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Magnetic braking of the present Sun

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ABSTRACT

Magnetic braking is essential for angular momentum transport in late-type stars which have convective envelopes. The mechanism may be entirely responsible for the slow rotation of the present Sun, on which a braking model is normally calibrated. Recent or current satellite missions such as *Helios* and *Ulysses* have jointly revealed a more complete picture of the solar corona, and more specifically of the solar wind. The wealth of these data at or near the solar minimum is valuable for constraining the dipolar solar braking model. In this paper, we use recently available observations (at or near the solar minimum) to constrain a solar magnetic braking model based on a dipolar field structure. It is found that the *Ulysses* data indicate a spherical Alfvén surface at high latitudes. We infer from a thermal wind model that it is located at 16 $R_\odot$, which is larger than the 12 $R_\odot$ deduced from the *Helios* data for the equatorial region near the solar minimum. It is also found that the braking model with a transition from a dipole to a split monopole field is generally consistent with *Ulysses* observations, provided that a linear relation between dead zone extent and dipole field strength is satisfied. Thus either the dipole field retains a sizeable dead zone but is much stronger than the standard value, $\sim$1 G, or the field has standard strength but an exceedingly small dead zone (< 2 $R_\odot$). The magnetic braking rate as constrained by *Ulysses* data is found to be 2.1×10^{30} dyn cm, which is about a quarter of the value deduced earlier from the Weber & Davis model and does not differ significantly from that deduced by Pizzo et al.

Key words: Sun: corona – Sun: magnetic fields – solar wind – stars: late-type – stars: rotation.

1 INTRODUCTION

Based on the earlier determination of the projection of rotational velocity $v \sin i$ of late-type stars, Skumanich (1972) derived an empirical relation which shows that the mean rotational velocity of solar-type G stars in the Pleiades (7×10^9 yr), the Hyades (8×10^8 yr) and the present Sun (4.5×10^9 yr) follows a relation $v_t \propto t^{-1/2}$, where t is the main-sequence age. This well-known *Skumanich relation* implies that the Sun must have had a quite rapid rotation in its earlier main-sequence life if it is a normal G-type star like those we see in the Pleiades and Hyades. The physical interpretation of the Skumanich relation is that solar-type stars have been undergoing continuous spin-down. Although other empirical relations predict different forms (see Benz, Mayor & Mermilliod 1984; Skumanich & MacGregor 1986; Mayor & Mermilliod 1991), the Skumanich relation seems to be the simplest and the most commonly used. However, the true solar rotation rate on the Zero Age Main Sequence (ZAMS) is not known. The Sun may belong to one of two categories: *slow rotators* which rotate less than 10–20 times faster than the present Sun, and *fast rotators* which have rotation rates near the centrifugal limit. There is some evidence, according to a possible rotational evolution model (Collier Cameron & Li 1994), that the early Sun could have been a fast rotator, but the evidence is not compelling.

The surface layer of the Sun (or solar-type stars) is convective, and the turbulent motion and rotation generate magnetic fields, thereby heating the corona and producing a magnetically controlled solar wind. because of the stiffening effect of a magnetic field, which tends to overcome the Coriolis force on a stellar wind and keep the plasma in corotation with a star, the loss of angular momentum per unit wind mass is far greater than that without a magnetic field. For a non-magnetic star with an outflowing stellar wind, the angular momentum is lost from the stellar surface and so the specific angular momentum per unit wind mass retains its value on the stellar surface. The presence of a magnetic field therefore greatly increases the effective ‘stellar radius’ – the Alfvén surface – and this mechanism is called *magnetic braking*, as first suggested by Schatzman (1962) and later formulated by Mestel (1967; 1968, hereafter M68) and Weber & Davis (1967, hereafter WD) in the frame of ideal magnetohydrodynamics (MHD). WD used observations made at 1 au at that time, and, assuming only a radical magnetic field (split monopole), the model predicted a solar spin-down torque of 7.55×10^{30} dyn cm. The braking rate is based on the adopted mass flux per sr $\rho v r^2 = 1.05 \times 10^{11}$ g s^{-1} and a solar angular velocity $\Omega = 3.0 \times 10^{-6}$ s^{-1}. The corresponding Alfvén surface is at 24.3 R$_\odot$. The model predicted a braking rate which is very close to that derived from the Skumanich relation, i.e. 8.5×10^{30} dyn cm, if we adopt the moment of inertia $I_\odot k^2$ $M_\odot$ $R_\odot^2$ with a standard value $k^2 = 0.1$. The magnetic field strength of the Sun predicted from the model is 2.31 G, based on the observed radial magnetic field 5γ at 1 au. The consistency between the WD model and the Skumanich relation establishes that the Sun may have indeed been braked continuously by magnetic braking. Mestel (1984) demonstrated that if a pure thermal wind, monopole field and linear dependence of magnetic field on rotation are adopted for the entire evolution of stellar spin, the WD model produces $-\dot{J} \propto \Omega^3$ which leads to the Skumanich relation.

The realistic field structure as used by WD cannot simply be a split monopole because the strong field on the surface (low plasma β) tends to close to form a closed field region, or *dead zone*, as first predicted and named by Mestel (1967). The dead zone does not contribute to magnetic braking, as plasma in the dead zone does not escape. From direct observations of solar eclipse images during activity minimum using the white-line coronagraph, and observations using the *Skylab* telescope and *Yohkoh* X-ray satellite, Mestel's dead zones are clearly seen. Mestel's subsequent work (see M68; Mestel 1984; Mestel & Spruit 1987, hereafter M87) takes this important feature into account. The importance of the dead zone adds a new ingredient to the braking model. It suggests that an increase of magnetic field alone leads to more closed field regions, limiting the efficiency of braking. The implication is straightforward: if one adopts a linear dependence of magnetic field on rotation $B_0 \propto \Omega$, then, realistically, braking must be smaller than in the WD model where no dead zone exists, given that other quantities are the same.

The other important feature, the effect of a centrifugally driven wind, that affects the magnetic braking significantly, was modelled by MS87. From the slow rotators to the fast rotators, they noticed that the wind begins to be more and more centrifugally driven, and so for given rotation and mass loss rates the centrifugal effect brings down the Alfvén surface, leading to a reduction of magnetic braking (cf. Li 1992). This effect significantly reduces magnetic braking for fast rotators compared with the WD model, which assumes both a monopolar field and a thermally driven wind. For the Sun, which we will focus on in the later sections, the wind is essentially thermal. In general, the MS87 model has a braking rate smaller than that of the WD model over the entire parameter domain.

The MS87 model includes more physics than the WD one, as it adopts a dipolar field structure and covers the centrifugal wind region. However, the predicted braking rate is smaller than for the WD model and therefore yields a spin-down law different from the Skumanich relation. The MS87 model is widely accepted and it has also been applied to cataclysmic variables, which are very rapidly rotating stars (Verbunt & Zwaan 1981).

A braking model contains a few key free parameters and so it is important to calibrate the model using a star like the Sun, for which we might have a better understanding of the plasma properties and magnetic fields. The early satellite missions gathered plasma data near the Earth's orbit; they helped constrain the plasma parameters of the solar wind and therefore reduced the free parameters of a solar braking model (see WD). The advantage of using space data to constrain a braking model is that we can avoid using the uncertain boundary conditions at the corona base.

The improved solar observations via *Helios* in the 1970s brought us more detailed observations in the solar ecliptic plane within 1 au. The data revealed, however, a braking rate smaller by about a factor of 4 when compared with WD's result for the current Sun (Pizzo et al. 1983). The corresponding Alfvénic distance is only half of that predicted from WD's result. These differences in value can arise from either the adoption of an inappropriately large magnetic field at 1 au in WD or current sheets and larger densities in the equatorial plane, which lead to a smaller observed Alfvénic distance. The *Helios* mission improved our knowledge of the solar wind in the solar ecliptic plane, but the properties of solar winds at high latitudes still remain unknown.

The current *Ulysses* polar mission has been revealing a three-dimensional picture of the solar wind beyond 1 au. It has so far enabled us to understand basic plasma parameters of the solar wind such as wind velocity, density, temperature and magnetic fields at the periods of minimum solar activity. Because the solar magnetic field at the solar minimum is the closest field to a dipolar structure in a solar cycle, these observations are particularly valuable for constraining a braking model which incorporates a dipolar magnetic field. One of the most important results is the confirmation that the mean poloidal magnetic flux and the mean mass loss flux are uniform at high latitudes above 20° (Forsyth et al. 1996; Goldstein et al. 1996). This indicates that a braking model which has a homogeneity in latitude at large distances, like the MS87 model, is basically correct.

In this paper, we attempt to examine the MS87 model as applied to the solar minimum in more detail by using the space observations currently available. We will first review and then refine the MS87 model for the Sun, and then show that the current *Ulysses* data indicate a spherical Alfvén surface at high latitudes, with a larger Alfvénic distance than that observed in the equatorial region. Thirdly, we examine the MS87 model as applied to the solar minimum. The motivation is that if realistic plasma conditions and magnetic field are adopted, one expects a consistency in the braking rate between observations and model predictions. As we will demonstrate, this is generally the case. However, there exists a quite well-constrained linear correlation between dead zone extent and dipole field strength, and it requires either a small (normal) dipole field strength with a small dead zone (< 2 R$_\odot$) or a large field with a large dead zone. Finally, we calculate the solar braking rate as constrained by *Ulysses* observations.

2 THE MS87 MODEL FOR THE SOLAR MINIMUM

The MS87 model included two essential features of a more realistic braking mechanism, i.e. a dipolar field structure and centrifugal wind acceleration. The formulation for the total braking rate is based on assuming a spherical Alfvén surface. This simple assumption is reasonable for slow rotators and seems to be consistent with the numerical work of Washimi & Sakurai (1993). To a first approximation, it is therefore also reasonable for fast rotators. The calculation of the braking rate follows the famous lever-arm analogy of plasma effective corotation up to the Alfvénic surface at r_A:

$$-\dot{J} = 2\int_0^{\pi/2} (\rho_A v_A)(\alpha r_A^2 \sin^2\theta)(2\pi r_A^2 \sin\theta)\mathrm{d}\theta$$
$$= 4\pi \int_0^{\pi/2} \alpha(\rho_A v_A r_A^2) r_A^2 \sin^3\theta \mathrm{d}\theta, \quad (1)$$

where ρ is the plasma density, v is the poloidal velocity, α the stellar

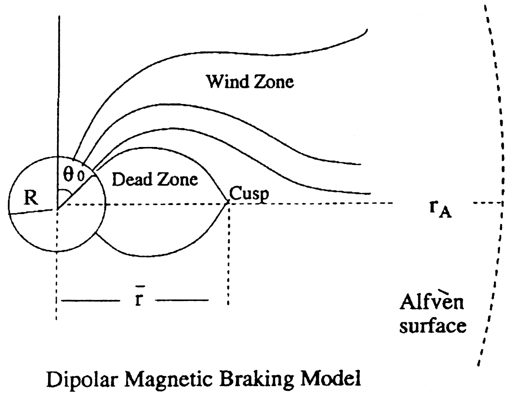

Figure 1. An illustration of the magnetic field lines in the axisymmetrical dipolar magnetic braking model (MS87). The magnetically confined region, the dead zone, ends at the cusp point at $r = \bar{r}$, and it is expected that the field in the wind zone starts to deviate from a dipolar structure at $r = \bar{r}$ and becomes a monopolar structure before or as it reaches the spherical Alfvén surface at r_A.

rotation rate and θ the coaltitude. The subscript A denotes quantities at the Alfvén surface. In the MS87 model, the dipolar field remains closed up to the cusp point (a breakup point for the closed field line) at $r = \bar{r}$, the dead zone extent is as illustrated in Fig. 1, and beyond that the field is assumed to be a split monopole. MS87 set

$$\rho_A v_A r_A^2 = \rho(\bar{r}) v(\bar{r}) \bar{r}^2 = (\rho_0)_w v_0 \left(\frac{\bar{B}}{B_0}\right) \bar{r}^2 = (\rho_0)_w v_0 R^2 (R/\bar{r}), \quad (2)$$

where the mass continuity equation per unit magnetic flux is used. As a result, the braking rate can be expressed as

$$-\dot{J} = \left[\frac{8\pi}{3} \alpha_\odot (\rho_0)_\odot R^4 v_0\right] K\left(\frac{\alpha}{\alpha_\odot}\right), \quad (3)$$

where

$$K\left(\frac{\alpha}{\alpha_\odot}\right) = (R/\bar{r})(r_A/R)^2 (\alpha/\alpha_\odot)^q, \quad (4)$$

R is the stellar radius and q is the index defining how the coronal base density depends on the surface magnetic field.

Although the former derivation may be quite accurate for a general braking model in evolutionary calculations, it has in fact overestimated the braking rate by approximately a factor of 2. For our use, this correction is important. In MS87, an assumption is made that beyond $\bar{r}$ the field becomes immediately a split monopole. In fact, we can made the more relaxed assumption that at the Alfvén surface the field becomes a split monopole. This means that there is a gradual transition from the dipole at $\bar{r}$ to a split monopole at the Alfvén surface. our derivation is as follows.

Using the mass continuity equation along the field line, we directly relate quantities at the stellar surface to those at the Alfvén surface:

$$\rho_A v_A r_A^2 = (\rho_0)_w v_0 \frac{B_A r_A^2}{B_{p0}} = (\rho_0)_w v_0 \frac{\Phi}{4\pi B_0 f(\theta)}, \quad (5)$$

where Φ is the total open flux responsible for braking, B_0 is the polar field strength and $B_{p0} = B_0 f(\theta)$ is the poloidal field strength at the stellar surface with

$$f(\theta) = \sqrt{1 - \frac{3}{4}\sin^2\theta}. \quad (6)$$

Since the flux must be conserved, irrespective of the complication near $r = \bar{r}$, we may evaluate it at the stellar surface:

$$\Phi = 4\pi B_0 \int_0^{\theta_0} \sin\theta R^2 \cos\theta d\theta = 2\pi B_0 R^2 \sin^2\theta_0 = 2\pi B_0 R^2 (R/\bar{r}), \quad (7)$$

where θ_0 is the colatitude of the last closed magnetic field line (see Fig. 1), and in the last step we used the dipole field line equation $\sin^2\theta/r =$ constant. Of course, the above calculation is based on an undisturbed dipole field up to $\bar{r}$. Combining (5), (6) and (7), we finally obtain

$$\rho_A v_A r_A^2 = \frac{1}{2} (\rho_0)_w v_0 R^2 \frac{(R/\bar{r})}{f(\theta_0)}. \quad (8)$$

Comparing (8) with (2), we find that our new derivation is smaller by a factor of 2 than tha tof the MS87 model [for braking limited to polar flux so $f(\theta_0) \simeq 1$]. The difference is created by the discontinuous jump in the MS87 model from a dipole field to a monopole field at $\bar{r}$. In the sphere $r = \bar{r}$, the total open flux of a dipole field is only half of that of a monopole field. In MS87, however, this correction is not made because the monopole field at $\bar{r}$ takes on the value of the stellar dipole field at the same location, and this artificial discontinuity leads to an overestimation of total open flux by roughly a factor of 2. In reality, the transition from dipole to monopole may occur gradually. As shown here, however, such a detail is irrelevant to the final result, given that the Alfvénic surface is at the monopole region. Using the correct expression (8), assuming a constant $(\rho_0)_w v_0$ and $f(\theta_0) \simeq 1$, the total braking rate is

$$-\dot{J} = \left[\frac{4\pi}{3} \alpha_\odot (\rho_0)_4 v_0\right] K\left(\frac{\alpha}{\alpha_\odot}\right), \quad (9)$$

where K is still given by (4). Note that (9) is a factor of 2 smaller than in MS87 for the same K.

Two important dimensionless parameters in the MS87 model are the ratios of magnetic pressure to gas pressure at the coronal base for both the wind and the dead zone regons:

$$\xi_w = \xi_w(\theta) = B_0^2/8\pi(\rho_0)_w a_w^2, \qquad \xi_d = B_0^2/8\pi(\rho_0)_d a_d^2, \quad (10)$$

where ξ_w can be a function of latitude as we will discuss later on, but ξ_d is assumed to be constant.

To estimate the wind speed at the Alfvén surface, thermal and centrifugal effects are considered in the MS87 model. For the Sun we neglect the centrifugal effect, so that

$$\left(\frac{v_A}{a_w}\right) = \left[\frac{v_{th}(r_A/R)}{a_w}\right]. \quad (11)$$

For a thermally driven wind, we can tabulate v_{th} at different distances (one may refer to the appendix in the MS87 paper for the numerical values). Once v_A is known, the continuity of mass leads to

$$4\pi \left[\frac{v_0(\rho_0)_w}{B_0 f(\theta)}\right] = \left(\frac{B_p}{v_p}\right)_{r_A} = \frac{\Phi/4\pi r_A^2}{v_A}. \quad (12)$$

Use of (7) in this expression then leads to

$$\left(\frac{r_A}{R}\right)^2 = \frac{\xi_w f(\theta)}{(v_A/a_w)(\bar{r}/R)(v_0/a_w)}, \quad (13)$$

which is about a factor of 2 smaller [for $f(\theta_0) \simeq 1$] on the right-hand side than the relation in the MS87 model. The difference comes from the aforementioned overestimation of open magnetic flux in the transition from the dipole to monopole in MS87. As noted, the numerical value of ξ_w is constrained by the angular momentum flux of the solar wind near the equatorial plane, which is observed by the

Helios spacecraft (Pizzo et al. 1983). Since the ecliptic region is associated with a current sheet and therefore involves magnetic reconnection caused by, for example, the tearing mode instability, the plasma motion is affected by friction, so we use the angular momentum flux calculated from the wind zone right above the equatorial region. As we will see, the difference is not significant since the estimated Alfvén surfaces at higher latitudes and in the equatorial region do not differ enormously.

3 OBSERVATIONAL CONSTRAINTS

The *Ulysses* mission reveals two basic observational facts: namely, the wind mass flux and radial magnetic fields at higher latitudes are uniform beyond 1 au. This implies that the mass loss rate per unit magnetic flux, η, must be a constant above the equatorial plane. As we know that $v_0/a_w \simeq 0.15$ for a dipolar field, then for a constant a_w we must assume the corona base density follows $\rho_0 \propto B_{p0} \propto f(\theta)$. All the plasma data gathered from this and previous missions enable us to examine a dipole–monopole model (MS87) in considerable detail.

3.1 The Alfvén surface at higher latitudes

The evaluation of the Alfvén surface $r_A = 12\ R_\odot$ in Pizzo et al. (1983) is for the solar ecliptic plane as based on the *Helios* mission. Because this mission was near the solar minimum, we do not expect significant differences compared with the solar minimum. At the high latitudes, however, the Alfvén surface can in principle be different. Direct observations of angular momentum flux at high latitudes are not yet available, but the current *Ulysses* observations may be used indirectly to infer the Alfvénic distance. We use the ideal MHD stellar wind model (M68; WD) to calculate the Alfvénic distance.

The property of a constant η reads

$$\eta = \frac{\rho v_p}{B_p} = \frac{\rho_A v_A}{B_{pA}}. \tag{14}$$

Assuming a monopole field beyond the Alfvénic surface, (14) leads to

$$\left(\frac{r_A}{R_\odot}\right) = \frac{(r/R_\odot)B_p(r)}{\sqrt{4\pi\rho v_p v_A}}. \tag{15}$$

We can obtain the same flux result by using the following relation for the angular momentum flux:

$$-\frac{\beta}{4\pi} = -\frac{\varpi B_\phi}{4\pi} + \eta\Omega\varpi^2. \tag{16}$$

The toroidal magnetic field is

$$\varpi B_\phi = \frac{4\pi\eta\alpha(\varpi^2 - \varpi_A^2)}{1 - \rho_A/\rho}. \tag{17}$$

In the limit $\varpi \gg \varpi_A$, we have

$$\frac{B_\phi}{B_p} \simeq -\frac{\alpha\varpi}{v_p}, \tag{18}$$

which is what is normally called *Parker's spiral*. This is generally confirmed in the *Ulysses* data (Forsyth et al. 1996). For the same approximation $\varpi \gg \varpi_A$, the angular velocity becomes

$$\frac{\Omega}{\alpha} = \frac{1 - \rho_A\varpi_A^2/\rho\varpi^2}{1 - \rho_A/\rho} \simeq \alpha(v_p/v_A - 1)\rho/\rho_A. \tag{19}$$

Combining (14), (16), (18) and (19), we arrive at an expression identical to (15).

The velocity v_A is assumed to be

$$v_A = Ca_w, \tag{20}$$

where C is a parameter determined by the nature of the wind: we assume a thermally driven wind. The mass flux is fixed from the observations, so $r_A/R_\odot \propto 1/\sqrt{v_A}$. Thus the temperature in the wind region becomes crucial in determining the Alfvén surface.

The exact temperature for the open field region is complicated but it is smaller than 2×10^6 K (e.g. Golub & Pasachoff 1997), which is the standard temperature chosen for the dead zone in MS87 and also for our analysis. However, we can estimate a reasonable wind temperature from some observational data. For example, the *Ulysses* data show that the temperature at heliocentric distance r follows a mean relation $r^{-0.9}$ (Goldstein et al. 1996) with a mean temperature 1.35×10^5 K at 1 au (Feldman et al. 1996). Combining these two values, the predicted temperature at 15 $R_\odot$ is 1.48×10^6 K. Thus we adopt 1.5×10^6 K, or $a_w^2 = 2.06\times10^{14}\ cm^2\ s^{-2}$. For such a temperature we find $r_A = 16.1\ R_\odot$, corresponding to $C = 2.48$ which is the extrapolated value of the thermally driven wind speed at $r/R = 16$. We therefore adopt $r_A = 16\ R_\odot$.

The above calculations are independent of latitude, and thus we conclude that the Alfvén surface at high latitudes is spherical with an Alfvénic distance $r = 16\ R_\odot$, which is larger than, but still close to, that in the equatorial region (12 $R_\odot$). Our result for the three-dimensional configuration of the Alfvén surface is qualitatively consistent with the numerical work of Washimi & Sakurai (1993). However, we must note that our result depends on v_A, the exact value of which is subject to the wind temperature and also the wind model. In the case of the wind temperature, because of the dependence $r_A \propto 1/v_A^{1/2}$, we do not expect a significant change in our result by adopting a different but reasonable a_w. In the case of the wind model, however, one may expect a modification to our result if the wind before reaching the Alfvén surface is not entirely thermally driven. Investigation of this aspect is interesting, but neglected here.

3.2 The dipole magnetic field strength

One of the uncertain quantities in the MS87 model is $[4\pi/3\alpha_\odot(\rho_0)_\odot R^4 v_0]$ $(= -\dot{J}_0)$. Physically, $-2J_0$ is the braking rate of a non-magnetic star by matter alone if we define $v_0\rho_0$ as a uniform radial mass loss from the whole spherical stellar surface, equivalent to the case of setting the Alfvén surface exactly at R. In fact, we have defined v_0 as the initial velocity along the field line, rather than radial, so when the Alfvén surface is on the stellar surface, the total radial mass loss flux from the stellar surface would be smaller than $\rho_0 v_0$. As a result, in this special hypothetical case, $-\dot{J}_0$ is very close to the total braking rate.

For the solar case, $-\dot{J}_0$ can be parametrized according to the density of the coronal base, but reliable values come from the observations in interplanetary space, namely the mass flux. Based on the early observations made in the ecliptic plane, the observed slow proton flux and fast proton flux at 1 au are $2.7\times10^8\ cm^{-2}\ s^{-2}$ and $3.9\times10^8\ cm^{-2}\ s^{-1}$ (Withbore 1989). The current *Ulysses* mission disclosed (Feldman et al. 1996) a similar result in the equatorial plane, but a smaller value at high latitudes. The mean number flux at high latitudes is $2.28\times10^8\ cm^{-2}\ s^{-1}$ and that in the ecliptic plane is $2.7\times10^8\ cm^{-2}\ s^{-2}$. Taking the latitude $\theta = 20°$ as the boundary between the lower and higher latitudes as indicated by the observations and assuming a symmetry about the rotational axis, we obtain the total wind mass loss rate from the current Sun, as inferred from the *Ulysses* mission, to be $1.14\times10^{12}\ g\ s^{-1}$ or

$1.81 \times 10^{-14}\ \mathrm{M_\odot\ yr^{-1}}$.

The space measurement of B_r determines the total open flux and therefore provides information about the surface dipole field strength. The exact value of the surface dipole field strength of course depends on having a model which considers the detailed transition from a stellar dipole to a monopole. Comprehensive numerical modelling on this aspect is important before we can fully make use of the observations. However, we have evidence that the field structure for the dead zone can be very close to a dipole, and, hence, using (7) to estimate the total open flux is a good approximation. First, a low plasma β magnetosphere argues for a dipole approximation. This means that the equatorial current sheet and body current in the stellar wind do not significantly alter the dipole field near the star. Secondly, the transition from a dipole to a monopole may occur gradually in a zone from the cusp point at $\bar{r}$ to r_A, so most of the body currents have a smaller effect on the stellar surface field. [We note that the modelling of Pneuman & Kopp (1971), who adopted a plasma $\beta = 1$, indicates a significant departure of the dead zone configuration from a dipole. As β is much smaller than unity in reality, our previous arguments are not altered.] The question is then how we determine the dead zone extent $\bar{r}$.

According to MS87, the dead zone extent, $x = \bar{r}/R$, is determined by the critical balance between the gas pressure in the dead zone and the magnetic pressure in the wind zone, i.e.

$$x^6 \exp(l_d/x) \exp\left[\frac{1}{2}\chi_\odot l_d (x^2 - 1/x)\right] = \frac{\xi_d}{4} \exp(l_d), \tag{21}$$

where the parameters (dimension ignored) are

$$a_d^2 = \Re T/\mu = 2.75 \times 10^{14}, \quad l_d = G\mathrm{M_\odot}/Ra_d^2 = 6.93,$$

$$\chi l_d = 1.10 \times 10^{-4}. \tag{22}$$

[Note that we have corrected the original incorrect value for χl_d in MS87 as noticed by McDermott & Taam (1989).] The right-hand side of (21) is a factor of 4 smaller than that in the MS87 model because we have considered a strict dipole field for the dead zone for consistency. We also note that the dipole field on the Sun is probably the net effect of much smaller scale magnetic fields (cf. Day 1998), and therefore ξ for the dipole field may be difficult to quantify. Different values of ξ for the dead zone and the wind zone would normally lead to a plasma pressure gradient along the stellar surface for a potential magnetic field. However, when the plasma β is small, as we might expect, the influence of the pressure imbalance on the assumed dipole field is not significant. For these reasons, we assume

$$\xi_d = b f(\theta) \xi_w. \tag{23}$$

The parameter b is thus the ratio of ξ_d to ξ_w at the pole. Lack of efficient thermal conduction across the magnetic field lines and lack of adiabatic expansion generally lead to a hotter gas in the dead zone. This suggests $b < 1$. The lower limit for b corresponds to $\bar{r} = \mathrm{R_\odot}$, which would be the case if the plasma $\beta = 1$ on the Solar surface. This is an unphysical situation since the Alfvén surface would then be roughly on the stellar surface, which is inconsistent with observations. Relations (21) and (23) lead to

$$x^6 \exp(l_d/x) \exp\left[\frac{1}{2}\chi_\odot l_d (x^2 - 1/x)\right] = \frac{b f(\theta_0)}{4} \xi_w \exp(l_d), \tag{24}$$

where $f(\theta)$ takes the value of $f(\theta_0)$ because (21) is achieved in the last closed field line. By substituting (12) in (24), we have

Table 1. The dead zone extent $x = \bar{r}/\mathrm{R_\odot}$ and the polar dipolar field strength B_0 for various ratios $b = \xi_d/\xi_w f(\theta_0)$, based on the standard parameters. The dipole field line equation, $\sin^2\theta_0 = 1/x$, relates the colatitude θ_0 to the dead zone extent.

b	1.0	0.5	0.2	0.1	0.05	0.042
x	5.98	4.96	3.79	3.00	2.21	1.00
B_0(G)	16.9	14.0	10.7	8.46	6.23	2.82

Table 2. As Table 1, with the difference that a hotter dead zone with $T = 4 \times 10^6$ K is adopted here.

b	1.0	0.5	0.2	0.1	0.05	0.042
x	2.99	2.48	1.90	1.51	1.11	1.00
B_0(G)	8.43	6.99	5.36	4.26	3.10	2.82

$$x^5 \exp(l_d/x)\left[\frac{1}{2}\chi_\odot l_d (x^2 - 1/x)\right] = \frac{b}{4}\left(\frac{v_A}{a_w}\right)\left(\frac{v_0}{a_w}\right)\left(\frac{r_A}{\mathrm{R_\odot}}\right)^2 \exp(l_d). \tag{25}$$

The dipole polar magnetic field strength can be calculated from the definition of ξ_w:

$$B_0^2 = 8\pi\rho_0 a_w^2 \xi_w = 8\pi\rho_0 v_0 \frac{a_w}{(v_0/a_w)} \xi_w, \tag{26}$$

which, with (13) and (14), can be expressed as

$$B_0 = 8\pi\eta \frac{a_w f(\theta_0)}{(v_0/a_w)} \xi_w = 8\pi\eta a_w \left(\frac{v_A}{a_w}\right)\left(\frac{r_A}{\mathrm{R_\odot}}\right)^2 \left(\frac{\bar{r}}{\mathrm{R_\odot}}\right). \tag{27}$$

The corresponding numerical value is

$$B_0 = 2.82\ x. \tag{28}$$

The dead zone extent x is calculated from (25), and then the model dipole field strength is calculated simply by (28). In Table 1, we have calculated several cases for different values of b.

We have also calculated a case with a hotter dead zone, 4×10^6 K, which is twice the standard value and in general exceeds the observed temperature in the active region (see Golub & Pasachoff 1997). The results are shown in Table 2.

Table 2 in general shows a much smaller dead zone when compared with Table 1, as expected. Although it is not physical, we have included the case with $x = 1$, a split monopole case, as the limit. However, we must point out that the dipole braking model becomes less accurate as the dead zone decreases. In the limit, the split monopole should be adopted rather than the dipole model, because the stellar surface is actually a monopole. The difference is a factor of 2: the surface monopole field is about half of B_0 in our model. Physically the difference is caused by the surface current on the stellar surface. If it is a monopole, the observationally inferred monopole field strength from the *Ulysses* data is $B_0 = 1.43$ G, which is very close to half of the value of 2.82 G predicted by the model. Once x becomes significantly large, the transition from dipole to split monopole naturally occurs at the wind region and the dipole field structure on the stellar surface is less affected, and so our model is more accurate. Given a dipole model with normal field strength, the dead zone must be very small, $< 2\ \mathrm{R_\odot}$.

The implication of Tables 1 and 2, or (28), is that the field strength and the dead zone extent are correlated: for a reasonable $x > 2$ the dipole field strength is generlaly much larger when compared with the normal value $\sim$1 G. The physical reason is perhaps that the observed wind mass flux is so large that a large Alfvénic distance could only be achieved by halving a rather large dipole field strength. Once ξ_w is fixed by observed quantities, the dead zone extent depends on the dead zone temperature and also b. We see from our calculations that the dead zone extent can vary greatly. However, that does not affect the relation between x and B_0. It seems that such a property is intrinsic.

Our conclusion is that either the realistic dipole field is indeed much larger than ~1 G, which is the value normally used, or the dead zone must be smaller than twice the solar radius. Since adopting a dipole field for the Sun is only approximately correct, our result may also suggest that the magnetic field for magnetic braking of the Sun at the solar minimum is not primarily from a potential dipole field, but significant open flux may come from the non-potential part arising from interactions between the complicated fields and the hot plasma. It is interesting to note in this connection an earlier simple analysis (Li 1994), which showed that a complicated field on the Sun may be sufficient to provide a braking rate similar to that caused by the mean dipole field. Since the *Ulysses* observations agree well with a split monopole beyond 1 au, we caution that the complicated fields, if indeed responsible for braking, may need to be reconnected well inside 1 au.

3.3 The braking rate at the solar minimum

The solar braking rate is readily calculated if we know the Alfvén surface across the whole solar wind. From the current data, we integrate the angular momentum flux in two separate regions, the lower latitude (0–20°) and higher latitude (20°–90°) regions. At lower latitudes we adopt a constant $r_A = r_{Alow} = (12\,R_\odot + 16\,R_\odot)/2 = 14\ R_\odot$ as the mean Alfvénic distance, and at higher latitudes a constant $r_A = r_{Ahigh} = 16\ R_\odot$. From (1), for the observed wind mass fluxes at both lower latitude $[(\rho v)_{1\ \mathrm{aulow}}]$ and higher latitude $[(\rho v)_{1\ \mathrm{auhigh}}]$, we obtain

$$-\dot{J}_\odot = \{-\dot{J}_{high}\} + \{-\dot{J}_{low}\}$$

$$= 4\pi\alpha(B_A r_A^2)\left[r_{Ahigh}^2\eta_{high}\int_0^{\theta_1}\sin^3\theta d\theta + r_{Alow}^2\eta_{low}\int_{\theta_1}^{\pi/2}\sin^3\theta d\theta\right]$$

$$= 4\pi_{1\ \mathrm{au}}^2 R_\odot^2\left\{\frac{2}{3}(\rho v)_{1\ \mathrm{auhigh}}\left(\frac{r_A}{R_\odot}\right)^2_{high} + \left(\cos\theta_1 - \frac{1}{3}\cos^3\theta_1\right)\right.$$

$$\left.\times\left[(\rho v)_{1\ \mathrm{aulow}}\left(\frac{r_A}{R_\odot}\right)^2_{low} - (\rho v)_{1\ \mathrm{auhigh}}\left(\frac{r_A}{R_\odot}\right)^2_{high}\right]\right\}, \quad (29)$$

where $\theta_1 = 70°$, $\cos\theta_1 = 0.342$, and η_{high} and η_{low} are the constant mass flux per unit magnetic flux tube at high and low latitudes respectively. We obtain from (29) $-\dot{J}_\odot = 2.1\times10^{30}$ dyn cm. This value is close to that inferred from Pizzo et al. (1983), and is about one quarter of the WD result. The braking rate leads to a contemporary spin-down time-scale

$$\tau = -\frac{J_\odot}{\dot{J}_\odot} = 3.6\times10^{10}\ \mathrm{yr}, \quad (30)$$

where we have assumed the standard parameter $k^2 = 0.1$ for the moment of inertia. This time-scale is about four times larger than that derived from the Skumanich relation.

4 CONCLUDING REMARK

The *Ulysses* mission enables us to obtain solar wind data at high latitudes, justifying a dipole–monopole model like MS87 as basically correct, at least for the solar minimum. Moreover, detailed observations have set stringent constraints on a braking model for the Sun. Close model–data comparison reveals a relation between the dead zone extent and the dipolar field strength. To be consistent with the *Ulysses* data, we either choose a smaller dead zone (< 2 $R_\odot$) with a normal dipole field (1–3 G) or a larger dipole field strength (> 5 G) with a larger dead zone (> 2 $R_\odot$), given a pure dipole braking model. We have also questioned whether a dipole field for a braking model is appropriate. Further examination of the relationship between the extent of the dead zone and magnetic field strength with the use of numerical modelling may help us to resolve how accurate a dipole braking model is for our present Sun.

The rate of spin-down of the Sun at its minimum seems to be smaller than that inferred from the Skumanich relation. Using the current solar spin-down time-scale, it seems difficult to evolve the spin of the Sun back to early times with a significant rotation rate even if we use the scaling set by the Skumanich relation. However, we are not certain whether other factors, for example the internal structure of the Sun, lead to overestimation of the braking rate if the Skumanich relation is used. Further investigation of these problems is important, as it relates to the question of whether the early Sun was a fast rotator or slow rotator. Here we only remark that observations in the young α Per cluster indicate that, for the solar-type stars, the distribution of $v \sin i$ is skewed towards a very low value (Stauffer 1991), and this implies that some young stars in α Per should be slow rotators. Thus the earlier Sun could in principle be a slow rotator, like those we see in α Per. In this connection, we cannot simply refute the suggestion that a smaller braking rate for the current Sun is a possible indication that the Sun was initially a slow rotator. Of course we will have a long way to go to reach a definitive answer.

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