Saha, Suvash, Sauret, Emilie, & Gu, YuanTong
(2014)
Magnetic convection heat transfer in an open ended enclosure filled with paramagnetic fluids.
Applied Mechanics and Materials, *553*, pp. 109-114.

https://doi.org/10.4028/www.scientific.net/AMM.553.109

Published in Applied Mechanics and Materials, Vol. 553 (2014) pp. 109--114.

Magnetic Convection Heat Transfer in an Open Ended Enclosure Filled with Paramagnetic Fluids

Suvash C. Saha[a], Emilie Sauret[b] and Y. T. Gu[c]

School of Chemistry, Physics & Mechanical Engineering
Queensland University of Technology, GPO Box 2434, Brisbane QLD 4001, Australia

[a]suvash.saha@qut.edu.au, [b]emilie.sauret@qut.edu.au, [c]yuantong.gu@qut.edu.au

Keywords: Paramagnetic fluid; thermomagnetic convection; microgravity condition; ramp heating; boundary layer.

Abstract. Numerical simulations of thermomagnetic convection of paramagnetic fluids placed in a micro-gravity condition ($g \approx 0$) and under a uniform vertical gradient magnetic field in an open ended square enclosure with ramp heating temperature condition applied on a vertical wall is investigated in this study. In presence of the strong magnetic gradient field thermal convection of the paramagnetic fluid might take place even in a zero-gravity environment as a direct consequence of temperature differences occurring within the fluid. The thermal boundary layer develops adjacent to the hot wall as soon as the ramp temperature condition is applied on it. There are two scenario that can be observed based on the ramp heating time. The steady state of the thermal boundary layer can be reached before the ramp time is finished or vice versa. If the ramp time is larger than the quasi-steady time then the thermal boundary layer is in a quasi-steady mode with convection balancing conduction after the quasi-steady time. Further increase of the heat input simply accelerates the flow to maintain the proper thermal balance. Finally, the boundary layer becomes completely steady state when the ramp time is finished. Effects of magnetic Rayleigh number, Prandtl number and paramagnetic fluid parameter on the flow pattern and heat transfer are presented.

1 Introduction

In presence of a gravitational field natural convection is everywhere in nature. Extensive studies have been conducted related to natural convection heat transfer in the field of oceanography, meteorology, geophysics, astrophysics, energy systems, material science and so on using analytical, experimental and numerical methods. The most popular study is to study the transient flow behaviour and heat transfer in a differentially heated cavity as it has fundamental interest in fluid mechanics and practical applications. The fundamental concept of buoyancy driven flows in differentially heated cavity is the working fluid close to the heated wall undergoes motion as a result of heat being transferred from the wall into the fluid. The fluid adjacent to the heated wall reduces its density and rises. In this way the thermal boundary layer develops and eventually the whole enclosure becomes thermally stratified. The driving force for natural convection is generally the density difference due to temperature difference between two fluid zones.

In a micro-gravity environment, there is no gravitational buoyancy acting on the fluid and natural convection phenomena are not present. However, if the fluid itself is subject to a magnetic field, it will experience a magnetic force, which depends on the magnetic susceptibility. The most important interest of studying this area is to understand the possibility of control of the convection phenomena and heat transfer rates in a terrestrial environment. Braithwaite et al. [1] reported enhancement or cancellation of gravitational convection due to a magnetic field for a solution of gadolinium nitrate in a shallow layer heated from below and cooled from above. Tagawa et al. [2, 3] derived a model equation for magnetic convection using a method similar to the Boussinesq approximation and carried out numerical simulations in a cubic cavity. Bednarz et al. [4, 5] have shown both numerically and experimentally how to enhance or suppress heat transfer by the application of a magnetic field. Very recently Saha [6] studied the effect of MHD and heat generation on natural convection flow in an open ended square cavity under microgravity condition.

In this study, the magnetic convection in an open ended square cavity due to ramp heating on the left vertical wall is considered. The detailed boundary layer development adjacent to the hot vertical wall is presented and the effects of magnetic Rayleigh number, γRa, Prandtl number, Pr, and paramagnetic fluid parameter, m, on the flow pattern and isotherms are discussed.

2 Mathematical Formulation

Under consideration is the transient behaviour of two-dimensional magnetic convection flow resulting from a motionless, isothermal ($\theta = \theta_0$), viscous incompressible, Newtonian fluid in the absence of gravity but in the presence of a magnetic field, which is shown in Fig. 1. The length of the square cavity is H and the righthand side boundary is open. The left vertical wall is heated with a ramp heating temperature. The top and bottom walls are adiabatic. Except for the right boundary, all the other boundaries are nonslip. It is also assumed that the flow is laminar. The fluid is assumed to be subject to a uniform, vertical gradient magnetic field. We further assume that this fluid has constant properties except that the variation of its density with temperature follows the Boussinesq approximation**.** In the presence of a magnetic field this magnetic buoyancy force acts as the driving force for the resultant magnetic convection. The problem is now similar to the gravitational natural convection flow in an open ended square cavity.

The development of the flow in an enclosure is governed by the following non-dimensional 2D Navier–Stokes and energy equations with the Boussinesq approximation:

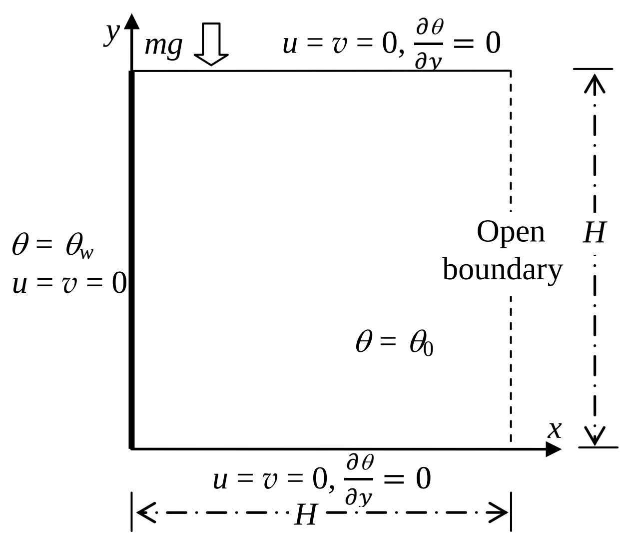


Fig. 1. Schematic of the physical problem and the coordinate systems. θ_w is the ramp wall temperature and T_0 is the temperature of the fluid at $\tau \le 0$.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\Pr}{(\gamma Ra)^{1/2}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial \tau}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\Pr}{(\gamma Ra)^{1/2}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{m\Pr}{2}\theta\,, \tag{3}$$

$$\frac{\partial \theta}{\partial \tau}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{(\gamma Ra)^{1/2}}\left(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}\right), \tag{4}$$

where u and v are the x-direction and y-direction velocity components, τ the time, p the pressure, θ the temperature, and γ the strength of the magnetic force.

Two governing parameters, called Prandtl number and Rayleigh number, are defined respectively as

$$\Pr=\frac{\nu}{\kappa}, \qquad Ra=\frac{g\beta\Delta T H^3}{k\nu}. \tag{5}$$

Where β, ν and κ are the thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid. The strength of the magnetic force and the dimensionless momentum parameter for paramagnetic fluid are defined respectively as

$$\gamma=\frac{\chi_0 b_0^2}{\mu_m g H} \quad \text{and} \quad m=1+\frac{1}{\beta T_0}\,, \tag{6}$$

Here, note that if only micro-gravity magneto-thermal convection conditions are considered ($g \approx 0$), then Ra becomes zero and γ becomes infinity. However, the product of Ra and γ is finite, which will be nominated as the appropriate parameter to characterise the magnetic effect [5],

$$\gamma Ra=\frac{\chi_0 b_0^2 \beta\Delta T H^2}{\mu_m \nu\kappa}. \tag{7}$$

The initial and boundary conditions imposed on the flow field are given by

$$\begin{array}{llll}
\tau<0 & u=v=0,\ \theta=0 & 0\le y\le 1 & 0\le x\le 1\\
\tau\ge 0 & y=0,1 & u=v=0,\ \dfrac{\partial\theta}{\partial y}=0 & 0\le x\le 1\\
 & x=0 & u=v=0,\ \theta_w=\min(1,\tau/\tau_p) & 0\le y\le 1\\
 & x=1 & \dfrac{\partial u}{\partial x}=-\dfrac{\partial v}{\partial y},\ \ \dfrac{\partial\theta}{\partial x}=\dfrac{\partial v}{\partial x}=0 & 0\le y\le 1
\end{array} \tag{8}$$

The average Nusselt number along the left vertical wall is given by the expression

$$Nu_{av}=\int_0^1 \frac{\partial\theta}{\partial x}\,dy \tag{9}$$

3. Numerical scheme and grid and time step dependence tests

Eqns (1) - (4) are solved along with the initial and boundary conditions (8) using finite element based software, COMSOL Multiphysics 4.3b. Three non-uniform grid sizes, 50×50, 100×100 and 150×150 with coarser grids in the core and finer grids concentrated in the proximity of three walls (except open end) were constructed for grid dependence tests. The time series of maximum vertical velocity along y = 0.5 is monitored and compared with different grid sizes and time steps. The maximum error among those results have been found to be 0.2%. This means that either grid system is able to capture the flow development and the heat transfer into this system. In consideration of better output, the grid system of 150×150 is adopted in this study.

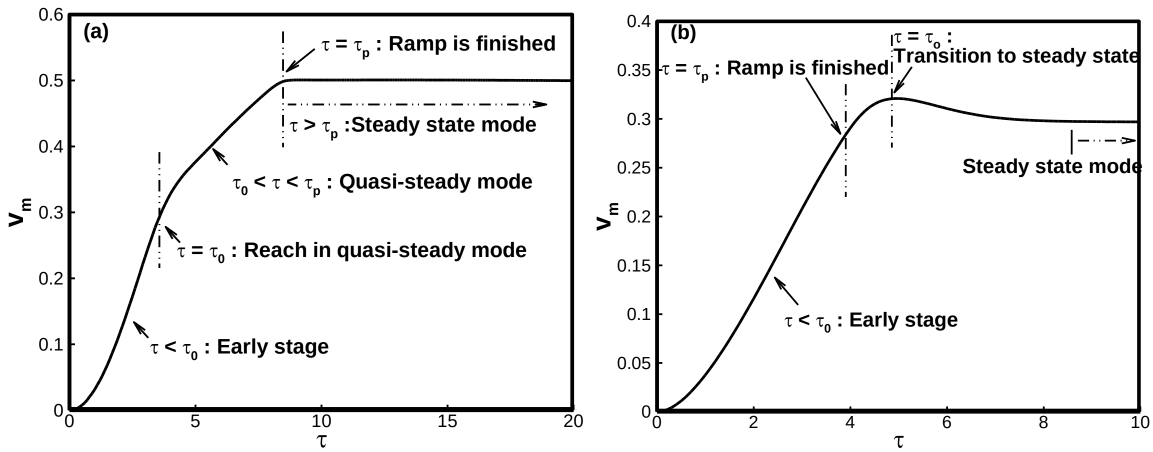


Fig. 2: Time evolution of the maximum velocity of the boundary layer recorded on $y = 0.5$ for (a) $\gamma Ra = 10^7$, Pr = 10, $m = 2$ and $\tau_p = 8$ and (b) $\gamma Ra = 10^4$, Pr = 10, $m = 2$ and $\tau_p = 4$.

4. Results and Discussions

The left vertical wall of the enclosure is subjected to a temperature boundary condition which follows a linear function up until some specified time (τ_p) and then remains constant. The time histories of the calculated maximum vertical velicity (v_m) adjacent to the left wall is plotted in Fig. 2 for two different magnetic Rayleigh numbers and different length of the ramp heating time (τ_p). Since the left wall is hot relative to the ambient fluid, the flow is laminar and stable so long as γRa is not too large. As a consequence, a natural convection boundary layer develops adjacent to the hot wall and continues to grow with increasing time. The development of the thermal boundary layer flow depends on the comparison of the time at which the ramp heating finishes and the time at which the thermal boundary layer completes its growth. If the ramp time is long compared with the steady state time, the layer reaches a quasi-steady stage as it is seen in Fig. 2a. Further increase in the heat input simply accelerates the flow to maintain the proper thermal balance. The overall flow development for this case may be characterized by the following: the early stage, the quasi-steady mode and the steady state mode which can be clearly identified in Fig. 2a. On the other hand, if the ramp is completed before the layer becomes steady the boundary layer grows as though the startup was instantaneous and eventually reaches a steady state, and thus there is no difference between the ramp and instantaneous start up cases (see Fig. 2b).

Figure 3 shows the time evolution of heat transfer as a form of Nusselt number through the left vertical wall for varying Magnetic Rayleigh number (Fig. 3a), Prandtl number (Fig. 3b) and the momentum parameter (Fig. 3c). It is found that for all cases the average Nusselt number increases with time and show several overshoot before it becomes steady state. The heat transfer rate for the variation of Magnetic Rayleigh number is significant. As expected, with increase of Magnetic Rayleigh number, the heat transfer also increases (See Fig. 3a). However, the variation of heat transfer is lower for the Prandtl number variation. Therefore, the heat transfer for different fluid medium does not affect much when the Pr > 1. However, when the dimensionless momentum parameter for paramagnetic fluid varies from 2 to 20 the heat transfer varies significantly.

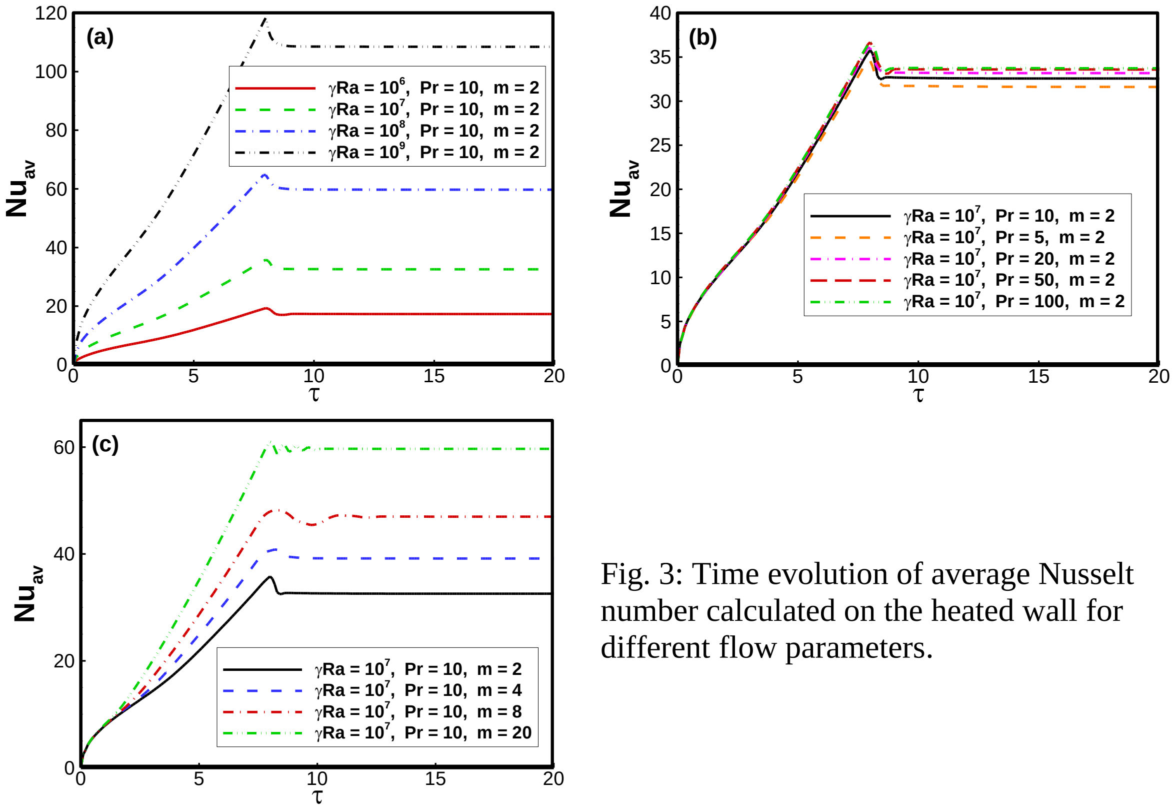
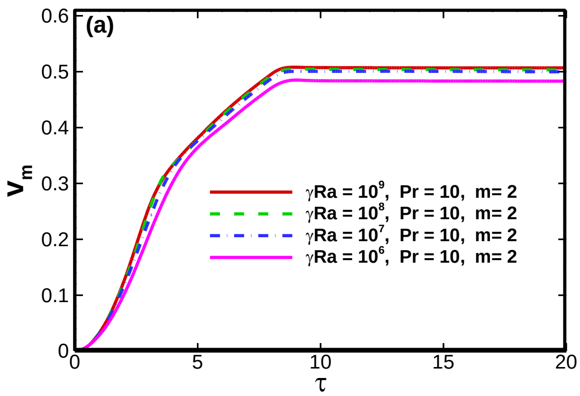


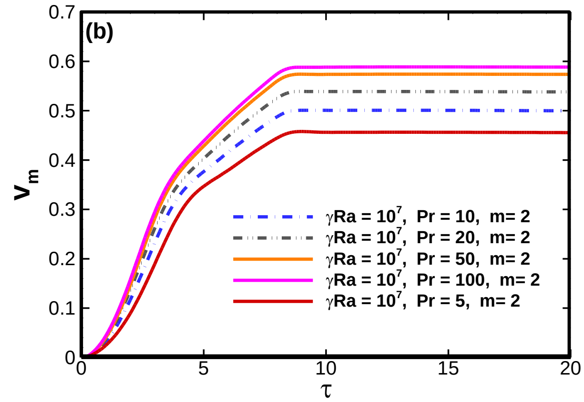
Fig. 3: Time evolution of average Nusselt number calculated on the heated wall for different flow parameters.

Time evolution of maximum vertical velocity calculated along the line perpendicular to the heated wall at mid point ($y = 0.5$) for different γRa, Pr and m are plotted in Fig. 4. It is observed that the vertical velocity increases for increasing values of all three parameters. It is also observed that the velocity strongly depends on the momentum parameter of the paramagnetic fluid compare to other two parameters. The variation of maximum vertical velocity is not significant for the variation of magnetic Rayleigh number.

5. Conclusions

A numerical simulation has been carried out for the thermo-magnetic convection in an open ended square cavity. The enclosure with non-instantaneous heating on left solid wall containing paramagnetic fluid has been placed in a micro-gravity condition ($g \approx 0$). A distinct magnetic convection boundary layer is developed by imposition of ramp temperature condition on the left vertical sidewall due to the effect of the magnetic body force generated on the paramagnetic fluid. The thermal convection of a paramagnetic fluid takes place even in the zero-gravity environments as a direct consequence of temperature differences occurring within the fluid placed within a magnetic field gradient. Effects of magnetic Rayleigh number, Prandtl number, and paramagnetic fluid parameter on the flow field and heat transfer are presented graphically. It is evident that by using a strong magnetic field we can also enhance or reverse the usual gravitational convection with different combinations of the two main body forces, gravitational and magnetic buoyancy forces that act together to drive thermo-magnetic convection of paramagnetic fluids.





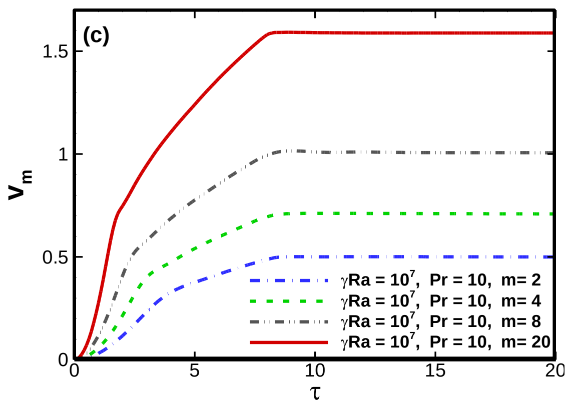


Fig. 4: Time evolution of maximum vertical velocity calculated along the line perpendicular to the heated wall at mid point (y = 0.5)

References

[1] D. Braithwaite, E. Beaugnon, R. Tournier, Magnetically controlled convection in paramagnetic fluid, *Nature*. 354 (1991) 667–673.

[2] Tagawa, T., Shigemitsu, R. and Ozoe, H., Magnetizing force modelled and numerically solved for natural convection of air in a cubic enclosure: effect of the direction of the magnetic field, *Int. J. Heat Mass Transfer*. 45 (2002) 267–277.

[3] T. Tagawa, A. Ujihara, H. Ozoe, Numerical computation for Rayleigh-Benard convection of water in a magnetic field, *Int. J. Heat Mass Transfer*. 46 (2003) 4097–4104.

[4] T. P. Bednarz, E. Fornalik, T. Tagawa, H. Ozoe, J. S. Szmyd, Experimental and numerical analyses of magnetic convection of paramagnetic fluid in a cube heated and cooled from opposing vertical walls, *Int. J. Therm. Sci.* 44 (2005) 933–943.

[5] T. P. Bednarz, W. Lin, J. C. Patterson, C. Lei, S. W. Armfield, Scaling for unsteady thermo-magnetic convection boundary layer of paramagnetic fluids of Pr > 1 in micro-gravity conditions, *Int. J. Heat Fluid Flow*. 30 (2009) 1157–1170.

[6] S. C. Saha, Effect of MHD and heat generation on natural convection flow in an open square cavity under microgravity condition, *Engineering Computations*. 30 (2013) 5-20.