenario analysis. It must be emphasized that these variables need not be those that add the most explanatory or forecasting power. The following variables were selected:

- **Gross Domestic Product:** GDP equals aggregate demand of an economy. Aggregate demand relates to sales of firms. Lower GDP growth means lower growth in sales of firms. The lower GDP growth is, the harder it is for firms to generate income through sales. Lower income thus increases the possibility that firms cannot meet their obligations and will default.
- **Interest Rate:** Firms often finance part of their activities by debt. The funding costs of firms are therefore positively related to interest rates. If interest rates are high, firms have higher costs and are more likely to default.
- **Exchange Rate:** The exchange rate is the price of the domestic currency expressed in terms of a foreign currency. Firms in sectors that have a great deal of international business are expected to be affected by exchange rates. However, the sign of the relationship varies. Business conditions of importing firms are positively affected by the exchange rate because imports become cheaper if the exchange rate is high. Business conditions of exporting firms are negatively affected by the exchange rate because exports become more expensive if the exchange rate is high.
- **Stock Market Return and Volatility:** Merton's theory predicts that the probability of default is negatively related to the stock market return and positively related to volatility. The stock market return and, thus, also volatility are variables that are popular in scenario analysis.
- **Oil Price:** Oil prices—or, more broadly, energy prices—affect the price of most of the products used by firms. Therefore, the cost of firms and thus their probability of defaulting are positively related to the oil price.

4.2 Interesting Properties of the Default Rate in the Netherlands

In this subsection we analyze the properties of the default rate in the Netherlands. We observe that the default rate in the Netherlands is persistent, negatively related to the business cycle, and correlated between sectors. No seasonal effects are observed.

The data set consists of defaults in companies in the Netherlands per quarter in the period from the first quarter (Q1) of 1983 and the second quarter (Q2) of 2006 (ninety-four quarters). The average economy default rate is about 0.23 percent per quarter, or just below 1 percent per year. The total number of firms in the economy varies between 408,665 (1983:Q1) and 652,367 (2006:Q2). Most firms in the Netherlands are rather small.

First of all, we examined if the economy default rate is persistent by estimating an autoregressive model of order 1 (AR(1)) with ordinary least squares (OLS), which results in

$$pd_{t,0} = 0.00 + .85pd_{t-1,0} + \upsilon_t. \tag{16}$$

The AR(1) model captures most of the serial correlation. The high coefficient of the first lag confirms that the default rate is persistent.

Second, the upper panel of figure 1 plots the economy default rate and $\Delta\%GDP_{t-1}$ against time. The relation is not obvious because GDP growth fluctuates a lot, while the default rate is persistent. Intuitively, however, one would expect a negative relation of the default rate with the business cycle. Accordingly, upon adding $\Delta\%GDP_{t-1}$ to (16) and carrying out a new estimate, we obtain

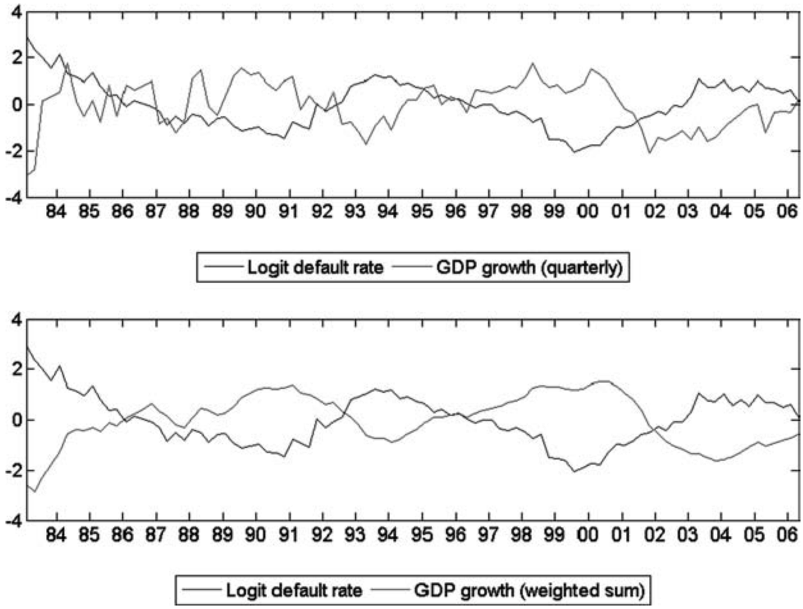
$$pd_{t,0} = .00 + .82pd_{t-1,0} - .00\Delta\%GDP_{t-1} + \upsilon_t \tag{17}$$

or, equivalently,

$$pd_{t,0} = \frac{.00}{1-.82} - .00\sum_{j=0}^{\infty} .82^j \Delta\%GDP_{t-1-j} + \sum_{j=0}^{\infty} .82^j \upsilon_{t-j}. \tag{18}$$

The intercept and the coefficient of GDP growth are rounded to 0.00. Nevertheless, they deviate significantly from zero. We can conclude

Figure 1. Economy Default Rate and GDP Growth



Notes: The upper panel plots $pd_{t,0}$ and $\Delta\% GDP_{t-1-j}$ against time. The lower panel plots $pd_{t,0}$ and $\sum_{j=0} .82^j \Delta\% GDP_{t-1-j}$ against time. All series are standardized to have a zero mean and unit variance.

that persistency actually implies that the default rate is related to a weighted sum of lags in GDP growth. This relation is shown in the lower panel of figure 1, which plots the economy default rate and $\sum_{j=0}^{19} 0.82^j \Delta\% GDP_{t-1-j}$ against time.

Third, with regard to the separate sectors, we found that all correlations of the default rates between sectors are significant at the 1 percent level. Thus, a highly significant correlation exists between the sectors.

Finally, we checked for seasonal effects of the default rate. The following simple model is estimated by OLS. In this model, the function 1_A is the indicator function for the event A.

$$pd_{t,i} = \sum_{j=1}^{4} \delta_j 1_{t\in \text{ quarter } j}(t) + \upsilon_t \tag{19}$$

We test $\delta_1 = \delta_2 = \delta_3 = \delta_4$. No indication of seasonal effects is found.

5. Estimation Results

5.1 Results for the Netherlands

First, we substitute the selected macroeconomic variables for $\mathbf{z}_t^*$ in $\tilde{pd}_{t,i} = \boldsymbol{\beta}_{i,0} + \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i} + \boldsymbol{\beta}_i^{*\prime}\mathbf{z}_{t-1}^* + \upsilon_{t,i}$ (see equation (10)) and identify which macroeconomic variables are significant. We find that the GDP growth and the oil price are related to the default rate in the Netherlands as well as, to a lesser extent, the interest and the exchange rate. The stock market return and volatility have no relation with the default rate.

Gross Domestic Product. The estimated coefficients of GDP growth have a negative sign, as expected. The "industry and mining," "transport, storage, and communications," "financial services," and "rental and corporate services" sectors are significant at the 1 percent level, which means they have the strongest link with the default rate. For the overall economy, the hypothesis of no relation with GDP growth is firmly rejected.

Interest Rate. Only the "construction" sector has a significant relation between the level of the short-term interest rate and the default rate. A reason why the results in this sector are different from those in other sectors is that construction firms are substantially affected by interest rates through another channel than the cost of debt; households find it easier to finance construction work on their homes when interest rates are low. This should affect, in particular, small construction firms. Indeed, about 86 percent of all construction firms have fewer than ten employees. The strong rejection of the hypothesis of equal coefficients supports the view that the "construction" sector is an exception. Demand for construction work is negatively related to interest rates, and it is interesting that the level of (and not the change in) the interest rate is significant. Apparently, people or firms react to the level of (and hardly to changes in) the interest rate.

We also tested the long-term instead of the short-term interest rate in $\mathbf{z}_t^*$. This yielded somewhat stronger but qualitatively similar results. Due to the correlation between the short-term and the long-term rate, it is not sensible to include both rates. Various term spreads can be included, however. They tested insignificant with a one-quarter lag. Given that the term structure forecasts GDP growth, the term structure does have forecasting power when used with a lag of several years.

Exchange Rate. The most significant sector is "transport, storage, and communications," as could be expected, particularly in the Netherlands. For the overall economy, the relation is moderately strong. No relation with the change in the exchange rate was found. Apparently, the level of the exchange rate is decisive in trading. This is remarkable since one would expect firms to become used to the level and only react to changes.

Stock Market Return and Volatility. The signs of the coefficients are both negative and positive, and in the case of both the return and volatility, the coefficients do not deviate significantly from zero. We may thus conclude that the default rate is unrelated to the stock market return and volatility. A reason for this is that the Dutch data set consists of mostly unlisted firms.

Oil Price. Most coefficients of the level of the oil price are significant. All coefficients have the expected sign. Again, the level of the oil price is apparently more important than the change. Furthermore, remarkably, the oil price is the only significant variable for which there is no statistical reason to doubt the hypothesis of equal coefficients. This suggests that the dependence of sectors on the oil price is equal.

Second, we estimate equation (10) without the stock market return and volatility, both of which were found to be insignificant. This leaves GDP growth, the short-term interest rate (level), the exchange rate (level), and the oil price (level) in $\mathbf{z}_t^*$. In general, the behavior of the variables is the same as noted above. Table 8 in appendix 2 lists the results.

Of this model, we measure short- and long-run effects of shocks in macroeconomic variables and their 95 percent confidence intervals. By applying the coefficients of the first lag in equation (14), we estimate that a year after a persistent shock, about 60–95 percent of the long-run effect is realized; see table 1.

Table 1. Short- and Long-Run Effects of Macroeconomic Shocks

	Lower Bound	Mean Effect	Upper Bound
GDP Growth: +.01 (Economy)	−2.38%	−1.40%	−.43%
Short Run	−14.91%	−9.01%	−3.11%
Long Run			
Short Rate: +.01 (Construction)	0.32%	1.52%	2.73%
Short Run	0.96%	6.60%	12.24%
Long Run			
Exchange Rate: +1%			
(Transport, Storage, and Communications)			
Short Run	0.75%	1.48%	2.22%
Long Run	1.39%	2.95%	4.51%
Oil Price: +10%			
(Transport, Storage, and Communications)			
Short Run	0.52%	1.79%	3.05%
Long Run	−6.46%	3.55%	13.57%

Notes: Short- and long-run effects (percentage changes) on the default rate of the economy or a certain sector are computed using (12) based on estimation results from table 8 in appendix 2. Upper and lower bounds of a 95 percent confidence interval are reported as well.

Although GDP growth shocks will be examined more closely in the next section, we can already note that the effects of GDP growth shocks are somewhat low. A persistent 3 percent decrease in GDP growth raises the long-run default rate by only 25 percent.

The explanatory power of the model can be assessed by comparing the variances of the macroeconomic variables, the latent systematic (ξ_t) disturbances, and the idiosyncratic ($\psi_{t,i}$) disturbances. The first lag is excluded from the decomposition because (i) it explains most of the variance and (ii) it is not independent from the macroeconomic variables.

Recall that $v_{t,i} = \sigma_{\xi,i}\xi_t + \sigma_{\psi,i}\psi_{t,i}$. We can rewrite (10) as

$$\tilde{pd}_{t,i} - \boldsymbol{\beta}_{i,0} - \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i} = \boldsymbol{\beta}_i^{*\prime}\mathbf{z}_{t-1}^* + \sigma_{\xi,i}\xi_t + \sigma_{\psi,i}\psi_{t,i}.$$

This leads to the following variance decomposition, given that the systematic and idiosyncratic disturbances are independent:

$$\mathrm{var}(\tilde{pd}_{t,i} - \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i}) = \boldsymbol{\beta}_i^{*\prime}\mathrm{var}\left(\mathbf{z}_{t-1}^*\right)\boldsymbol{\beta}_i^* + \sigma_{\xi,i}^2 + \sigma_{\psi,i}^2$$

or, equivalently,

$$\frac{\boldsymbol{\beta}_i^{*\prime}\mathrm{var}\left(\mathbf{z}_{t-1}^*\right)\boldsymbol{\beta}_i^*}{\mathrm{var}(\tilde{pd}_{t,i} - \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i})} + \frac{\sigma_{\xi,i}^2}{\mathrm{var}(\tilde{pd}_{t,i} - \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i})} + \frac{\sigma_{\psi,i}^2}{\mathrm{var}(\tilde{pd}_{t,i} - \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i})} = 1. \tag{20}$$

The three fractions are referred to as the macroeconomic, the latent systematic, and the idiosyncratic parts, and are presented in table 2. More variance is explained by the latent systematic part than by the macroeconomic part. Although the macroeconomic variables were not selected because of their explanatory power, this result does illustrate the difficulty of finding relevant systematic variables.

5.2 Sensitivity Analysis

An advantage of the model used in this paper is that it is easy to implement for other countries. We received data from Austria. The default frequency data are from the Austrian business information provider and debt collector Kreditschutzverband (KSV). The KSV database provides us with time series of insolvencies and the total number of firms in most NACE branches at a quarterly frequency. All other data are obtained from the Austrian Statistics Bureau and the IMF International Financial Statistics. We ran the model from 1991 onward.

The results are not the same as the results for the Dutch data series. It seems that the model is country specific. No significant relationship was found between the macroeconomic variables and the default rate. In other words, the model did not detect a relation between the macro economy and the default rate. This does not mean that the model of this research is incorrect. A reason might be the relatively short period we ran the model with. It might also be that another model is appropriate for the Austrian case.

Table 2. Variance Decomposition

	Macroeconomic Part	Latent Systematic Part	Idiosyncratic Part
Industry and Mining	22%	20%	58%
Construction	13%	35%	52%
Trade and Repair, Consumer Products	12%	51%	38%
Catering	10%	29%	60%
Transport, Storage, and Communications	33%	23%	44%
Financial Services	25%	27%	48%
Rental and Corporate Services	27%	42%	31%
Other	8%	13%	79%
Average	19%	30%	51%

Note: This table shows the variance decomposition (20) based on the model estimated in table 8 in appendix 2.

6. Scenario Analysis of Zero GDP Growth

In this section, results from the scenario analysis are presented. We will examine the default behavior in 2007, given an unfavorable macroeconomic scenario of zero GDP growth in the third and fourth quarters of 2006. We will compare it with the average of a base scenario and the average of all 2.5 percent worst cases of the base scenario.

A framework for stress testing the credit exposure to macroeconomic shocks was developed on the basis of Virolainen (2004). In this framework, stress tests are conducted by comparing the average result of a stressed scenario, where an artificial adverse macroeconomic development is introduced, with that of the average of a base scenario, where no adverse shock takes place. Estimated averages of the default rates for each sector corresponding to the stressed and base scenarios are obtained from simulating a large number of future default rates by applying a Monte Carlo method. This is partly governed by the simulated future path of the macroeconomic variables.

For the scenario analysis, we use the macroeconomic model (10) with only GDP included in $\mathbf{z}_t$. This allows us to examine a GDP growth scenario without the need to make assumptions on the other macroeconomic variables. In formula,

$$\begin{aligned}\tilde{pd}_{t,i} &= \boldsymbol{\beta}_{i,0} + \boldsymbol{\beta}_{i,1}\tilde{pd}_{t-1,i} + \boldsymbol{\beta}_{i,2}\Delta\% GDP_{t-1} + \upsilon_{t,i} \\ \upsilon_{t,i} &\overset{iid}{\sim} \left(0, \sigma^2_{\xi,i} + \sigma^2_{\psi,i}\right). \end{aligned} \tag{21}$$

Table 3 shows the estimation results. The coefficients of GDP growth are somewhat closer to zero than in the model where more macroeconomic variables were included. This is consistent with macroeconomic theory, which states that an increase in GDP lowers the default rate but also leads to higher interest rates and an appreciating exchange rate, which in turn causes the default rate to rise.

A model to forecast the behavior of GDP growth is also required. It seems that an AR(1) model fits GDP growth quite well. Let $\boldsymbol{\gamma}$ be a parameter vector and σ_ν a non-negative parameter.

$$\begin{aligned}\Delta\% GDP_t &= \boldsymbol{\gamma}_0 + \boldsymbol{\gamma}_1 \Delta\% GDP_{t-1} + \nu_t \\ \nu_t &\overset{iid}{\sim} \left(0, \sigma^2_\nu\right)\end{aligned} \tag{22}$$

Estimation with OLS using data over the period 1978:Q1 to 2006:Q2 (114 observations) leads to $\Delta\% GDP_t = .01 + .61 \Delta\% GDP_{t-1} + .02$. All parameters are significantly different from zero at the 1 percent level.

To apply the Monte Carlo simulation, we need to draw realizations of the disturbances $\upsilon_{t,i}$ in (21) and ν_t in (22). We assume the disturbances $\upsilon_{t,i}$ and ν_t follow, after standardizing, a standardized t-distribution with df degrees of freedom. The probability density function (pdf) of a standardized t-distribution evaluated at a real number x is given by

$$\frac{\Gamma\left(\frac{df+1}{2}\right)}{\Gamma\left(\frac{df}{2}\right)\sqrt{(df-2)\pi}}\left(1+\frac{x^2}{df-2}\right)^{\frac{df+1}{2}}.$$

Table 3. Estimated Parameters Model (10) with GDP Growth Only

	Intercept	First Lag	GDP Growth	Systematic s.d.	Idiosyncratic s.d.
Industry and Mining	−1.49***	0.73***	−2.43***	0.05***	0.10***
Construction	−0.86***	0.85***	−0.83	0.07***	0.10***
Trade and Repair, Consumer Products	−0.92***	0.85***	−1.23**	0.07***	0.07***
Catering	−0.54***	0.91***	−1.16	0.09***	0.13***
Transport, Storage, and Communications	−1.69***	0.70***	−2.93***	0.08***	0.13***
Financial Services	−0.50***	0.89***	−2.67***	0.09***	0.09***
Rental and Corporate Services	−1.44***	0.76***	−2.64***	0.09***	0.06***
Other	−2.32***	0.68***	−1.14	0.05***	0.16***
Economy	−0.84***	0.86***	−1.46***	NA	0.08***
Pooled	−0.75***	0.86***	−1.47***	0.07***	0.11***
(P-Value Equal Coefficients)	0.0000	0.0000	0.2619	0.0315	0.0000

Notes: Model (10) is estimated with only GDP included in z_t. All estimated parameters are reported. Pooled results are obtained by estimating the sector model under the restriction $\beta_{1,j} = \ldots = \beta_{8,j}$ or $\sigma_{j,1} = \ldots = \sigma_{j,8}$ for certain j while allowing the other parameters to differ per sector. These restrictions are tested and the p-values are reported. The 1, 5, and 10 percent levels are denoted ***, **, and *, respectively.

Table 4. Sample Kurtosis and Degrees of Freedom of Disturbances in Equations (21) and (22)

	Sample Kurtosis	Degrees of Freedom
Sector Model	4.31	8.60
Economy Model	4.12	9.38
GDP Model	5.49	6.41

Notes: The sector and economy models refer to $v_{t,i}$ $\forall_i\{1,\ldots,8\}$ and $v_{t,0}$ in (21), respectively; the GDP model refers to v_t in equation (22). The sample kurtoses and the degrees of freedom of the fitted t-distributions are shown.

The standardized t-distribution with *df* degrees of freedom allows us to adjust the kurtosis, which is important for worst-case scenarios. Moreover, the degrees of freedom are set equal to match the sample kurtoses. Table 4 shows the sample kurtoses of the disturbances and the degrees of freedom of the fitted t-distributions. The kurtoses are pooled for the sector model because they differ between the sectors.

$$\frac{3df-6}{df-4}$$

To analyze the scenarios, we generate 200,000 paths of the logit default rate using equations (21) and (22), given certain starting values for $\tilde{pd}_{t,i}$ and $\Delta\% GDP_t$. Disturbances $v_{t,i}$ and ν_t are generated by multiplying draws from the t-distributions by their respective standard deviations $\sqrt{\sigma^2_{\xi,i}+\sigma^2_{\psi,i}}$ and σ_ν. Finally, we invert equation (5) to find the default rate:

$$pd_{t,i}=\frac{\exp(\tilde{pd}_{t,i})}{1+\exp(\tilde{pd}_{t,i})}.$$

Based on the average of the simulations, the average default rate of the year 2007 is defined as

$$\bar{pd}_{2007,i}=\frac{1}{4}\sum_{t=1}^{4}pd_{2007.t,i}.$$

Table 5. Base Scenario

	Default Rate
Industry and Mining	0.31%
Construction	0.26%
Trade and Repair, Consumer Products	0.22%
Catering	0.31%
Transport, Storage, and Communications	0.29%
Financial Services	0.83%
Rental and Corporate Services	0.20%
Other	0.06%
Economy	0.22%

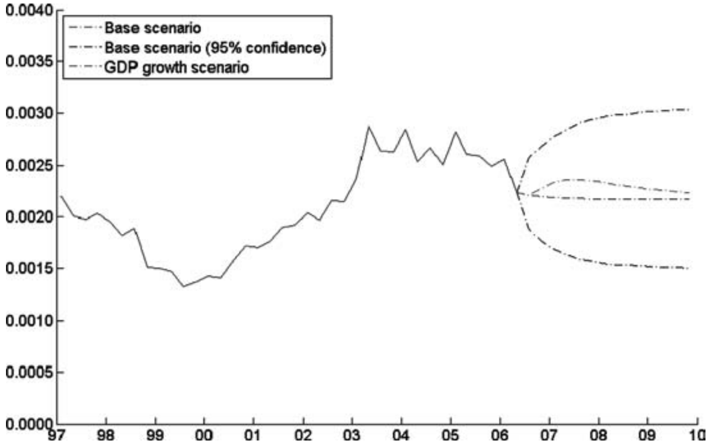
Note: This table shows the average generated $\bar{pd}_{2007,i}$ given the base scenario.

Base Scenario. The expected average 2007 default rate is computed without making assumptions on what will happen after 2006:Q2. We do so by setting the starting values for $\tilde{pd}_{t,i}$ and $\Delta\%GDP_t$ equal to the known values from 2006:Q2, generating the logit default rates and computing $\bar{pd}_{2007,i}$. The results in table 5 serve as a benchmark for the stress scenario. The average economy default rate is plotted against time in figure 2. It remains approximately constant because both the default rate and GDP growth were already close to their long-run averages in 2006:Q2.

The 2.5 Percent Worst Cases. Table 6 compares the 2.5 percent worst cases of $\bar{pd}_{2007,i}$ (the 0.975th percentile of the base scenario) with the overall average of the base scenario. The percentage difference and its 95 percent confidence interval are reported. The 2.5 percent worst cases of the base scenario result in a 31–62 percent rise in the default rate (depending on the sector). The "industry and mining" and "trade and repair, consumer products" sectors have relatively the smallest difference in default rate. The "catering" and "financial services" sectors have a relatively large difference in the default rate.

Zero GDP Growth. The zero GDP growth scenario assumes GDP growth to be zero in 2006:Q3 and 2006:Q4. After 2006:Q4, GDP growth develops in accordance with equation (22). Logit

Figure 2. Forecasting the Economy Default Rate



Notes: The figure shows (i) the average and (ii) the .025th and .975th percentiles of the generated economy default rates given the base scenario, as well as (iii) the average generated economy default rate given the zero GDP growth scenario plotted against time.

default rates evolve according to equation (21), with the $\tilde{pd}_{t,i}$ from 2006:Q2 as the starting value. We generate the logit default rates and compute $\bar{pd}_{2007,i}$. Table 7 compares the average generated $\bar{pd}_{2007,i}$ of the zero GDP growth scenario to the base scenario. The percentage difference and its 95 percent confidence interval are reported. The confidence interval captures uncertainty in the percentage difference of the expected effect caused by uncertainty in the estimated parameters of models (21) and (22). The effect of the GDP growth scenario in 2007 is a 4–15 percent rise in the default rate, depending on the sector. The effects are surprisingly small even if we look at the upper bounds. Note that, in accordance with the estimation results from table 3, the “industry and mining,” “transport, storage, and communications,” “financial services,” and “rental and corporate services” sectors are affected most by the zero GDP growth scenario.

The question arrises as to whether this small effect is realistic in comparison with historical results. During the period 1983–91, there were about three brief, sharp drops in GDP growth (figure

Table 6. The 2.5 Percent Worst-Case Scenario

	Lower Bound	% Difference Default Rate	Upper Bound
Industry and Mining	25%	33%	41%
Construction	31%	39%	48%
Trade and Repair, Consumer Products	23%	31%	39%
Catering	54%	62%	70%
Transport, Storage, and Communications	35%	43%	51%
Financial Services	43%	51%	59%
Rental and Corporate Services	27%	35%	43%
Other	32%	40%	48%
Economy	20%	28%	36%

Notes: This table shows the percentage difference between the .975th percentile of all generated $\bar{pd}_{2007,i}$ and the average generated $\bar{pd}_{2007,i}$ given the base scenario. The percentage difference is an estimate because of uncertainty in the estimated parameters of models (21) and (22). Upper and lower bounds of a 95 percent confidence interval for the percentage difference are reported.

1). In these cases, the default rate did not react visibly. However, during the more lengthy GDP growth slowdowns of 1991–93 and 2000–03, the default rate approximately doubled. It appears that the default rate only reacts substantially to long-lasting GDP growth developments.

The average of the 2.5 percent worst-case scenarios is a lot worse than the average of the zero GDP growth scenarios. For most sectors, the 2.5 percent worst-case scenario is three to four times as bad as the zero GDP growth scenario. For the "construction," "catering," and "other" sectors, which are relatively insensitive to GDP growth, the worst-case scenario is more than nine times as bad. Figure 2 plots the average economy default rates. The figure clearly shows that the 2.5 percent worst-case scenario is a lot worse than the zero GDP growth scenario.

Table 7. Zero GDP Growth Scenario

	Lower Bound	% Difference Default Rate	Upper Bound
Industry and Mining	5%	10%	16%
Construction	−1%	4%	10%
Trade and Repair, Consumer Products	1%	6%	12%
Catering	1%	7%	12%
Transport, Storage, and Communications	7%	12%	18%
Financial	10%	15%	21%
Rental and Corporate Services	6%	12%	18%
Other	−1%	4%	10%
Economy	2%	8%	13%

Notes: This table shows the percentage difference between the average generated $\bar{pd}_{2007,i}$ given the zero GDP growth scenario and the base scenario. The percentage difference is an estimate because of uncertainty in the estimated parameters of models (21) and (22). Upper and lower bounds of a 95 percent confidence interval for the percentage difference are reported.

7. Discussion and Conclusion

The focus of this paper is to (i) assess which macroeconomic variables are related to the default behavior of Dutch firms and (ii) assess the default behavior given two quarters of zero GDP growth in the third and fourth quarters of 2006. We will discuss these two aspects in turn.

To assess which macroeconomic variables are related to the default behavior of Dutch firms, we studied GDP growth, interest rates, exchange rates, stock market returns and volatility, and oil prices. A convincing negative relation with the default rate and GDP growth was found. The relation with the oil price is also significant in several sectors. Furthermore, there is some indication of a positive relation with the (short-term) interest rate for the “construction” sector and with the (logarithm of the real) exchange rate for the "transport, storage, and communications" sector, the "financial services" sector, and the "rental and corporate services" sector. No relation with stock market return and volatility was found. Remarkably, for the interest rate, exchange rate, and oil price, it is not the change but the level of the variables that is significant to the default rate.

For the overall economy, the relations with the default rate and the macroeconomic variables are stable through time. The macroeconomic relations with the sector default rates are mostly unstable, except for the oil price. A reason for the instability is that results amongst sectors can differ according to the growth opportunities of the sector of economic activity to which firms belong, the sector's degree of internationalization, and its dependence on other sectors.

The first lag of the logit default rate has a highly significant coefficient. This implies that the effect of persistent macroeconomic shocks gradually increases over time. Without taking account of the lagged default rate, the macroeconomic variables explain, on average, about a fifth of the variance of the default rate. A latent factor affecting all sectors explains about 30 percent, and the rest is explained by sector-specific disturbances. Other literature mainly confirms the results on GDP growth and, to a limited extent, interest and exchange rates. Furthermore, the stock market is often found to be related, but always for firms listed on a stock exchange.

To assess what the behavior would be for another country, we ran the model with Austrian data. We can conclude that each country has its own dynamics. For the Dutch data, the results showed a relationship between the macro economy and the default rate. For Austrian data, the model showed no significant relationship between the macroeconomic variables and the default rate.

The effect in Dutch default behavior given two quarters of zero GDP growth in the third and fourth quarters of 2006 is that in 2007, there is a 4–15 percent rise of the default rate, depending on the sector. Historic recessions of similarly short duration are in accordance with these small numbers: the default rate does not visibly react to short recessions. However, historic recessions lead to higher long-run effects of more persistent recessions. It can be concluded that a short recession of two quarters does not influence the default rate significantly.

Concluding, a stress-test scenario of two quarters of zero GDP growth, as required by Basel II, might underestimate the true risk. We would advise to do the stress test with a more severe scenario to gain a better estimate of the true risk.

Appendix 1. Literature

- Couderc and Renault (2005) estimate the default rate of firms listed in the S&P index over the period 1981–2003 by means of a continuous time model. They also investigate lags of variables. They show that past economic conditions are of prime importance in explaining probability changes: current shocks and long-term trends jointly determine default probabilities. Significant variables are the stock market return and volatility, the term and credit spread, and GDP growth.
- Carling et al. (2002) estimate a duration model to explain the survival time to default for borrowers in the business loan portfolio of a major Swedish bank over the period 1994–2000. The model takes both firm-specific characteristics and the prevailing macroeconomic conditions into account. The output gap, the yield curve, and consumers' expectations of future economic development have significant explanatory power for the default risk of firms.
- Koopman and Lucas (2005) estimate the default rate of U.S. firms over the period 1933–97 for a general class of periodic unobserved-components time-series models with stochastic trend, seasonal, and cycle components. They take into account the correlation between stochastic cycle effects. GDP growth is found to be significant.
- Fiori, Foglia, and Ianotti (2006) find that the explanatory power of macro factors for defaults is relatively limited, but that a residual cross-section correlation of default rates suggests the presence of contagion effects from the impact of sector-specific risk on the default rates of other sectors.
- Jakubik (2006) estimates the default rate of Finnish firms over the period 1988–2003 by means of a linear vector autoregressive model. Jakubik found GDP growth to be significant, interest rates to be somewhat insignificant, and the exchange rate to be significant for the trading sector.

- Hamerle, Liebig, and Scheule (2004) estimate the default rate of German firms over the period 1991–2000 by means of a discrete time model, also including firm-specific variables. They show that systematic variables make a latent factor insignificant. They find that the inclusion of variables that are correlated with the business cycle improves the forecasts of default probabilities. Asset and default correlations depend on the factors used to model default probabilities. They conclude that correlations and default probabilities should always be estimated simultaneously. GDP growth is found to be significant.
- Lucas et al. (2006) study the relation between the credit cycle and macroeconomic fundamentals using rating transition and default data of U.S. corporations from Standard and Poor's over the period 1980–2005. They conclude that many of the variables thought to explain the credit cycle turn out to be insignificant. The main exceptions are GDP growth and, to some extent, stock market returns and stock market return volatilities. Their economic significance appears low, however.
- Kavvathas (2001) assesses the potential of conditioning on economy-wide state variables in improving the forecasting of the Credit Rating Transition Probability Matrix over the period 1981–98. He finds that an increase in nominal short-term, long-term, and real interest rates, a lower equity return, and a higher equity return volatility are associated with higher relative downgrade intensities.
- Vlieghe (2001), using UK data over the period 1975–99, suggests that the substantial rise in the number of defaults during the recession in the early 1990s mainly reflected deteriorating company finances, including a marked buildup of indebtedness. In the subsequent recovery, however, rising GDP relative to trend and other macroeconomic factors seem to have had greater explanatory power than changes in company finances in accounting for the fall in the rate of corporate liquidations to its currently low level.
- Virolainen (2004), using Finnish data over the period 1986–2003, finds a significant relationship between corporate sector default rates and macroeconomic factors, including GDP, interest rates, and corporate indebtedness.

Appendix 2. Results

Table 8. Estimated Coefficients of the Parsimonious Specification

	Intercept	First Lag	GDP Growth	Short Rate	ln(ER)	ln(Oil) Price)	Systematic s.d.	Idiosyncratic s.d.
Industry and Mining	−3.41***	.67***	−2.75***	.97*	.28	.06	.10***	.06***
Construction	−1.77	.77***	−1.24	1.52**	.05	.03	.09***	.07***
Trade and Repair, Cons. Products	−3.39***	.78***	−1.25**	.71	.40*	.07*	.06***	.07***
Catering	−4.84**	.84***	−.50	.30	.71*	.16**	.13***	.09***
Transport, Storage, and Comm.	−10.27***	.50***	−3.22***	−.58	1.48***	.18***	.12***	.08***
Financial Services	−4.79**	.83***	−2.89***	−.18	.78**	.12**	.10***	.07***
Rental and Corp. Services	−4.71***	.69***	−3.07***	1.18**	.55**	.08*	.07***	.08***
Other	−2.82	.59***	−1.15	.33	−.10	.06	.15***	.06***
Economy	−2.61**	.84***	−1.40***	.71	.32*	.05*	.08***	00
Pooled	−3.23***	.79***	−1.77***	.78*	.32*	.07**	.10***	.07***
P-Value Equal Coefficients	.0003	.0000	0.421	.0011	.0001	.1305	.0000	.1153

Notes: Model (10) is estimated with the macroeconomic variables selected in section 5 included in z_t. All estimated parameters are reported. Pooled results are obtained by estimating the sector model under the restriction $\beta_{1,j} = \ldots = \beta_{8,j}$ or $\sigma_{j,1} = \ldots = \sigma_{j,8}$ for certain j while allowing the other parameters to differ per sector. These restrictions are tested and the p-values are reported. The 1, 5, and 10 percent levels are denoted ***, **, and *, respectively.

References

Carling, K., T. Jacobson, J. Lindé, and K. Roszbach. 2002. "Capital Charges under Basel II: Corporate Credit Risk Modeling and the Macro Economy." Working Paper No. 142, Sveriges Riksbank.

Chan-Lau, J. A. 2006. "Fundamentals-Based Estimation of Default Probabilities: A Survey." IMF Working Paper No. 06/149.

Couderc, F., and O. Renault. 2005. "Times-to-Default: Life Cycles, Global and Industry Cycle Impact." FAME Research Paper Series, No. 142.

Fiori, R., A. Foglia, and S. Ianotti. 2006. "Estimating Macroeconomic Credit Risk and Sectoral Default Rate Correlations for the Italian Economy." Working Paper, Banca d'Italia.

Hamerle, A., T. Liebig, and H. Scheule. 2004. "Forecasting Credit Portfolio Risk." Discussion Paper Series 2: Banking and Financial Studies, No. 1, Deutsche Bundesbank.

Hoggarth, G., A. Logan, and L. Zicchino. 2005. "Macro Stress Tests of UK Banks." *BIS Papers* 22 (April): 392–408.

Jakubik, P. 2006. "Does Credit Risk Vary with Economic Cycles? The Case of Finland." IES Working Paper No. 2006/11.

Kavvathas, D. 2001. "Estimating Credit Rating Transition Probabilities for Corporate Bonds." Paper presented at the American Finance Association Annual Conference, New Orleans, LA, January 5–7.

Koopman, S. J., and A. Lucas. 2005. "Business and Default Cycles for Credit Risk." *Journal of Applied Econometrics* 20 (2): 311–23.

Lucas, A., A. Monteiro, S. J. Koopman, and R. Kräussl. 2006. "Credit Cycles and Macro Fundamentals." Working Paper, Tinbergen Institute.

Virolainen, K. 2004. "Macro Stress Testing with Macroeconomic Credit Risk Model for Finland." Discussion Paper No. 18/2004, Bank of Finland.

Vlieghe, G. W. 2001. "Indicators of Fragility in the UK Corporate Sector." Working Paper No. 146, Bank of England.