MAGNETIC AND ELECTRIC PHENOMENA IN THE SUN'S ATMOSPHERE ASSOCIATED WITH SUNSPOTS

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Summary

The magnetic field in the Sun's atmosphere near a sunspot, and the electric field resulting from changes in the magnetic field, are computed for an isolated sunspot growing in a non-conducting, and also in a uniformly conducting, medium. For a conductivity of 10^{-8} e.m.u., which applies in the absence of a magnetic field, it is shown that the velocity of propagation of the magnetic field is very slow; the conductivity must be less than 10^{-13} e.m.u. if the field is to penetrate to distances of the order of 3×10^9 cm. in the time of growth of the sunspot. It is shown, however, that a rapid propagation can be brought about in the atmosphere by the decrease in conductivity perpendicular to the magnetic field. The discussion is extended to include fields resulting from the growth of bipolar groups.

If the magnetic field due to other sources is negligible the current density is low, as the electric field is everywhere perpendicular to the magnetic field, in which direction the conductivity is low. When an external magnetic field is taken into consideration very different results are obtained, and currents can circulate. For a general magnetic field inclined to the Sun's surface, the currents are of two types: (*a*) a current sheet in which all the current passes through the neighbourhood of a neutral point in the magnetic field, (*b*) a volume current, all of which passes through a sunspot. This latter type is possible because the conductivity across the magnetic field increases with depth inside a sunspot. Both types of current are discussed in some detail. The present theory forms the basis of a theory of flares which will be presented subsequently.

Introduction

The magnitudes of the magnetic and electric fields in the chromosphere and corona near growing sunspots, and the associated currents, are discussed in this paper.

It is usually assumed that the magnetic field above a sunspot is similar to that from either a vertical magnetic dipole or a magnetic pole, as convenient, without examination of the rate of propagation of the field into the atmosphere. Normally the atmosphere is a good electrical conductor and, as the velocity of propagation is then very small, the above assumption needs investigation. While magnetic fields can be transported by a bulk movement of conducting material, and large movements of matter do occur in the Sun's atmosphere, our discussion is restricted to the case of a static atmosphere. A mechanism whereby electromagnetic fields can penetrate to considerable distances in a reasonably short time is given in some detail.

Consideration of the magnetic fields in the atmosphere shows that there are certain conducting paths along which current can flow if the magnetic flux linkages through them vary. Attempts are made to estimate the magnitudes of the resulting currents.

The treatment is exploratory, rather than exhaustive. In subsequent papers, theories of phenomena observed in the Sun's atmosphere, including chromospheric flares and one type of prominence, will be developed on the basis of the conclusions derived here.

Propagation of Electromagnetic Fields in the Sun's Atmosphere

Magnitudes of the Fields Assuming a Non-conducting Atmosphere.—Associated with sunspots are magnetic fields of the order of 2000 gauss (**1**), the observed direction of the field being vertical at the centre of the sunspot and horizontal at the outer edge of the penumbra. In general, the flux from the sunspot passes to some other part of the Sun's surface and does not return directly to the same sunspot, although there are rare exceptions to this. If the magnetic flux through the surface of the sunspot varies, as during its growth, an electric field must be induced in the neighbourhood. The magnetic and electric fields depend in an important manner on the conductivity of the atmosphere, but as a guide to subsequent discussion the fields will first be evaluated assuming the atmosphere to be non-conducting.

For simplicity we shall here consider two models of an isolated sunspot: (i) a current in a circle of fixed radius b equal to that of the sunspot and having an axial magnetic field H_0 equal to that observed at the centre; (ii) a semi-infinite cylindrical current sheet of the same radius and with the same axial magnetic field at its end. In both cases, the sunspot axis will be assumed vertical.

In case (i), the magnetic field H in the plane of the current is perpendicular to the Sun's surface and is given to sufficient accuracy by

$$H = b^3 H_0 / 2a^3$$

at a distance a from the centre of the sunspot, where a is large compared with b. The induced electric field E is everywhere perpendicular to the magnetic field, the lines of electric force being circles coaxial with the sunspot and parallel to the Sun's surface. In the plane of the current,

$$E = (b^3/2a^2)(dH_0/dt).$$

In case (ii), the lines of magnetic force near the sunspot are approximately radial from its centre. H is given to sufficient approximation by

$$H = H_0 b^2/2a^2,$$

while

$$E = (b^2/2a)(dH_0/dt),$$

the lines of electric force again being circles coaxial with the sunspot.

To illustrate the average values of E and final values of H around a growing sunspot, it will be assumed that H_0 grows at a constant rate to 2000 gauss in 50 hours, b being constant and equal to one-hundredth of the Sun's radius (i.e. approximately 7×10^8 cm.). Values of E and H in a plane containing the sunspot, as computed from the above formulae, are given in Table I. The magnitudes are similar for the two models.

Magnitudes of the Fields Assuming an Atmosphere of Uniform Conductivity.—The rate of propagation of an electromagnetic field through a medium depends on its conductivity. The Sun's atmosphere is normally a conductor, and its properties have been discussed by Cowling (**2**), who has shown that in the presence of a magnetic field, the conductivity becomes anisotropic; the conductivity

TABLE I

Electric and Magnetic Fields Surrounding a Sunspot in a Non-conducting Medium

Model	Distance from Centre of Sunspot (cm.)	Magnetic Field (gauss)	Electric Field (e.m.u.)
Circular current	3.5×10^9	8	1.55×10^5
	1.75×10^9	64	6.2×10^5
Cylindrical current sheet	3.5×10^9	40	7.7×10^5
	1.75×10^9	160	15.6×10^5

For a spot of diameter one-hundredth that of the Sun, the above distances correspond to 5 and 2·5 times the spot radius from the centre of the spot. The magnetic fields are for an axial magnetic field of 2000 gauss, and the electric fields are the average induced fields when the magnetic field grows uniformly to 2000 gauss in 50 hours. The results apply only in the plane of the Sun's surface.

at right-angles to the magnetic field decreases as the latter increases in strength. Thus a discussion of the effects of the real atmosphere becomes complex, and we shall consider first the case of an ideal atmosphere of constant conductivity.

The circular current sunspot model is used and we shall consider only its magnetic moment which may be represented during the growing stage by $\frac{1}{2}b^3H_0 \sin \omega t$. The fields will be regarded as those due to a magnetic dipole situated at the centre of the sunspot and having a moment equal to the imaginary part of $\frac{1}{2}b^3H_0e^{-i\omega t}$. Thus we require the fields due to a dipole whose moment is zero before time $t=0$, and has the above value subsequently, but as the sunspot moment can only be represented in the above way for the first quarter period, the results will be valid only during this time.

The electromagnetic field surrounding a continuously oscillating magnetic dipole in a homogeneous isotropic conducting medium is well known (3). In a plane containing the dipole, and perpendicular to its axis,

$$E = \mathscr{I} k^2(\mu/\epsilon)(1/a + i/ka^2)Ce^{-i\omega t^*},$$

$$H = \mathscr{I}(1/a^3 - ik/a^2 - k^2/a)Ce^{-i\omega t^*},$$

where $\omega/2\pi$ and C are the frequency and the amplitude of the magnitude moment of the dipole, σ and μ are the conductivity and permeability of the surrounding medium, $\epsilon = \epsilon_0 + i\sigma/\omega$ is its complex dielectric constant, $k^2 = \epsilon_0\mu\omega^2 + i\omega\mu\sigma$, $\omega t^* = \omega t - ka$, and a is the distance from the dipole; the symbol $\mathscr{I}$ indicates that the imaginary part of the whole of the expression is to be taken. In the above equations, all terms are in unrationalized (e.g. c.g.s. electromagnetic) units. In practice ω is extremely small, a value of 5.6×10^{-6} sec.$^{-1}$ corresponding to a sunspot growth time, from first appearance to maximum size, of 78 hours. (This value yields the same maximum value of E for $\sigma = 0$ as the simple treatment of the preceding section.) Thus unless σ is negligibly small (less than 5×10^{-27} e.m.u.), $\epsilon = i\sigma/\omega$ and $k^2 = i\omega\mu\sigma$.

We shall first discuss the magnetic field H, concerning which there is some observational evidence. The original function for the magnetic moment of the sunspot may be replaced by

$$\mathscr{I}\frac{C}{2\pi i}\int_L \frac{e^{st}}{s+i\omega}ds,$$

where $is/2\pi$ may be treated as a component frequency in the original function, and the path of integration L is a line parallel to, and to the right of, the imaginary axis, and extending from $\gamma - i\infty$ to $\gamma + i\infty$(4). Then

$$H = \mathscr{I}(C/2\pi a^3 i)\int_L \{1 - ika - k^2a^2\}\{\exp(st + ika)\}/(s + i\omega)\,.\,ds$$
$$= \mathscr{I}(C/2\pi a^3 i)\int_L \{1 + \alpha s^{\frac{1}{2}} + \alpha^2 s\}\{\exp(st - \alpha s^{\frac{1}{2}})\}/(s + i\omega)\,.\,ds,$$

where $\alpha = a(\mu\sigma)^{\frac{1}{2}}$. Now

$$\int_L \exp(st - \alpha s^{\frac{1}{2}})/(s + i\omega)\,.\,ds =$$
$$\pi i e^{-i\omega t}[\exp\{(1 - i)\alpha(\omega/2)^{\frac{1}{2}}\}\operatorname{erfc}\{\alpha/2t^{\frac{1}{2}} + (1 - i)(\omega t/2)^{\frac{1}{2}}\}$$
$$+ \exp\{-(1 - i)\alpha(\omega/2)^{\frac{1}{2}}\}\operatorname{erfc}\{\alpha/2t^{\frac{1}{2}} - (1 - i)(\omega t/2)^{\frac{1}{2}}\}],$$

which we may write as $\pi i f(\alpha, t)$, where

$$\operatorname{erfc} z = 2\pi^{-\frac{1}{2}}\int_z^\infty \exp(-x^2)dx.$$

Also

$$\int_L \alpha s^{\frac{1}{2}}\{\exp(st - \alpha s^{\frac{1}{2}})\}/(s + i\omega)\,.\,ds = -\alpha\pi i\,\frac{df(\alpha, t)}{da}$$
$$= -\alpha\pi i(1 - i)(\omega/2)^{\frac{1}{2}}e^{-i\omega t}[\exp\{(1 - i)\alpha(\omega/2)^{\frac{1}{2}}\}\operatorname{erfc}\{\alpha/2t^{\frac{1}{2}} + (1 - i)(\omega t/2)^{\frac{1}{2}}\}$$
$$- \exp\{-(1 - i)\alpha(\omega/2)^{\frac{1}{2}}\}\operatorname{erfc}\{\alpha/2t^{\frac{1}{2}} - (1 - i)(\omega t/2)\}] + 2i\pi^{\frac{1}{2}}\alpha t^{-\frac{1}{2}}\exp\{-\alpha^2/4t\},$$

which we may write as

$$\pi i[-\alpha(1 - i)(\omega/2)^{\frac{1}{2}}g(\alpha, t) + 2\alpha(\pi t)^{-\frac{1}{2}}\exp\{-\alpha^2/4t\}].$$

Again,

$$\int_L \alpha^2 s\exp(st - \alpha s^{\frac{1}{2}})/(s + i\omega)\,.\,ds = \pi i\alpha^2\frac{df(\alpha, t)}{da}$$
$$= \pi i[-i\alpha^2\omega f(\alpha, t) + \alpha^3\pi^{-\frac{1}{2}}t^{-\frac{3}{2}}\exp\{-\alpha^2/4t\}].$$

Hence

$$H = \mathscr{I}(C/a^3)[\tfrac{1}{2}f(\alpha, t)(1 - i\alpha^2\omega) - (\alpha/2)(1 - i)(\omega/2)^{\frac{1}{2}}g(\alpha, t)$$
$$+ (\alpha/2)(2 + \alpha^2/t)(\pi t)^{-\frac{1}{2}}\exp(-\alpha^2/4t)].$$

It may be noted that the first factor is simply the amplitude of the magnetic field in a non-conducting atmosphere.

Values of $\operatorname{erfc} z$ for a complex z having a positive real part may be obtained from a table given by Born (5), while values for z having a negative real part are readily obtained from the relation

$$\operatorname{erfc} z + \operatorname{erfc}(-z) = 2.$$

Values of H for $\omega = 5{\cdot}6 \times 10^{-6}$ sec.$^{-1}$ and $t = 10^5$ sec., and for various values of K, where $\alpha = K(2\omega)^{\frac{1}{2}}t$, are given in Table II. a is then K times as great as the distance travelled at the so-called "phase velocity" in time t. H is expressed as a fraction of the magnetic field expected at time t in a non-conducting atmosphere. It will be noticed that the magnetic field does not differ greatly from the latter, provided $K < 3$, but is negligible when $K > 3$. Thus the magnetic field grows as if propagated at about three times the "phase velocity" $(2\omega/\mu\sigma)^{\frac{1}{2}}$, of a wave of angular frequency ω.

TABLE II

Electric and Magnetic Fields Surrounding a Magnetic Dipole in a Uniformly Conducting Medium

$K\left(=\frac{a}{t}\left\{\frac{\mu\sigma}{2\omega}\right\}^{\frac{1}{2}}\right)$	1	2	3	4	5
$H/H_{\sigma=0}$	1·18	0·82	0·30	0·057	0·0057
$E/E_{\sigma=0}$	0·98	0·60	0·20	0·035	0·0034

The values given above are the ratios of the actual magnetic and electric fields in a plane perpendicularly bisecting the axis of a magnetic dipole situated in a homogeneous isotropic conducting medium, to the corresponding fields when the medium is a non-conductor. The dipole has zero moment before time $t=0$, and moment $C=C_0 \sin \omega t$ subsequently. K gives the ratio of the distance of the point considered from the dipole to the distance travelled by a disturbance at the "phase velocity" $(2\omega/\mu\sigma)^{\frac{1}{2}}$. The particular values for which the above table applies are $\omega=5{\cdot}6\times 10^{-6}$ sec. and $t=10^5$ sec.

If the magnetic field is to penetrate to distances of the order $3{\cdot}5\times 10^9$ cm. in the time of growth of the sunspot, $2{\cdot}8\times 10^5$ sec., then σ must not exceed about 10^{-13} e.m.u. It is generally accepted that the magnetic field from a sunspot penetrates to distances greater than this, and if this be so, the effective conductivity of the atmosphere has an upper limit of 10^{-13} e.m.u. On the other hand, it is known that in the absence of a magnetic field, the electrical conductivity is much greater than this, being of the order of 10^{-8} e.m.u. This immediately raises the question whether the magnetic field can penetrate into the atmosphere in a reasonable time, a matter to be discussed in following sections.

As with the magnetic field, the electric field due to the growing sunspot is given by

$$E=\mathscr{I}(C\mu^{\frac{1}{2}}/2\pi a^2 i)\int_L\{ak^2+ik\}\{\exp(st+ika)\}/\epsilon^{\frac{1}{2}}(s+i\omega)\,.\,ds,$$

which reduces to

$$E=\mathscr{I}(C\mu\omega/a^2)[(i/2)f(\alpha,t)-(1+i)(\alpha/2)(\omega/2)^{\frac{1}{2}}g(\alpha,t)$$
$$+\{(2i-\alpha^2/2\omega t^2)/2(\pi t)^{\frac{1}{2}}\}\exp(-\alpha^2/4t)],$$

where the first factor is simply the amplitude of the induced field in a non-conducting atmosphere.

Values of E for $\omega=5{\cdot}6\times 10^{-6}$ sec.$^{-1}$ and $t=10^5$ sec., and for various values of K, are also given in Table II, E being expressed as a fraction of the induced electric field expected in a non-conducting atmosphere.

The Electrical Conductivity of the Sun's Atmosphere.—Cowling (2) has shown that as regards electrical conductivity the solar chromosphere and corona behave as highly ionized gases, and has expressed the electron conductivity, when the electric and magnetic fields are mutually perpendicular, in the form

$$\sigma^{\rm I}+i\sigma^{\rm II}=(rZT^{-\frac{3}{2}}-i\,8{\cdot}6\times 10^3 HT/p)^{-1}\text{ e.m.u.},$$

where $\sigma^{\rm I}$ and $\sigma^{\rm II}$ are the direct conductivity (i.e. in the direction of the electric field) and transverse conductivity (i.e. in a direction mutually perpendicular to the electric and magnetic fields) respectively, Z is the mean degree of ionization (equal to unity for hydrogen), T the electron temperature (deg. K.), and p the electron pressure. r is a quantity which varies but little over a wide range of electron concentrations and temperature, and for the chromosphere its value is approximately $1{\cdot}1 \times 10^{14}$.

The values given by the above formulae are approximations only; a better approximation is obtained when σ^{II}/σ^{I} is small by putting, for hydrogen, $Z = 0{\cdot}51$. Cowling's formula does not include the positive ion conductivity, which is of considerable importance here, and a mean free path treatment including electron and positive ion conductivities is given below.

To find the current, we follow the normal practice of resolving the electric field along and perpendicular to the magnetic field. The respective conductivities are denoted by σ_0 and σ^{I}, while the transverse conductivity, in a direction perpendicular to the plane containing the electric and magnetic fields, is denoted by σ^{II}. The conductivity σ_0 is effectively the electron conductivity, and is given by

$$\sigma_0 = ne^2\tau/m,$$

where n is the number of electrons per c.c., e and m are the electron charge and mass, and τ, the mean free time between collisions of electrons with protons, is given by

$$\tau = u^3/\pi\beta^2 NZ^2,$$

where u is the mean electron velocity, N the number of positive ions per c.c., and β a constant, equal to $1{\cdot}57 \times 10^9$ cm.^{3}/sec.^{2} (**6**). Thus putting $n = N$ and $Z = 1$,

$$\sigma_0 = e^2u^3/\pi\beta^2 m,$$

which has the value $4{\cdot}88 \times 10^{-9}$ e.m.u. if u corresponds to 5750 deg. K., as compared with Cowling's more accurate value of $7{\cdot}8 \times 10^{-9}$ e.m.u. when $Z = 0{\cdot}51$.

The direct conductivity σ^{I} is given by

$$\sigma^{I} = \frac{ne\omega\tau}{H(1+\omega^2\tau^2)} + \frac{Ne\Omega\tau}{H(1+\Omega^2\tau^2)},$$

where the first and second terms are the contributions from electrons and positive ions respectively, $\omega = eH/m$ and $\Omega = e_pH/M$, e_p and M being the proton charge and mass (**7**). For small values of H, the proton contribution is negligible, but for large values of H it predominates. If $n = N$, it is found on substituting for ω, Ω and τ that σ^{I} is a function of H/n; values of σ^{I} are given in Table III for $T = 5750$ deg. K.

The transverse conductivity σ^{II} is given by

$$\sigma^{II} = \frac{ne\omega^2\tau^2}{H(1+\omega^2\tau^2)} - \frac{Ne\Omega^2\tau^2}{H(1+\Omega^2\tau^2)}.$$

When H is small, so that $\Omega^2\tau^2 \ll 1$, the proton contribution is negligible, but when $\Omega^2\tau^2 \gg 1$, the terms become numerically equal, and σ^{II} vanishes. We shall assume that σ^{II} plays little part in the general behaviour of the electromagnetic field in the atmosphere, although it may be of importance in other respects. It will be neglected for the remainder of this paper.

The electron concentration, required for the evaluation of σ, has been given by Cillié and Menzel (**8**) as

$$n = 3{\cdot}9 \times 10^{11} \exp(0{\cdot}77 \times 10^{-8}x) \text{ per c.c.},$$

where x is the height above the base of the chromosphere. The corresponding electron pressure, at a temperature 5750 deg. K., is

$$p = 0{\cdot}30 \exp(0{\cdot}77 \times 10^{-8}x) \text{ dynes/cm.}^2.$$

TABLE III

The Electrical Conductivity of the Chromosphere

H/n gauss cm.^{3}	Electron Conductivity e.m.u.	Proton Conductivity e.m.u.	σ^{I} e.m.u.
0	$4 \cdot 9 \times 10^{-9}$	$2 \cdot 7 \times 10^{-12}$	$4 \cdot 9 \times 10^{-9}$
10^{-12}	$4 \cdot 5 \times 10^{-9}$	$2 \cdot 7 \times 10^{-12}$	$4 \cdot 5 \times 10^{-9}$
$3 \cdot 2 \times 10^{-11}$	$2 \cdot 5 \times 10^{-9}$	$2 \cdot 7 \times 10^{-12}$	$2 \cdot 5 \times 10^{-9}$
10^{-11}	$4 \cdot 7 \times 10^{-10}$	$2 \cdot 7 \times 10^{-12}$	$4 \cdot 7 \times 10^{-10}$
$3 \cdot 2 \times 10^{-10}$	$5 \cdot 2 \times 10^{-11}$	$2 \cdot 7 \times 10^{-12}$	$5 \cdot 5 \times 10^{-11}$
10^{-10}	$5 \cdot 2 \times 10^{-12}$	$2 \cdot 7 \times 10^{-12}$	$7 \cdot 9 \times 10^{-12}$
$3 \cdot 2 \times 10^{-9}$	$5 \cdot 2 \times 10^{-13}$	$2 \cdot 7 \times 10^{-12}$	$3 \cdot 2 \times 10^{-12}$
10^{-9}	$5 \cdot 2 \times 10^{-14}$	$2 \cdot 6 \times 10^{-12}$	$2 \cdot 7 \times 10^{-12}$
$3 \cdot 2 \times 10^{-8}$	$5 \cdot 2 \times 10^{-15}$	$2 \cdot 1 \times 10^{-12}$	$2 \cdot 1 \times 10^{-12}$
10^{-8}	$5 \cdot 2 \times 10^{-16}$	$7 \cdot 1 \times 10^{-13}$	$7 \cdot 1 \times 10^{-13}$
$3 \cdot 2 \times 10^{-7}$	$5 \cdot 2 \times 10^{-17}$	$9 \cdot 3 \times 10^{-14}$	$9 \cdot 3 \times 10^{-14}$
10^{-7}	$5 \cdot 2 \times 10^{-18}$	$9 \cdot 7 \times 10^{-15}$	$9 \cdot 7 \times 10^{-15}$

The conductivities given in the above table are based on the assumption that the chromosphere behaves as a highly ionized gas at a temperature of 5750 deg. K.

Electromagnetic Fields in an Ionized Atmosphere.—We shall now consider the propagation of an electromagnetic field into a static ionized medium, the discussion being restricted to the simple case where the induced electric field is perpendicular to the magnetic field; as, for example, the case where the magnetic field is due to no other cause than the growing sunspot.

If throughout the region under consideration there be a magnetic field of sufficient intensity, the conductivity of the Sun's atmosphere in the direction of the induced electric field, i.e. perpendicular to the magnetic field, will be so low that changes in the field, such as postulated above, will be propagated rapidly. On the other hand, the rate of propagation is very small when the magnetic field is small, and at first sight it might appear that the magnetic field could not increase sufficiently, in the time available, to allow rapid propagation. It is necessary, therefore, to find the rate of propagation of a magnetic field into a medium whose conductivity depends on the magnitude of the field. A simple and rigorous treatment seems impracticable, but we may be able to infer the general nature of the solution from a consideration of a simpler problem, namely, the one dimensional propagation of a magnetic field, whose boundary conditions are initially $H = H_z$, $a < 0$ and $H = 0$, $a > 0$, into a medium of constant conductivity. In cases with which we are concerned, H_z will be of the order of hundreds of gauss close to a sunspot and of the order of tens of gauss farther away. The solution may be obtained by a method similar to that used earlier in this paper, or from analogy with a formally similar and well-known problem in heat conduction. The solution is $H = H_z \operatorname{erfc}(\alpha/2t^{\frac{1}{2}})$, where, as before, $\alpha = a(\mu\sigma)^{\frac{1}{2}}$. Let $\alpha/2t^{\frac{1}{2}} = h$ when H has some given value. Then $(d\alpha/dt)_{H \text{ constant}} = h^2/2\alpha$ is a measure of the velocity of propagation of any particular value of H and is infinite for $a = 0$. If a be large, the velocity of propagation into a medium in which $\sigma = 10^{-8}$ e.m.u. is so small that the magnetic field of a sunspot could not penetrate an appreciable distance during its time of growth. However, we see from this problem that when the gradient of H is large the rate of propagation is large.

When we take into account the reduction in conductivity perpendicular to the magnetic field (which is the only direction of importance) caused by fields greater than about one gauss, the value of $h^2/2\alpha$ yields large velocities of propagation wherever σ is very small. At the same time the gradient of the magnetic field for H less than one gauss is maintained at such a value that the velocity of propagation equals that for the higher fields, so that the governing factor in determining the velocity is the value of σ for a large magnetic field.

For illustration purposes, the rate of penetration, which is of the order of $3(2\omega/\mu\sigma)^{\frac{1}{2}}$, may be given for a typical case, $\omega = 5{\cdot}6 \times 10^{-6}$ per sec., $\mu = 1$, $H = 100$ gauss (which corresponds to $\sigma = 7{\cdot}1 \times 10^{-13}$ where $n = 10^{10}$ per c.c.). The velocity is then $1{\cdot}2 \times 10^4$ cm./sec. For higher values of H (i.e. closer to the sunspot) or for smaller values of n (i.e. higher in the chromosphere) the velocity will be greater.

It is worth noting that if the effects of mass movements be excluded, magnetic fields cannot be propagated to any appreciable distance beneath the Sun's surface during the time of growth of the sunspot; for the electron pressure is so high there that the conductivity is scarcely affected by magnetic fields of the magnitude expected in the case of a non-conducting surrounding medium. Since the conductivity is at least as great as 10^{-8} e.m.u., the "phase velocity" is no more than 33 cm./sec., and in a day the field penetrates, at three times this velocity, a distance of only about 86 km.

In that region of the atmosphere where H exceeds a few gauss, the electric field is essentially equal to that for similar conditions in a non-conducting atmosphere, and this is simply the induced electric field. The current density at any point is small because the conductivity in the direction of the electric field is small. Outside this region, E is very small, so that again the current density is small. It follows that under such conditions no significant amount of energy can be acquired from these fields by electrons.

The Influence of the Sun's Surface on the Fields.—So far we have discussed the fields as set up by a magnetic dipole situated in a uniform medium. Sunspots, however, appear near the boundary of two media which are individually reasonably uniform in electrical properties but differ markedly from one another; viz. the Sun's atmosphere, in which the conductivity is a function of the magnetic field strength, and the body of the Sun where, except perhaps within sunspots, the conductivity is generally unaffected by the magnetic field. The flux returning to the Sun from the atmosphere cannot rapidly penetrate the surface, as magnetic fields are propagated only very slowly through the body of the Sun. Thus the flux cannot be represented everywhere by that from a vertical magnetic dipole. Surface currents are set up which, at least during the growth of the sunspot, just neutralize at the surface any vertical component of the magnetic field due to the sunspot; at the same time these currents affect the magnetic and electric field in the atmosphere.

Usually sunspots occur in bipolar groups, and the effect of the conducting surface is to prevent the flux from one sunspot returning to the Sun elsewhere than at one or more other sunspots of opposite polarity. The magnetic field away from the immediate neighbourhood of the sunspots is almost the same as that due to two isolated poles on the Sun's surface. To the same approximation, the electric field may also be regarded as due to equal simultaneous changes in the strengths of the two poles. Further we may assume that, away from the immediate neighbourhood of the sunspots, the electric field is due to a magnetic dipole whose axis lies in the plane of the surface, and whose magnetic moment U is equal to that of the spot group. In this case, assuming a non-conducting atmosphere,

$$E = U\mu\omega \cos \omega t \,.\, (\sin\theta)/a^2,$$

where θ is the angle between the axis of the dipole and the radius vector from the centre of the dipole. U is given simply by $lb^2H_0/2$, l being the separation between the centres of the two sunspots. Putting $\mu = 1$,

$$E = lb^2H_0\omega \cos \omega t \,.\, (\sin\theta)/a^2.$$

In conformity with data previously used, put $H_0 = 2000$ gauss, $\omega = 5{\cdot}6 \times 10^{-6}$ sec.$^{-1}$, $b = 7 \times 10^8$ cm., $l = 1{\cdot}5 \times 10^9$ cm., $a =$ (i) $1{\cdot}75 \times 10^9$ cm., (ii) $3{\cdot}5 \times 10^9$ cm. Then for $t = 0$, $E = 1{\cdot}3 \times 10^6 \sin\theta$ and $3{\cdot}4 \times 10^5 \sin\theta$ e.m.u. respectively. In this case, the lines of electric force are circles coaxial with the axis of the spot group, and thus rise vertically from the surface of the Sun.

Since the induced electric and magnetic fields are still mutually perpendicular, no appreciable currents can result from the above electric field, and also the fields are propagated through the Sun's atmosphere at the same rates as discussed in the previous sections.

Currents in the Sun's Atmosphere

The Influence of the Sun's General Magnetic Field.—Thus far we have considered orthogonal electric and magnetic fields. If there be superimposed on the field of a growing sunspot a steady magnetic field of some other origin, then the induced electric field remains unchanged but the magnetic field is altered, and the two need no longer be mutually perpendicular. The induced electric field may indeed have a strong component in the direction of the magnetic field, in which direction the conductivity is high. Such a state of affairs may be brought about by the Sun's general magnetic field (which is about 50 gauss at the poles and 25 gauss at the equator (**9**)) or may be due to other sunspots in the same or a nearby spot group.

Under these conditions, the rate of propagation of the sunspot's magnetic field into the Sun's atmosphere needs re-examination. This complex problem cannot be treated simply, and we first investigate the types of current paths which may occur on the assumption that changes in the magnetic field are propagated rapidly. We then estimate the resistances of various current paths and finally examine the rate of propagation of changes in the magnetic field.

The general magnetic field is permanent and therefore we assume that it passes through the Sun's surface unchanged in magnitude or direction. It is presumably roughly north-south and inclined to the surface. As it is of the same magnitude as the sunspot's magnetic field at distances of the order of 3×10^9 cm. from the sunspots, the two fields exactly neutralize one another at a point at some such distance, and the point where this occurs will be called a neutral point.

Some of the lines of force from the sunspot can now lead down into the body of the Sun, as is illustrated in Fig. 1, which represents a vertical plane containing a neutral point and a sunspot. The general field below the Sun's surface remains constant and this limits the number of lines of force which can pass from the sunspot down into unit area of the surface of the Sun. If we consider

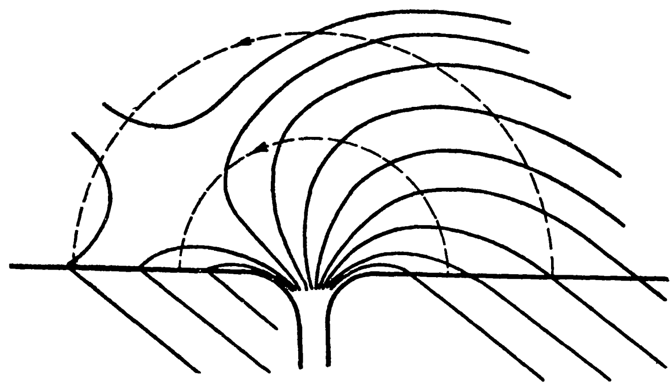

FIG. 1.—*Section through neutral point and sunspot, showing lines of magnetic force.*
A section, by a vertical plane, of the magnetic field resulting from the Sun's general magnetic field and that due to a bipolar spot group. The section, which is perpendicular to the axis of the spot group, includes a neutral point and its associated sunspot. Note that the magnetic field remains unchanged below the Sun's surface. The direction of the induced electric field is indicated by broken lines.

a spot group containing two sunspots of equal and opposite polarity, the arrangement of the lines of force will depend on the orientation of the external constant field to the axis of the group. This will probably be approximately 90° in the case of the Sun's general field and, in a plane containing the neutral point and spot group axis, the lines of force will be as shown in Fig. 2. Discussion of the currents will be limited to the case of relatively weak sunspots where the lines

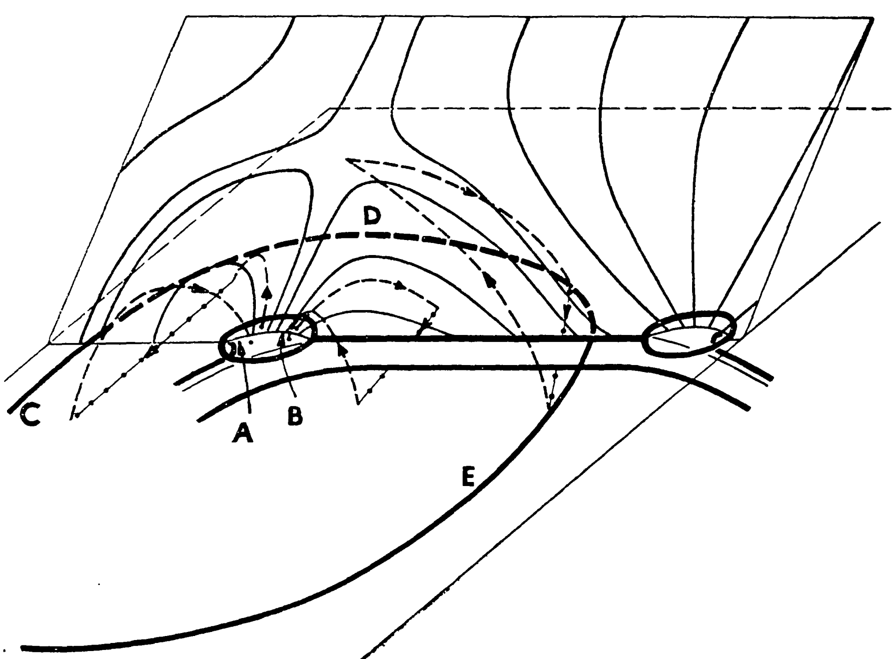

FIG. 2.—*Magnetic field and current distribution in a growing bipolar spot group.*
The lines of magnetic force are shown in a plane containing the Sun's general magnetic field, and intersecting the two sunspots (the "dividing" plane). Portion of the sub-surface toroidal magnetic field is also shown. The sunspots are relatively weak, so that their flux is captured by the general field. Typical "neutral point" and "spot" currents, as well as their relative directions, are also given. The line AB is the locus of points on the surface of the sunspot from which lines of magnetic force return to the Sun along the axis of the spot group. The curve CDE is the locus of points where lines of magnetic force from near the neutral point meet the Sun's surface, and is thus the line along which the neutral point current sheet meets the surface.

of force from the spots are all captured by the Sun's general field. This certainly does not happen with all spot groups, nor is it certain that it happens with any. Nevertheless, some of the lines of force from every spot group are captured by the general field, and the following discussion applies equally to these groups.

Currents Through a Neutral Point.—Since in the atmosphere currents must follow the direction of the magnetic field, unless the latter be weak, complete conducting circuits can occur only in limited ways. One such circuit may commence at the Sun's surface, and coincide with lines of magnetic force except close to the neutral point, near which it passes; beyond, it again coincides with the magnetic field down to the Sun's surface. Below the surface the conductivity is unaffected by the general magnetic field, and the circuit may be completed by subsurface paths on either side of the sunspot. Owing to the great cross-section available, the subsurface resistances are effectively zero.

Rather than evaluate the total e.m.f. around such a circuit from field theory, we shall find it more convenient for the remainder of this paper to estimate it from the rate of change of magnetic flux through the circuit. The question then arises, can the magnetic flux through any such circuit change?

Hale and Nicholson's remarks on the growth of a spot group have been summarized by Chapman as follows (**1**). "A typical spot group begins as two small spots, or groups of spots of opposite polarity, nearly in the same latitude, and 3° or 4° apart in longitude. The two principal spots grow rapidly and separate in longitude to a distance of 10° or more; the rear spot attains its maximum area in 3 or 4 days; the leader attains a larger maximum area in 7, 8 or 9 days. Many smaller spots develop within the group, mostly near the two main ones. The dipole moment of the group is greatest when the spots are largest and furthest apart".

Owing to the high conductivity of the Sun's surface surrounding a spot it may be expected that during its growth, there would be set up in the immediate neighbourhood currents resulting in the return of magnetic flux immediately outside the spot. Such a field in the opposite direction to that in the centre of the sunspot has not been reported and is presumably non-existent. We shall assume, therefore, that in the absence of a general magnetic field, the flux from a sunspot would pass entirely to one or more other sunspots.

The current distribution in the atmosphere depends in an important manner on the means whereby the sunspot is created, so that it is desirable to consider this. We shall, in fact, evaluate the currents on an assumed mechanism for the creation of sunspots, although the results are reasonably general; for any other mechanism, currents of the same type and magnitude will result, although their location and direction may be different. The mechanism adopted, however, must be in conformity with the observed growth of sunspots.

The exact nature of the origin of a sunspot is still not clear. There is good reason to believe that the magnetic field of the sunspot has been in existence beneath the Sun's surface for many years before the appearance of the spot. Alfvén (**10**) and Walén (**11**), for example, have proposed a theory in which the original magnetic field is approximately toroidal, the current creating the magnetic field flowing in a circle around every right section of the toroid. When such a toroid reaches the surface of the Sun, the magnetic field is thought to break through to form two spots. Alfvén attributes the breaking through the surface to the decrease in density there, but from our previous discussion, a decrease in conductivity, such as occurs perpendicular to a magnetic field in the atmosphere, seems essential. In either case, the magnetic field during the formation of the spots is probably somewhat as shown in Fig. 3, where some of the lines of force have already broken through to form two sunspots. As further lines of force break through the surface, they add to the flux from the sunspots. We shall now use this as our sunspot model, although the following arguments can readily be modified to suit any other model of a growing sunspot.

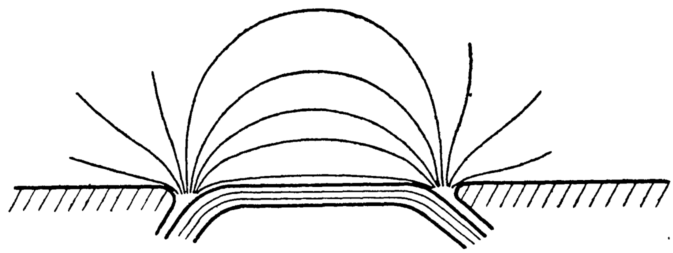

FIG. 3.—*Vertical cross-section through bipolar spot group during growth.*
The magnetic field near a sunspot group during its growth is presumably somewhat as shown above. Part of the original toroidal magnetic field is still below the surface. As lines of force break through, they give rise to an increase in the flux from the sunspots.

Although all atmospheric conducting paths of the type described above pass close to the neutral point, they are not confined to one fixed direction, for near the neutral point the magnetic field is symmetrical about the line joining the neutral point and sunspot; in general, circuits come in to the neutral points from all directions perpendicular to this line.

We shall now discuss the case in which the general magnetic field is perpendicular to the axis of the spot group (i. e. the line joining the two sunspots), and inclined to the Sun's surface at an angle ψ (see Fig. 2). Since the conductivity at right-angles to the magnetic field is high only when the field is weak, there will be only a small region near the neutral point where the current density perpendicular to the magnetic field is high. The current is elsewhere directed along lines of magnetic force, and assumes the form of a thin sheet. This is illustrated in part in Fig. 4.

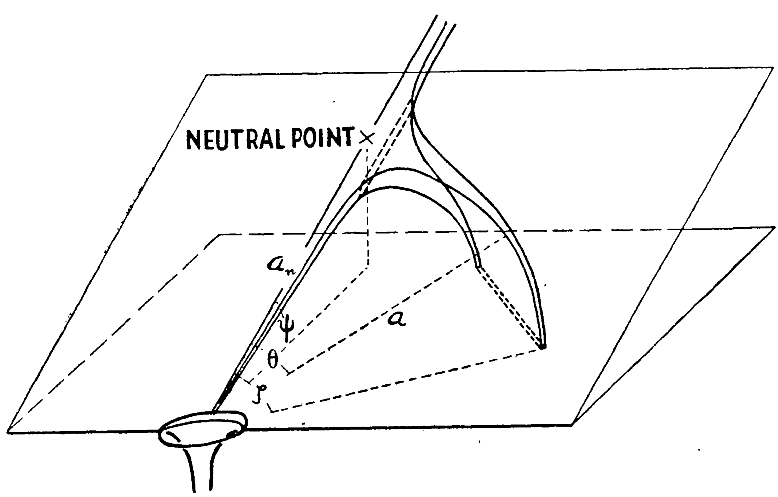

FIG. 4.—" *Neutral* " *point current sheet.*
Portion of a neutral point current sheet is shown above, the inclined plane lying in the direction of the Sun's general magnetic field and meeting the Sun's surface in a line containing the axis of the spot group.

With a bipolar group produced by a subsurface magnetic field parallel to the surface and which breaks through the surface as in our adopted model, the induced electric field in a non-conducting atmosphere would take the form of semicircles, concentric with the axis of the spot group, i.e. the line joining the two sunspots, and would be directed in the same sense around all these semicircles.

In considering the real atmosphere, we find it convenient to define the plane containing the Sun's general magnetic field and the axis of the bipolar group as the " dividing plane ". Then since the lines of magnetic force act as conductors, the components of the induced field along the lines of magnetic force reverse direction as we pass from lines of magnetic force on one side of the dividing plane to the other. This may be seen from Fig. 1, where the magnetic field is shown in a section passing through a sunspot and perpendicular to the axis of the bipolar group. Consequently currents flow *towards* the neutral point on one side of the dividing plane, and *away* from the neutral point on the other.

Alternately, it may be seen from Fig. 2 that any circuit to the right of the neutral plane (the vertical plane containing the neutral point and its associated sunspot) which is linked with the toroid below the surface and has paths from the surface to near the neutral point, one on each side of the dividing plane, will have a flux linkage through it which, because of loss from the toroid, decreases as the sunspots grow. The above current cannot be short-circuited immediately above the toroid since, if the magnetic flux is to escape, the conductivity there must be low. Any similar circuit to the left of the neutral plane may be completed along or beneath the Sun's surface, and through this the flux linkages increase. In both circuits the currents on the same side of the dividing plane are directed both towards or both away from the neutral point.

The e.m.f. around such a circuit to the right of the neutral plane, both paths of which lie close to and on either side of the dividing plane, will be given by $\pi b^2 H_0 \cos \omega t$, the rate of change of flux through the sunspot. We shall assume here that the penetration of the magnetic field through the current sheet takes place sufficiently rapidly for the above formula to hold. The penetration is, in fact, very rapid once the magnetic field penetrates the surrounding atmosphere, and this more important factor is discussed later in this paper.

To obtain the resistance around such a path, we shall treat for convenience the case in which the magnetic field is similar to that from an isolated magnetic pole in a uniform general field G. Then

$$H = G\{(a_n^2/a^2 - \cos\theta)^2 + \sin^2\theta\}^{\frac{1}{2}},$$

where θ is the angle between the lines joining the sunspot to the neutral point and the point concerned, while a_n is the distance of the neutral point from the sunspot (see Fig. 4).

Over the greater part of this circuit, $a \doteqdot a_n$, and

$$H \doteqdot 2G \sin\theta/2.$$

Along the length of any current filament, the cross-sectional area A varies inversely as the magnetic field strength, provided this field guides the current. Thus $A = P/\sin(\theta/2)$, P being a parameter defining any current tube. Near the neutral point the magnetic field cannot guide the current, and we suppose this happens for $\theta < \theta_1$. In general θ_1 is small and, in evaluating the resistance, may be assumed zero. Then the resistance R along any current tube, to the position $\theta = \zeta$, is easily found to be

$$R = 2a_n(1 - \cos \zeta/2)/\sigma_0 P.$$

To estimate the current density near the neutral point, we deduce from Cowling's conductivity formula that at right-angles to the magnetic field the conductivity is roughly constant for $H/p < 2$, but is quite small for $H/p \gg 2$. We suppose the current to be directed along lines of magnetic force when H/p is greater than 2; for smaller values of H/p, the conductivity is assumed independent of the magnetic field. This assumption neglects the leakage of current across the magnetic field when $H/p > 2$, and leads to a lower limit for the current density near the neutral point.

We may now calculate the cross-section of the region near the neutral point which carries currents across lines of magnetic force. The lines of force which limit the conducting path in the current sheet are those along which the minimum value of the magnetic field is $2p$, where p is the electron pressure at the neutral point. The locus of points at which the magnetic field is minimum along a line of magnetic force passing near the neutral point is the cone

$$2\Delta a = a_n \theta,$$

where $\Delta a = a_n - a$. On this cone

$$H = 3^{\frac{1}{2}} G\theta$$

(the numerical factor does not quite agree with that used in the approximation for obtaining R, but this is of little importance).

The cross-sectional area A of the entire conducting sheet to the right of the neutral plane and above the dividing plane is approximately $2(\Delta a)^2$ near the neutral point. Thus for the whole of this sheet,

$$P = AH/2G = 2a_n^2 \, p^3/3G^3$$

and

$$R_1 = 3G^3(1 - \cos \zeta_1/2)/\sigma_0 a_n p^3,$$

ζ_1 being a mean value of ζ for points where the current sheet on one side of the dividing plane enters the Sun. Similarly R_2, the resistance of the path on the other side of the dividing plane, is given by a similar formula in which ζ_1 is replaced by the corresponding mean value of ζ, viz. ζ_2. The total resistance $R = R_1 + R_2$.

The total current I to the right of the neutral plane is given roughly by putting $\zeta_1 = \zeta_2 = \pi/2$; then

$$I = \pi b^2 H_0 \cos \omega t \, . \, \sigma_0 a_n p^3/6G^3(1 - \cos \pi/4)$$

which, using data previously given, and putting $a_n = 3{\cdot}5 \times 10^9$ cm. and $t = 0$, becomes $2{\cdot}2 \times 10^{13} p^3$ e.m.u.

To the left of the neutral plane (see Fig. 2) the induced current is rather smaller in magnitude. The current density, in fact, decreases gradually from a maximum near the dividing plane on the right of the neutral plane, to zero near the dividing plane on the left of the neutral plane.

The current density near the neutral point is approximately $i/2(\Delta a)^2 = 1{\cdot}7 \times 10^{-3} p$ e.m.u., and the corresponding electric field is $1{\cdot}7 \times 10^5 p$ e.m.u. The cross-section of the conductor decreases away from the neutral point, and hence the current densities and electric fields inside the conductor increase correspondingly.

Currents Passing into Sunspots.—A current system of a different type can occur in the atmosphere near a sunspot group if the currents can pass through a sunspot itself. If this be so, currents could rise from the surface of a sunspot and flow along lines of magnetic force until they descend to the Sun's surface. Other currents could rise from the Sun's surface at other places and flow to the sunspot along lines of magnetic force (Fig. 2). Whether or not such currents can flow depends on the conductivity of the sunspot material perpendicular to the spot's internal magnetic field.

It is usually thought that sunspots contract below the surface, and that the magnetic field there is considerably increased. Chapman has estimated (**1**) that the field may reach 5×10^8 gauss, although the validity of his argument has recently been questioned (**12**). Walén's (**11**) treatment indicates a much smaller maximum value of about 10,000 gauss at $2{\cdot}5 \times 10^9$ cm. below the surface.

Although these magnetic fields are high, the temperature and electron pressure increase rapidly beneath the Sun's surface. It is difficult to estimate the departure of the temperature and electron pressure inside a sunspot from normal conditions at the same depth below the Sun's surface, and in the absence of better information we shall assume them identical. The conductivity at a depth of 10^9 cm. is then found to be approximately

$$\sigma^{\mathrm{I}} + i\sigma^{\mathrm{II}} = (4{\cdot}2 \times 10^6 - i26{\cdot}5H)^{-1} \text{ e.m.u.},$$

by putting $Z=3$ in Cowling's conductivity formula, and using Eddington's formula (**13**) for the pressure (mainly electron pressure) and temperature inside a polytrope for which $n=3$. σ^{I} is not significantly affected by H unless the latter be of the order of $1{\cdot}5 \times 10^5$ gauss. Thus currents of the above type seem quite feasible, crossing the sunspot's internal magnetic field at a depth depending on the strength of the latter, but in any case not far below the surface.

A complete conducting circuit can be formed by a tube of magnetic force in the atmosphere (one end of the tube passing into the sunspot, the other into the Sun's surface elsewhere), together with a return circuit through the wall of the sunspot. Alternately, a complete conducting circuit may be formed by two tubes of magnetic force in the atmosphere, along one of which the current passes into the sunspot; if inside the sunspot the resistance between the two tubes is small compared with that between either of the tubes and the wall of the sunspot, then the current may flow between the two tubes within the spot, while the body of the Sun, whose conductivity is high, can complete the circuit between the other ends of the tubes. The sense and order of magnitude of the current in the atmosphere are the same in the two cases, and we will therefore discuss only the latter case. Two such circuits are shown in Fig. 2. The current on one side of the dividing plane is all directed into the sunspot, while on the other side it is all directed out from the sunspot, the senses being the same as with neutral point currents.

We proceed to evaluate the resistances of the various elements of such circuits. For simplicity, a circuit is considered to consist of three sections, (*a*) the parts which are approximately radial from the sunspot, (*b*) the circuit through the sunspot itself, and (*c*) the remainder of the circuit. The resistance of (*c*) in fact may be neglected in comparison with those of (*a*) and (*b*).

For (*a*) we assume radial lines of force leaving the sunspot at $a = a_n/10$ extend to $a = a_n$. The resistance is $R_a = 18/\sigma_0 a_n \Delta X$, where ΔX is the solid angle of the tube. The resistance R_b through the sunspot is difficult to estimate, but may be given in terms of the length s of a cylindrical conductor of constant conductivity equal to the value of σ_0 in the chromosphere, whose resistance equals that of the sunspot, and whose cross-section is constant and equal to that of the tube entering the sunspot, i.e. $a_n^2 \Delta X/100$. If the spot funnels in rapidly so that the magnetic field increases by a factor of 10^3 just below the surface, and if the depth of the circuit into the sunspot is 10^9 cm., then $s = 2 \times 10^{12}$ cm. roughly. This value is little better than a guess. The total resistance R around a circuit is given by

$$R = (18 + 100s/a_n)/\sigma_0 a_n \Delta X.$$

As in the previous section, we shall discuss the case in which the external constant magnetic field is perpendicular to the axis of the bipolar group and inclined to the surface at an angle ψ.

The magnetic flux through a circuit of the type described above decreases as the spot grows, provided the circuit threads under the subsurface magnetic field of the toroid, and thus induced currents will flow around it. Some idea of their magnitude can be obtained from the following rough analysis.

Let AB (Fig. 2) be the locus of points on the sunspot, lines of magnetic force from which eventually meet the Sun's surface on the axis of the spot group, i.e. immediately above the subsurface toroid. Then the lines of magnetic force emanating from AB lie in the dividing plane, and on one side of this plane the direction of the current is into the sunspot; on the other side of the plane, it is out of the sunspot. To a first approximation, the current is symmetrical about this plane.

The value of s for current filaments close to the dividing plane will be less than for filaments well away from it, for the path across the magnetic field inside the sunspot will be least in the former case. Thus the current will be greatest close to this plane.

Before evaluating the currents, we must return to a problem previously left in abeyance, namely whether in the presence of a general field the magnetic field due to a sunspot can escape into the Sun's atmosphere in a reasonable time. We have seen earlier that the velocity of propagation of the electromagnetic field near a magnetic dipole is of the order of $3(2\omega/\mu\sigma)^{\frac{1}{2}}$. This expression may not be valid near the origin of a magnetic disturbance, but we shall assume here that it is valid everywhere.

At the dividing plane the electric field is perpendicular to the magnetic field and thus cannot give rise to any currents. (We here neglect Hall currents.) Thus the magnetic field is propagated rapidly in this plane. It is not so easy to predict the rate of propagation in other directions, but we shall suppose that this is governed by some effective value of σ. In an isotropic medium of conductivity σ_e, the current paths are parallel to the electric field and are concentric with the axis of the bipolar spot group, while the resistance of a current tube of cross-section ΔY, radius z, is $2\pi z/\sigma_e \Delta Y$. We assume the effective conductivity σ_e, of the real atmosphere, at a point where the magnetic field and induced electric field are parallel (e. g. in a plane perpendicular to the dividing plane and containing the axis of the spot group, and at a distance z from the latter) to be such that the total resistance around a current tube having the same cross-section ΔY at this point also equals $2\pi z/\sigma_e \Delta Y$. Then if this point be distant b from the sunspot through which the current passes, $\sigma_e = 2\pi z\sigma_0 a_n/(18+100s/a_n)b^2$. The values of σ_e/z for $b=a_n$ and the corresponding penetration velocities are given in Table IV.

TABLE IV

Circuits through Sunspots

s (cm.)	σ_e/z (e.m.u.)	Penetration velocity $\times z^{\frac{1}{2}}$ (cm.$^{\frac{3}{2}}$ sec.$^{-1}$)	E (e.m.u.)
10^7	10^{-18}	$3\cdot3\times10^6$	$2\cdot7\times10^5$
10^8	$8\cdot6\times10^{-19}$	$3\cdot6\times10^6$	$2\cdot3\times10^5$
10^9	$3\cdot8\times10^{-19}$	$5\cdot4\times10^6$	$1\cdot0\times10^5$
10^{10}	$5\cdot9\times10^{-20}$	$1\cdot4\times10^7$	$1\cdot6\times10^4$
10^{11}	$6\cdot3\times10^{-21}$	$4\cdot3\times10^7$	$1\cdot7\times10^3$
10^{12}	$6\cdot3\times10^{-22}$	$1\cdot4\times10^8$	$1\cdot7\times10^2$
10^{13}	$6\cdot3\times10^{-23}$	$4\cdot3\times10^8$	$1\cdot7\times10$

In the above table, s is the equivalent length of path through a sunspot, σ_e the effective conductivity at a point distant z from the spot group axis in a plane perpendicular to the dividing plane, the penetration velocity being measured at this point. The electric field E corresponds to a distance $a_n=3\cdot5\times10^9$ cm. from the spot and for a circuit everywhere close to the dividing plane. E varies approximately inversely as the square of the distance from the spot, so that at the edge of the sunspot it may be one hundred times as great as indicated. E is computed for $\omega=5\cdot6\times10^{-6}$ sec.$^{-1}$, $H_0=2000$ gauss, $\cos\omega t=1$.

It is clear from these data that for filaments lying near the dividing plane, the field may escape in a time of 10^5 sec. even if s be small; but if $z=10^9$ cm., s must be greater than 10^{13} cm. for the field to escape in this time.

Using the values originally adopted for the rate of spot growth, the current I in any tube not too far from the dividing plane will be given roughly by

$$I=6\times10^{17}\Delta X/(18+2\cdot9\times10^{-8}s) \text{ e.m.u.};$$

the current density at a distance a will be

$$6\times10^{17}/a^2(18+2\cdot9\times10^{-8}s) \text{ e.m.u.};$$

and in the direction of the lines of magnetic force the electric field supporting this current density is

$$6\times10^{25}/a^2(18+2\cdot9\times10^{-8}s) \text{ e.m.u.};$$

values of the electric field for $a=3\cdot5\times10^9$ cm. are also given in Table IV.

Discussion.—We have discussed the induced electromotive forces caused by the changing magnetic fields of sunspots and have shown that in certain neighbouring regions of the chromosphere and corona whose locations depend in part on the configuration of the total magnetic field near sunspots there will be both electric fields and electric currents. The electric fields may be of the order of 10^5 e.m.u., i. e. 10^{-3} volts per cm. Free electrons under the influence of such fields can readily acquire, owing to their long mean free paths, energy sufficient to cause excitation of hydrogen and other atoms. The localization of these phenomena in the neighbourhood of sunspots suggests a basis of an explanation of solar flares. A quantitative discussion, which will be published shortly, shows that many of the essential phenomena of flares can be so explained. The theory also leads to possible explanations of other observed phenomena, for example a certain type of prominence and the emission of radiation in the one-metre band.

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1946 *December* 2 *and* 1947 *December* 17.

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