Mon. Not. R. astr. Soc. (1988) **231**, 535–547

Magnetic braking and the evolution of cataclysmic binaries

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Accepted 1987 October 12. Received 1987 September 11

Summary. We study the evolution and the period distribution of cataclysmic variables using a magnetic braking law proposed recently by Mestel & Spruit. The application of this law (with a suitable choice of parameters) gives a predicted period distribution that matches the observed one. The computed mass-transfer rates are in much better agreement with the observations than those resulting from the application of the prescription of Verbunt & Zwaan.

We also show that the observed accumulation of AM Her systems with periods in the range 114–115 min can be explained in terms of evolution, as a result of the adiabatic expansion of the secondary when accretion resumes at the end of the period gap provided that the masses of the white dwarfs in AM Her systems are in a restricted range. We examine finally the recent claim by Lamb & Melia that when the magnetic white dwarf synchronizes with the orbit, the system detaches, and enters a prolonged gap. We show that the proposed mechanism does not work, and that synchronization has a relatively small effect on the evolution of these systems.

1 Introduction

Cataclysmic variables are low-mass binary systems in which the accreting compact star is a white dwarf and the secondary a low-mass star that fills its Roche lobe. At periods greater than about 10 hr, the secondary must be evolved, as the system is quite wide, but below ~10 hr, there is good evidence that the secondary lies close to the main sequence. The orbital period distribution of these systems shows two prominent features: the almost complete absence of systems below ~80 min, and between ~2 and 3 hr. It has long been recognized that these characteristics are probably the result of the secular orbital evolution of the systems.

While the nuclear evolution of the secondary drives the secular evolution of long-period systems, which therefore expand, it is well known that short-period systems with main-sequence companions contract under the influence of orbital angular momentum losses which more than compensate for the effects of mass transfer. At periods below ~3 hr, these losses are due to gravitational radiation. The minimum period at 80 min is the result of the secondary becoming partially degenerate as its mass drops below about $0.1\,M_\odot$ (Paczyński 1981; Paczyński & Sienkiewicz 1981; Rappaport, Joss & Webbink 1982). However, the accretion rates observed at periods greater than 3 hr are too large to be accounted for by gravitational radiation, and one has to invoke an additional mechanism to drive mass-transfer rates an order of magnitude greater than predicted by gravitational radiation. Braking by a magnetically coupled stellar wind from the secondary is a very promising candidate, but unfortunately a complete theory of this mechanism is not yet available. Verbunt & Zwaan (1981) have given a semi-empirical theoretical estimate of the mass-transfer rate, based on the Skumanich (1972) law fitting the observed equatorial rotation velocity as a function of age for isolated stars; the accretion rates $\dot{M}$ they find are of the correct order of magnitude, although the dependence of $\dot{M}$ on orbital period is too steep. In a recent paper, Mestel & Spruit (1987) give a theoretical estimate of the braking torque, taking into account the existence of a 'dead zone' of closed magnetic field lines, through which no wind flows. We will see that the application of this law leads to a dependence of $\dot{M}$ on orbital period in much better agreement with observation.

The period gap between 2 and 3 hr can be explained if this magnetic braking mechanism stops or is severely reduced at 3 hr, for some reason (Spruit & Ritter 1983; Rappaport, Verbunt & Joss 1983; Ritter 1985a). The effect of mass transfer is to drive the secondary star slightly out of equilibrium, as the mass transfer time-scale (identical to the angular momentum loss time-scale) is comparable to or shorter than the Kelvin–Helmholtz time of the secondary. When magnetic braking is switched off, the time-scale for angular momentum losses becomes much longer than the thermal time, and the secondary contracts and comes back to equilibrium. It no longer fills its Roche lobe, and accretion ceases, until the contraction of the system under the effect of gravitational radiation or any residual magnetic braking brings the system into contact again.

Cataclysmic variables having a strongly magnetic white dwarf form a distinct subclass. Their period distribution is not statistically different from that of non-magnetic systems; however, the period distribution of AM Her systems, in which the white dwarf rotates synchronously with the orbit is significantly different from that of non-magnetic systems. Out of 12 AM Hers, nine have periods below 2 hr, while only 29 per cent of non-magnetic systems in Ritter's catalogue (1987a) are below the gap. On the other hand, the intermediate polars (IPs), i.e. the non-synchronous magnetic systems, usually have periods greater than 3 hr, although their period distribution is compatible with that of non-magnetic systems. A straightforward interpretation of this is that IPs evolve into AM Her systems (King, Frank & Ritter 1985). Lamb & Melia (1987a, b) have suggested that some magnetic systems become synchronous only when close to the minimum period, and that, as a result of the transfer of angular momentum from the white dwarf to the orbit, the system detaches and enters a prolonged detached phase which ends after the relatively massive secondary has cooled and has become fully degenerate. We will show that this mechanism does not work, and that synchronization has relatively little effect on the evolution of these systems.

Another striking feature of the period distribution of AM Hers is the accumulation of systems between 113 and 115 min (Morris *et al.* 1987). Although AM Hers and non-magnetic systems have compatible period distributions *below the gap*, such an accumulation is highly significant. We propose that it results from the emergence of systems from the period gap. When accretion resumes, the secondary reacts adiabatically during a thermal time, and therefore expands; the period increases by about one minute, and decreases afterwards. We will also see that this explanation requires that the masses of the primaries are rather similar.

In Section 2, we compute the secular evolution of cataclysmic variables using the Mestel and Spruit law; in Section 3, we study in detail the emergence of systems from the gap, and address the problem of the accumulation of systems at 114–115 min; in Section 4, we consider the effects of synchronization of magnetic systems on their evolution.

2 Magnetic braking

The essential ingredients of the Mestel & Spruit (1987) model are a standard stellar wind theory, and a simple, but realistic model for the magnetic field which takes into account the existence of a 'dead zone' of closed field lines, through which no wind flows. The extent of this dead zone increases with the magnetic field strength; this effect strongly modifies the dependence of the braking with field strength. With these hypotheses, the angular momentum losses $\dot{J}$ due to magnetic braking are:

$$-\dot{J}=\left[\frac{8\pi}{3}\,\Omega_\odot \varrho_{0,\odot} R^4 c_\mathrm{w}\,\frac{v_0}{c_\mathrm{w}}\right]\frac{R}{\bar{r}}\left(\frac{r_\mathrm{A}}{R}\right)^2\left(\frac{\Omega}{\Omega_\odot}\right)^{1+n} \tag{1}$$

where $\Omega_\odot=2.5\times10^{-6}\,\mathrm{s}^{-1}$ is the angular rotation velocity of the Sun, $\varrho_{0,\odot}=4.0\times10^{-16}\,\mathrm{g\,cm}^{-3}$ the density at the base of the solar wind, R the radius of the star, $c_\mathrm{w}=1.66\times10^7(T/T_\odot)^{1/2}\,\mathrm{cm\,s}^{-1}$ the sound speed at the base of the wind, T the coronal temperature and $T_\odot$ that of the Sun, v_0 the wind velocity at its base, that equals $0.15\,c_\mathrm{w}$ in the case of the Sun, r_A the Alfvén radius at which the wind decouples from the magnetic field and $\bar{r}$ the extent of the dead zone in which matter is trapped by closed magnetic field lines, and n describes the variations of the density ϱ_0 at the base of the wind as a function of Ω, as $\varrho_0/\varrho_{0,\odot}=(\Omega/\Omega_\odot)^n$. X-ray observations of stellar coronae tend to favour $n=1$ (Mestel & Spruit 1987), with however some uncertainties in this parameter. The Alfvén radius is given by:

$$\left(\frac{r_\mathrm{A}}{R}\right)^2\left[4\,\ln\frac{2}{l_\mathrm{d}}\,\frac{r_\mathrm{A}}{R}+7.3\times10^{-5}\left(\frac{\Omega}{\Omega_\odot}\right)^2\left(\frac{r_\mathrm{A}}{R}\right)^2\left(\frac{R}{R_\odot}\right)^2\right]^{1/2}=\frac{1600}{\bar{r}/R}\left(\frac{\Omega}{\Omega_\odot}\right)^{2p-n} \tag{2}$$

where

$$l_\mathrm{d}=6.93\left(\frac{M}{M_\odot}\right)\left(\frac{R_\odot}{R}\right)\left(\frac{T_\odot}{T}\right) \tag{3}$$

and the extent of the dead zone $\bar{r}=xR$ is given by:

$$x^6\,\exp\left(\frac{l_\mathrm{d}}{x}\right)\,\exp\left[7.25\times10^{-5}\left(\frac{\Omega}{\Omega_\odot}\right)^2\frac{T_\odot}{T}\left(\frac{R}{R_\odot}\right)^2\left(x^2-\frac{1}{x}\right)\right]=\zeta_\odot\left(\frac{\Omega}{\Omega_\odot}\right)^{2p-n}\exp\,(l_\mathrm{d}) \tag{4}$$

where $\zeta_\odot$ is the ratio of magnetic pressure to gas pressure at the base of the solar corona, here taken equal to 60, and p describes the variations of the secondary magnetic field as a function of Ω, i.e. $B/B_\odot=(\Omega/\Omega_\odot)^p$; p is usually taken as unity, although there is no compelling theoretical reason for doing so. The coronal temperature is assumed to be proportional to the virial temperature, i.e. $T/T_\odot=(M/M_\odot)(R_\odot/R)$.

This value for the angular momentum losses was used to determine the secular evolution, using a double polytrope code following the method of Rappaport *et al.* (1983) (see also Rappaport *et al.* 1982), with slight differences described in the Appendix. In particular, Rappaport *et al.* use the fitting formulae derived by Stellingwerf (1975) to determine the opacities at the interface between the radiative and convective zone. This formula, initially derived to study stellar pulsations, is inappropriate at low temperatures and high densities (greater than $1\,\mathrm{g\,cm^{-3}}$) as found in the central regions of low-mass stars. The opacities found are greater by a factor 2 to 4 at densities greater than $1\,\mathrm{g\,cm^{-3}}$ for the temperatures of interest here, as compared with the tables of Cox & Tabor (1976). We therefore multiplied the value given by the fit of Stellingwerf (1975) by $(1+\varrho^4)/(1+\alpha\varrho^4)$, where ϱ is the density in CGS units. We considered α as a free parameter, accounting for the uncertainties both in the opacities, and also in the modelling of the secondary itself [in particular concerning the matching of the entropy of the convective envelope with that at the stellar surface, see e.g. Paczyński & Sienkiewicz (1983)]. It should be realized that the details of the stellar structure are relatively unimportant once it is accepted that the period gap is, for some reason, the result of the cessation of magnetic braking when the orbital period reaches 3 hr: the lower side of the gap determines the secondary mass, while the upper edge imposes the secondary radius, and hence the departure from thermal equilibrium as well as the accretion rate, above the gap. α must, however, be in the range $2\leqslant\alpha\leqslant 4$. We repeat that the magnetic braking mechanism is turned off or sharply reduced as soon as the radiative core disappears.

The results of our computations are shown in Fig. 1. We took $p=1$, and determined n and α so as to obtain a gap starting at 3 hr and ending at 2 hr. This gave $n=0.6$ and $\alpha=2.9$. The white dwarf mass M_1 was kept fixed at $1\,M_\odot$; all of the accreted mass was assumed to be lost in nova explosions with the same specific angular momentum as that of the white dwarf. We also show, for comparison, the results obtained using the magnetic braking law of Verbunt & Zwaan (1981):

$$\dot{J}=2.66\times10^{-37} f_{\mathrm{vz}}^{-2} r_{\mathrm{g2}}^2 M_2 R_2^4 \left(\frac{P}{1\,\mathrm{hr}}\right)^{-3}\ \mathrm{g\ cm^{-2}\ s^{-2}} \tag{5}$$

where f_{vz} is a parameter of the order of unity, r_{g2} the gyration radius of the secondary, and M_2 and R_2 are its mass and radius. It should be noted that the secondary wind is magnetically coupled to the convective envelope only, and not to the radiative interior. Therefore, the relevant value of the radius of gyration that enters the Verbunt & Zwaan law should, in the absence of any other coupling mechanism, be the one of the convective envelope, defined as:

$$r_{\mathrm{g2}}^2=\frac{2}{3}\int_{\substack{\text{convective}\\\text{envelope}}} \frac{r^2\,dm}{M_2 R_2^2} \tag{6}$$

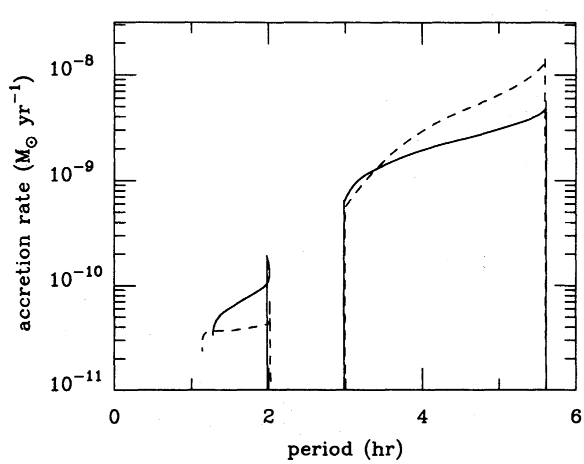

Figure 1. Accretion rate versus period in the case of Mestel & Spruit (1987) law (solid line) as compared with the case of Verbunt & Zwaan (1981) (dashed line). The primary mass is in both cases $1\,M_\odot$, and the parameters n and p entering the braking law are $n=1$ and $p=0.6$. Magnetic braking can either be turned off when the secondary star becomes fully convective (dashed line), or severely reduced (solid line).

that becomes smaller with increasing mass (and hence binary period); for fully convective stars, one would have $r_{g2}^2=0.2$, while for the $0.7\,M_\odot$ main-sequence stars considered here at the beginning of the evolution, $r_{g2}^2=0.03$. This effect would lead to a sharp decrease of magnetic braking when the radiative core radius exceeds 70 per cent of the total stellar radius, which would cause difficulties in explaining the observed mass-transfer rates of systems with periods of the order of 10 hr. Because of the presence of other coupling mechanisms between the radiative core and the convective envelope, r_{g2} will be intermediate between the value corresponding to a fully convective star and that given by equation (6). For simplicity, we have taken here r_{g2} as constant and equal to 0.45 (note that r_{g2} does not enter the Mestel & Spruit law). We have also used $f_{vz}=0.84$, with a new value of α, $\alpha=2.57$, in order to obtain a period gap starting at 3 hr and ending at 2 hr. Fig. 1 shows that the dependence of the accretion rate with period according to the Mestel & Spruit law is much flatter than that obtained using the law of Verbunt & Zwaan, in agreement with observations (Patterson 1984).

We have carried out numerical computations of zero-age main-sequence stars with the value of α determined previously. The results are in reasonable agreement with the results obtained with more elaborate stellar evolution codes (Copeland, Jensen & Jørgensen 1970; Grossman, Hays & Graboske 1974). We find for example that the star becomes fully convective when its mass reaches $0.28\,M_\odot$, to be compared with a more accurate value of $0.35\,M_\odot$; the value obtained using Stellingwerf law for the opacities is $0.44\,M_\odot$. Main-sequence stars have hotter interiors than those of the secondaries in binary systems, and the correction, α, to Stellingwerf opacities one should take is therefore smaller than the value we have taken here. Our models give a better description of secondaries out of thermal equilibrium than of main-sequence stars.

We also used equation (1) with a multiplicative coefficient f, taking $n=1$ and $p=1$, and obtained a curve almost indistinguishable from that given in Fig. 1, with $f=2.5$. Other sets of parameters n and p were used, and the resulting accretion rates were similar. The secondary magnetic field is:

$$B=650\left(\frac{P}{1\ \mathrm{hr}}\right)^{-1}\ \mathrm{G}. \qquad (7)$$

The wind mass-loss rate, given by $\dot{M}_{\rm wind}=-\dot{J}/\Omega r_A^2$ is shown in Fig. 2 for $p=1$ and $n=0.6$, as a function of orbital period. It is seen that for $P=4$ hr, one obtains $\dot{M}_{\rm wind}=2.6\times10^{-11}\,M_\odot\,\mathrm{yr}^{-1}$; the value of the magnetic field, as deduced from equation (7), is $B=160$ G. These values are quite reasonable, in comparison with those required by the model of Verbunt & Zwaan (1981), i.e. $B=100$ G, and $\dot{M}_{\rm wind}\geqslant10^{-10}\,M_\odot\,\mathrm{yr}^{-1}$.

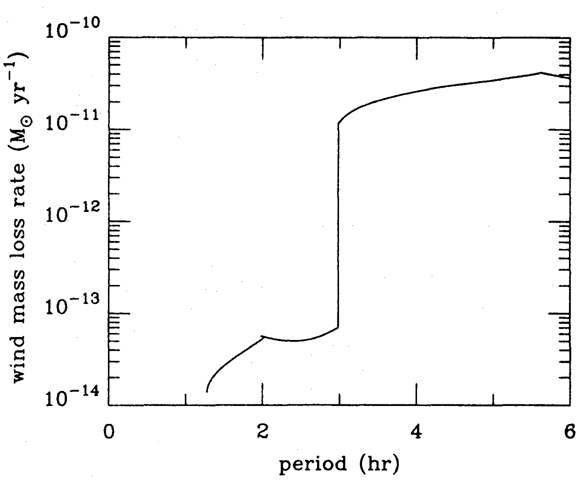

Figure 2. Secondary wind mass-loss rate as a function of period. The parameters of the braking law are the same as in Fig. 1; the primary mass is $1\,M_\odot$.

The reason for the weak dependence of accretion rate versus period can be understood in the following way. To an accuracy better than 10 per cent, the Mestel & Spruit (1987) law can be fitted by a power law for secondary masses in the range $0.2\,M_\odot \leqslant M_2 \leqslant 0.8\,M_\odot$:

$$\dot{J}=2.37\times10^{36}\left(\frac{M_2}{M_\odot}\right)^{1.54}\left(\frac{R_2}{R_\odot}\right)^{-0.27} \tag{8}$$

(here, $p=1$ and $n=1$), and by

$$\dot{J}=6.91\times10^{35}\left(\frac{M_2}{M_\odot}\right)^{1.457}\left(\frac{R_2}{R_\odot}\right)^{-0.025} \tag{9}$$

for $p=1$, and $n=1.4$, and by

$$\dot{J}=1.61\times10^{35}\left(\frac{M_2}{M_\odot}\right)^{1.36}\left(\frac{R_2}{R_\odot}\right)^{0.26} \tag{10}$$

for $p=1$ and $n=0.5$. If one makes the assumption that the secondary is on the main sequence, its radius is proportional to its mass, and therefore to the orbital period P. $\dot{J}$ varies therefore as P^β, with β equal to 1.27, 1.43, and 1.52 in the three above cases respectively, while in the case of the Verbunt & Zwaan (1981) law $\beta=2$.

Finally, we stress that the reason for the cessation of magnetic braking is not at all understood. It has been suggested that when the radiative core of the secondary disappears magnetic field lines are no longer rooted at the base of the convective zone and the dynamo effect is no longer efficient. This attractive possibility is not, however, based on any theoretical calculation. In addition, the disappearance of the magnetic field might not be sufficient to cause a period gap for AM Her systems, as the stellar wind from the secondary, caught in the magnetic field of the primary, still carries off angular momentum from the system (Schmidt, Stockman & Grandi 1986). Finally, a relatively strong secondary magnetic field is required to maintain corotation in AM Her systems (Campbell 1985; Hameury *et al.* 1987). One might suggest that the wind itself disappears (Hameury *et al.* 1987), which, in Mestel & Spruit's (1987) model, translates into a decrease of the index n. We therefore studied the effect of changing n from 1.4 to 0.5 when the period reaches 3 hr (which is, in our case, the point at which the radiative core disappears). The results are displayed in Fig. 1. It is seen that the gap has indeed the same extent in period, but because magnetic braking still has a significant effect, individual systems cross it more quickly, and the accretion rates below the gap can be extrapolated from the values above the gap. This seems to be in agreement with observations (Patterson 1984). The minimum period at about 80 min is also slightly increased, but its exact value depends so strongly on the surface opacities of the cool, low-mass secondary (Paczyński & Sienkiewicz 1983) that this shift is of no importance.

3 The 114-min peak in the period distribution

When the secondary starts transferring matter to the white dwarf, it reacts adiabatically during a fraction of the Kelvin–Helmholtz time of the convective envelope, until it has departed enough from equilibrium. During this short phase, the star expands if the convective zone is substantially greater than the radiative interior, i.e. if the secondary mass is lower than $0.6\,M_\odot$ for a main-sequence star (Rappaport *et al.* 1983). The radius of a fully convective star is given by:

$$R_2=KM_2^{-1/3} \tag{11}$$

where K is a coefficient proportional to the specific entropy of the secondary, that varies according to:

$$\frac{\dot{K}}{K}=\frac{L_2-L_{\rm nuc}}{L_2 t_{\rm KH}} \tag{12}$$

where L_2 is the luminosity of the secondary, $L_{\rm nuc}$ the energy generation rate by thermonuclear reactions, and $t_{\rm KH}$ the Kelvin–Helmholtz time. When leaving the period gap, $L_2=L_{\rm nuc}$. L_2 varies on a time-scale $t_{\rm KH}$, and $L_{\rm nuc}$ on the accretion time-scale $t_{\rm acc}$, defined as $t_{\rm acc}=M_2/\dot{M}$. Therefore, K remains constant for a fraction of the shortest of $t_{\rm KH}$ and t_j, the low-mass secondary star expands slightly, and the orbital period, proportional to $R_2^{3/2}M_2^{-1/2}$ increases until a sufficient luminosity deficit has been established to cause a decrease in K; the star then contracts, with R_2 approximately proportional to M_2, and the period decreases until the minimum period is reached. Therefore, during a short interval, the derivative of the orbital period with respect to time is very small, which leads to an accumulation of systems at that period. The discovery probability $p_{\rm dis}$ is given by:

$$p_{\rm dis}\propto \dot{M}^{3/2}\left(\frac{dP}{dt}\right)^{-1} \tag{13}$$

assuming that the detection is flux-limited, which is probably true for AM Her systems, as they are generally found by their X-ray emission. In Fig. 3, we plot the $\dot{M}-P$ diagram in the vicinity of $P=2$ hr, and it is seen that the period first increases by 3 min before decreasing, which leads to the discovery probability given in Fig. 4, summed over 1-min intervals. Here, we have chosen $M_1=0.75\,M_\odot$, in agreement with observations (see e.g. Mukai & Charles 1987). It appears that the period discovery probability is approximately seven times greater in the period interval 114–115 min than in the other intervals. We predict that the probability of an AM Her system below the gap to be in the 113–115 min period range is 0.34; the expected number of systems in that interval is thus 3.4±1.8, i.e. to within 1.5σ of the observed value. This calculation, however, is based on a given primary mass of 0.75 $M_\odot$. In reality, a spread in the primary masses would result in a spread of the accumulation period. From our numerical calculations, we estimate that the primary masses must be kept fixed to within 20 per cent. One can imagine several possibilities for such a small spread of primary masses of AM Her systems. For example, selection effects tend to favour high-mass systems; these effects are important in non-magnetic systems (Ritter & Burkert 1986), and are even stronger for X-ray-selected objects: the X-ray luminosity varies as $M_1^{7/3}$, and

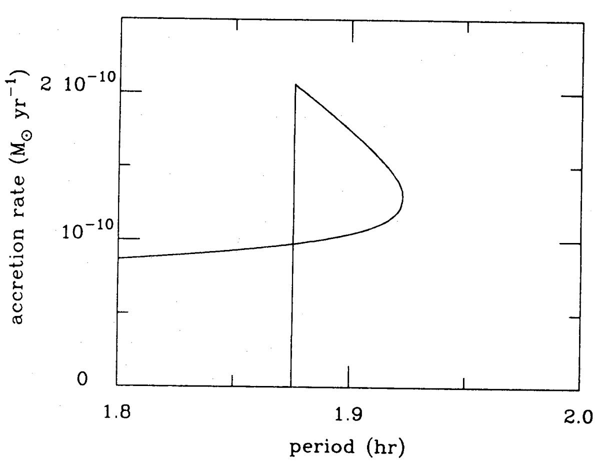

Figure 3. Enlargement of Fig. 1 in the region P~2 hr. We have adopted the hypothesis that the index n changes suddenly when the period reaches 3 hr, i.e. that the period gap is due to a strong decrease of the secondary stellar wind. The primary mass is now taken to be 0.75 $M_\odot$, consistent with that of AM Her systems.

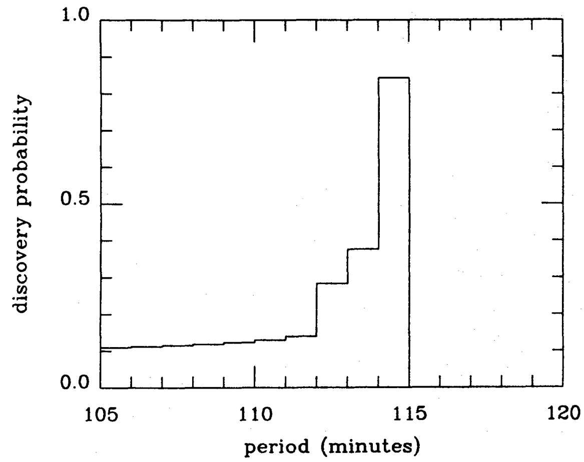

Figure 4. Discovery probability of systems with periods around 2 hr, as given by equation (13). The parameters of the evolution are the same as in Fig. 3.

the discovery probability as $M_1^{7/2}$. If the actual distribution of primary masses is flat with a sharp cut-off, the observed distribution will be peaked at the maximum mass with a spread of 18 per cent. It is indeed also possible that the intrinsic spread of primary masses in AM Her systems is very small.

There are three possible reasons for the accumulation to appear only in the case of AM Her systems. It might first be that the period distribution of AM Her systems and that of non-magnetic systems derive from the same parent distribution, as is indicated by the Kolmogorov–Smirnov test (Schmidt & Liebert 1987). The apparent discrepancy of the distribution of magnetic and non-magnetic systems would then be a statistical effect. However, a K–S test is not very sensitive for testing this type of difference. Another possibility is that the spread of primary masses is much larger in the case of non-magnetic systems than it is in AM Her systems, as the discovery probability of X-ray-selected systems is much more sensitive to M_1 than that of optically selected variables. Finally, it is possible that, as a result of the capture of the secondary stellar wind by the primary magnetic field, $\dot{M}$ is larger in AM Hers, and so the excursion in period at the emergence of the gap is larger, and also the accumulation at 2 hr. It is also possible that the three effects combine equally.

It should finally be noted that the discussion in this section is valid only if the increase of the secondary radius is greater than its scale height, since it is assumed that R_2 is identical to the Roche lobe radius, although they differ by a few atmospheric scale heights. We also require that the onset of mass transfer is almost instantaneous in units of the Kelvin–Helmholtz time, so that the secondary reacts adiabatically. This is true for the low-mass secondaries considered here, but it need not be the case for more massive stars, as shown by detailed numerical calculations by Mazzitelli, D'Antona & Ritter (1988, in preparation). The condition that ΔR_2 is greater than the atmospheric scale height H can be written as:

$$\frac{\Delta P}{P} > \frac{3}{2}\frac{H}{R} = \frac{3}{2}\frac{T_{\rm surf}}{T_{\rm v}} \tag{14}$$

where $T_{\rm surf}$ is the stellar surface temperature, and $T_{\rm v}=GM_2m_{\rm p}/R_2$ the virial temperature. One therefore requires a fractional period change by at least a few 10^{-4}, which is easily fulfilled in our case.

4 Synchronization-induced period gap

It has recently been proposed by Lamb & Melia (1987a, b) that when the white dwarf synchronizes with the orbit in a magnetic system, a prolonged gap ensues. If synchronization occurs close to the minimum period, the secondary could contract and reach the degenerate branch, thus having a much smaller radius. The system would enter into contact again only at a period significantly shorter than the minimum period, and an ultra-short system would therefore be formed. However, as noted by Ritter (1987b), the separation increases only by a small amount. The angular momentum given to the orbit is:

$$\Delta J = r_{g1}^2 M_1 R_1^2 (\Omega - \omega) \tag{15}$$

where ω is the rotation frequency of the white dwarf just before synchronization, R_1 is the white-dwarf radius, and r_{g1} its gyration radius. For $M_1 \leqslant 0.95\, M_{CH}$, $r_{g1} \simeq 0.452 + 0.0853 \log(1 - M_1/M_{CH})$, M_{CH} being the Chandrasekhar mass, while $r_{g1} = 0.275$ for $M_1 = M_{CH}$ (Ritter 1985b). The separation of the system varies by an amount Δa given by:

$$\frac{\Delta a}{a} = 2\,\frac{\Delta J}{J} = 2(1+q) r_{g1}^2 \left(\frac{R_1}{a}\right)^2 \left(\frac{\omega}{\Omega} - 1\right) \tag{16}$$

where $q = M_1/M_2$, which is of the order of $3 \times 10^{-5} (\omega/\Omega - 1)$ for an orbital period of 2 hr and $R_1 = 5 \times 10^8$ cm. A firm upper limit can be set by assuming that the primary rotates at the break-up velocity; one obtains:

$$\frac{\Delta a}{a} < 2 q^{1/2} (1+q)^{1/2} r_{g1}^2 \left(\frac{R_1}{a}\right)^{1/2} \tag{17}$$

which is still much smaller than unity. This upper limit is far too conservative, since the magnetized white dwarf cannot accrete if the spin period is greater than the value at which centrifugal forces balance gravitation at the Alfvén radius; for magnetic moments as low as 10^{33} G cm^3, and accretion rates of 10^{16} g s^{-1}, the minimum period is 9 min (Hameury *et al.* 1987); $\Delta a/a$ is then of the order of a few 10^{-4} for a primary with a radius of 5×10^8 cm, i.e. just larger than a scale height. If the magnetic moment is greater than the value used here, or if the white dwarf is not rotating at the maximum allowed spin (note that the intermediate polars, which are not thought to have stronger magnetic fields than AM Her systems, have spin periods usually greater than 15 min), the relative change in separation is even smaller.

In order to detach the system, one requires that the variation of R_L, ΔR_L, is greater than one atmospheric scale height of the secondary, H_p, i.e. that

$$\frac{\Delta R_L}{H_p} = 7.85 \times 10^{-4}\, \frac{R_2}{H_p}\, q^{2/3} (1+q)^{1/3} r_{g1}^2 \left[\left(\frac{M_1}{M_{CH}}\right)^{4/3} - 1\right] \left(\frac{P}{1\ \mathrm{hr}}\right)^{-4/3} \left(\frac{\Omega}{\omega} - 1\right) \geqslant 1 \tag{18}$$

where we have used Nauenberg's (1972) mass–radius relation for white dwarfs:

$$\frac{R_1}{R_\odot} = 1.12 \times 10^{-2} \left[\left(\frac{M_1}{M_{CH}}\right)^{-2/3} - \left(\frac{M_1}{M_{CH}}\right)^{2/3}\right]^{1/2}. \tag{19}$$

For 0.1 $M_\odot$ secondaries, R_2/H_p is of the order of 2000, and the system would then detach only if the primary has a relatively low mass, or if it is rotating rather rapidly.

Even assuming that accretion ceases, a prolonged gap will not follow. The Roche lobe R_L varies in time according to:

$$\frac{\dot{R}_L}{R_L} = 2\,\frac{\dot{J}}{J} + \xi_L\, \frac{\dot{M}_2}{M_2} \tag{20}$$

where ξ_L is a function of the primary and secondary masses. The secondary radius varies as:

$$\frac{\dot{R}_2}{R_2}=\left(\frac{\dot{R}_2}{R_2}\right)_h+\xi_2\frac{\dot{M}_2}{M_2} \tag{21}$$

where the adiabatic mass–radius exponent ξ_2 is a function of M_2, and $(\dot{R}_2/R_2)_h$ represents the contraction of the secondary due to cooling. From these two equations, one deduces the rate at which the Roche lobe catches the secondary:

$$\frac{d}{dt}\left[\ln\frac{R_L}{R_2}\right]=\frac{\dot{R}_L}{R_L}-\frac{\dot{R}_2}{R_2}=\left[2\frac{\dot{J}}{J}-\left(\frac{\dot{R}_2}{R_2}\right)_h\right]+(\xi_L-\xi_2)\frac{\dot{M}_2}{M_2}. \tag{22}$$

During the mass-transfer phase, the left-hand side of equation (22) is zero, which determines $\dot{M}_2$; it is easy to show that the mass transfer is stable only if $\xi_L-\xi_2<0$, and the term in brackets is therefore negative. When the secondary detaches from the Roche lobe, $\dot{M}_2=0$, and the term in brackets remains unchanged on a Kelvin–Helmholtz time. $d/dt(R_L-R_2)$ is therefore negative, and the Roche lobe catches the secondary in a time:

$$\tau_{\text{detached}}=\frac{\Delta a}{a}\left[\frac{2}{\tau_j}-\frac{1}{\tau_{\dot{R}}}\right]^{-1} \tag{23}$$

where $\tau_{\dot{R}}=(R_2/|\dot{R}_2|)_h$ and t_j is the angular momentum loss time-scale (identical to t_{acc} during the mass-transfer phase). Note that $\tau_{\dot{R}}$ is different from $\tau_{K\text{-}H}$, as $\tau_{K\text{-}H}$ is proportional to the stellar luminosity, and $\tau_{\dot{R}}$ is proportional to the luminosity deficit; for the periods considered here, $\tau_{\dot{R}}\simeq 2\tau_{K\text{-}H}$. The detached phase lasts much less than the time-scale required for the secondary to contract and reach the degenerate sequence, $\tau_{\dot{R}}$, except if the system is extremely close to being unstable, namely if:

$$\frac{2}{\tau_j}-\frac{\Delta a}{a}\frac{1}{\tau_j}<\frac{1}{\tau_{\dot{R}}}<\frac{2}{\tau_j} \tag{24}$$

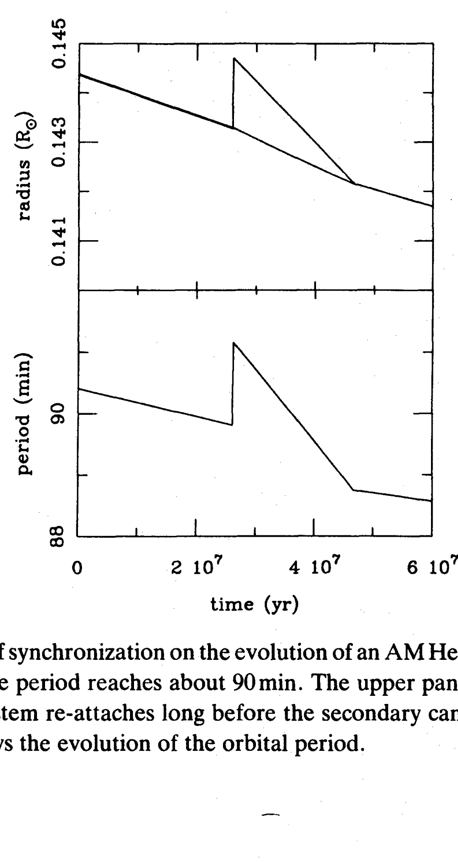

Figure 5. Example of the effect of synchronization on the evolution of an AM Her system. The separation is suddenly increased by 1 per cent when the period reaches about 90 min. The upper panel shows the variation of R_L (upper curve) and R_2 with time; the system re-attaches long before the secondary can contract and reach the degenerate sequence. The lower panel shows the evolution of the orbital period.

which, for the small values of $\Delta a/a$ resulting from synchronization, is very unlikely. An example of standard evolution of a system in which separation has suddenly been increased by 10^{-2} is shown in Fig. 5; it is seen that the Roche lobe catches the secondary after 2×10^7 yr, at an orbital period one minute shorter than that before the system detached. For smaller values of Δa, that are more likely, the effect would be hardly noticeable.

This conclusion depends on the assumption that the angular momentum losses do not depend on the mass-transfer rate; this is true for both gravitational radiation and magnetic braking. Even if $\dot{J}$ were to depend linearly on $\dot{M}$, the same conclusion would hold, as the same derivation applies; one would have to invoke very strong non-linearities in order to obtain the effect claimed by Lamb & Melia (1987a, b).

The Lamb & Melia scenario could work if the angular momentum losses could be switched off when the system approaches the minimum period, for a time long enough for the secondary to contract and reach the degenerate branch. However, gravitational radiation is *always* efficient, and any additional mechanism extracting angular momentum must be relatively inefficient in comparison with gravitational radiation, if the minimum period is to remain close to 80 min. It seems therefore difficult to form ultra-short-period systems this way. More plausible mechanisms can be found, for example, in Tutukov *et al.* (1985) or Nelson, Rappaport & Joss (1986), but they all imply that the secondary is hydrogen-poor (with hydrogen abundances below 0.1).

5 Conclusion

We have shown that the application of the magnetic braking law of Mestel & Spruit (1987) to the secular evolution of cataclysmic variables can give accretion rates in good agreement with observation; in particular the dependence of $\dot{M}$ with period is much flatter than that obtained using the semi-empirical law of Verbunt & Zwaan (1981). The required values of the surface magnetic field and wind mass-loss rate of the secondary are also reasonable, of the order of 100 G and a few $10^{-11} M_\odot \mathrm{yr}^{-1}$ respectively. We explain the observed peak at 114 min in the period distribution of AM Her systems as a result of their evolution. Our model requires that the primaries' masses are very similar in these systems, which could, for instance, be the result of selection effects that are different in magnetic and non-magnetic systems, thus explaining the difference in their distribution below the gap. We finally show that the recent claim of Lamb & Melia (1987a, b) that magnetic systems enter a prolonged detached phase when synchronizing with the orbit is unlikely to be correct.

Acknowledgments

HR acknowledges support by the Deutsche Forschungsgemeinschaft, grant Ku 474/8-2.

References

Campbell, C. G., 1985. *Mon. Not. R. astr. Soc.*, **215**, 509.
Copeland, H., Jensen, J. O. & Jørgensen, H. E., 1970. *Astr. Astrophys.*, **5**, 12.
Cox, A. N. & Tabor, J. E., 1976. *Astrophys. J. Suppl.*, **31**, 271.
Grossman, A. S., Hays, D. & Graboske, H. C., Jr, 1974. *Astr. Astrophys.*, **30**, 95.
Hameury, J. M., King, A. R., Lasota, J. P. & Ritter, H., 1987. *Astrophys. J.*, **316**, 275.
King, A. R., Frank, J. & Ritter, H., 1985. *Mon. Not. R. astr. Soc.*, **213**, 185.
Lamb, D. Q. & Melia, F., 1987a. *Astrophys. Space Sci.*, **131**, 511.
Lamb, D. Q. & Melia, F., 1987b. In: *The Physics of Accretion onto Compact Objects*, eds Mason, K. O., Watson, M. G. & White, N. E., p. 113, Springer-Verlag, Berlin.
Mestel, L. & Spruit, H. C., 1987. *Mon. Not. R. astr. Soc.*, **226**, 57.
Morris, S. L., Schmidt, G. D., Liebert, J., Stocke, J., Gioia, I. & Maccacaro, T., 1987. *Astrophys. J.*, **314**, 641.

Mukai, K. & Charles, P. A., 1987. *Mon. Not. R. astr. Soc.,* **226,** 209.
Nauenberg, M., 1972. *Astrophys. J.,* **175,** 417.
Nelson, L. A., Rappaport, S. A. & Joss, P. C., 1986. *Astrophys. J.,* **304,** 231.
Paczyński, B., 1981. *Acta Astr.,* **31,** 1.
Paczyński, B. & Sienkiewicz, R., 1981. *Astrophys. J.,* **248,** L27.
Paczyński, B. & Sienkiewicz, R., 1983. *Astrophys. J.,* **268,** 825.
Patterson, J., 1984. *Astrophys. J. Suppl.,* **54,** 443.
Rappaport, S., Joss, P. C. & Webbink, R. F., 1982. *Astrophys. J.,* **253,** 908.
Rappaport, S., Verbunt, F. & Joss, P. C., 1983. *Astrophys. J.,* **275,** 713.
Ritter, H., 1985a. *Astr. Astrophys.,* **145,** 227.
Ritter, H., 1985b. *Astr. Astrophys.,* **148,** 207.
Ritter, H., 1987a. *Astr. Astrophys. Suppl.,* in press.
Ritter, H., 1987b. *Astr. Astrophys.,* submitted.
Ritter, H. & Burkert, A., 1986. *Astr. Astrophys.,* **158,** 161.
Schmidt, G. D. & Liebert, J., 1987. *Astrophys. Space Sci.,* **131,** 549.
Schmidt, G. D., Stockman, H. S. & Grandi, S. A., 1986. *Astrophys. J.,* **300,** 804.
Skumanich, A., 1972. *Astrophys. J.,* **171,** 565.
Spruit, H. C. & Ritter, H., 1983. *Astr. Astrophys.,* **124,** 267.
Stellingwerf, R. F., 1975. *Astrophys. J.,* **195,** 441.
Tutukov, A. V., Fëdorova, A. V., Ergma, E. V. & Yungel'son, L. R., 1985. *Sov. Astr. Lett.,* **11,** 52.
Verbunt, F. & Zwaan, C., 1981. *Astr. Astrophys.,* **100,** L7.

Appendix

We use here the same notations as Rappaport *et al.* (1983). The secondary of mass M and radius R has a radiative core of radius QR, and is described by two polytropes of indices 3 and 3/2. The specific entropy of the convective envelope K varies as:

$$\frac{\dot{K}}{K}=\frac{L_i+L_2-L}{H_2(Q)GM^2/R} \tag{A1}$$

where L_i is the radiative flux through the interface between the radiative core and the convective envelope, L_2 the nuclear luminosity generated in the envelope, and L the surface luminosity. We use a modification of Stellingwerf's (1975) fitting formula for the opacities, as described in the text:

$$\varkappa_i=\varkappa_0\,\frac{1+\varrho_i^4}{1+\alpha\varrho_i^4} \tag{A2}$$

where $\varkappa_0$ is the value given by Stellingwerf, ϱ_i the density at the interface in g cm^{-3}, and α a parameter in the range $2\leqslant\alpha\leqslant4$ that both improves the fit of Stellingwerf and takes into account the inaccuracy of the description of the star by two polytropes. H_2 is fitted by:

$$H_2=\tfrac{3}{7}\exp\,(0.180958\,Q-2.606764\,Q^2-1.061038\,Q^3-4.417689\,Q^4-6.016268\,Q^5+0.382751\,Q^6). \tag{A3}$$

Q varies according to:

$$\frac{\dot{Q}}{Q}=\left(\frac{dH}{dQ}+\frac{H}{f}\frac{df}{dQ}\right)^{-1}\left\{\left(1-\frac{3H}{7}\right)\frac{\dot{M}}{M}+\frac{R}{GM^2}\left[L_1+(L_2-L)\left(1-\frac{H}{H_2}\right)\right]\right\} \tag{A4}$$

where f is a function of Q. $\dot{Q}/Q$ can be computed using the following fits:

$$\frac{1}{Q^4}\frac{1}{f}\frac{df}{dQ}=17.71851-2.77833\,Q-1.43361\,Q^2 +\exp\,(-0.71293-9.80098\,Q+53.02404\,Q^2-51.23385\,Q^3) \quad \text{(A5)}$$

$$H-\tfrac{3}{7}=Q^5(1.15605-0.24066\,Q+1.85336\,Q^2-25.33256\,Q^3+85.56914\,Q^4 -111.29891\,Q^5+49.13757\,Q^6) \quad \text{(A6)}$$

$$\frac{1}{Q^4}\frac{dH}{dQ}=5.75609+0.50200\,Q-23.02642\,Q^2+50.47171\,Q^3-26.16610\,Q^4 -\exp\,(-31.875089+107.10502\,Q-115.44264\,Q^2+42.311065\,Q^3) \quad \text{(A7)}$$

$$\frac{H-H_2}{Q^3}=4.83890-0.50527\,Q-15.54831\,Q^2+18.29942\,Q^3-6.36135\,Q^4. \quad \text{(A8)}$$

All these fits are more accurate than 10^{-2} in the range $0\leqslant Q\leqslant 0.8$. For all other quantities, we followed the prescription of Rappaport *et al.* (1983).