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MAGNETIC OPTICS FOR PROTON RADIOGRAPHY*

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Abstract

High energy protons of 10 to 50 GeV can be used to radiograph dense objects. Because the transmitted beam particles undergo multiple coulomb scattering (MCS) in the object, a magnetic lens system is used to focus the particles exiting each point of the object onto a distant image plane. Without the lens, the MCS would seriously blur the radiographic image. Correlations can be introduced in the illuminating beam to cancel a major part of the chromatic and geometric aberrations in the lens, while providing locations inside the lens where the rays are sorted by MCS angle. This allows the introduction of angle cut apertures to aid material identification. The requirement for a matched multistage lens system with successively smaller angle-cut apertures leads to the use of minus-identity (-I) lenses, in which the angle sorting is in the longitudinal mid plane of the lens, and the exit beam correlations are the same as the input correlations. A single stage -I lens has been successfully tested at Brookhaven with 10-GeV protons and another is being used in dynamic experiments with 0.8-GeV protons at Los Alamos. The resolution achievable at higher energies is briefly surveyed.

1 PROTON RADIOGRAPHY

A proton beam passing through a dense homogeneous object of material (Z,A) is depleted by removal collisions, with an attenuation length $\lambda_T = (n\sigma_T)^{-1}$ dependent on the density n of nuclei, and the total nuclear cross section $\sigma_T \propto A^{2/3}$. The surviving protons undergo millions of small coulomb scatterings that produce a roughly gaussian angular distribution at the exit surface of the object. The half-width, in milliradians, of this multiple coulomb scattering (MCS) exit cone [1] is

$$\phi_c = \frac{14.1}{p\beta}\sqrt{\frac{L}{L_R}}\left[1+\frac{1}{9}log_{10}\left(\frac{L}{L_R}\right)\right], \qquad (1)$$

where p is the beam momentum in GeV/c, L is the path length through the object, and $L_R \propto Z^2/A$ is the radiation length of the material. Appropriate weighted averages for the path length ratios are applicable to real inhomogenous objects. The proton radiographic image is formed by a magnetic lens downstream of the object. The lens refocuses the MCS exit cone from each point of the object back to a corresponding point in the detector plane. A second requirement on the magnetic optics is to provide a location where the rays are sorted by MCS angle, so an aperture can block the larger angles. Points in the image where the aperture will pass the entire exit cone would be unaffected, but other points, having an angular width greater than the aperture acceptance, would be recorded with reduced intensity.

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A series of radiographs made with different angle cut apertures thus opens the possibility of discriminating material type as well as density.

2 THE IDENTITY LENS

The first-order transfer matrix **R** of an imaging lens maps a particle with initial coordinates (x,θ) to a final position $x_f = R_{11}x + R_{12}\theta$. Point-to-point imaging means $R_{12} = R_{34} = 0$, so the final position is independent of the initial angle. This requires at least a quadrupole doublet. One-to-one (inverted) imaging, meaning $R_{11} = R_{33} = -1$, requires at least a triplet, but quadruplets are preferred because they can produce the same beam excursions in both planes.

A quadrupole beamline is reflection symmetric if its second half is the mirror image of the first half, meaning the sequence of drifts and quads is traversed in the reverse order and with the opposite signs for the magnetic fields. The symmetry forces $R_{34} = R_{12}$, and $R_{43} = R_{21}$ so a reflection symmetric lens always focuses simultaneously in both planes. It also forces $R_{33} = R_{22}$, and $R_{44} = R_{11}$, so the 1:1 condition occurs simultaneously in both planes as well.. Russian quadruplets [2,3] of the form +A-B+B-A are an example of reflection symmetry.

If the first half of the beamline is itself reflection symmetric, then the second half is the same as the first, and the lens consists of two identical cells. In this case, if the transfer matrix of one cell is **M**, that of the whole lens is given (in each plane) by the 2x2 matrix identity

$$\mathbf{R} = \mathbf{M}^2 = -\mathbf{I}\,det(\mathbf{M}) + \tau\mathbf{M}, \qquad (2)$$

where $\tau \equiv Tr(\mathbf{M}) \equiv M_{11} + M_{22}$ is the trace of $\mathbf{M}$. The determinant $det\,\mathbf{M} = 1$ for beamline matrices. When the trace $\tau = 0$, we have an identity lens with **R=-I**.

An identity lens preserves beam correlations because the output phase space is an inverted copy of the input. Therefore multistage systems can be built by simply stacking identical identity lenses in series. Since **-I** commutes with everything, including a drift matrix, an identity lens actually images an entire 3D volume surrounding the object.

In the simplest case, the cell **M** is a doublet, defined by only four parameters: the outside drifts f, the inside drift s between the quads, and the length and strength $(L, \pm G)$ of the two quads. Any one of these can be adjusted to set $\tau = 0$.

3 CHROMATIC CANCELLATION

The chromatic aberration coefficients define the final position errors for particles off the design momentum. The matrix elements in general are functions of the fractional momentum deviation $\Delta \equiv \delta p/p$, so the final position of an off-momentum ray with initial coordinates (x,θ) is

$$x_f = R_{11}(\Delta)x + R_{12}(\Delta)\theta. \qquad (3)$$

Suppose now that the initial beam is strongly correlated, as if it came from a (virtual) point source a distance L upstream of the entry plane of the lens. Such a beam would lie along the line $\theta = wx$ in phase space, where $w = L^{-1}$ is the beam correlation coefficient. Real particles exit the object at angle $\theta = wx + \varphi$, where φ represents all deviations from the correlation line due to both MCS in the object and non-zero emittance in the illuminating beam. To first order in Δ, the final position of this particle is (primes indicate momentum derivatives):

$$x_f = R_{11}x + R_{12}\theta + (R_{11}' + wR_{12}')x\Delta + R_{12}'\varphi\Delta + \cdots \quad (4)$$

If we choose the correlation coefficient w so that

$$R_{11}' + wR_{12}' = 0, \quad \text{or} \quad w = -R_{11}'/R_{12}' \quad (5)$$

all position dependent chromatic aberrations vanish. The contribution of the $R_{11}' \equiv T_{116}$ (TRANSPORT notation) term is canceled by the principal coefficient $C_x \equiv R_{12}' \equiv T_{126}$. With $R_{12} = 0$, and $R_{11} = -1$, the final position is

$$x_f = -x + C_x\varphi\Delta. \quad (6)$$

The remaining chromatic aberration depends only on the deviation angle φ. We call a beam with this correlation chromatically matched. K. Brown [4] has pointed out that $R_{11}'=T_{116}$ vanishes if the diagonal elements of **M** are zero. This requires 3 or 4 quads per cell, but has the advantage that parallel illumination with w=0 is the matched beam.

4 ANGLE SORTING

The angle cut aperture must be placed where the rays are completely sorted by MCS angle. This always occurs in the midplane of a chromatically matched identity lens. If **M** is the transport matrix of the first cell, Eq.(2) gives the matrix elements of the whole lens for off-momentum particles:

$$R_{jk}(\Delta) = -\delta_{jk} + \tau(\Delta)M_{jk}(\Delta). \quad (7)$$

The momentum derivatives of these at the origin $\Delta = 0$ are

$$R_{jk}' \equiv \left.\frac{dR_{jk}}{d\Delta}\right|_{\Delta=0} = M_{jk}\frac{d\tau}{d\Delta} + \tau\left.\frac{dM_{jk}}{d\Delta}\right|_{\Delta=0} = \tau' M_{jk}, \quad (8)$$

because the lens is focused with $\tau(0) = 0$. Therefore

$$R_{11}' + wR_{12}' = \tau'(M_{11} + wM_{12}) = 0, \quad (9)$$

meaning that after the first cell, the position of the rays depends only on φ, not on x. This is angle sorting:

$$x_{mid} = (M_{11} + wM_{12})x + M_{12}\varphi = M_{12}\varphi. \quad (10)$$

5 THE ILLUMINATING BEAM

The required phase space correlations are provided by a matching section just upstream of the object. This section must also expand the beam to fully illuminate the field-of-view. The expansion is provided by either a diffuser, or a quadrupole lens, which increases the angular size of the beam as it enters a long drift. Other, larger quadrupoles then adjust the beam parameters to best fit the required correlation line $\theta = wx$ in each plane. In the usual beam moments notation $\langle x\theta \rangle \equiv -\alpha\varepsilon$ and $\langle x^2 \rangle \equiv \beta\varepsilon$, where ε is the beam emittance. The final β must be set to produce the required beam size. Then α is adjusted to minimize the mean square deviation from the correlation line. The minimum is $\langle \phi^2 \rangle = \beta/\varepsilon$, obtained when $\alpha = -w\beta$. The residual incident emittance angles are added in quadrature to the multiple coulomb scattering angles produced by the object.

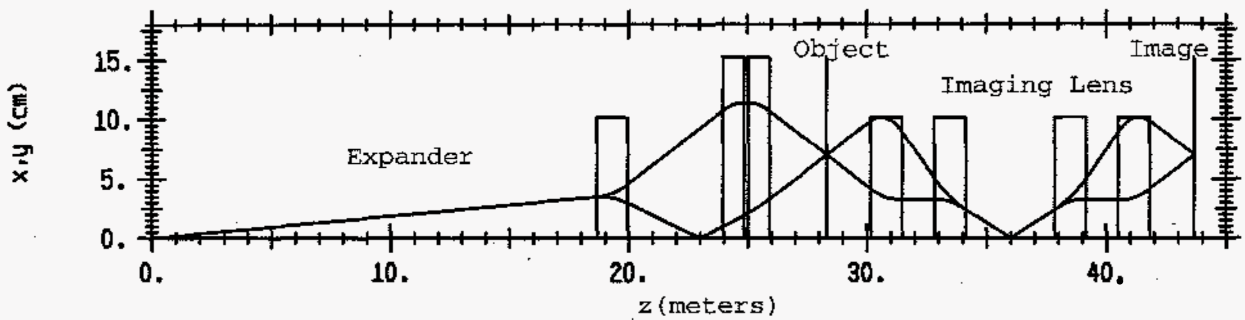

Figure 1: Beam Expander and Identity Lens set up in the A1 beam line at the AGS.

6 THE BROOKHAVEN EXPERIMENT

The first test of such a system was carried out in Experiment 920 at Brookhaven National Laboratory in July 1996. Available 8Q48 quadrupoles running near their maximum field of 12.5 T/m were set up as an identity lens. Another 8Q48 and a pair of 12Q30 quads were used for the matching lens to properly illuminate the object out to 7 cm from the axis. Fig.1 shows the layout, with boxes indicating the location and size of the quadrupoles, and vertical lines marking the object and image planes. A 3 mm tantalum plate was used to spread the 10 GeV test beam into an 18.6m drift. Two sigma beam envelopes in x and y from this diffuser are shown. Fig.2 shows a log-ratio attenuation image of a tungsten step wedge taken with this lens. The image is 15 cm square, bounded by the intersection of two elongated ellipses, which are the images of the limiting beam pipe from different locations in the x and y planes.

7 FIELD OF VIEW

The field-of-view (FOV) is an important measure of the performance of a magnetic lens. Here we define the FOV as the maximum initial x or y coordinate that a particle can have and still travel all the way through the lens without striking a limiting aperture. This clearly depends on the initial angle $\theta = wx + \varphi$, making the FOV a function of the

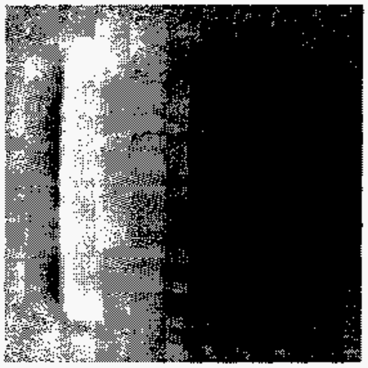

Figure 2: Experimental Image of a Tungsten Step Wedge.

deviation angle φ. Fig 3 shows matched rays scattered at $\varphi=0$, $\pm 5mR$ and $\pm 10mR$ from points spaced at 2 cm in the object plane of the Brookhaven test lens. Note the angle

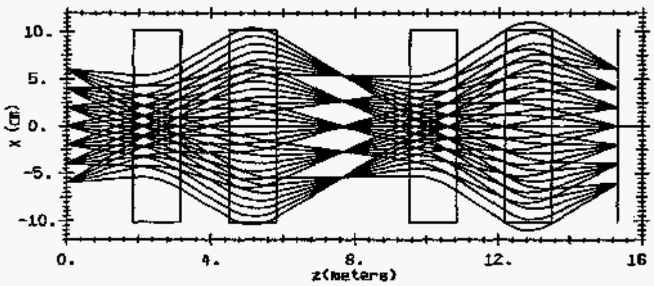

Figure 3: X-plane Ray Traces in the BNL A1 Identity Lens

sorting at the midplane. The x-position at z of a ray with initial coordinates (x, φ) is given [5] by the "sinelike" and "cosinelike" rays $S(z) \equiv M_{12}(z)$, and $C(z) \equiv M_{11}(z)$ characterizing the map out to z. The ray that just reaches the limiting aperture $a(z)$ at z is

$$x_{ap}(z) = (a(z) - S(z)\varphi)/(C(z) + wS(z)) \qquad (11)$$

The FOV is the largest possible x for each φ. This is the smallest $x_{ap}(z)$ for any z, and may be obtained by a numerical search. Fig. 4 shows this function for the BNL

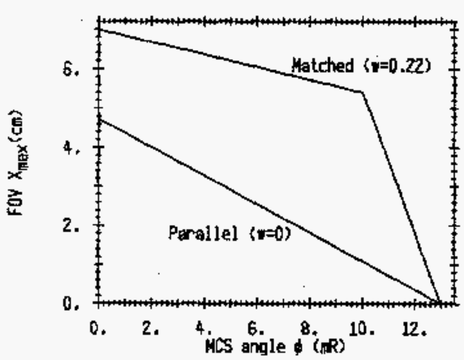

Figure 4: Field of View of BNL Lens (cm)

lens with and without the matching correlation. The endpoint of both curves at 13 mR is the maximum angle the lens will transmit from a point on axis. Without correlations, the FOV at $\varphi=10$ milliradians is only 1.1 cm. With them, it increases to 5.4 cm, about half of the actual quad aperture.

8 HIGH ENERGY LENS SURVEY

To assess the ultimate performance of magnetic lenses for proton radiography, we note that the dominant lens error is chromatic aberration of the form $C_x\varphi\Delta$, and both ϕ and Δ scale inversely with the beam momentum. Better resolution is therefore expected with higher beam energy. The chromatic coefficients (C_x, C_y) scale with the overall length of the imaging lens. Shortening the lens requires higher field gradients, suggesting the use of superconducting quadrupoles. The FOV scales with the aperture or pole tip radius a of the quadrupoles. For an estimate of lens performance at high energy, assume a pole tip field of B=5 Tesla, a focal standoff drift of f=2m, and no space between the quads in a cell. For aperture a, the gradient will be $G=B/a$ T/m. We adjust quad length L to achieve focus, and plot in Fig. 5 the chromatic error $C_x\varphi\Delta$ vs. a (roughly the FOV diameter) for a series of energies: 10, 20, 30, 50, and 100 GeV. The chromatic error scale is for a reference object which produces $\varphi=10$mR and $\Delta=0.01$ RMS at 10 GeV. The error rises with aperture because G decreases, requiring longer quads, but is below 0.1 mm with a 15cm FOV for this object at 50 GeV.

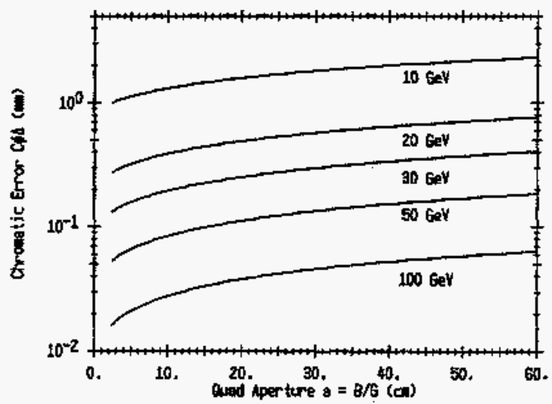

Figure 5. Scaled Chromatic Blur vs. Quad Aperture

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