Magnetization of left-handed metamaterials*

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(Dated: Received 20 May 2006; revised 6 June 2006)

We propose a possible mechanism for the generation of magnetic fields in negative refraction index composite metamaterials. Considering the propagation of a high-frequency modulated amplitude electric field in a left-handed material (LHM), we show that the ponderomotive interaction between the field and low-frequency potential distributions leads to spontaneous generation of magnetic fields, whose form and properties are discussed.

PACS numbers: 41.20.Jb, 78.20.Ci, 42.25.Bs, 42.70.Qs
Keywords: Electromagnetic waves, left-handed materials, negative refractive index, nonlinear phenomena, ponderomotive force.

A great deal of recent research work has focused on *negative refraction index* (NRI) materials, i.e. composite metamaterials which are characterized by a negative value of both the dielectric permittivity ϵ and magnetic permeability μ, in addition to a negative value of the refraction index $n = -\sqrt{\epsilon\mu}$. Maxwell's laws for electromagnetic (EM) wave propagation in such media imply a left-handed orthogonal triad vector relation between the electric field, the magnetic field and the EM wave-related energy flow (Poynting vector), hence the term *left-handed* (LH) materials (LHM), widely in use. Following the original LH medium concept first introduced as early as in 1945 by Mandelstam [1], and later elaborated by Pafomov [2] and Veselago [4] in the 1960s, recent theoretical studies [5–7] have suggested how these unique physical properties could be realized in artificially built materials, and experiments subsequently confirmed those predictions [8, 9] (see also Refs. [10] and [11] for recent reviews). A number of theoretical works followed those studies [12–14], and exciting applications were suggested in order to exploit the singular behavior of LHM (beam re-focusing, inversion of Snell's law and of the Doppler shift effect, backward Cherenkov radiation, to mention only a few [15]). Nonlinear properties of left-handed metamaterials have also been investigated by Zharov *et al.* [16].

In this Brief Report, we consider the propagation of a high-frequency (HF) electric field (HEF) in LHM. We show that the ponderomotive coupling between the HEF and the background medium generates ambipolar electric fields, which can separate charges and produce currents that are responsible for spontaneous generation of magnetic fields in metamaterials with negative index of refraction.

Let us consider a high-frequency electric field $\mathbf{E} = (1/2)\,\mathcal{E}\,(\mathbf{r},t)\,\exp(i\mathbf{kr} - i\omega t) + \text{c.c.} = |\mathcal{E}\,(\mathbf{r},t)|\,\cos(\mathbf{kr} - \omega t + \theta)$ propagating in a left-handed medium; here, ω and $\mathbf{k}$ denote the field frequency and the wave vector, respectively; θ is an initial (real) phase and c.c. stands for the complex conjugate). Adopting the general formalism of Washimi and Karpman [17], the slowly varying field (complex) amplitude $\mathcal{E}$ is related to a ponderomotive mean force $\mathbf{F_p}$ exerted on the medium slow motion, which may be expressed as

$$\mathbf{F_p} = \mathbf{F_p}^{(s)} + \mathbf{F_p}^{(t)}\,, \tag{1}$$

where $\mathbf{F_p}^{(s)}$ and $\mathbf{F_p}^{(t)}$ are related to space and time variations of the field intensity $|\mathcal{E}|^2$

$$\mathbf{F_p}^{(s)} = \frac{\epsilon - 1}{16\pi}\,\nabla|\mathcal{E}|^2\,, \tag{2}$$

$$\mathbf{F_p}^{(t)} = \frac{1}{16\pi}\frac{1}{\omega^2}\,\frac{\partial[\omega^2(\epsilon - 1)]}{\partial\omega}\,\frac{\partial|\mathcal{E}|^2}{\partial t}\,\mathbf{k}\,. \tag{3}$$

Here, $\epsilon = (ck/\omega)^2$ is the dielectric permittivity. We notice that $\mathbf{F_p}^{(t)}$ is parallel to the propagation direction, while the direction of $\mathbf{F_p}^{(s)}$ depends on the intensity inhomogeneity of the HF electromagnetic waves. Neglecting the electron inertia, the (cold) plasmon fluid equation of motion leads to a force balance equation of the form

$$-n_0 e\tilde{\mathbf{E}} + \mathbf{F_p} = 0\,, \tag{4}$$

where n_0 denotes the mean electron number density and e is the magnitude of the electron charge. Combining equations (1) and (4), the slowly varying effective electric field $\tilde{\mathbf{E}}$ which is felt by the electrons is expressed as

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$$\tilde{\mathbf{E}} = \frac{1}{n_0 e}\mathbf{F_p} = -\nabla\phi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\,, \tag{5}$$

where ϕ and $\mathbf{A}$ denote a pair of scalar and vector potentials, respectively, given by

$$\phi \;=\; -\frac{\epsilon - 1}{16\pi n_0 e}\,\nabla|\mathcal{E}|^2\,, \tag{6}$$

$$\mathbf{A} \;=\; -\frac{c}{16\pi n_0 e}\,\frac{\mathbf{k}}{\omega^2}\,\frac{\partial[\omega^2(\epsilon-1)]}{\partial\omega}\,|\mathcal{E}|^2\,, \tag{7}$$

where c is the speed of light in vacuum. Therefore, a slowly varying magnetic field $\tilde{\mathbf{B}} = \nabla\times\mathbf{A}$ is generated by the electron current associated with charge separation. The magnetic field $\tilde{\mathbf{B}}$, whose direction is transverse to the wave propagation direction, is given by [18]

$$\tilde{\mathbf{B}} = -\frac{c}{16\pi n_0 e}\,\frac{1}{\omega^2}\,\frac{\partial[\omega^2(\epsilon-1)]}{\partial\omega}\,\nabla\times(\mathbf{k}\,|\mathcal{E}|^2)\,. \tag{8}$$

In the following, we shall evaluate the magnitude of the generated magnetic field $\tilde{\mathbf{B}}$ for a left-handed material composed by a lattice of thin conducting rectilinear wires and split ring resonators (SRRs) [6, 8], which are, respectively, responsible for the negative value of the medium dielectric permittivity ϵ_{eff} and magnetic permeability μ_{eff}. Making use of the generic dispersion relation for this material, which is given in Ref. [8], we have

$$\epsilon = \frac{(\omega^2-\omega_p^2)(\omega^2-\omega_b^2)}{\omega^2(\omega^2-\omega_0^2)}\,. \tag{9}$$

Here, ω_p is a cutoff ("plasma") frequency, valid for EM wave propagation along the wire axis, whose value is related to wire geometry (use of sufficiently thin wires results in ω_p in the microwave frequency range or even less). Energy dissipation (losses) is (are) neglected here. The characteristic frequency ω_b is given by $\omega_0/\sqrt{1-F} > \omega_0$, where ω_0 and F are positive constants, which depend on the material structure. The above dispersion relation (first derived for SRRs in the absence of wires) refers to an incident polarized EM wave magnetic field $H_\parallel$, which is parallel to the split ring axis; in this case, the frequency gap is related to the negative value of μ_{eff} [8], and the propagation frequency band is restricted between ω_0 and ω_b (we note that the propagation is characterized by a negative group velocity and a bandwidth which is independent of the plasma frequency $\omega_p > \omega_b$). An other case of interest is when $H_\perp$ is perpendicular to the SRR axis (whence the gap is related to the negative ϵ_{eff}), which is shown in Ref. [8] to be approximately (assuming $\omega_b \simeq \omega_0 \ll \omega_p$) characterized by a dispersion relation of identical form [upon setting $\omega_b^2 \to \omega_f^2 = (1/\omega_0^2 + 1/\omega_p^2)^{-1} < \omega_0^2$ therein]. The propagation band then extends from ω_f up to ω_0, and the bandwidth strongly depends on the plasma frequency ω_p (in fact grows wider as ω_p decreases). Let us therefore retain the generic form of the dispersion relation (9) in the following.

Equation (9) implies that

$$\omega^2(\epsilon-1) = \frac{\omega^2(\omega_0^2-\omega_p^2-\omega_b^2)+\omega_p^2\omega_b^2}{\omega^2-\omega_0^2}\,, \tag{10}$$

which readily leads to

$$\frac{\partial[\omega^2(\epsilon-1)]}{\omega^2\partial\omega} = -\frac{2(\omega_b^2-\omega_0^2)(\omega_p^2-\omega_0^2)}{\omega(\omega^2-\omega_0^2)^2}\,. \tag{11}$$

Substituting (10) into Eq. (7), we obtain

$$\tilde{\mathbf{B}} = \frac{c}{8\pi n_0 e}\,\frac{(\omega_b^2-\omega_0^2)(\omega_p^2-\omega_0^2)}{\omega(\omega^2-\omega_0^2)^2}\,\nabla\times(\mathbf{k}\,|\mathcal{E}|^2)\,. \tag{12}$$

The electron Larmor rotation, due to the induced magnetic field $\tilde{\mathbf{B}}$, is associated with the gyrofrequency $\omega_c = e\tilde{B}/mc$ (m is the electron mass); the latter is found from Eq. (12) as

$$\omega_c = \frac{1}{8\pi n_0 m}\,\frac{(\omega_b^2-\omega_0^2)(\omega_p^2-\omega_0^2)}{\omega(\omega^2-\omega_0^2)^2}\,\frac{k}{L}\,|\mathcal{E}|^2\,, \tag{13}$$

where L is a characteristic scalelength associated with the gradient ∇ (i.e. essentially expressing the spatial variation of $|\mathcal{E}|^2$). The ratio between the electron gyro and EM wave frequencies is

$$\frac{\omega_c}{\omega} = \frac{(\omega_b^2-\omega_0^2)(\omega_p^2-\omega_0^2)}{2(\omega^2-\omega_0^2)^2\omega^2}\,\frac{k u_0^2}{L}\,, \tag{14}$$

where we have denoted the electron quiver speed $u_0 = e|\mathcal{E}|/m\omega_{p,e}$, and the electron plasma frequency $\omega_{p,e} = (4\pi n_0 e^2/m)^{1/2}$. Equations (12) to (14) express a general result, regarding the $H_\parallel$ parallel-to-SRR case (see above). We note that spontaneously generated magnetic field is directed along $\nabla\times(\mathbf{k}\,|\mathcal{E}|^2)$, given the ordering $\omega_0 < \omega < \omega_b < \omega_p$.

For $H_\perp$ transverse to the SRR axis, Eq. (12) reduces to [18]

$$\tilde{\mathbf{B}} \approx \frac{c}{8\pi n_0 e}\,\frac{\omega_p^2\,(\omega_f^2-\omega_0^2)}{\omega(\omega^2-\omega_0^2)^2}\,\nabla\times(\mathbf{k}\,|\mathcal{E}|^2)\,. \tag{15}$$

That the generated magnetic field here lies in the direction *opposite to* $\nabla\times(\mathbf{k}\,|\mathcal{E}|^2)$, given that $\omega_f < \omega < \omega_0 \ll \omega_p$, for the propagation. Also note that the gyrofrequency changes sign here [cf. (13), upon shifting ω_b ($> \omega_0$) to ω_f ($< \omega_0$) therein; see the definitions above], implying an inverse sense of electron rotation.

As an illustration of the above assertions, we may consider a set of typical values for the $H_\perp$ polarization case (cf. [8]), given by $\omega_f = 4.2$ GHz, $\omega_0 = 4.6$ GHz and $\omega_p = 13$ GHz; the permitted propagating frequency bandwidth is about 400 MHz. Taking $u_0 \simeq 10^{-3}c$, $k = 2\pi/\lambda \simeq 14.7m^{-1}$ (for a propagating EM wave frequency $\omega = 4.4$ GHz) and $L \simeq 10\lambda$, we obtain $\omega_c \simeq 5.6$ kHz, i.e. $\omega_c/\omega \simeq 1.27 \times 10^{-6}$, or higher (we recall that ϵ has a pole at $\omega = \omega_0$, so that the magnetic field strength could be substantial in its vicinity; see Fig. 1a). Similar values are obtained in the permitted frequency band, i.e. for ω in $[4.2, 4.6]$ GHz (see above); cf. Fig. 1. However, this is only a rough estimation of the correction to the HEF associated magnetic field; lower or higher values might also be obtained, depending on the value of $\alpha = L/\lambda$ characterizing the HEF modulation (recall that the modulation length L, here taken to be equal to 10λ, might as well be lower or higher, and so could the value of $\beta = u_0/c$). This fact might essentially affect the refraction index of the LH medium (e.g. via a nonlinear "Kerr-like" field amplitude dependence mechanism), which would in turn influence propagation properties of EM waves and the medium's anticipated "transparency" to them.

It may be added, for rigor, that a similar magnetization mechanism has been proposed in the past as a plasma diagnostics method by Gradov and Stenflo [19]. The nonlinear generation of magnetic fields was also considered by Stenflo and Shukla [20], in the context of the filamentation instability in magnetoactive semiconductors [see Eq. (10) in Ref. [20]].

In conclusion, we have shown that the propagation of modulated electromagnetic wave packets in a negative refraction index material, which is composed of two superposed lattices of thin conducting wires and split ring resonators (according to a widely accepted recent theoretical suggestion), results in spontaneous generation of magnetic fields, due to the ponderomotive coupling of the slow electron fluid motion to the HF electric field. Explicit expressions for the generated magnetic field have been calculated in terms of intrinsic material parameters. The present work is the first prediction of the magnetization of LH media due to the ponderomotive force of electromagnetic waves. The latter could, in turn, be scattered off by randomly distributed magnetic fields, and could be used for diagnostic purposes.

Acknowledgments

I.K. is grateful to Professor Greg Morfill at Max-Planck-Institut für extraterrestrische Physik (Garching, Germany) for the award of a fellowship, in the period during which this research was carried out. Partial support from the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 591 entitled "Universelles Verhalten Gleichgewichtsferner Plasmen: Heizung, Transport und Strukturbildung" is also gratefully acknowledged.

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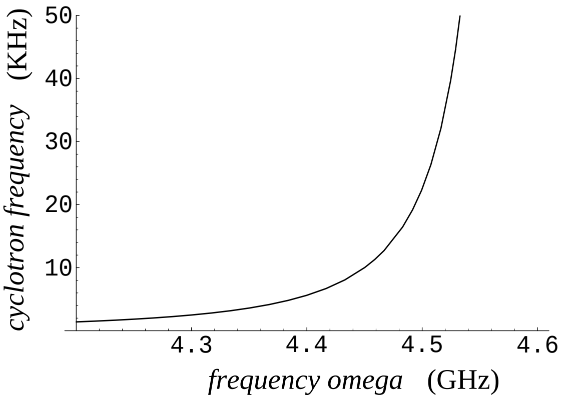
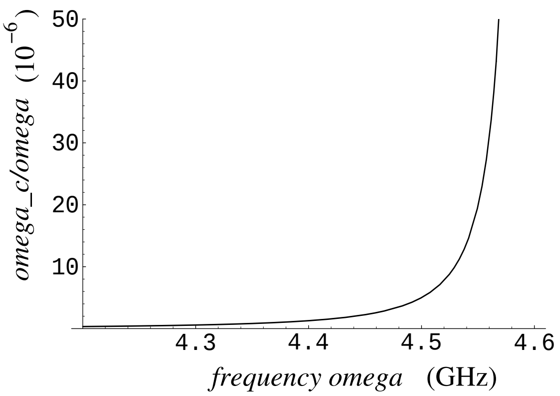


(a)



(b)

FIG. 1: (a) The (absolute) value of the electron gyrofrequency ω_c, which is associated with spontaneously generated magnetic field (in kHz) is depicted vs. the propagating EM wave frequency ω (in GHz), in the "transparent" frequency range. (b) The ratio ω_c/ω (in 10^6 axis units) is depicted vs. the propagating EM wave frequency ω (in GHz).