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Magnetic Susceptibility for the Half-Filled Hubbard Model

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In a previous paper[1] we calculated the magnetization curve for the one-dimensional half-filled Hubbard model. The Hamiltonian of the Hubbard model is

$$\mathcal{H}=-T\sum_{\sigma}\sum_{\langle ij\rangle}c_{i\sigma}^{\dagger}c_{j\sigma}+I\sum_{i}n_{i\uparrow}n_{i\downarrow}\,,\quad T>0\,.$$

We mistook the calculation of magnetic susceptibility for $I>0$. Our previous result $\chi=\mu_0{}^2I/\pi^2T^2$ is valid only at the limit $I/T\to\infty$. The correct result is

$$\chi=\frac{\mu_0{}^2}{\pi T}I_0\left(\frac{2\pi}{U}\right)\Big/I_1\left(\frac{2\pi}{U}\right). \tag{1}$$

Here $U=I/T$, μ_0 is the magnetic moment of an electron and is equal to $g\mu/2$ in the notation of reference 1). I_0 and I_1 are modified Bessel functions. At the limit $U\to 0+$ this coincides with Pauli's paramagnetic susceptibility $\chi=\mu_0{}^2/\pi T$.

In a recent paper Ovchinikov[2] gave the spectrum of spin waves of this system. The dispersion relation*) is

$$q=\frac{2\pi}{U}\int_{\Lambda_0}^{\infty}d\Lambda\int_{-\pi}^{\pi}\frac{dk}{2\pi}\times\operatorname{sech}\frac{2\pi(\Lambda-\sin k)}{U}-\pi\,(\text{or } 0), \tag{2a}$$

$$\varepsilon=\frac{4T}{U}\int_{-\pi}^{\pi}dk\frac{\cos^2 k}{2\pi}\operatorname{sech}\frac{2\pi(\Lambda_0-\sin k)}{U}, \tag{2b}$$

where Λ_0 is a parameter which moves from $-\infty$ to $+\infty$. This spectrum is plotted in Fig. 2. It has the double periodicity as the spin-wave spectrum of one-dimensional antiferromagnetic Heisenberg model.[3] With the use of (2a) and (2b) one can calculate the group velocity at $q\approx 0$ or $q\approx\pi$. Taking the limit $\Lambda_0\to\infty$ in (2a) and (2b) Ovchinikov obtained

$$v_s=\left|\frac{d\varepsilon}{dq}\right|=2TI_1\left(\frac{2\pi}{U}\right)\Big/I_0\left(\frac{2\pi}{U}\right). \tag{3}$$

Compairing with Eq. (1) we find a relation between the magnetic susceptibility and the velocity of the spin waves:

$$\chi=\frac{2\mu_0{}^2}{\pi v_s}. \tag{4}$$

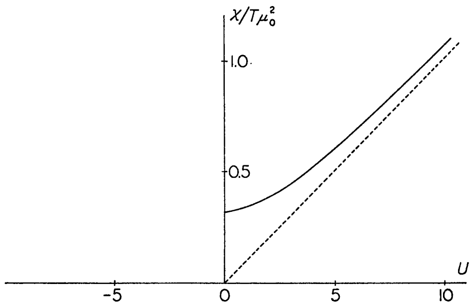

Fig. 1. Magnetic susceptibility χ is plotted vs $U=I/T$. In the case $U<0$ χ is zero. The dashed line is the asymptote at $U\to\infty$.

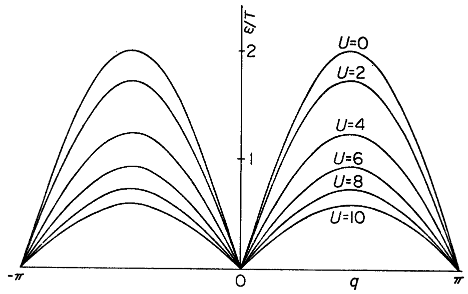

Fig. 2. Spin wave spectrum.

*) The author also gave the same result independently of his thesis, which was submitted to University of Tokyo (1969).

1) M. Takahashi, Prog. Theor. Phys. **42** (1969), 1098 [Erratum: **43** (1970), 860].
2) A. A. Ovchinikov, Zh. Eksp. i Teor. Fiz. **57** (1969), 2137.
3) J. des Cloizeaux and J. J. Pearson, Phys. Rev. **128** (1962), 2131.