

Magnetic spots on hot massive stars

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ABSTRACT

Context. Hot luminous stars show a variety of phenomena in their photospheres and winds which still lack clear physical explanation. Among these phenomena are photospheric turbulence, line profile variability (LPV), non-thermal emission, non-radial pulsations, discrete absorption components (DACs) and wind clumping. It has been argued that a convection zone close to the stellar surface could be responsible for some of these phenomena. This convective zone is caused by a peak in the opacity associated with iron-group elements and is referred to as the “iron convection zone” (FeCZ).

Aims. Assuming dynamo action producing magnetic fields at equipartition in the FeCZ, we investigate the occurrence of subsurface magnetism in OB stars. Then we study the surface emergence of these magnetic fields and discuss possible observational signatures of magnetic spots.

Methods. Simple estimates are made using the subsurface properties of massive stars, as calculated in 1D stellar evolution models.

Results. We find that magnetic fields of sufficient amplitude to affect the wind could emerge at the surface via magnetic buoyancy. While at this stage it is difficult to predict the geometry of these features, we show that magnetic spots of size comparable to the local pressure scale height can manifest themselves as hot, bright spots.

Conclusions. Localized magnetic fields could be widespread in those early type stars that have subsurface convection. This type of surface magnetism could be responsible for photometric variability and play a role in X-ray emission and wind clumping.

Key words. dynamo – stars: activity – stars: massive – stars: atmospheres – stars: magnetic field – convection

1. Introduction

There is growing direct and indirect evidence that magnetic fields are present at the surface of massive stars. Direct measurements have been obtained using the Zeeman effect for O and early B stars (see e.g., Petit 2010, for a review). As the number of known magnetic massive stars is small, these observations have not yet been able to answer the question of whether magnetism in these stars is fundamentally different from that in the intermediate-mass stars, i.e. late B and A stars (the so-called Ap and Bp stars; see e.g., Mathys (2009) for a review, or Mestel & Landstreet (2005) for a broader review of stellar magnetism), or just a continuation of the same phenomenon. Ap and Bp stars – accounting for some percentage (~5–10%) of main-sequence A and late B stars – display a magnetic field which is static, in the rotating frame (these stars appear to rotate as solid bodies), large-scale, and with strengths of 200 G to 100 kG. These fields are thought to be “fossil” remnants left over from formation: magnetohydrostatic equilibria which survive for the entire stellar lifetime (Cowling 1945; Braithwaite & Nordlund 2006; Duez et al. 2010). However, since direct detection methods are biased toward the discovery of large scale magnetic fields, small-scale fields generated by a contemporary dynamo may be present in the rest of the population. Whilst in A stars even small-scale fields would have to be rather weak to escape detection (Petit et al. 2010), the detection limits on small-scale fields in more massive stars are much higher (e.g. Schnerr et al. 2008).

In addition there is a variety of indirect evidence for such magnetic fields at the surface of OB stars, coming from observations of surface and wind phenomena. For example,

discrete absorption components (DACs) and line profile variability (LPV) have been often associated with the presence of surface magnetic fields, even if non radial pulsation (NRP) is also a possibility (e.g., Cranmer & Owocki 1996; Fullerton et al. 1996; Kaper et al. 1997; Henrichs et al. 2005). Interestingly these phenomena seem to be ubiquitous among early type stars (e.g., Howarth & Prinja 1989; Kaper et al. 1996; Fullerton et al. 1996; Prinja et al. 2002). Therefore one may wonder if magnetic fields in massive stars are more widespread than previously thought. Moreover, theoretical considerations point to a difference between massive and intermediate-mass stars. In the latter, a dynamo would have to operate either in the radiative envelope, powered by differential rotation (Spruit 2002) which itself would have to be sustained by some physical process; or in the convective core, in which case the resulting field would have to make its way upwards through the radiative envelope (Moss 1989; MacGregor & Cassinelli 2003) which seems to take place only on an embarrassingly long timescale. In contrast to this, recently Cantiello et al. (2009, hereafter C09) studied the properties of convective regions in the envelopes of hot, massive stars, caused by an opacity peak associated with iron (Iglesias et al. 1992). These convective zones (hereafter FeCZ), which are not present in intermediate-mass stars, could host a dynamo, producing magnetic fields visible at the surface.

The reality and importance of subsurface convective layers in early-type stars has been recently stressed by observations of stellar surface properties. The idea that the observed microturbulence is caused by the presence of the FeCZ (C09) is supported by recent observations (Fraser et al. 2010; Przybilla, priv. comm.). In the Sun stochastic convective motions excite

Table 1. Properties of the outer layers in a 20 M_{\odot} and 60 M_{\odot} model main-sequence stars of solar metallicity at ages 6.41×10^6 yr and 2.37×10^6 yr respectively, corresponding to about 90% and 80% of the main-sequence lifetime.

M_{ini} M_{\odot}	R_{\star} R_{\odot}	R_{FeCZ}^a R_{\odot}	ΔR_{FeCZ}^b R_{\odot}	H_p^c R_{\odot}	v_c^d km s^{-1}	ρ^e g cm^{-3}	ΔM_{FeCZ}^f M_{\odot}	ΔM_{top}^g M_{\odot}	τ_{turn}^h days	τ_{conv}^i days	\dot{M}^j $M_{\odot} \text{ yr}^{-1}$
20	10.46	10.20	0.28	0.08–0.24	10.74	7.4×10^{-8}	3.6×10^{-6}	5.8×10^{-7}	0.53	18 250	7.3×10^{-8}
60	22.04	21.34	2.84	0.23–1.93	69.26	6.2×10^{-9}	1.6×10^{-5}	9.8×10^{-7}	0.61	1570	3.7×10^{-6}

Notes. ^(a) Radial coordinate of the top of the FeCZ. ^(b) Radial extension of the FeCZ. ^(c) Pressure scale height at top/bottom of the FeCZ. ^(d) Maximum of the convective velocity inside the FeCZ. ^(e) Density at v_c . ^(f) Mass contained in the convective region. ^(g) Mass in the radiative layer between the stellar surface and the upper boundary of the convective zone. ^(h) Convective turnover time, $\tau_{\text{turn}} := H_p/v_c$. ⁽ⁱ⁾ Time that a piece of stellar material spends inside a convective region, $\tau_{\text{conv}} := \Delta M_{\text{FeCZ}}/\dot{M}$. ^(j) Mass-loss rate.

non-radial pulsations (Ulrich 1970; Leibacher & Stein 1971). In analogy with the Sun, it has been suggested that solar-like oscillations are excited in hot, massive stars by the FeCZ (C09, Belkacem et al. 2010). Intriguingly, non-radial pulsations that are compatible with stochastic excitation from subsurface convection have recently been found by CoRoT in both a B (Belkacem et al. 2009) and an O star (Degroote et al. 2010), but see also Balona et al. (2011).

C09 pointed out that the FeCZ could host dynamo action very close to the surface. Simulations of turbulent convection in the presence of shear and rotation show that dynamo action is possible and can produce magnetic fields reaching equipartition on scales larger than that of the convection (e.g., Käpylä et al. 2008; Cantiello et al. 2011; Guerrero & Käpylä 2011). In this paper we consider the emergence of magnetic fields produced in a subsurface convective layer. In the following section we briefly review the properties of the FeCZ, before examining in Sect. 3 the magnetic fields produced within them and mechanisms for those fields to reach the surface. In Sect. 4 we look at observable effects on the surface, and finally we discuss the results and conclude in Sects. 5 and 6.

2. Subsurface convection

The occurrence and properties of the iron convection zone were studied in detail by C09. Here we summarize their findings and describe some of the fundamental properties of the outer regions of hot massive stars. The evolutionary calculations used in this paper are the same as in C09. We refer to that paper and to Brott et al. (2011) for a detailed discussion of the models and the stellar evolution code with which these are computed.

The peak in the opacity that causes the FeCZ occurs at around 1.5×10^5 K. In the envelope of a massive star this region remains relatively close to the surface during the whole main sequence. When the star expands to become a supergiant, the surface temperature decreases and the convective zone moves downwards in radius, further from the surface. For photospheric temperatures $\leq 10^4$ K, partial ionization of H and He drive convection directly at the surface. Therefore we limit the discussion of the properties of the FeCZ to spectral classes O and B (and partially A), which lack this additional convective layer¹.

The presence of subsurface convection depends also on the luminosity and metallicity of the star. At solar metallicity the FeCZ is present in models above $\sim 10^{3.2} L_{\odot}$, while at the metallicities of the LMC and SMC the thresholds are $\sim 10^{3.9} L_{\odot}$ and $\sim 10^{4.2} L_{\odot}$ respectively. Atomic diffusion and radiative acceleration can in principle lower these limits (Richer et al. 2000).

¹ The discussion in Sect. 3 is more general, and can be applied to any radiative star with a convective region below the surface.

Due to the very low densities in the outer layers of early type stars ($\rho \sim 10^{-8} \text{ g cm}^{-3}$), these convective regions contain very little mass despite their large radial extent. In Table 1 we show the properties in the outer layers of Galactic (solar metallicity) O/B stars above the luminosity threshold. Values for the size of the convection zone and of the radiative layer above are shown for models of a 20 and a 60 M_{\odot} at ages 6.41×10^6 yr and 2.37×10^6 yr respectively. These correspond to the calculations discussed in detail in C09, where is possible to see how the extension and location of subsurface convection changes during the MS evolution (See their Figs. 2 and 3).

In the FeCZ the transport of energy by convective motions is relatively inefficient: radiation dominates and transports most of the total flux (e.g., about 95% in the 60 M_{\odot} model of Table 1). This is because the density is very low and the mean free path of photons correspondingly long. In this situation convection is significantly superadiabatic, and the actual gradient $\nabla \equiv \text{dln } T/\text{dln } P$ has to be explicitly calculated from the mixing length equations (e.g. Kippenhahn & Weigert 1990). C09 found velocities ranging from 1 to 10's of km s^{-1} in the FeCZ of OB stars, where the sound speed is usually of the order 100 km s^{-1} .

In addition, it is informative to estimate the values of the diffusivities in the FeCZ. As usual in non-degenerate stars, the thermal diffusivity is the largest:

$$\chi = \frac{K}{\rho c_p} = \frac{4acT^3}{3\rho^2 c_p \kappa}, \quad (1)$$

where a , c_p and κ are the radiation constant, the specific heat at constant pressure and the Rosseland mean opacity. In the 20 and 60 M_{\odot} models χ is around 5×10^{17} and $8 \times 10^{19} \text{ cm}^2 \text{ s}^{-1}$ respectively. The momentum diffusivity (kinematic viscosity) is (in contrast to lower mass stars) also dominated by photons, and is given in this case by

$$\nu = \frac{4aT^4}{15c\kappa\rho^2} \quad (2)$$

where in the 20 and 60 M_{\odot} models it is approximately 6×10^9 and $10^{12} \text{ cm}^2 \text{ s}^{-1}$. Finally, the magnetic diffusivity as given by Spitzer's expression (Spitzer 1962)

$$\eta = \frac{\pi^{1/2} m_e^{1/2} Z e^2 c^2}{\gamma_E 8 (2k_B T)^{3/2}} \ln \Lambda \approx 7 \times 10^{11} \ln \Lambda T^{-3/2} \text{ cm}^2 \text{ s}^{-1} \quad (3)$$

which is roughly $10^5 \text{ cm}^2 \text{ s}^{-1}$ in both models (assuming a fully ionized H plasma and a value $\ln \Lambda = 10$ for the Coulomb logarithm). Note that photons have a negligible effect on the magnetic diffusivity. The magnetic Prandtl number ($\text{Pr}_m \equiv \nu/\eta$) is consequently 10^5 to 10^7 . We can compare these numbers to the

values usually discussed in the solar dynamo ($\text{Pr}_m \sim 10^{-2}$ in the tachocline) and in the Galactic dynamo contexts (the ISM has typically $\text{Pr}_m \sim 10^{12}$).

3. Magnetic spots

The occurrence of convection zones close to the surface of hot massive stars opens a new scenario. Magnetic fields could be readily produced by dynamo action and reach the surface via magnetic buoyancy (C09). Below we discuss this hypothesis.

3.1. Dynamo action

In an astrophysical plasma a dynamo is a configuration of the flow which is able to generate a magnetic field and sustain it against ohmic dissipation. Depending on the scale of the resulting magnetic field respect to the scale of kinetic energy injection, dynamos are usually divided into small and large scale. In large scale dynamos the field has correlation length bigger than the forcing scale in the flow, while small scale dynamos result in magnetic fields with correlation scale of order or smaller than the forcing scale. Small scale dynamos can occur in non-helical turbulent flows, while anisotropic flows (e.g., shear flows) are required for a large scale dynamo. In the mean field approach, large scale dynamos are often divided into $\alpha\Omega$ and α^2 , depending on the role of the Ω and α effect in regenerating the toroidal and poloidal components of the field. We refer to [Brandenburg & Subramanian \(2005\)](#) for a very nice review of astrophysical dynamos.

The FeCZ is a turbulent layer close to the surface of a massive star and as such could be the site of a small scale dynamo (e.g., [Moss 1994](#)). Moreover main sequence massive stars usually rotate rapidly. A typical equatorial rotational velocity of 150 km s^{-1} (typical for Galactic B-type stars, e.g. [Dufon et al. 2006](#)) corresponds to a rotational period of the order of the convective turnover timescale (Rossby number is in the range 1...10), therefore an efficient α^2 or $\alpha\Omega$ -dynamo could be possible. Assuming magnetic fields at equipartition with the kinetic energy of the convective motion, gives magnetic fields up to $\sim 2 \text{ kG}$. This is supported by simulations of turbulent convection in the presence of rotation and shear, which show dynamo excitation with magnetic fields reaching equipartition on scales larger than the scale of convection ([Käpylä et al. 2008](#); [Cantiello et al. 2011](#)).

Hence dynamo action in the FeCZ could depend on parameters like the stellar rotation (as measured by the Rossby number) and the shear profile in the region of interest. The scale of the magnetic field is depending on the type of dynamo occurring in the relevant layers. The dynamo may also be affected by a (presumably fossil) large-scale magnetic field penetrating upwards from the radiative zone below (see Sect. 5). However it is interesting to note that many of the photospheric and wind phenomena observed in hot massive stars are ubiquitous (e.g., [Howarth & Prinja 1989](#); [Kaper et al. 1996](#); [Fullerton et al. 1996](#); [Prinja et al. 2002](#)), in contrast to fossil fields which have been found at the surfaces of some (as yet badly measured) fraction of massive stars. As it is difficult to predict the exact rotational properties of the plasma inside the FeCZ, here we will not focus on a detailed study of subsurface dynamo action. As a preliminary study of subsurface magnetism we will just follow the results of [Cantiello et al. \(2009, 2011\)](#), and assume that magnetic fields at equipartition are produced in the FeCZ. In Fig. 1 we show the

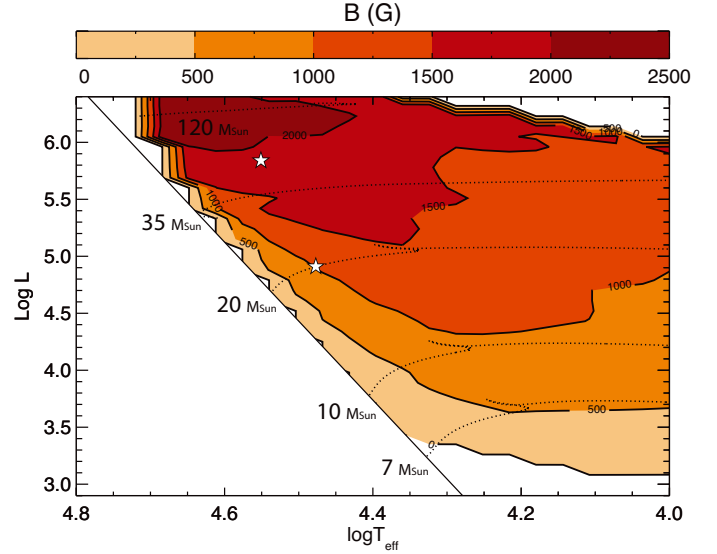


Fig. 1. Values of maximum magnetic fields (in Gauss) in the FeCZ, as a function of the location in the Hertzsprung-Russell (HR) diagram. This plot is based on evolutionary models between $5 M_{\odot}$ and $120 M_{\odot}$ for solar metallicity (some evolutionary tracks shown as dotted lines). The amplitude of magnetic fields is calculated assuming equipartition with the kinetic energy of convective motion. The full drawn black line roughly corresponds to the zero age main sequence. The two star symbols correspond to the location of the 20 and 60 M_{\odot} models discussed in the text and in C09.

expected maximum magnetic field inside the FeCZ, assuming equipartition of magnetic and kinetic energy:

$$\frac{B_{\text{eq}}^2}{8\pi} = \frac{1}{2} \rho v_c^2, \quad (4)$$

where we adopted the maximum value of the ρv_c^2 inside the FeCZ (as computed in the non-rotating models of C09). Due to the higher power dependency on the velocity, the location of the maximum of ρv_c^2 always roughly corresponds to the maximum of v_c . Values of magnetic fields calculated using the average convective velocity and density typically differ by less than 30%.

3.2. Magnetic buoyancy

There are various possible mechanisms which can bring a magnetic field generated in the FeCZ through the overlying radiative layer to the surface, and it is a useful exercise to compare their timescales. As the title of this section suggests, it turns out that among the mechanisms studied here, the dominant one is magnetic buoyancy.

The magnitude of the mass loss from massive stars is such that material resides in the outer, radiative layer for just a short time. Assuming a rate of $\sim 10^{-7} M_{\odot} \text{ yr}^{-1}$, this time is $\sim 1 \text{ yr}$.

Magnetic field could also be brought to the surface by Ohmic diffusion. The timescale over which it produces effects over the relevant radial length scale, i.e. the pressure scale height H_p , is

$$\tau_{\text{Ohm}} \sim \frac{H_p^2}{\eta} \sim \left(\frac{H_p}{10^{10} \text{ cm}} \right)^2 \left(\frac{T}{10^5 \text{ K}} \right)^{3/2} 10^7 \text{ yr}, \quad (5)$$

where η is the magnetic diffusivity. We shall now see that this timescale and that of advective transport are much longer than the buoyancy timescale.

Buoyancy causes magnetic features to rise through a radiative zone because magnetic field provides pressure without contributing to density. Since the sound and Alfvén timescales are shorter than the thermal/buoyancy timescale (the buoyant rise is subsonic and sub-Alfvénic), we can assume that a magnetic feature is in pressure equilibrium with its surroundings. Calling the sum of gas and radiation pressure inside and outside the feature P_i and P_e we have

$$P_e = P_i + P_{\text{mag}}, \quad (6)$$

where the value of the magnetic pressure P_{mag} depends on the geometry of the feature. For instance, in a self-contained structure it is $B^2/24\pi$, and in a flux tube of fixed length it is $B_{\text{ax}}^2/8\pi$ where B_{ax} is the component along the axis of the tube². Note the implicit assumption that the size of the structure l is much smaller than the scale height H_p ; it is quite likely (see below) that in fact $l \approx H_p$ but this will make a difference just of factors of order unity. Now, in the absence of thermal diffusion the feature reaches an equilibrium where $\rho_i = \rho_e$, made possible by the internal temperature being lower than the external $T_i < T_e$. With the addition of a *small* thermal diffusion, the feature absorbs heat from its surroundings and rises quasistatically upwards through surroundings of increasing entropy. The speed of this rise is given by:

$$v_{\text{therm}} \sim \frac{2\chi H_p}{l^2 \beta (\nabla_{\text{ad}} - \nabla)} \cdot \frac{1}{4 - 3\alpha} \quad (7)$$

where $\beta \equiv P_e/P_{\text{mag}}$, l is the size of the feature, ∇ and ∇_{ad} have their usual definitions and take values $\nabla = \nabla_{\text{rad}} \approx 0.24$ and $\nabla_{\text{ad}} \approx 0.25$ in the radiative zones between the FeCZ and the photosphere, and χ is the thermal diffusivity given in (1).

If on the other hand thermal diffusivity is *large*, the feature rises so fast that the speed is limited by aerodynamic drag. In this regime the speed is independent of thermal conductivity, and we can make the approximation that $T_i = T_e$; note that unlike above, in this regime we have $\Delta\rho \equiv \rho_e - \rho_i \neq 0$. The buoyant force as given by Archimedes' principle is balanced by the drag force,

$$\frac{1}{2} C_d A \rho_e v_{\text{drag}}^2 = V g \Delta\rho \quad (8)$$

where A and V are the cross-sectional area (projected onto a horizontal plane) and volume of the magnetic feature, and C_d is the drag coefficient whose value depends on geometry. This can be rearranged to

$$v_{\text{drag}}^2 \approx \left(\frac{2V}{C_d H_p A} \right) \frac{P_{\text{mag}}}{\rho_e}. \quad (9)$$

Since the length scale of the feature $l \approx V/A$, the term in brackets is approximately equal to l/H_p^3 . The remaining part is approximately the square of the Alfvén speed, which one can alternatively express in terms of the sound speed c_s and the ratio of total to magnetic pressure, β :

$$v_{\text{drag}} \sim v_A \left(\frac{l}{H_p} \right)^{1/2} \sim \frac{c_s}{\beta^{1/2}} \left(\frac{l}{H_p} \right)^{1/2}. \quad (10)$$

Putting the numbers into (7) and (10) gives

$$v_{\text{therm}} \approx \left(\frac{l}{H_p} \right)^{-2} \beta^{-1} 10^{10} - 10^{13} \text{ cm s}^{-1}, \quad (11)$$

$$v_{\text{drag}} \approx \left(\frac{l}{H_p} \right)^{1/2} \beta^{-1/2} 10^7 - 3 \times 10^7 \text{ cm s}^{-1}, \quad (12)$$

where ranges in the numbers come from the difference between 20 and 60 M_\odot models and from differences between top and bottom of the surface radiative layer. It is clear that unless $l \gg H_p$, for which the approach above breaks down anyway, or the field is implausibly weak (and therefore undetectable and unimportant), v_{therm} is larger and the rise is therefore limited by drag. For a magnetic feature with $l \sim H_p$ and $\beta \sim 100$ (a conservative estimate), the time taken to rise one scale height is of order 2 h in both models, much shorter than the advective and Ohmic timescales.

This is in contrast to the thermal-diffusion-limited regime which was studied by MacGregor & Cassinelli (2003) to check the possibility that magnetic fields generated in a convective core could rise to the surface. In that case the timescale appears to exceed the main sequence lifetime unless very small-scale features are used. Moreover Mullan & MacDonald (2005) argue that the inclusion of compositional gradients may suppress the buoyancy, making this scenario unlikely⁴.

The situation in massive star surface layers is in some sense more similar to that of flux tubes in the solar convection zone in that the rise is limited by drag forces, although the drag force in the solar convection zone does not scale simply as v^2 but depends also on the convective motion. A stationary flux tube in the solar convection zone experiences a net downwards aerodynamic force once the average of upwards- and downwards-moving convective cells has been taken, with the result that there is a field-strength threshold below which the tube moves downwards despite its intrinsic buoyancy.

Once a magnetic field element reaches the surface of the star it would be useful to estimate its lifetime t_{spot} . This is not trivial, as the photosphere of OB stars is complicated by the presence of strong winds and, likely, turbulence and shear. However we can put some limits on the lifetime.

As we discussed at the beginning of this section, if the material in the radiative layer above the FeCZ has a mass ΔM , it will be removed by mass loss in a time $\Delta M/\dot{M}$. This sets an upper limit for the time a magnetic element, which is not anchored to the convective zone, can exist at the surface: this time is about 8 years for the 20 M_\odot and 3 months for the 60 M_\odot model. Of course this holds true only as long as the field is not annihilated by dissipative processes like magnetic reconnection. If the magnetic element is anchored to the convective zone, it can exist at the surface as long as the stellar wind does not remove all the mass contained in both the radiative layer *and* the FeCZ. It turns out that this limits t_{spot} to be less than 50 years (20 M_\odot model) and 4 years (60 M_\odot model). We refer to Table 1 for the data used for these simple estimates. These are solid upper limits, and it is likely that such features would survive for a much shorter time, owing to turbulence, shear and magnetic reconnection. As a lower limit for t_{spot} we could assume the time a feature of size l takes to cross the photosphere while rising with a velocity v_{drag} . For $l \sim H_p$ this gives a timescale of the order of hours.

² See Braithwaite (2010) for a discussion of this point and a review of the stability of flux tubes.

³ If the feature is spherical, $C_d \approx 0.75$ (Churazov et al. 2001).

⁴ Mullan & MacDonald (2005) also discuss the possibility of buoyant rise of magnetic fields generated in the radiative zone by the Tayler-Spruit dynamo. However it is not clear how this dynamo process actually works (Braithwaite 2006; Zahn et al. 2007).

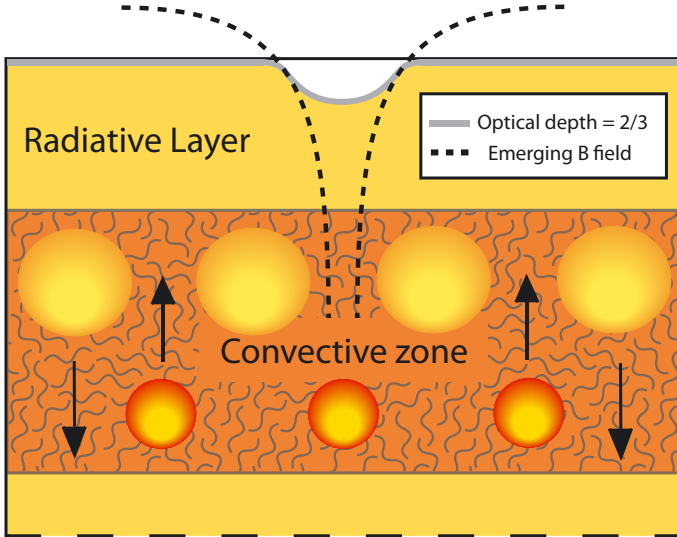


Fig. 2. Schematic representation of the effect of an emerging magnetic element at the surface of a hot massive star. A magnetic field (dashed line) rising from the subsurface convection zone threads the radiative layer and reaches the stellar photosphere. The solid, grey line shows the location of the stellar surface, as defined by the value $2/3$ for the optical depth. Notice that inside the magnetic field this line reaches deeper (and hotter) layers of the star, as compared to regions not affected by the field.

Finally, note that it is not inconceivable that the magnetic dynamo does not saturate properly because of the finite time the gas spends in the FeCZ. However, we consider this unlikely, since the convective turnover times (\sim hours – a day) appear to be very much shorter than the aforementioned mass-flux timescales of 50 and 4 years.

4. Observable effects

Let us first estimate the likely field strength at the photosphere. Whilst one might expect it to be proportional to the field strength produced in the convective layer, the difference in field strength of a magnetic feature between the top and bottom of the radiative layer depends on its geometry. In a self-contained magnetic feature (a “blob” or “plasmoid”) the field strength scales as $B \propto \rho^{2/3}$, while a horizontal flux tube of fixed length has $B_{ax} \propto \rho$ and $B_h \propto \rho^{1/2}$ where B_{ax} and B_h are the axial and “hoop” components. Knowing that $P \sim \rho^{4/3}$ (approximately, since $\nabla \approx 1/4$) in the radiative layer, we see that these two scenarios give $\beta = \text{const.}$ and $\beta \propto P^{-1/2}$ respectively. In contrast, if the central section of a flux tube rises to the surface while its ends are still in the convective layer, plasma can flow downwards along the tube and the field strength at the surface can in principle reach equipartition with the surrounding (gas plus radiation) pressure, as we see happening in the solar convection zone. Assuming equipartition magnetic field in the convective layer gives up to ≈ 2 kG at the base of the radiative layer, and $\beta \approx 100$. The density contrast across the radiative layer is around 50 and 100 in the $20 M_{\odot}$ and $60 M_{\odot}$ models respectively, which gives field strengths at the photosphere of 40–150 G and 20–100 G respectively in the blob and horizontal tube cases. The photospheric field could be significantly greater if a tube arches upwards and fluid is allowed to flow along it – the limit should then be of the order of equipartition with the photospheric pressure (about 300 G) or somewhat more, since the photosphere (i.e. the $\tau = 2/3$ level) in the tube is lower (see Fig. 2 and Sect. 4.1).

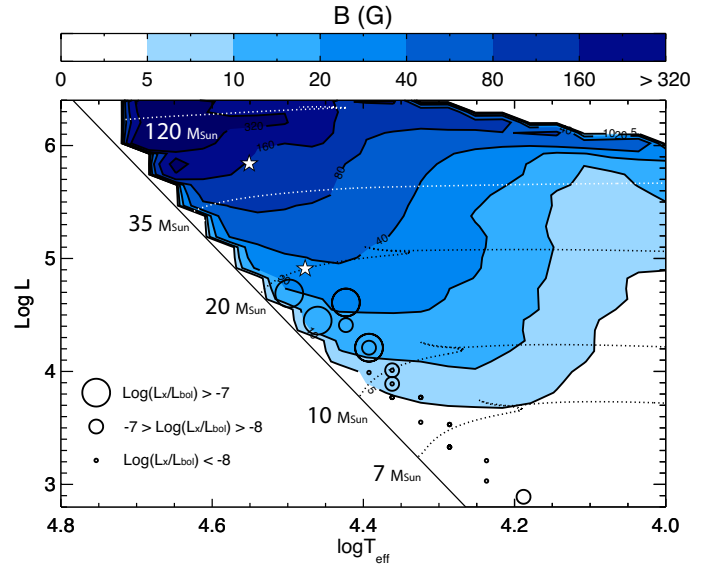


Fig. 3. Minimum values of expected surface magnetic fields (in Gauss) as a function of location in the HR diagram. This plot is based on evolutionary models between $5 M_{\odot}$ and $120 M_{\odot}$ for solar metallicity (some shown as dotted lines). Surface magnetic fields are calculated scaling the equipartition fields in the FeCZ (see Fig. 1) with $\rho^{2/3}$ (see discussion in Sect. 4). The full drawn black line roughly corresponds to the zero age main sequence. Circles correspond to the observed stars in the Cohen et al. (1997) sample, with symbol size coding different range of L_x/L_{bol} , as explained in the legend. The two star symbols correspond to the location of the 20 and $60 M_{\odot}$ models discussed in the text and in C09.

In Fig. 3 we show the expected amplitude of surface magnetic fields emerging from the FeCZ, assuming a scaling $B \propto \rho^{2/3}$. We argue that this represents a lower limit for the magnetic field that should be found at the surface of hot massive stars, if equipartition of kinetic and magnetic energy is reached in their FeCZ. It might seem plausible that magnetic features rising through the radiative layer partially annihilate each other on the way up. However, if these magnetic bubbles have a size at the bottom of the radiative layer comparable to the local scale height, they only have to rise through a distance about three times their own size to reach the photosphere, expanding as they do so. They rise at approximately the Alfvén speed, and since any reconnection would presumably take place at some fraction of the Alfvén speed, there is limited time available to destroy much of the original flux. It seems more likely that significant reconnection does take place *above* the photosphere.

While these fields appear to have very small amplitude, their role at the stellar surface should not be underestimated. In Fig. 3, the ratio of the magnetic to wind pressure ($\eta \approx B_{eq}^2 R_*^2 / M v_{\infty}^2$, [ud-Doula & Owocki 2002](#)) is of the order of unity over the whole region where magnetic fields are present. This means that already such low magnetic fields can alter the dynamic of the stellar wind, for example contributing to seed instabilities in the flow.

4.1. Surface bright spots and photometric variability

Once a magnetic field element reaches the stellar photosphere, one would like to know what effect this can have on observable properties of the star. In the Sun, magnetic fields of amplitude up to ~ 3 kG emerge at the surface as sunspots. The main effect of such magnetic fields is to locally inhibit convection. As the energy flux below the surface of the Sun is mainly transported

by convection, this produces a decrease of the flux at the surface, resulting in a visible dark spot. In contrast, the hot massive stars considered here have radiative surfaces and the effect of an emerging magnetic field is very different.

Under the simple assumption of magnetohydrostatic equilibrium we can make use of Eq. (6). Assuming again that also the gas temperature inside and outside is the same ($T_e = T_i$), we obtain once more that the density inside the magnetic element is lower than outside. Since we are at the stellar surface, this makes the element more “transparent”, i.e. photons can escape from deeper down the star compared to outside the magnetic flux. This is somehow similar to what occurs in the solar faculae, which are regions of emerging magnetic fields with a scale much smaller than the scale of convection. In that case Keller et al. (2004) found, using MHD calculations, that “The opacity in the magnetic flux concentration is strongly reduced owing to its low density and temperature and thus provides a clear sight straight through the magnetic field onto the adjacent nonmagnetic granule”. Their calculations basically verified the explanation for solar faculae proposed by Spruit (1976, 1977).

Photons emerging from deeper down the star are emitted from regions where the temperature is higher. The temperature stratification, assuming that the magnetic pressure is much smaller than the gas + radiation pressure (i.e. $\beta \gg 1$), is given by the radiative gradient ∇_{rad}

$$\nabla_{\text{rad}} \equiv \left(\frac{d \ln T}{d \ln P} \right)_{\text{rad}}.$$

Therefore it is easy to estimate the temperature difference between the region of the star visible through the magnetic element and outside of it:

$$\frac{\Delta T}{\Delta P} = \frac{T \Delta(\ln T)}{P \Delta(\ln P)} \approx \frac{T}{P} \nabla_{\text{rad}}$$

$$\frac{\Delta T}{T} \approx \frac{\nabla_{\text{rad}}}{\beta}. \quad (13)$$

This means that regions with an emerging magnetic field will look hotter. In the case of the FeCZ, assuming the minimum value of magnetic fields shown in Fig. 3, preliminary estimates give temperature differences up to a few hundred K. In the case of a surface magnetic field in equipartition with the photospheric pressure (gas plus radiation) Eq. (13) is not strictly correct, as the magnetic field can affect the radiative gradient. Nevertheless, assuming for a moment that the correction to this relation is small, one expects temperature differences up to a few kK.

Since the stellar luminosity is related to the effective temperature through the relation $L = 4\pi R^2 \sigma T^4$, magnetic fields in stars having radiative surfaces will produce bright spots. We can estimate the local luminosity contrast between the magnetic bright spot and the non magnetic regions of the stellar surface

$$\frac{\Delta L_{\text{loc}}}{L_{\text{loc}}} \approx 4 \left(\frac{\Delta T}{T} \right) \approx \frac{4 \nabla_{\text{rad}}}{\beta}. \quad (14)$$

Temporal changes in the observed stellar luminosity might be expected, as a result of the appearance/disappearance of such a bright spot. In a rotating star the luminosity would also decrease (increase) when the spot moves out of (into) the visible part of the stellar disc. It is difficult at this stage to give a firm estimate of the total expected change in luminosity, since this will depend not only on the intensity of the magnetic field, but also on the

geometry and number of magnetic spots. Also, it is not clear exactly where the extra luminosity in the spots should come from, in terms of underluminous areas. Nevertheless, if we assume a circular spot of radius r , with filling factor $f = (r/R)^2$, where R is the stellar radius, then the effect on the stellar luminosity can be estimated if r is known. A simple approach, based on preliminary models of subsurface convection (Cantiello et al. 2011), is to equate r to the pressure scale height in the FeCZ. These simulations show dynamo action producing magnetic fields on scales comparable to or larger than the scale of convection, i.e. the local pressure scale height H_p . We obtain:

$$\frac{\Delta L}{L} = \frac{4 \nabla_{\text{rad}}}{\beta} \left(\frac{H_p}{R} \right)^2, \quad (15)$$

which, for typical values of subsurface convection, gives $\Delta L/L \sim 10^{-5}$. This is the relative change in luminosity caused by the appearance/disappearance of one magnetic spot of about 100 G with size $0.2 R_\odot$ at the surface of a hot massive star. Recall that these stars have radii around $10 R_\odot$.

Of course spots with stronger magnetic fields and bigger filling factors are certainly possible. Such spots would lead, most likely, to larger changes in the integrated light from the star. An estimate of the amplitude of the variability as function of filling factor and surface strength is much more difficult in this case, as one can no longer consider the magnetic element thin (in a thermal sense). MHD calculations of magnetic flux tubes in a radiative environment are required to study the details of this problem.

Photometric variability at the level of 10^{-3} mag and with timescales of the order of days seems to be widespread in O stars (Balona 1992). Using CoRoT to perform high-precision photometry, Degroote et al. (2010) found solar-like oscillations in a young O-type star (HD 46149). These are modes with a finite lifetime that are believed to be excited by the presence of subsurface convection (Belkacem et al. 2010). Intriguingly, this system also shows a low-frequency variability with time scale of days and amplitude of order 10^{-3} mag. The authors could not relate this variability to any type of pulsations. Instead they argue that this could be the signature of spots, stellar winds or chemical inhomogeneities modulated by the stellar rotation (Degroote et al. 2010). Pápics et al. (2011) obtained high-precision photometric observations of the B0.5IV star HD 51756 with CoRoT. While no solar-like oscillations were identified in this case, they found cusp-like features in the light-curve. These features have amplitudes of order 10^{-3} mag, and recur on a time scale of days. Similar to HD 46149, Pápics et al. (2011) propose as a possible source for this variability photospheric features like spots or chemical inhomogeneities, modulated by stellar rotation.

4.2. X rays

OB stars have been found to emit X rays (Harden et al. 1979). O stars follow the relation $L_x/L_{\text{bol}} \sim 10^{-7}$ (see e.g. Pallavicini et al. 1981) while B star X-ray luminosities show a lot of scatter (Meurs et al. 1992; Berghoefer et al. 1997). Among possible physical mechanisms for the production of X rays are magnetic coronae (Cassinelli & Olson 1979; Waldron & Cassinelli 2007, 2009) and the intrinsic instability of line-driven winds (Lucy & White 1980; Owocki et al. 1988; Feldmeier 1995).

The L_x/L_{bol} relation only applies for $L_{\text{bol}} > 10^{38}$ erg s⁻¹, i.e. earlier than B0-B1 type stars. It breaks down at lower luminosities, with the ratio L_x/L_{bol} two orders of magnitude smaller at B3 than at B1 (Cassinelli et al. 1994; Cohen et al. 1997). This is

shown in Fig. 3, where we plot the data from Cohen et al. (1997) together with the theoretical expectation for the minimum values of surface magnetic fields (assuming dynamo action in the FeCZ). While there could be different explanations for such a correlation, it is interesting to note that in the sample stars with $L_x/L_{\text{bol}} > -7$ fall in a region of the HR diagram where larger values of surface magnetic fields are predicted. Recently Gagné et al. (2011) have studied with *Chandra* the X-ray emission from OB stars in the Carina Complex. They also found that most of their O stars follow the L_x/L_{bol} relation, while this breaks down for B stars. Interestingly they identify a group of B stars with high X-ray emission that cannot be explained by a distribution of ordinary coronal, pre main-sequence companions. A possibility is that in O stars wind embedded shocks dominate and are responsible for the L_x/L_{bol} relation, while the observed X-ray luminosity in B stars is mainly due to the presence of subsurface generated magnetic fields.

5. Discussion

We showed that, among advection, magnetic diffusion and magnetic buoyancy, the most likely process that can bring to surface magnetic fields generated by dynamo action in the FeCZ is magnetic buoyancy. However there could be other ways for a magnetic field to escape the subsurface convection region and reach the stellar surface. For example, Warnecke & Brandenburg (2010) and Warnecke et al. (2011) studied the magnetic flux produced by a turbulent dynamo in Cartesian and spherical geometry, respectively. They found that magnetic flux can rise above the turbulent region without the need of magnetic buoyancy. This appears to be related to the release of magnetic tension, which leads to the relaxation and emergence of the field. Convective overshoot might also contribute to the transport of magnetic flux out of the FeCZ. However, given the fact that the FeCZ usually sits a few pressure scale heights from the stellar surface, this effect is likely marginal.

Some fraction of massive stars display large-scale, apparently fossil fields of strengths (1–3 kG) comparable to the FeCZ equipartition strengths derived above. A sufficiently strong field will suppress convection, forcing an increase in temperature gradient so that the entire energy flux can be transported radiatively. However, it is not obvious where the field strength threshold should be; one might naïvely expect convection to be suppressed by a field of greater energy density than the convective motion, but the work of Gough & Tayler (1966), Moss & Tayler (1969) and Mestel (1970) suggests that in that case the temperature gradient would simply steepen further above the adiabatic gradient until convection resumes, and that to suppress convection completely, a field above (approximate) equipartition with the *thermal* energy would be required. However, this work considers a situation where the energy flux is almost entirely convective, such as in the bulk of the solar convective zone. In the FeCZ context the situation is different, in that only a small fraction of the stellar heat flux is carried by convection, and that the temperature gradient is already significantly above adiabatic. It is plausible therefore that a fossil field of a few kG could indeed suppress convection, which would have consequences on observables connected to the presence of subsurface convection (e.g. microturbulence, C09). One can make an analogy here with the intermediate-mass stars (A and late B), where fossil fields do appear to suppress or at least significantly reduce the weak helium-ionisation convection at the photosphere which manifests itself as microturbulence (Shulyak, priv. comm.)

C09 proposed the FeCZ as a possible unifying mechanism for the formation (or seed) of different observational phenomena at the surface of hot, massive stars. The presence of turbulent subsurface convection appears to be able to trigger at least three different classes of physical phenomena: running waves (g-modes and p-modes, e.g., Goldreich & Kumar 1990, C09), solar-like oscillations (e.g., C09; Belkacem et al. 2010) and magnetic fields (C09; this work). Each of these perturbations can, in principle, results in observable effects at the surface and/or in the stellar wind. Estimating the amplitude, length-scale and time-scale of the surface perturbations produced by each mechanism is of primary importance to establish the connection between FeCZ and observable phenomena. For example wind clumping could be produced by small-scale velocity and/or density fluctuations at the stellar surface. These perturbations could excite the line-driven instability (Runacres & Owocki 2002) closer to the stellar surface than originally predicted from the theory. This would reconcile theory with observations of the radial stratification of wind clumping (Puls et al. 2006). However both running waves and localized magnetic fields could be responsible for such a perturbation at the base of the wind. To understand which one dominates, hydrodynamic simulations of the line-driven instability are required. These calculations need to include, as boundary conditions at the base of the stellar wind, the different velocity and density perturbations predicted in the case of running waves and surface magnetic fields. While simple estimates for these perturbations exist (C09 and this paper), realistic MHD simulations of the FeCZ and the radiative layer above it are necessary. Understanding the exact role and properties of each perturbation is also fundamental to explain the puzzling co-existence of small- and large- scale phenomena observed at the surface and in the wind of hot, massive stars (e.g., microturbulence/macroturbulence and wind clumping/DACs). An effort is clearly required to improve existing MHD calculations of subsurface convection (Cantiello et al. 2011).

The presence of surface magnetic fields might have implications for the evolution of OB stars. This is because, for wind confinement parameters $\eta \gtrsim 1$, the magnetic field can in principle affect the stellar wind mass-loss and angular momentum-loss rates. For large scale fields at the surface, the impact on angular-momentum loss has been studied (Weber & Davis 1967; ud-Doula et al. 2009). In the case of the magnetic star σ Ori E the theoretical spin-down is in agreement with observations (Townsend et al. 2010). The effect of large scale fields on the angular momentum evolution of main sequence massive stars has been investigated by Meynet et al. (2011). Their results seem to show that only fields above a few hundred Gauss can have a substantial impact. For a given surface field amplitude, higher order multipoles (i.e. less coherent fields, as expected in the case of dynamo action in the FeCZ) result in a reduced angular momentum-loss. Moreover in the case of small-scale fields a lot of the flux may reconnect and disappear fairly close to the surface, below where most of the wind acceleration takes place. Therefore we expect magnetic fields produced in the FeCZ to have only a modest effect on the angular momentum evolution of massive stars. Nevertheless further investigation of their geometry and amplitude is required to confirm this statement.

6. Conclusions

We have shown for the first time that emerging magnetic fields produced in the FeCZ of hot, massive stars could reach the stellar surface. The surface fields expected are localized and with amplitudes smaller than a few hundred Gauss.

As pointed out by [Henrichs et al. \(2005\)](#), a number of unexplained observational phenomena in hot, massive stars are consistent with surface magnetism. These phenomena include the photometric variability and X-ray emission discussed above, line profile variability ([Fullerton et al. 1996](#); [Morel et al. 2004](#)) and discrete absorption components (DACs, e.g., [Cranmer & Owocki 1996](#); [Kaper et al. 1997](#)). The magnetic fields emerging from the FeCZ could be widespread in OB stars at solar metallicity, and play an important role in some or even all these phenomena. The metallicity dependency of the FeCZ is such that these effects would weaken and eventually disappear with decreasing metallicity (C09).

Direct detection of magnetic fields emerging from the FeCZ has to overcome the problem that, for complex fields, opposite line-of-sight magnetic polarities and their respective opposite circular polarization signals do cancel when integrated over the stellar disc. However if the star is rotating fast enough, it might still be possible to detect such fields using the Zeeman Doppler Imaging technique ([Semel 1989](#)). While direct measurement of FeCZ-generated magnetic fields is challenging, we argue that a viable indirect approach is to use high-precision photometry – indeed we show that magnetic fields at the stellar surface of radiative stars should produce bright spots. The finite lifetime of the magnetic spots, together with rotational modulation, could leave a detectable signature in the light curve of hot, massive stars. This kind of variability seems to be present in some CoRoT targets ([Degroote et al. 2010](#); [Pápics et al. 2011](#)), and might be used to gather preliminary information on the magnetic field geometry. These information could be used to assess the feasibility and the requirements of a targeted direct detection campaign.

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