Interdependence between Structural Engineering and Construction Management
Edited by Ozevin, D., Ataei, H., Modares, M., Gurgun, A., Yazdani, S., and Singh, A.

ISBN: 978-0-9960437-6-2

MACROMECHANICAL ANALYSIS OF SPHERICALLY VOIDED BIAXIAL CONCRETE SLABS

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The use of spherically voided biaxial concrete slab (SVBS) system, which uses hollow plastic balls as infill material, has increased widely because of its reduced weight-to-strength and weight-to-stiffness ratios when compared to solid concrete slabs. However, SVBS is a heterogeneous composite structure in which building a representative continuum model poses a significant challenge. To mitigate this challenge, the feasibility of determining the macromechanical structural behavior of spherically voided biaxial concrete slabs is studied using plate theories, aided by mechanical properties that were determined from a homogenization process of the representative volume element (RVE). This paper presents numerical analysis results of SVBS using both Mindlin-Reissner (thick) and Kirchhoff-Love (thin) plate theories. The results from both theories predicted the slab behavior reasonably well and they were within 10% of each other with the exception of the prediction of the twisting moment. Possible explanation of this deviation is provided in the paper.

Keywords: Plate theory, RVE, Mindlin-Reissner, Kirchhoff-Love.

1 INTRODUCTION

The use of spherically voided biaxial concrete slab (SVBS) system, which uses hollow plastic balls as infill material, has increased in recent years because of its reduced weight-to-strength and weight-to-stiffness ratios when compared to solid concrete slabs. The inherently high flexural rigidity of SVBS systems mostly comes from the two solid facing plates separated by a hollow core (Voros 2011, Mota 2013). Figure 1 shows a typical biaxially reinforced concrete flat slab system that uses grids of hollow plastic balls as void formers.

A great deal of analytical research on modeling of solid, ribbed, and hollow core reinforced concrete slabs has been conducted over the past decades. However, research on the analytical modeling of spherically voided biaxial concrete slabs is relatively limited. Generally, there are three fundamentally different approaches to obtain continuum mechanics based constitutive equations for predicting the structural behavior (responses) of an SVBS system. These include phenomenological, numerical, and micromechanics approaches.

The phenomenological approach is based on conducting experiments on test specimens whose dimensions are large enough for determining stiffness properties. The stiffness properties obtained from the experiments are then used to determine parameter estimates for an assumed constitutive model in predicting the macroscopic/structural behavior of the slab. This approach certainly offers the best advantage for isotropic materials, because the response characteristics of the materials can be well-described based on few tests (Ali and Urgessa 2012, Ali and Urgessa 2014a). However, the anisotropic and non-homogeneous nature of the SVBS system greatly magnifies the challenge involved in the experimental determination of the stiffness properties. To describe a full 3-dimensional constitutive model of the SVBS, its mechanical properties in all orthogonal planes have to be obtained, which is difficult to obtain using testing results only.

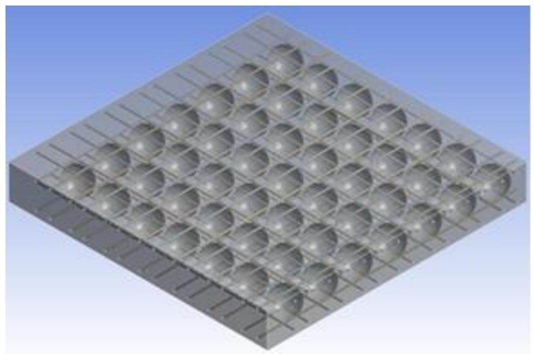

Figure 1. SVBS system.

The numerical approach is based on a detailed finite element analysis, which is a powerful technique for carrying out structural analysis. However, conducting 3D finite element analysis on a large (full-scale) SVBS has its own challenges because the geometry is complex and it needs contact algorithms between different constituents. Limited finite element simulations that investigated the structural capacities and failure mechanisms of SVBS systems under different load configurations, without conducting expensive experimental tests, are reported in Schnellenbach-Held and Pfeffer (2002), Abramski *et al. (*2010), and Ali and Urgessa (2014b). The results from these studies confirmed that SVBS systems are cost-effective when compared to conventional solid concrete slabs.

The computational effort of analyzing an SVBS system using a detailed three dimensional finite-element can be greatly reduced by the third analytical approach: modeling it as an equivalent thick-orthotropic plate through the process of micromechanical homogenization. Figure 2 illustrates this computational approach. First, a representative volume element (RVE) containing all the geometric and constitutive information of the SVBS is extracted. Then the axial, flexural and torsional properties of the RVE are determined based on the application of suitable boundary conditions that generate characteristic deformation and rotation modes. Once the homogenization process is complete, the mechanical properties of the RVE are used in conducting macromechancial analysis of the entire SVBS system.

2 MICROMECHANICAL HOMOGENIZATION

The objective of micromechanical homogenization is to establish the macroscopic behavior of a heterogeneous system by capturing the effective (overall) characteristics of its heterogeneity. The heterogeneous material is then replaced by a homogeneous material whose global characteristics are a good representation of the heterogeneous system (Urgessa and Casey 2013). In the homogenization process used here, the RVE is subjected to eight linearly independent deformations (including displacements and rotations). For each of the eight deformation/rotation cases, the equivalent reaction forces and moments at the constrained boundaries of the RVE are computed from the FE-model analysis using ANSYS (Figure 3).

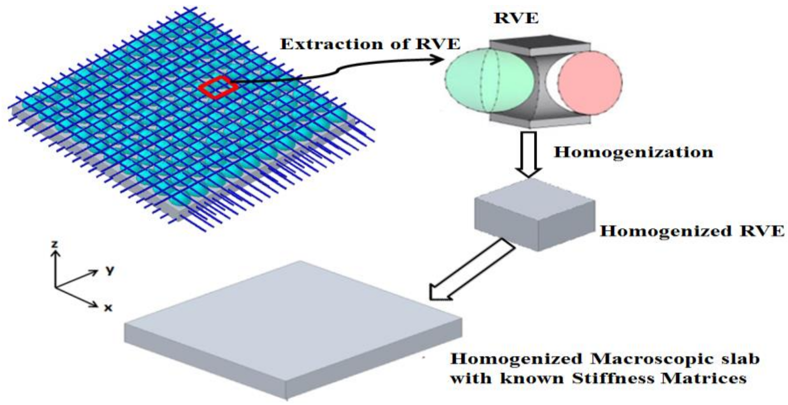

Figure 2. Framework for homogenization.

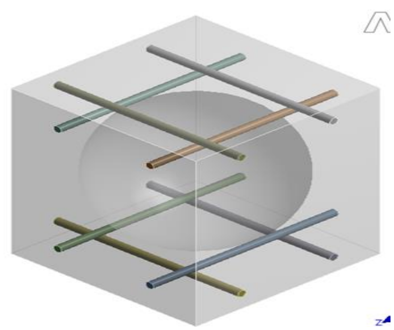

Figure 3. RVE showing concrete, rebar, and hollow composite core.

Then, based on the FE analysis results, the equivalent material properties such as the stiffness coefficients of the RVE are determined. These coefficients are similar to what are commonly referred to as ABDE stiffness matrices in mechanics of composite materials (Challagulla *et al.* 2010).

3 MACHROMECHANCIAL ANALYSIS

A macromechanical analysis of an example SVBS system was conducted using ABDE stiffness matrices determined from micromechanical modeling of RVE. A 340 mm thick SVBS slab having a length of 12 m and a width of 10 m was subjected to a vertical uniform distributed load of 20 kPa. The RVE selected for micromechncial characterization was 300 mm by 300 mm in the planar directions. Once the ABDE stiffness matrices of the RVE were characterized, a macromechancial analysis of the slab was conducted using classical plate theories.

Based on the dimensions of SVBS, the typical thickness to span ratio can be as high as 4%. Therefore, the displacement based first order shear deformation theory (FSDT), also known as Mindlin-Reissner or thick plate theory, was incorporated in the analysis. In FSDT, the effect of the transverse shear force on the deformation of the SVBS is included and the transverse shear strain distribution is assumed to be constant through the plate thickness (Cecchi *et al.* 2007). A shear correction factor was applied to account for the strain energy due to shear deformation. The other widely cited plate theory, Kirchhoff-Love theory (KLT) or thin plate theory, which was expected to under-predict deflections and stresses was also included in the study for comparison purposes (Reddy 2006).

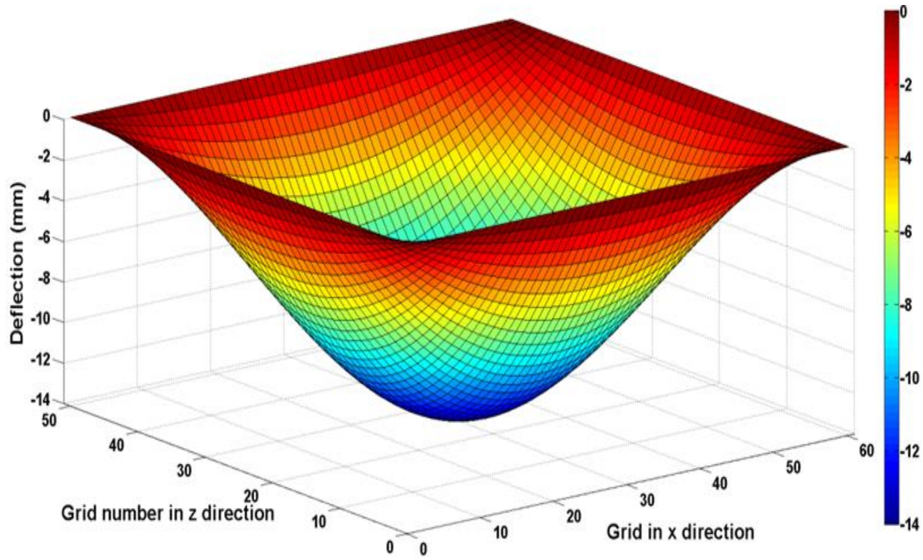

Figure 4. Mid-plane deflection using FSDT: Maximum deflection = 14 mm.

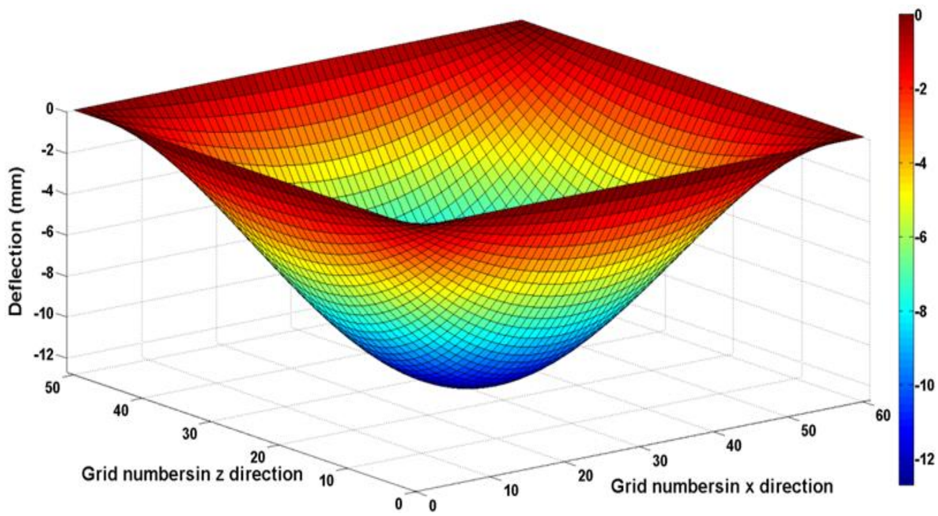

Figure 5. Mid-plane deflection using KLT: Maximum deflection = 12.5 mm.

A MATLAB program was written that determines the mid-plane deflection, bending moments in both planar directions, twisting moment, maximum bending stresses in both planar directions, and in-plane shear stress of the homogenized slab using both FSDT and KLT. Figure 4 and Figure 5 show sample outputs for the mid-plane deflection of the SVBS system using FSDT and KLT, respectively.

Results from KLT were found to be smaller than FSDT by about 10% because of the fact that KLT does not take into account the effect of transverse shear and transverse normal stresses.

This observation was true for all response parameters (deflection, bending moment and bending stress) with the exception of twisting moment in which the observed difference was about 20%. The latter can be explained by the fact that in KLT elements were assumed to remain perpendicular to the mid-plane, yet equilibrium requires that planar stress components would be present causing these elements to twist substantially.

4 CONCLUSIONS

This paper presented macromechanical analysis of a SVBS system through the process of micromechanical homogenization of RVE. The heterogeneity of the SVBS was captured by mechanical properties extracted from a homogenous RVE. Once the homogenization process was complete, two of the classic plate theories were used to determine structural responses. The results from both theories predicted the slab behavior reasonably well and results were within 10% of each other with the exception of the prediction of the twisting moment.

Acknowledgments

This research was supported by the Presidential Scholarship at George Mason University. The authors are grateful for the support.

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