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Magnetar oscillations pose challenges for strange stars

Anna L. Watts[1]★ and Sanjay Reddy[2]★

[1]*Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany*

[2]*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

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ABSTRACT

Compact relativistic stars allow us to study the nature of matter under extreme conditions, probing regions of parameter space that are otherwise inaccessible. Nuclear theory in this regime is not well constrained: one key issue is whether neutron stars are in fact composed primarily of strange quark matter. Distinguishing the two possibilities, however, has been difficult. The recent detection of seismic vibrations in the aftermath of giant flares from two magnetars (highly magnetized compact stars) is a major breakthrough. The oscillations excited seem likely to involve the stellar crust, the properties of which differ dramatically for strange stars. We show that the resulting mode frequencies cannot be reconciled with the observations for reasonable magnetar parameters. Ruling out strange star models would place a strong constraint on models of dense quark matter.

Key words: equation of state – stars: magnetic fields – stars: neutron – stars: oscillations – X-rays: stars.

1 INTRODUCTION

The properties of compact stars offer the best opportunity to study the phase diagram of quantum chromodynamics (QCD) at extreme densities, a region that is poorly constrained. One hypothesis, based on the possibility that strange quark matter (SQM) could be absolutely stable, is that compact stars are made almost entirely of deconfined SQM (Witten 1984). In this scenario, terrestrial matter made of nuclei is only metastable, albeit with an essentially infinite lifetime because it would require multiple weak interactions to convert normal matter to SQM (Weber 2006). However, the extreme ambient conditions characteristic of compact stars can facilitate the conversion of ordinary matter to SQM on short enough time-scales so that some or all compact stars could be strange stars. Distinguishing strange stars from neutron stars, however, has proved difficult. Signatures such as potentially smaller radii, and a supposed inability to glitch, are fraught with serious observational and theoretical uncertainty.

New observations, however, could change this. Certain magnetars (compact stars with magnetic fields $\geqslant 10^{14}$ G), the soft gamma repeaters (SGRs), exhibit gamma-ray flares powered by field decay (Duncan & Thompson 1992). Timing analysis of rare giant flares, which have peak luminosities 10^{44}–10^{46} erg s^{-1} and decaying tails lasting several minutes, has recently revealed high-frequency quasi-periodic oscillations (QPOs). The 2004 giant flare of SGR 1806−20 shows QPOs at 18, 26, 30, 92, 150 and 625 Hz and higher (Israel et al. 2005; Watts & Strohmayer 2006; Strohmayer & Watts 2006), whilst the 1998 giant flare from SGR 1900+14 has QPOs at 28, 53, 84 and 155 Hz (Strohmayer & Watts 2005). The most promising model involves seismic vibrations, triggered by a starquake associated with the giant flare. The lowest frequency 18- and 26-Hz QPOs fit predictions for Alfvén modes of the core. For the higher frequencies, attention has focused on toroidal shear modes of the crust (or their global magneto-elastic equivalent). These have frequencies that are, for neutron stars, a good match to the observations (McDermott, van Horn & Hansen 1988; Duncan 1998; Piro 2005; Glampedakis, Samuelsson & Andersson 2006; Samuelsson & Andersson 2007). The 28–155 Hz QPOs would be $n = 0$ modes (no radial node) with differing angular quantum number l. The 625-Hz QPO is consistent with being the $n = 1$ first radial overtone.

The likely dependence on crust properties is very exciting. Originally, strange stars were expected to be devoid of a solid crust and were characterized by an ultra-dense quark liquid extending up to the surface (Haensel, Zdunik & Pichon 1986). Such bare strange quark stars could not account for torsional shear oscillations since there is no solid region in the vicinity of the surface. However, strange stars can have solid crusts. One possibility (Alcock, Farhi & Olinto 1986) is that the strange star has a thin crust of normal nuclear material extending down to neutron drip at density $\rho \approx 4 \times 10^{11}$ g cm^{-3} (for a neutron star the crust extends beyond neutron drip to $\rho \approx 10^{14}$ g cm^{-3}), suspended above liquid SQM by an enormous electric field. A more recent model (Jaikumar, Reddy & Steiner 2006) posits a crust in which nuggets of SQM are embedded in a uniform electron background. The strange star crusts have different shear speeds and are thinner than neutron star crusts: both factors affect shear mode frequencies, crust thickness ΔR being of particular importance to the radial overtones (Hansen & Cioffi 1980). Magnetar seismology may therefore offer a robust means of distinguishing strange stars from neutron stars.

★E-mail: anna@mpa-garching.mpg.de (ALW); reddy@lanl.gov (SR)

2 MODEL

Most previous studies computing torsional shear modes of the stellar crust assume free slip over the fluid core, but if the strong magnetic field couples crust and core together then one should instead consider global magneto-elastic perturbations. Recent work on the coupled problem, however, suggests that the vibrations most likely to be observed have very similar frequencies to those computed in the uncoupled problem (Glampedakis et al. 2006; Levin 2007).[1] The natural crust frequencies are therefore physically relevant, so for simplicity we neglect coupling and apply zero traction boundary conditions (Carroll et al. 1986).

Following Piro (2005) we use a plane-parallel geometry with constant gravitational acceleration $g = GM/R^2$ in the vertical direction z, M and R being the mass and radius of the star. The Newtonian equations of hydrostatic equilibrium determine the crust density profile. Using a slab rather than spherical geometry allows us to incorporate the magnetic field without difficulty; we assume a constant field $\boldsymbol{B} = B\hat{z}$. For pure toroidal shear modes, which are incompressible and have no vertical component of displacement, the perturbation equations for the horizontal displacement ξ reduce to

$$\frac{(\mu\xi')'}{\rho} + v_{\rm A}^2\xi'' + \left[\omega^2\left(1+\frac{v_{\rm A}^2}{c^2}\right) - \frac{(l^2+l-2)\mu}{\rho R^2}\right]\xi = 0, \qquad (1)$$

where μ is the shear modulus and $v_{\rm A} = B/(4\pi\rho)^{1/2}$ is the Alfvén speed. Shear speed $v_{\rm s} = (\mu/\rho)^{1/2}$. We have assumed a periodic time-dependence $\exp({\rm i}\omega t)$, ω being the frequency. We subsequently correct for gravitational redshift to obtain the observed frequency. Primes indicate derivatives with respect to z. We have used $\nabla_\perp^2\xi = -[(l+2)(l-1)/R^2]\xi$ to mimic a spherical geometry, l being the standard angular quantum number. The scaling with l differs from that used by Piro (2005) but gives better agreement between slab and spherical models in the zero field limit (McDermott et al. 1988; Samuelsson & Andersson 2007). The shear modulus μ (Strohmayer et al. 1991) is

$$\mu = \frac{0.1194}{1+0.595(\Gamma_0/\Gamma)^2}\frac{n_{\rm i}(Ze)^2}{a}. \qquad (2)$$

Here Z is the atomic number of the ions, $n_{\rm i}$ is the density of ions and $a = (3/4\pi n_{\rm i})^{1/3}$ is the average inter-ion spacing. The parameter $\Gamma = (Ze)^2/ak_{\rm B}T$, where T is the temperature and $\Gamma = \Gamma_0 = 173$ marks the point at which the solid lattice melts to form an ocean (Farouki & Hamaguchi 1993), and we use this to determine the upper boundary of the crust. Crust temperature in the tail of a giant flare is not well constrained: observations set a lower limit of 10^7 K (Tiengo et al. 2005), but theory suggests that it could be as high as $\sim 10^9$ K (Lyubarsky, Eichler & Thompson 2002), so we examine the range $T = 10^7$–10^9K.

We consider stellar models with $M = 1.2$–$2.5\ {\rm M}_\odot$ and $R = 8$–15 km, subject to causality constraints, to cover the full range of possible strange star parameter space (Page & Reddy 2006). Both thin nuclear and nugget crust models are studied (Alcock et al. 1986; Jaikumar et al. 2006). We model magnetic fields in the range $B = 10^{12}$–10^{15} G. The strong field affects only the very outermost low-density layers of the crust (Harding & Lai 2006): we estimate the corrections to computed mode frequencies to be less than 1 per cent. Energetic arguments show that global crust structure, including the suspended crust model, is unaffected.

For the thin nuclear crust there is some uncertainty in the composition of neutron-rich nuclei, which will be constrained by future rare isotope experiments. We therefore survey a range of equations of state (Haensel & Pichon 1994; Rüster, Hempel & Schaffner-Bielich 2006).

The nugget crust is composed of a lattice of strange quark nuggets embedded in a background degenerate electron gas. The electrons contribute to the pressure, and the nuggets to the energy density. The density within the crust is given by $\rho = x\epsilon_0$, where $\epsilon_0 = n_{\rm quark}\mu_{\rm q}$ is the energy density of quark matter inside nuggets and depends on the density $n_{\rm quark} \approx 1\ {\rm fm}^{-3}$ and chemical potential $\mu_{\rm q} \approx 300$ MeV at which stable quark matter vanishes. x is the volume fraction occupied by nuggets and is given by

$$x = \frac{\mu_{\rm e}^3}{3\pi^2(n_{\rm Q} - \chi_{\rm Q}\mu_{\rm e})}, \qquad (3)$$

where $\mu_{\rm e}$ is the electron chemical potential, $n_{\rm Q}$ is the electric charge density of the quark nugget and $\chi_{\rm Q}$ is its charge susceptibility (Jaikumar et al. 2006). The quark matter parameters are poorly known and can only be determined within the context of specific models. In the Bag model, $n_{\rm Q} = m_{\rm s}^2\mu_{\rm q}/2\pi^2$ and $\chi_{\rm Q} = 2\mu_{\rm q}^2/\pi^2$. Further, requiring SQM to be absolutely stable and simultaneously requiring normal nuclei to be metastable restricts $\mu_{\rm q}$ to a narrow range centred around $\mu_{\rm q} \sim 300$ MeV. Thus, within the Bag model, the remaining uncertainty is parametrized through the effective strange quark mass $m_{\rm s}$ which we expect to be in the range 150–250 MeV.

The spherical nugget phase occupies most of the crust; we will neglect the small region at the base of the crust containing the pasta phase (see below). In computing shear modulus (equation 2), the quantity

$$Z = \frac{4\pi}{3} f R_{\rm n}^3(n_{\rm Q} - \chi_{\rm Q}\mu_{\rm e}) \qquad (4)$$

is the charge of the nugget, where $R_{\rm n} = y\lambda_{\rm d}$ is the typical nugget size, $\lambda_{\rm d} = 1/(4\pi\alpha\chi_{\rm Q})^{1/2}$ being the Debye screening length and α the fine structure constant. The factor f is a correction due to screening inside nuggets, $f \approx 3(y - \tanh y)/y^3$, where $y \approx 1.6$ (Alford et al. 2006) gives $f \approx 0.5$ for the larger droplets. The quantity $a = R_{\rm n}/x^{1/3}$ is the average inter-nugget distance, and $n_{\rm i} = 3/(4\pi a^3)$ is the density of nuggets.

The variation of $\mu_{\rm e}$ with depth is given by integrating the equations of hydrostatic equilibrium. Using the limits $x = 0$ and $x = 1$, Jaikumar et al. (2006) followed this procedure to estimate ΔR. In reality the region with $x \gtrsim 0.5$ that contains the pasta phase occupies a tiny region with thickness $\lesssim$1 m. Further, with increasing x the free energy gain of the heterogeneous state becomes negligible and even a small surface tension can disfavour the large-x region. For these reasons we set $x \simeq 0.5$ at the base of the crust. The value of x at the top of the crust is set by the melting condition: for the parameters examined, it lies in the range $x \sim 10^{-12}$–10^{-4}.

Fig. 1 shows the variation of $v_{\rm s}$ and $v_{\rm A}$ with depth for examples of crust models (compare to fig. 1 of Piro 2005). The shear speed in the nugget crust is smaller than in the thin nuclear crust. This can be understood by noting that at constant pressure $v_{\rm s} \sim \sqrt{Z^{5/3}/A}$, where A denotes the baryon number. Further, and unlike in the nuclear case, both Z and Z/A of the nuggets decrease rapidly with depth.

[1] The recent calculation by Levin (2007) is particularly interesting, as it examines the time-dependence of the interaction between the crust and the core in response to an initial perturbation. The behaviour is complicated, with time-dependence that is not pure oscillatory, but results in periodic amplification at the natural crust frequencies.

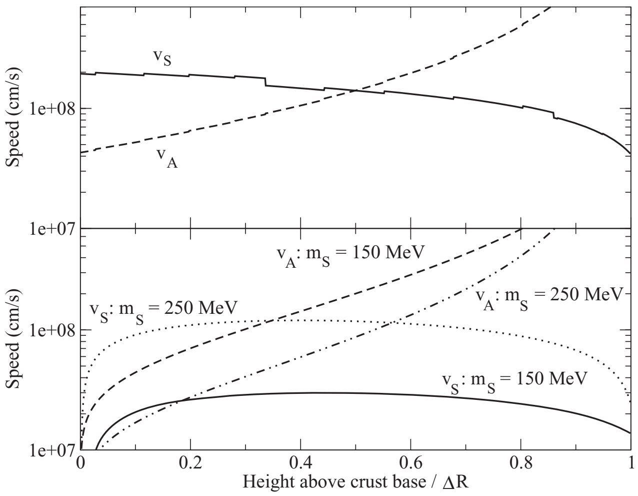

Figure 1. Shear and Alfvén velocities (v_s, v_A) in the crust for stars with $M = 1.4\,M_\odot$, $R = 12$ km, $T = 10^8$K and $B = 10^{14}$ G. Top: thin nuclear crust (crust thickness $\Delta R/R = 5.4$ per cent). Bottom: crust with nuggets, for $m_s = 150$ MeV ($\Delta R/R = 0.7$ per cent) and $m_s = 250$ MeV ($\Delta R/R = 6.1$ per cent).

3 RESULTS

Our results are in good agreement with analytic estimates (McDermott et al. 1988; Duncan 1998; Piro 2005; Samuelsson & Andersson 2007). Overall, frequencies scale with v_s. The $n = 0$ mode frequencies are almost independent of ΔR, and varying B and T changes frequency by no more than a few Hz. The frequencies scale as $[(l + 2)(l - 1)]^{1/2}$, making it difficult to fit a mode sequence with an 18- or 26-Hz fundamental. Following previous studies, we assume that these represent global Alfvén modes, and search for a fundamental at ≈30 Hz. The radial overtones vary little with l, but depend strongly on ΔR, frequencies tending to increase as the crust thins. Crust thickness is set in part by compactness. Increasing temperature also tends to push overtone frequencies up as the outer layers of the crust melt. The overtones also depend strongly on B, since v_A exceeds v_s at magnetar field strengths.

For the thin nuclear crust models, the residual uncertainty in the equation of state leads to variations of at most a few per cent, and in what follows we quote results based on Haensel & Pichon (1994). For the parameter space studied, the frequency of the $n = 0, l = 2$ mode lies in the range 26–54 Hz. To obtain a fundamental $\leqslant 30$ Hz requires high mass ($M \geqslant 2.4\,M_\odot$ for all radii, or $M \geqslant 2.2\,M_\odot$ and $R \geqslant 14$ km), pushing the limits of strange star parameter space (Page & Reddy 2006).

The overtone frequencies are all high. Even at $B = 10^{12}$ G, the lowest frequency is ≈725 Hz, for the model with the thickest crust ($M = 1.2\,M_\odot$, $R = 15$ km, $\Delta R/R = 8$ per cent). For $B > 10^{14}$ G, the lower limit on overtone frequency is higher still, at ≈1100 Hz. There is no model that would allow an overtone at 625 Hz. Fig. 2 illustrates the effects of varying B and T on the frequencies of both the $n = 0$ and $n = 1$ modes.

For the nugget crust models there is additional uncertainty in the value of m_s. Increasing m_s increases shear speed, and for the $n = 0$ modes frequency scales directly with m_s. However, in general v_s is lower than for the nuclear crust models. For the parameter space studied, the frequency of the $n = 0, l = 2$ fundamental lies in the range ≈1–21 Hz. There is no model that would permit a fundamental in the range 28–30 Hz.

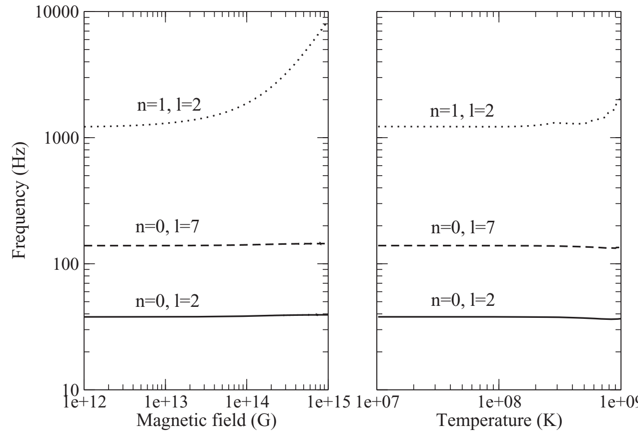

Figure 2. Thin nuclear crust mode frequencies (for a star with $M = 1.4\,M_\odot$, $R = 12$ km). Left: effect of varying magnetic field, for fixed $T = 10^8$ K. Right: effect of varying temperature, for fixed $B = 10^{12}$ G. Crust thickness $\Delta R/R$ falls from 5.4 per cent at $T = 10^8$ K to 2.1 per cent at $T = 10^9$ K.

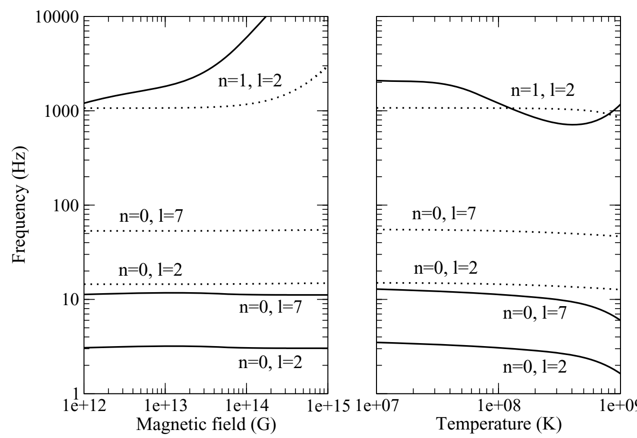

Figure 3. Nugget crust mode frequencies (for a star with $M = 1.4\,M_\odot$, $R = 12$ km) for strange quark mass $m_s = 150$ MeV (solid) and $m_s = 250$ MeV (dotted). Left: effect of varying magnetic field, for fixed $T = 10^8$ K. Right: effect of varying temperature, for fixed $B = 10^{12}$ G. The minimum in frequency for the overtone occurs at higher temperatures for $m_s = 250$ MeV.

The range of possible frequencies for the $n = 1$ overtones is extremely large. Crust thickness increases dramatically with m_s, and this effect can in fact compensate for the change in v_s. For the highest value of m_s, $\Delta R/R$ is comparable to or larger than the thin nuclear crust; for the lowest values, however, it can be an order of magnitude smaller. The temperature dependence is complex. As T increases from 10^7 K the frequency drops as v_s drops, the effect being more pronounced than for the nuclear crust. However, at high enough T, crust thinning accelerates dramatically. This can offset the effects on v_s and cause the frequency to rise again. At $B = 10^{12}$ G, frequencies for the $n = 1$ overtone can be as low as ≈260 Hz. Frequencies do, however, rise as B increases, and by $B = 10^{14}$ G only a few very extreme models permit an overtone frequency as low as 625 Hz ($M = 1.2\,M_\odot$, $R = 15$ km, $m_s \approx 250$ MeV, $T \geqslant 9 \times 10^8$ K). Fig. 3 illustrates the effects of varying B, T and m_s on the $n = 0$ and $n = 1$ modes.

4 DISCUSSION

For neutron star models, torsional shear modes, or their global magneto-elastic equivalents, are a good fit for the observations. For strange stars, the situation is much more difficult. A thin nuclear crust model can give a fundamental in the right range if stellar mass is large, but the overtone frequencies are far too high. The nugget crust model permits a wider range of frequencies because of the uncertainty in the strange quark mass m_s. The lowest order $n = 0$ modes could explain some of the QPOs in the range 18–150 Hz, but because the fundamental frequencies are so low it is difficult to fit a mode sequence, given the expected scaling with l (Samuelsson & Andersson 2007). For the radial overtones there are regions of parameter space at high temperature with modes at the right frequency, but only for magnetic fields at least an order of magnitude lower than those inferred for magnetars. At magnetar field strengths only models at the very limits of parameter space permit an overtone frequency as low as 625 Hz. The other constraint on the nugget crust is the sensitivity of the overtones to temperature fluctuations (greater for this model than for the thin nuclear crust or neutron star crusts). The observations indicate that the 625-Hz QPO has high coherence and lasts for several hundred seconds, a period during which temperature could vary substantially (Strohmayer & Watts 2006; Watts & Strohmayer 2006).

We conclude that the frequencies of toroidal shear modes in strange star crusts have serious difficulty explaining the QPO frequencies observed during magnetar hyperflares. If the results of Glampedakis et al. (2006) and Levin (2007) hold true, this conclusion will not change greatly when coupling to the core is included in the calculation. However, these results must now be verified using more sophisticated models that include a realistic stellar geometry, field configuration, and general relativistic corrections. There may also be other types of modes in the right frequency range (Chugunov 2006), although these should be harder to excite and detect. We note that there are alternative non-seismic models for the QPOs, but that these models have serious difficulties (for a discussion see Watts & Strohmayer 2007).

The clear distinction between the theoretical predictions for neutron star and strange star crust models is extremely promising. It offers a robust, largely model-independent means of distinguishing strange stars from neutron stars, something that has in the past been lacking. This type of study should lead to rapid progress in constraining the equation of state of compact stars. Ruling out the strange star hypothesis would directly impact the phase diagram of QCD at finite chemical potential. It would offer a strong constraint on models of dense quark matter, indicating that the deconfinement transition is not significantly lowered by the dynamics of the strange quarks. One important implication is that multiply strange hadronic states such as the H-dibaryon in the terrestrial context are less likely. In addition, constraints on the models at finite chemical potential should be relevant for finite-temperature extensions of these models which have been employed to describe heavy-ion experiments such as the Relativistic Heavy-Ion Collider (RHIC). Perhaps most importantly, ruling out strange stars directly from observations is the only way to ascertain that terrestrial matter is stable.

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