# MAGNETIC BRAKING OF COLLAPSING INTERSTELLAR CLOUDS 

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(Received 1975 November 11; in original form 1975 August 4)

## SUMMARY

The purpose of this investigation is to show that recourse to anisotropic compression along a magnetic field is not a necessary condition for star formation within large collapsing interstellar gas clouds. Although such a scenario would certainly eliminate the magnetic field and angular momentum problems associated with isotropically collapsing clouds, we believe there is sufficient observational and theoretical evidence to warrant the present study of magnetically braked, isotropically collapsing gas clouds. We do not attempt to disprove the hypothesis of anisotropic compression, but offer instead, a reasonable alternative.

We examine angular momentum transfer from magnetically braked, cool interstellar gas clouds of $10^{2}, 10^{3}$ and $10^{4} M_{\odot}$. Magnetic torques acting on a contracting, rotating cloud, permeated by a frozen-in magnetic field coupling the cloud to the galactic field of the surrounding interstellar medium, produce kinks in the galactic field lines which are radiated away in the form of hydromagnetic waves, thereby rotationally decelerating the cloud. Initially, the braking constrains the clouds to co-rotate with the galactic background until just prior to the epoch of magnetic field uncoupling, when the braking mechanism becomes inefficient and the clouds contract conserving angular momentum thereafter. Centrifugal forces are always kept well below gravity, so that the clouds undergo complete gravitational collapse on the magnetically diluted dynamic free-fall time scale. Our results are shown to be consistent with observations of stellar rotational velocities, and, also, with the angular momentum of the protosun.

## I. INTRODUCTION

The fact that young stars are frequently found in clusters, suggests that stars are formed by a fragmentation process which occurs during the gravitational collapse of large interstellar gas clouds, formed by a thermo-gravitational instability. However, large-scale frozen-in magnetic fields and angular momentum present problems to the theory of star formation within contracting clouds. Mestel \& Spitzer (1956) have shown that as long as the galactic magnetic field is frozen into a contracting cloud, the magnetic pressure sets a lower limit to the mass that can remain gravitationally bound. If the cloud collapses isotropically, then fragmentation into smaller typical protostellar masses would be forbidden. An interstellar cloud will also possess angular momentum by virtue of local galactic rotation. If the cloud
conserves angular momentum while contracting isotropically, the centrifugal forces will increase faster than the gravitational forces, and the angular velocity $\omega$ increases $\sim R^{-2}, R$ being the cloud radius, resulting in rotational instability. Assuming fragmentation were at all possible in such a cloud, stars would then rotate much faster than is observed. Evidently then, some means must be found for uncoupling the magnetic field and removing angular momentum from contrating interstellar clouds if star formation is to occur within these clouds.

Mestel \& Spitzer (1956) and Nakano \& Tademaru (1972) have shown that the magnetic field problem is only temporary. Ambipolar diffusion allows the field to uncouple from the gas when the cloud density is sufficient to screen external sources of ionization, thus reducing the conductivity of the gas. In this paper, we concern ourselves with the angular momentum problem, and examine the efficiency of transferring angular momentum from a collapsing magnetic cloud to the surrounding interstellar medium via magnetic-dipole radiation of hydromagnetic waves. This suggested mechanism to relieve the angular momentum problem relies upon the temporary magnetic field problem; it is operative only so long as the cloud remains magnetically coupled to the galactic field of the surrounding interstellar medium. We must therefore consider, in some detail, the process of ambipolar diffusion, since the time scale for angular momentum transfer will be determined by the ambipolar diffusion time.

The following set of assumptions shall be adopted throughout the present investigation:
(1) The strength of the large-scale galactic magnetic field $B_{0}=3 \mu \mathrm{G}$, and $\omega_{0}=10^{-15} \mathrm{~s}^{-1}$ is taken to be the angular velocity of the Galaxy at a distance from the galactic centre presently occupied by the Sun.
(2) The time scale for collapse is given by a modified free-fall time scale,

$$
\begin{equation*}
t_{\mathrm{c}}=\left(3 \pi / 32 \rho G^{\prime}\right)^{1 / 2}, \tag{I}
\end{equation*}
$$

where $\rho$ is the mass density of the cloud, and $G^{\prime}$ is the magnetically diluted gravitational constant which can be shown to be $G^{\prime}=G\left(\mathrm{r}-\mathcal{M}_{0} /\left|\Omega_{0}\right|\right), \mathscr{M}_{0}$ and $\Omega_{0}$ being, respectively, the initial magnetic and gravitational energies of the cloud, both of which increase $\sim R^{-1}$. For our purposes here, we take as a physically realistic ratio, $\left|\Omega_{0}\right| / M_{0}=2$, although it will be shown in conclusion that our results are quite insensitive to a particular choice of $\left|\Omega_{0}\right| / \mathscr{M}_{0}$.
(3) The time scale for angular momentum transfer is determined by the ambipolar diffusion time $t_{\mathrm{D}}$, since the magnetic torque which brakes a cloud is no longer operative once the field uncouples from the cloud and its surroundings. The time scale for ambipolar diffusion is (Nakano \& Tademaru 1972; Mestel \& Spitzer 1956)

$$
\begin{equation*}
t_{\mathrm{D}}=8 \pi R^{2} n_{\mathrm{i}} n_{\mathrm{H}} m_{\mathrm{H}}\langle\sigma \nu\rangle B^{-2}, \tag{2}
\end{equation*}
$$

where $n_{1}$ and $n_{\mathrm{H}}$ are the number densities of ions (protons) and neutral hydrogen (the primary constituent of interstellar clouds), respectively, $m_{\mathrm{H}}$ is the mass of the hydrogen atom, $\sigma$ and $\nu$ are, respectively, the collision cross-section and relative velocity of ions and atoms, and $B$ is the average magnetic field strength within the cloud. So long as $t_{\mathrm{D}}>t_{\mathrm{C}}$, the field remains essentially frozen-in, and the cloud is able to lose angular momentum. Uncoupling occurs at an epoch in the collapse sequence when $t_{\mathrm{D}} \simeq t_{\mathrm{c}}$. Afterwards, the cloud contracts conserving angular momentum.
(4) The clouds are uniform and spherically symmetric and the collapse is isotropic. Although a seemingly artificial postulate, observations of magnetic field strengths in interstellar clouds are consistent with the hypothesis that the field is frozen into the gas and, in particular, that $B \sim \rho^{2 / 3}$, as expected for an isotropic spherical contraction, for which we have $\rho \sim R^{-3}$ and $B \sim R^{-2}$ (Verschuur 1970). Spitzer (1968a) has argued that a preferential one-dimensional gas flow down along the magnetic and rotation axes is not very efficient at low densities because the gravitational forces increase slowly in comparison with the thermal pressure of the gas. Unless a cloud is initially a cylinder elongated in the direction of the galactic field, this would probably result in stellar clusters having disk shapes rather than being approximately spherical as is observed. Furthermore, the rotational axis will, in general, be aligned roughly perpendicular to the magnetic field since observations indicate that the galactic field runs longitudinally along the spiral arms of the Galaxy. This being the case, an anisotropic contraction seems hardly possible since magnetic forces will oppose gas flow down the rotational axis, whereas centrifugal forces inhibit streaming down along the magnetic field lines.
(5) The magnetic stresses on a contracting, rotating cloud will produce kinks in the galactic field lines which are radiated away in the form of hydromagnetic waves. The cloud is thereby rotationally decelerated and loses angular momentum. While there is some uncertainty as to the rate of power loss from such a cloud by hydromagnetic waves, we adopt the formula of Kulsrud (1971). While Kulsrud's formula would not apply strictly during the initial collapse stages, it should apply reasonably well in the later stages. Also, the formula has the virtue of underestimating the rate of power loss by up to a factor of 10 in cases for which it applies (Kulsrud 1971) since Kulsrud ignored the magnetic-dipole radiation loss in both the Alfvén and slow magnetosonic mode. Kulsrud's formula reads

$$
\begin{equation*}
P \simeq 0 \cdot 2 \beta_{\infty}{ }^{1 / 2} B_{0} B R^{3} \omega \tag{3}
\end{equation*}
$$

where $\beta_{\infty} 1 / 2 \simeq \mathrm{I}$ is the ratio of thermal to magnetic pressure in the surrounding medium, $B_{0}$ and $B$ are, as before, the magnetic field strengths of the (constant) background galactic field which is taken to be the initial field strength of the cloud, and of the cloud's average field, respectively, and $\omega$ is the angular velocity of the cloud. In equation (3), we have assumed implicitly that the cloud's magnetic field structure approximates a dipole, and further, that it is aligned nearly perpendicular to the rotational axis.
(6) The frozen-in magnetic field constrains a cloud to rotate uniformly, since the magnetic energy density, $B^{2} / 8 \pi$, is always greater than the turbulent kinetic energy density, $\rho u^{2} / 2$, which tends to tangle the magnetic lines of force. Thus, we ignore turbulence within the clouds.
(7) The collapse proceeds isothermally with cloud temperatures between 10 and 20 K , in agreement with observations made by Heiles (1971) and the calculations of Fleck (1974).
(8) The thermal pressure of a cloud can be ignored in comparison to the cloud's magnetic pressure, since for typical interstellar clouds, $B^{2} / 8 \pi>n k T$. For an isotropic contraction, the magnetic pressure $\sim \rho^{4 / 3}$ increases faster than the thermal gas pressure $\sim \rho$, since $T$ remains constant.

In short, we consider the magnetically diluted, isotropic, isothermal, free-fall gravitational collapse of a cool, spherical, uniform, interstellar gas cloud which is magnetically braked, losing angular momentum prior to the uncoupling of its
frozen-in magnetic field via ambipolar diffusion. Initially, the magnetic flux is conserved (the ' magnetic field problem ') while angular momentum is not. As the field detaches from the cloud and its surrounding medium, magnetic flux is no longer conserved, whereas angular momentum is strictly conserved thereafter.

We do not speculate as to how the initial clouds form. It will be shown in the following section that initial hydrogen number densities would have to range from $10 \mathrm{~cm}^{-3}$ for a $10^{4} M_{\odot}$ cloud, up to approximately $100 \mathrm{~cm}^{-3}$ for a smaller $10^{2} M_{\odot}$ cloud in order that the clouds be gravitationally unstable. Observationally such clouds are known to exist and a variety of mechanisms (e.g. thermal instability) can account for cloud formation.

## 2. ANGULAR MOMENTUM TRANSFER

In order to investigate the efficiency of angular momentum transfer from collapsing interstellar clouds, we calculate in this section, the angular velocity of a magnetically-braked cloud throughout the collapse sequence, and compare this with the angular velocity the cloud would have if angular momentum had been conserved throughout the collapse. The rate of rotational energy loss $P$ due to the braking effect of the magnetic torque $\tau$ is simply

$$
\begin{equation*}
P=-\tau \omega . \tag{4}
\end{equation*}
$$

Denoting time derivatives with the usual dot notation, we have $\tau=J$, where $J=0.4 m M_{\odot} R^{2} \omega$ is the angular momentum of a uniform, spherical cloud with mass $m$ ( $m$ being dimensionless solar mass units). Using equation (3) for the power loss via hydromagnetic waves, and writing $B=B_{0}\left(R_{0} / R\right)^{2}$ for the case of isotropic compression, with subscripts denoting initial values, we obtain from equation (4) with $\tau=J$, an expression for the deceleration of a cloud,

$$
\begin{equation*}
\dot{\omega}=-B_{0}^{2} R_{0}^{2} / 2 m M_{\odot} R-2 \omega \dot{R} / R \tag{5}
\end{equation*}
$$

where $\left.\dot{R}=\sqrt{2 G^{\prime} m M_{\odot}\left(\mathrm{I} / R-\mathrm{I} / R_{0}\right.}\right)$ is the velocity of collapse. Note that for negligible braking, i.e. no power loss ( $P=0$ ), this expression properly reduces to the statement of angular momentum conservation.

Maximum initial cloud radii $R_{0}$ required to initiate gravitational collapse can be found by equating the initial magnetic and (negative) gravitational energies of a cloud, $B_{0}{ }^{2} R_{0}{ }^{3} / 6$ and $3 G\left(m M_{\odot}\right)^{2} / 5 R_{0}$, respectively. In this paper, we consider clouds with $10^{2}, 10^{3}$ and $10^{4} M_{\odot}$, that is, we take $m=10^{2}, 10^{3}$ and $10^{4}$. Observations seem to preclude masses much larger than these. We find, setting $B_{0}=3 \mu \mathrm{G}$ : $R_{0}\left(m=10^{2}\right)=5.7 \times 10^{18} \mathrm{~cm}, R_{0}\left(m=10^{3}\right)=1.8 \times 10^{19} \mathrm{~cm}$ and $R_{0}\left(m=10^{4}\right)=$ $5.7 \times{ }^{10} 0^{19} \mathrm{~cm}$. These values correspond to initial cloud densities $n_{0}$ ranging from approximately $10 \mathrm{~cm}^{-3}$ for $m=10^{4}$, up to $100 \mathrm{~cm}^{-3}$ for the smaller $10^{2} M_{\odot}$ clouds. All of the above conditions can be realized in the interstellar medium (cf. Lequeux 1974).

Results of the numerical integration of equation (5) are displayed graphically in Fig. I below.

We see that initially, magnetic braking constrains the clouds to co-rotate with the Galaxy ( $\omega=\omega_{0}$ ), with angular momentum being efficiently transferred to the surrounding interstellar medium. Quite abruptly, however, $\omega$ increases as the collapse proceeds, until $\omega \sim R^{-2}$ with angular momentum being conserved prior to the uncoupling epoch. Evidently, the braking mechanism, although still opera-


Fig. I. Angular velocity $\omega\left(s^{-1}\right)$ as a function of radius $R(c m)$ for $10^{2}, 10^{3}$ and $10^{4} M_{\circ}$ clouds. Initial cloud radii $R_{0}$, and uncoupling radii $R_{u}$ and angular velocity $\omega_{u}$ are also indicated for the three cloud masses considered.
tive since the magnetic field remains frozen-in, is inefficient during the advanced stages of collapse.

The integrations were terminated at the uncoupling epoch, since, in the absence of inhibiting magnetic pressures, a cloud becomes unstable against fragmentation into protostellar-mass objects according to the Jeans instability, when $n_{H} \gtrsim \mathrm{I}^{2} T^{3} / m^{2}$. The magnetic field becomes essentially uncoupled when $t_{\mathrm{D}} \simeq t_{\mathrm{C}}$ ( $c f$. Mestel \& Spitzer 1956; Nakano \& Tademaru 1972). Writing $B=B_{0}\left(R_{0} / R\right)^{2}$ and $\rho=\mu m_{\mathrm{H}} n_{\mathrm{H}}=3 m M_{\odot} / 4 \pi R^{3}$ for uniform, spherical, isotropically contracting clouds, we obtain upon equating expressions (1) and (2), the radius of a cloud at uncoupling,

$$
\begin{equation*}
R_{\mathrm{u}}=\mathrm{I} \cdot 7 \times \mathrm{Io}^{-33} R_{0} 8 / 3 / m n_{1}{ }^{2 / 3} \mathrm{~cm} \tag{6}
\end{equation*}
$$

where we have taken $\mu=1.38$ for the mean molecular weight of the interstellar medium, and $\langle\sigma \nu\rangle=2 \times 10^{-9} \mathrm{~cm}^{3} \mathrm{~s}^{-1}$ (Osterbrock 1961). It follows that, here, $n_{1}$ is now the ion density at uncoupling.

Ions are mainly supplied by the ionization of hydrogen by cosmic rays and $K^{40}$ radioactivity; ultraviolet radiation (Werner 1970) and X-rays (Nakano \& Tademaru 1972) are screened in the peripheral regions of a cloud. Nakano $\&$ Tademaru have calculated the ionization rate by cosmic rays. They find $\xi_{\mathrm{CR}}=6 \cdot \mathrm{I} \times \mathrm{IO}^{-18} \exp (-\rho R / 66) \mathrm{s}^{-1}$. Cameron (1962) has estimated the $\mathrm{K}^{40}$ ionization rate to be $\xi_{\mathrm{K}}=1.4 \times 1 \mathrm{o}^{-21} \mathrm{~s}^{-1}$. Thus, we expect the ion density to be
initially determined primarily by cosmic rays until they become effectively screened when cloud densities reach $n_{\mathrm{H}} \simeq 10^{9} \mathrm{~cm}^{-3}$, after which, ions are supplied at a constant rate by $\mathrm{K}^{40}$. Assuming equilibrium between ionization and recombination, we can write

$$
\begin{equation*}
R_{\mathrm{e}}+n_{\mathrm{H}} R_{\mathrm{g}}=n_{\mathrm{H}}\left(\xi_{\mathrm{CR}}+\xi_{\mathrm{K}}\right), \tag{7}
\end{equation*}
$$

where, assuming $n_{\mathrm{e}} \simeq n_{\mathrm{i}}, R_{\mathrm{e}}=2 \times \mathrm{Io}^{-11} n_{\mathrm{i}}^{2}$ is the rate of radiative recombination of electrons and ions (Zanstra 1954), and $R_{\mathrm{g}}=2.2 \times 10^{-17} n_{\mathrm{i}}$ is the radiative recombination rate of electrons and ions upon grains (Spitzer 1968b). Solving the equilibrium equation, (7), for $n_{\mathrm{i}}$, and expanding for the case of typical uncoupling densities $n_{\mathrm{H}} \simeq 10^{7} \mathrm{~cm}^{-3}$, we find

$$
\begin{equation*}
n_{\mathrm{i}} \simeq 4.53 \times \mathrm{ro}^{16}\left(\xi_{\mathrm{CR}}+\xi_{\mathrm{K}}\right) \mathrm{cm}^{-3} . \tag{8}
\end{equation*}
$$

Substitution of this expression into equation (6) yields a transcendental equation for $R_{\mathrm{u}}$ which can be solved by iteration.

Magnetic screening of incoming cosmic rays (Fleck 1974) may reduce the ion densities as calculated from equation (8). Cosmic rays propagating through an untangled magnetic field will stream along the field lines in a helical motion, so that not all particles entering a magnetic cloud will have trajectories carrying them into the central regions. A minimum constant ion density $n_{i}=6.34 \times 10^{-5} \mathrm{~cm}^{-3}$ is maintained, however, by $\mathrm{K}^{40}$ radioactivities (see equation (8) with $\xi_{\mathrm{CR}}=0$ for complete cosmic ray screening). With this low ion density, the ambipolar diffusion time is short, and one expects uncoupling of the magnetic field to occur at lower densities. This may explain the absence of very high fields in some dust clouds with $n_{\mathrm{H}} \gtrsim \mathrm{IO}^{4} \mathrm{~cm}^{-3}$.

Since the initial angular momentum of a protostar is established at the uncoupling epoch, at which time a cloud is expected to fragment (hence, the ' epoch of fragmentation'), it is of interest to calculate certain cloud parameters at the time of uncoupling. These appear below in Table I. We have considered the two extreme cases of ionization, namely, strong cosmic ray ionization at a rate determined by $\xi_{\mathrm{CR}}$, and weak ionization by $\mathrm{K}^{40}$ only, at a rate given by $\xi_{\mathrm{K}}$, bearing in mind that the cosmic ray rate is probably the more realistic of the two $; \mathrm{K}^{40}$ ionization providing a workable lower limit.

For the case of ionization by $\mathrm{K}^{40}$ only, all clouds co-rotate throughout the entire collapse sequence. This is understandable since they uncouple early before magnetic braking becomes inefficient. Although the CR cloud sizes at uncoupling appear to be smaller than observations of star clusters seem to indicate, it must be remembered

## Table I

Relevant parameters of $10^{2}, 10^{3}$ and $10^{4} M_{\odot}$ clouds at the uncoupling epoch for strong cosmic ray ionization ( $C R$ ), and weak $K^{40}$ ionization ( $K^{40}$ )

| $m\left(M_{\odot}\right)$ |  | $R_{0}$ (cm) | $R_{\mathrm{u}}(\mathrm{cm})$ | $\omega_{u}\left(s^{-1}\right)$ | $\omega_{\mathrm{u}}{ }^{j=0}\left(\mathrm{~s}^{-1}\right)$ | $F_{\mathrm{g}} / F_{\mathrm{c}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $10^{2}$ | CR | $5 \cdot 7 \times 10^{18}$ | $\mathrm{I} \cdot 8 \times \mathrm{IO}^{16}$ | $3.9 \times 10^{-15}$ | $\mathrm{I} \cdot 0 \times 10^{-10}$ | $1.5 \times 10^{8}$ |
|  | $\mathrm{K}^{40}$ | $5.8 \times 10^{18}$ | $\mathrm{I} \cdot \mathrm{I} \times 10^{18}$ | $\omega_{0}$ | $2.6 \times 10^{-14}$ | $9.8 \times 10^{3}$ |
| $10^{3}$ | CR | $\mathrm{I} \cdot 8 \times 10^{19}$ | $5.2 \times 10^{16}$ | $4.1 \times 10^{-14}$ | $1 \cdot 2 \times 10^{-10}$ | $5 \cdot 6 \times 10^{5}$ |
|  | $\mathrm{K}^{40}$ | $\mathrm{I} \cdot 8 \times 10^{19}$ | $2.4 \times 10^{18}$ | $\omega_{0}$ | $5.7 \times 10^{-14}$ | $9.9 \times 10^{3}$ |
| 104 | CR | $5.7 \times 10^{19}$ | $1.5 \times 10^{17}$ | $4.2 \times 10^{-13}$ | $1 \cdot 4 \times 10^{-10}$ | $2.1 \times 10^{3}$ |
|  | $\mathrm{K}^{40}$ | $5.7 \times 10^{19}$ | $5 \cdot 1 \times 10^{18}$ | $\omega_{0}$ | $1.2 \times 10^{-13}$ | $9.8 \times 10^{3}$ |

that as the more massive stars ' turn on', their radiation will heat the surrounding gas causing a protocluster to expand and possibly lose mass through thermal outgassing. It is a well-known principle of mechanics that if a system in equilibrium loses half of its original mass, expansion to infinity will be inevitable. Angular velocity at uncoupling for a cloud conserving angular momentum, $\omega_{u}{ }^{\dot{j}=0}$, is tabulated for comparison with $\omega_{\mathrm{u}}$ for braked clouds. Only the smallest CR clouds approximately corotate with the Galaxy throughout the entire collapse, being rotationally decelerated by almost five orders of magnitude. Evidently, the braking is less efficient for the more massive clouds, which is not surprising in view of the shorter ambipolar diffusion time for these larger clouds. The ratio $F_{\mathrm{g}} / F_{\mathrm{c}}=G m M_{\odot} /$ $R_{\mathrm{u}}{ }^{3} \omega_{\mathrm{u}}{ }^{2}$ indicates the important result that centrifugal forces are kept well below gravity in all cases considered. However, Fleck (1974) has shown this not to be the case for very large ( $m \gtrsim 10^{5}-10^{6} M_{\odot}$ ) clouds, which might explain the low occurrence of such clouds in nature.

In the following section, our results are shown to be consistent with observations of stellar rotation on the main sequence, and also with the angular momentum of the protosun.

## 3. OBSERVATIONAL EVIDENCE FOR ANGULAR MOMENTUM TRANSFER

Noting the values of $\omega_{u}{ }^{j=0}$ in Table I, a simple calculation shows that a mainsequence star would rotate with velocities close to that of light if it formed from a collapsing interstellar gas cloud which conserved angular momentum prior to fragmentation. Of course, it appears doubtful that such a star would ever reach the main sequence because of ensuing rotational instability which would certainly halt further contraction perpendicular to the rotational axis. As a test of our present hypothesis of magnetic braking, we calculate and compare with observations, the equatorial rotational velocities of early-type main-sequence stars, and the angular momentum of the protosun.

Assuming no angular momentum loss during pre-main-sequence contraction, the angular momentum of a protostar $J_{*}$ is established at the time of fragmentation (uncoupling), that is, $J_{* u}=J_{*}$. Thus, it becomes possible to predict mainsequence rotation and angular momentum in terms of cloud parameters at the uncoupling epoch. We write $J_{* \mathrm{u}}=0.4 m_{* \mathrm{u}} M_{\odot} R_{* \mathrm{u}}{ }^{2} \omega_{* \mathrm{u}}$ for a uniform, spherical, protostellar fragment at uncoupling, with mass $m_{* \mathrm{u}}$, radius $R_{* \mathrm{u}}$, and angular velocity $\omega_{* \mathrm{u}}$, and $J_{*}=\alpha m_{*} M_{\odot} R_{*} v_{*}$ for a uniformly rotating main-sequence star with mass $m_{*}$, radius $R_{*}$, gyration constant $\alpha$, and equatorial rotational velocity $v_{*}$. A frozen-in magnetic field constrains a cloud to rotate uniformly, so that a fragmenting protostar will initially rotate at roughly the same rate as its parent cloud at the uncoupling epoch. Thus, we set $\omega_{* u} \simeq \omega_{\mathrm{u}}$. Furthermore, since the clouds are assumed uniform, the initial density of a fragment will be identical to the cloud density at uncoupling, and we write accordingly, $R_{* \mathrm{u}}{ }^{2}=R_{\mathrm{u}}{ }^{2}\left(m_{* \mathrm{u}} / m\right)^{2 / 3}$. With negligible mass loss during pre-main-sequence contraction, $m_{* \mathrm{u}} \simeq m_{*}$, so that for $J_{* \mathrm{u}}=J_{*}$, we arrive at

$$
\begin{equation*}
v_{*}=\left(4 \times 10^{-6} R_{\mathrm{u}}{ }^{2} \omega_{\mathrm{u}} / \alpha R_{*}\right)\left(m_{*} / m\right)^{1 / 3} \mathrm{~km} \mathrm{~s}^{-1} . \tag{9}
\end{equation*}
$$

Aveni (1974, private communication) has estimated the mean mass of star clusters and associations to be slightly less than $1 \circ^{3} M_{\odot}$. Using $\omega_{\mathrm{u}}$ and $R_{\mathrm{u}}$ from Table I for a $10^{3} M_{\odot}$ CR cloud, and taking main-sequence values for $m_{*}$ and $R_{*}$ from

Allen (1974), letting $\alpha \simeq 0 \cdot 1$ for early-type stars, we find: $v_{*}\left(\mathrm{O}_{5}\right)=42 \mathrm{I}$, $v_{*}(\mathrm{Bo})=614, v_{*}(\mathrm{~B} 5)=610$ and $v_{*}(\mathrm{Ao})=583 \mathrm{~km} \mathrm{~s}^{-1}$. Although considerably higher than the $\left\langle v_{*} \sin i\right\rangle$ observed by Abt \& Hunter (1961) for early-type stars, the agreement is surprisingly good in view of the uncertainties and approximations which enter into the calculations. The sharp drop in $v_{*}$ at $\mathrm{O}_{5}$, and gradual decline at Ao from a maximum near Bo-B5, also matches the observations. Equation (9) also predicts smaller rotational velocities for stellar aggregates of smaller mass, in agreement with a preliminary observation made by Abt (1970).

Equation (9) implicitly assumes angular momentum conservation during pre-main-sequence contraction, and thus predicts anomalously high rotational velocities for stars of spectral types later than A5. These stars are believed to be rotationally decelerated by a stellar wind or a magnetic torque which transfers a considerable fraction of angular momentum to a planetary system, with the total angular momentum of the ' solar nebula' remaining constant (Hoyle 1960). Within the framework of Hoyle's model for the formation of our solar system, we calculate the angular momentum of the protosun $J_{\odot p}$ prior to angular momentum transfer to the surrounding protoplanetary material. With the same simplifying assumptions incorporated into equation (9), we obtain, taking $m_{\odot p} \simeq \mathrm{I}$,

$$
\begin{equation*}
J_{\odot \mathrm{p}}=0.4 M_{\odot} R_{\mathrm{u}}{ }^{2} \omega_{\mathrm{u}} m^{-2 / 3} \mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-1} \tag{io}
\end{equation*}
$$

Again, assuming cosmic ray ionization, taking $\omega_{\mathrm{u}}$ and $R_{\mathrm{u}}$ from Table I for $m=10^{3}$, we find $J_{\odot p}=9 \times 10^{50} \mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-1}$. This is to be compared with the present angular momentum of the solar system, $4 \times 10^{50} \mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-1}$, and with Hoyle's estimate of the original angular momentum of the protosun (the Sun and planets plus approximately 0.0 or $M_{\odot}$ of hydrogen and helium that must be added to make up the normal solar abundances of these elements in the planets), $J_{\odot p} \simeq 4 \times 10^{51} \mathrm{~g} \mathrm{~cm}^{2} \mathrm{~s}^{-1}$.

## 4. CONCLUDING REMARKS

Based on the present investigation, we conclude that the ' angular momentum problem' associated with isotropically collapsing rotating clouds, may be greatly alleviated by a magnetic braking mechanism which transfers angular momentum from a cool magnetic cloud to the surrounding interstellar medium via hydromagnetic waves, prior to uncoupling of the frozen-in magnetic field via ambipolar diffusion and subsequent fragmentation of the cloud into protostellar-mass objects.

Centrifugal forces never become important enough ( $F_{\mathrm{g}} / F_{\mathrm{c}} \gg \mathrm{I}$ ) to inhibit the isotropic contraction of clouds with $10^{2}, 10^{3}$ and $10^{4} M_{\odot}$. The time scale of the magnetically diluted collapse is thus determined uniquely by the ratio $\left|\Omega_{0}\right| / \mathcal{M}_{0}$, ignoring thermal pressure during these early pre-opaque stages. Interestingly enough, our results are practically insensitive to this somewhat uncertain ratio. A larger initial magnetic energy will lengthen to collapse time, resulting in an earlier uncoupling of the magnetic field. This being the case, a cloud will have a slightly larger uncoupling radius and a correspondingly smaller angular velocity. However, the product $R_{\mathrm{u}}{ }^{2} \omega_{\mathrm{u}}$, which determines the angular momentum of mainsequence stars, remains roughly constant.

That the braking is less efficient for large clouds might explain the low occurrence of massive ( $m>10^{5} M_{\odot}$ ) clouds in nature. The time scale for angular momentum transfer (determined by the ambipolar diffusion time scale) in such clouds would become drastically shortened by an increased time scale for collapse brought on by the increasing centrifugal forces.

Calculations based on the magnetic braking hypothesis are consistent with observations of stellar rotation and angular momentum. Indeed, without angular momentum transfer, it appears doubtful that stars could form within large interstellar clouds without recourse to anisotropic contraction.

## ACKNOWLEDGMENTS

This paper was condensed from a thesis submitted by one of us (RCF) to the Department of Astronomy at the University of South Florida in partial fulfillment of the requirements for the MA degree. (JHH was the thesis supervisor.) Special thanks to Albert Rust of the Mathematics Department for many enlightening discussions and for writing the computer code for the numerical integrations.

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