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# MAGNETIC PROPERTIES OF METAMATERIAL COMPOSED OF CLOSED RINGS

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**Abstract**—The magnetic properties of the metamaterial composed of both periodic and aperiodic closed rings are studied. Experimental results validate that metamaterials with $0 < \mu < 1$ can be non-dispersive in a wide frequency range. The magnetic properties are insensitive to disorders of the closed rings, e.g., the position disorders and the size disorders. The related causality issue is also discussed.

## 1. INTRODUCTION

Natural magnetic materials such as ferrite usually show magnetic activities at frequencies fewer than gigahertz. These magnetic materials have been widely used in applications like sensors, power conversion, magnetic recording, screening, data storage, and quantum devices, etc. In most of these natural materials, e.g., permalloy, steel, and ferrite, etc., the effective permeability is much higher than 1. This means the density of the lines of force in these magnetic materials is much higher than that in air for the same applied magnetic force. In recent years, metamaterials were proposed pushing the permeability from positive to negative [1–7]. The operating frequencies of the magnetic metamaterials have also been extended from the microwave frequencies to optical frequencies [8]. Because

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metamaterials with negative permeability are usually realized with subwavelength magnetic resonators, they are highly dispersive and negative permeability are therefore only limited in a small frequency band [9]. Compared with metamaterials with negative effective permeability, i.e., $\mu < 0$, metamaterials with $0 < \mu < 1$ have some specific applications, such as invisibility cloak [10–14], but are less investigated. In fact, $0 < \mu < 1$ can be achieved by non-dispersive metamaterials over a wide frequency band. One of the example is the closed rings, which have been theoretically studied by Pendry and his co-workers [15]. Some researchers have also studied the properties of the closed rings, but focusing on their resonant regions [16–18], in particular their electrical resonant regions, rather than the non-resonant regions where they exhibit specific magnetic properties, i.e., $0 < \mu < 1$.

In this paper, the magnetic properties of the closed rings are experimentally studied. Both periodic and aperiodic closed rings are realized and measured. Their effective permeabilities are retrieved from a waveguide based retrieval method [19]. The experimental results validate that metamaterial with $0 < \mu < 1$ can be non-dispersive in a wide frequency range. In addition, we show that compared with resonator based metamaterial, closed rings are less sensitive to disorders, e.g., the position disorders and the size disorders.

## 2. METAMATERIAL WITH PERIODIC CLOSED RINGS

The first metamaterial sample is realized with a periodic arrangement of closed square rings as shown in Fig. 1. The square rings are printed on an FR4 substrate of thickness $d = 1\,\text{mm}$ in the $y$ direction, and of relative permittivity $\epsilon_s = 4$ (measured at 2 GHz). The dimensions of

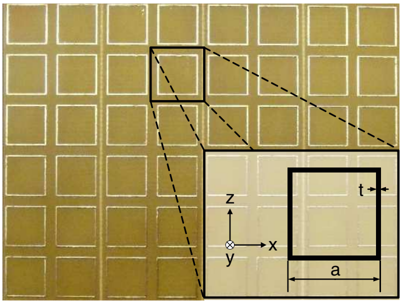


**Figure 1.** A slide of the metamaterial sample composed of periodic square closed rings.

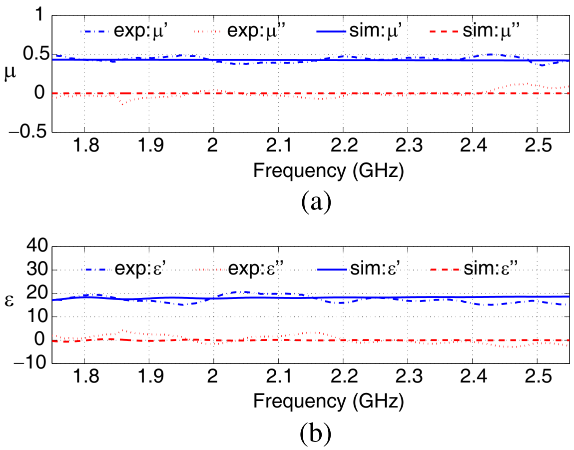


**Figure 2.** Retrieved $\mu_y$ and $\epsilon_x$ or $\epsilon_z$ of the metamaterial composed of square closed rings.

the sample in Fig. 1 are: $a = 7\,\mathrm{mm}$, $t = 0.2\,\mathrm{mm}$. The metamaterial has a periodicity of $p_x = 9\,\mathrm{mm}$ along the $x$ direction, of $p_y = 1\,\mathrm{mm}$ along the $y$ direction, and of $p_z = 9\,\mathrm{mm}$ along the $z$ direction. The effective parameters of the metamaterial sample are retrieved by a waveguide based retrieval method [19]. The experimental setup is similar to that shown in [19]. A WR-430 ($a = 109.22\,\mathrm{mm}$, $b = 54.61\,\mathrm{mm}$) waveguide operating in the $\mathbf{TE}_{10}$ mode with the range of $1.72 \sim 2.61\,\mathrm{GHz}$ is used in the experiment. The scattering parameters are recorded by an Agilent 8722ES network analyzer. Two independent measurements with different orientations of sample are measured in the waveguide [19]. Based on the two sets of the measured scattering parameters, we can inversely calculate the effective parameters of the sample using the waveguide based retrieval algorithm [19]. The effective permeability along the $y$ direction is shown in Fig. 2(a). In the figure captions, $\mu'$ and $\epsilon'$ are the real parts of the permeability and permittivity, respectively; $\mu''$ and $\epsilon''$ are the imaginary parts of the permeability and permittivity, respectively; exp and sim represent the experimental and simulation results, respectively. We see that the permeability of the sample is around 0.45 over the whole operating frequency band of the waveguide. The retrieved effective permeability from the simulation results is shown for comparison. We see they agree well with each other. As the measurement in waveguide cannot get the data at the frequencies below its cut-off frequency, we also simulated the structure under normal incident wave, the results show that the permeability is around 0.45 and is almost constant over a wide frequency range starting from 0 GHz.

The magnetic activity of the closed rings has been theoretically predicted in Ref. [15]. It was shown that the relative permeability of the closed rings is $\mu_y = 1 - F$, where $F$ is the fraction volume of the periodic unit cell in the $xz$ plane occupied by the interior of the closed ring [15]. From the parameters shown in Fig. 1, we can get $F = \frac{(a-2t)^2}{p_x p_z} = 0.54$ for the square closed ring. Therefore, the effective permeability is $\mu_y = 1-F = 0.46$, in good agreement with the retrieved results from the experiment measurement.

From the results, we see that the metamaterial composed of closed rings is almost non-dispersive with $0 < \mu < 1$ over a wide frequency range. At first glance, we may think wave will exceed the light speed as the relative permeability is smaller than 1. However, the retrieved effective permittivity of the closed rings is around 18, as shown in Fig. 2(b). The refractive index, $n_{x,z} = \sqrt{\mu_y \epsilon_{z,x}} = 2.87$, is therefore larger than 1, indicating that the wave travelling inside of the closed rings will not exceed the speed of the light. These results validate the theoretical prediction on the electrical properties of the closed rings [15].

## 3. METAMATERIAL WITH RANDOMIZED CLOSED RINGS

In order to see how sensitive the magnetic properties of the closed rings to the disorders, we realized five samples with randomized distributed circular closed rings for measurements. The slides of the first four samples are shown in Fig. 3. The closed rings, with the inner

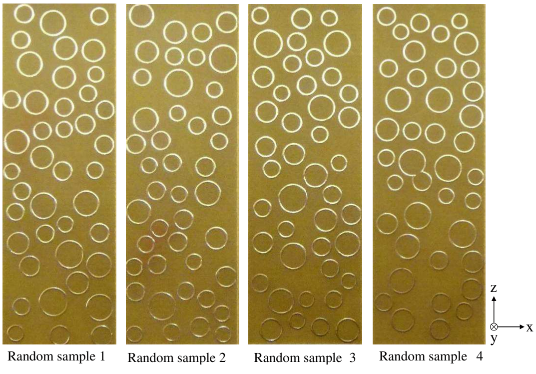


**Figure 3.** Slides of four randomized closed rings with different fraction volume.

radius randomly ranging from 1 mm to 2 mm, are randomly positioned on the slides. The width of the metallic strips for all the closed rings are 0.3 mm. The slides are periodically arranged along the $y$ direction, forming a solid state metamaterial. In these four samples, the total area enclosed by the rings in the $xz$ plane are $306.120\,\text{mm}^2$, $295.263\,\text{mm}^2$, $310.697\,\text{mm}^2$, and $315.271\,\text{mm}^2$, respectively. The area of the substrates for all the four samples is $18\,\text{mm} \times 54\,\text{mm}$. Therefore the fraction volume of the substrate in the $xz$ plane occupied by the interior of the closed rings for the four samples are $F_1 = 0.3149$, $F_2 = 0.3038$, $F_3 = 0.3196$, and $F_4 = 0.3244$, respectively. The effective permeabilities of the four random samples predicted from theory are: $\mu_y^{(1)} = 0.6851$, $\mu_y^{(2)} = 0.6962$, $\mu_y^{(3)} = 0.6804$, and $\mu_y^{(4)} = 0.6756$, respectively. The retrieved effective permeability are shown in Figs. 4(a)–(d), where the theoretical value are also indicated in the left side of each sub-figure. We see that the experimental results are in good agreement with the theoretical predictions. The imaginary part of the permeability is close to zero, indicating the loss can be almost neglected.

In addition, we realized the fifth random sample (Random Sample 5), which is composed of the four slides shown in Fig. 3 by randomly arranging these slides along the $y$ direction with the same ratio. Compared with those of Random Sample 1 to 4, the closed rings of Random Sample 5 are not only distributed randomly along the $x$ and $z$ directions, but also distributed randomly along the $y$ direction. Theoretically, the averaged fraction volume of the metamaterial sample in the $xz$ plane is $F_5 = (F_1 + F_2 + F_3 + F_4)/4 = 0.3157$, yielding an effective permeability of $\mu_y^{(5)} = 0.6843$. The measured retrieval permeability is shown in Fig. 4(e). We see the curve is around 0.6843, in agreement with the predicted theoretical value.

From the above five randomized metamaterial samples, we see that the position disorders and the size disorders have little effect on the magnetic permeability of the closed rings. This is because the size of the closed rings are around $\lambda/50$, which is much smaller than the operating wavelength. It is difficult for the operating wavelength to see the disorders of the geometric details in such a small scale. As the closed rings work in a non-resonant region, the local field is very uniform, indicating the scattering loss is not as strong as that of split-ring resonators [20–22] and plasmonic particles [23]. The magnetic properties of the closed rings are therefore much less sensitive to disorders than those of split-ring resonators.

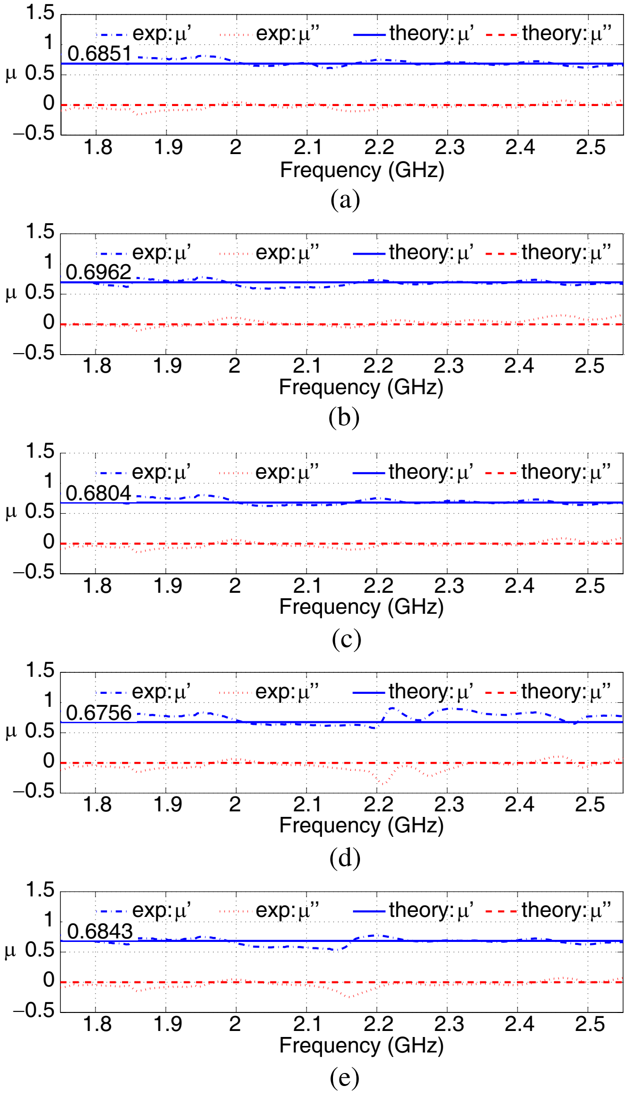


**Figure 4.** Retrieved $\mu_y$ of the five randomized metamaterials composed of circular closed rings.

## 4. CAUSALITY

It should be noted that all passive medium should not violate the causality requirement, i.e., the Kramers and Kronig relations [24–26]. If loss may be neglected, for $\epsilon(\omega)$, the causality requires the permittivity satisfy the following constraint: $\frac{d\epsilon(\omega)}{d\omega} > 0$, and $\frac{d\epsilon(\omega)}{d\omega} > 2(1-\epsilon(\omega))$. Summing these two inequalities, we obtain the following constraint: $\frac{d[\omega\epsilon(\omega)]}{d\omega} > 1$. If we apply the same causality requirements to $\mu(\omega)$, we find the permeability of the closed ring does not meet the causality requirements as it is less than unity and is almost non-dispersive for a wide spectrum starting from 0 GHz. In Ref. [26], Landau et al. pointed out that the causality formula for $\mu(\omega)$ should be slightly different from those of $\epsilon(\omega)$ because $\mu(\omega)$ can cease to be physically meaningful at relatively low frequencies, the Kramers and Kronig formula must extend only to frequencies where $\mu(\omega)$ is still meaningful and no longer variable. The constraint requirement therefore can be written as $\frac{d\mu(\omega)}{d\omega} > 0$, and $\frac{d\mu(\omega)}{d\omega} > 2(\mu(\omega_1) - \mu(\omega))$, where $(0, \omega_1)$ is the frequency band of the metamaterial that $\mu(\omega)$ is physically meaningful, here is the long wavelength region of the metamaterial composed of closed rings. Summing the above two formula, we can get $\frac{d[\omega\mu(\omega)]}{d\omega} > \mu(\omega_1)$. As stated in Ref. [26], $\mu(\omega_1)$ can be less than unity in the low frequencies. This point was also verified by a D.C. magnetic metamaterial [27] which shows an effective $\mu(0)$ smaller than unity in the static magnetic field, i.e., $\omega = 0$. The effective permeability of the metamaterial composed of the closed rings is therefore physically meaningful.

If $\omega_1 \to \infty$, i.e., the long wavelength region covers a wide frequency band from 0 to $\infty$, the causality requires that $\mu(\omega)$ should tend to unity in the limit $\omega \to \infty$ because the inertia of electrons prevent the materials from being polarized in the high frequencies. At very high frequencies, we need to take in mind that the length scale of the closed rings should also be scaled down correspondingly in order for the closed rings to work in higher frequencies. When the size of the closed rings is much smaller than the visible wavelength, the inertia of the electrons cannot be neglected. In this case, the domination of the two kinds of inductance, the geometric inductance, $L_g$, which scales linearly with the ring size, and the inertial inductance, $L_i$, which comes from the finite mass of the electrons in the metal and scales in inverse proportion to the ring size, is switched [28–30]. A more suitable expression for the closed rings is $\mu = 1 - F\frac{L_g}{L_g+L_i}\frac{\omega^2}{\omega^2+i\Gamma}$, which can be derived from Ref. [28] by setting an infinite large capacitance. Because $L_i$ becomes increasingly dominant over $L_g$ when the ring size is decreased, $\mu(\omega)$ tend to unity in the limit $\omega \to \infty$, without violating the causality constraint. This also indicates that in order to get a good magnetic activity, i.e., $\mu \ll 1$, it is highly recommended to use closed rings with large size to reduce the influence of $L_i$ as long as the long wavelength region is met.

## 5. CONCLUSION

In conclusion, the magnetic properties of the metamaterial composed of both periodic and aperiodic closed rings are studied. Experimental results show that metamaterials with $0 < \mu < 1$, can be non-dispersive in a wide frequency range. We show that metamaterials realized with closed rings are insensitive to disorders, such as the position disorders and the size disorders. As we know, metamaterial with $0 < \mu < 1$ is very important in achieving invisibility cloak, the experimental work in this paper is therefore significantly meaningful for cloak applications.

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