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MAGNETO-GRAVITY WAVES AND THE HEATING OF THE SOLAR CORONA *

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ABSTRACT

It is generally believed that the heating of the solar corona is caused by waves originating in the photosphere and propagating into the corona where their energy is dissipated. The medium through which these waves propagate is in general permeated by magnetic fields complicating the behavior of this propagation considerably. ~~We have therefore analyzed~~ ~~the~~ Wave motions in a plasma permeated by constant magnetic and gravitational fields were analyzed. ~~In~~ general, three wave modes were found~~, which we called~~ the ~~t mode, mode and the Alfvén mode~~. Each ~~mode~~ of these was ~~found to be~~ strongly coupled to each of the three kinds of motion; acoustic, gravity, and hydromagnetic. However, the Alfvén mode was found to be separable from the dispersion relation and therefore independent of compressibility and gravity. The local dispersion relation is derived and expressed in nondimensional form independent of the constants that describe a particular atmosphere. From the dispersion relation one can show that rising waves propagate either with a constant or a growing wave amplitude depending on the magnitudes and directions of the gravitational field, magnetic field and the wave vector. The variation of the density with height is taken into account by a generalized W.K.B. method. Equations are found which give the height at which wave reflection occurs, giving the upper bound for possible wave propagation.

MAGNETO-GRAVITY WAVES AND THE HEATING OF THE SOLAR CORONA

I. INTRODUCTION

The extremely high temperatures of the solar corona are generally believed to be due to the transfer of energy from the convection zone by waves. The waves that have been considered are acoustic waves, gravity waves and hydromagnetic waves. One very special situation of magneto-gravity waves has been treated by YU[1]. However, in a magnetized plasma atmosphere, it is in principle not correct to consider either one of these modes independently of the others. All modes interact with each other and must be considered simultaneously. Therefore, we have investigated plasma wave propagation within a magnetized atmosphere of infinite conductivity under the influence of gravity in the magneto-hydrodynamic (M.H.D.) approximation, which is valid for low-frequency waves. We only consider the propagation of these waves and therefore neglect dissipative effects arising from viscosity, electrical resistivity and heat conductivity.

In the equilibrium state, we allow for the exponential dependence in height (z-direction) of the density and pressure, as is certainly the case for an isothermal atmosphere. In the second part of this paper, this variation of density with height in the magnetic field term in the equation of motion is taken into account by a generalized W.K.B. method.

The addition of a magnetic field complicates the problem, not only by the introduction of another wave mode, but also due to the fact that the magnetic field is a vector, thereby creating a third direction, which can be aligned arbitrarily with respect to the gravity vector $\underline{g}$ and the wave vector $\underline{k}$.

2. FUNDAMENTAL EQUATIONS

The fundamental equations necessary to describe the wave motions are (in Gaussian units)

Euler equation

$$\rho\left[\frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \nabla)\,\underline{v}\right] = -\nabla p + \rho \underline{g} - \frac{1}{4\pi}\,\underline{H} \times (\nabla \times \underline{H}) \tag{2.1}$$

In this equation $\underline{v}$ is the velocity vector of a particle of the oscillating medium, p is the pressure, ρ the density, $\underline{g}$ the gravity vector, and $\underline{H}$ the magnetic field vector. For convenience we choose $\underline{g}$ to be along the negative z-direction (downward), and we choose to orient the coordinate system so that the arbitrarily directed unperturbed magnetic field in the absence of wave motion has no y-component.

Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{v}) = 0 \tag{2.2}$$

Second law of thermodynamics (adiabatic approximation)

$$\frac{ds}{dt} = \frac{\partial s}{\partial t} + \underline{v} \cdot \nabla s = 0 \;, \tag{2.3}$$

where s is the specific entropy.

Equation of state (differential form)

$$d\rho = \left.\frac{\partial \rho}{\partial s}\right|_p ds + \left.\frac{\partial \rho}{\partial p}\right|_s dp \tag{2.4}$$

Ohm-Maxwell equation

$$\frac{\partial H}{\partial t} = \nabla \times (\underline{v} \times \underline{H}) \quad . \tag{2.5}$$

For the equilibrium conditions for an ideal gas at constant temperature, the variation of density and pressure with height is exponential. Thus, we have at equilibrium

$$\rho_{eq} = \rho_o \ e^{-z/h}$$

and (2.6)

$$p_{eq} = p_o \ e^{-z/h} \quad ,$$

where $h = a^2/g\gamma$ is the "scale height".

3. LINEARIZATION

By assuming that all deviations of the perturbed quantities from their equilibrium values are small, we may put

$$\rho(\underline{r},t) = \left[\rho_o + \rho'(\underline{r},t)\right] e^{-z/h}$$

$$p(\underline{r},t) = \left[p_o + p'(\underline{r},t)\right] e^{-z/h}$$

$$s(\underline{r},t) = s_{eq} + s'(\underline{r},t)$$

and

$$\underline{H} = \underline{H}_0 + H'(\underline{r},t) \qquad , \tag{3.1}$$

where ρ_0 , p_0 , and $\underline{H}_0$ are constant. The quantities ρ', p', s' and $\underline{H}'$, together with the velocity $\underline{v} = \underline{v}(\underline{r},t)$ are considered to be perturbing quantities of first order.

By considering the equilibrium conditions and the linearisation of the basic equations, eqs. (2.1), (2.2), (2.3), (2.4), and (2.5), and upon taking the time derivative of the Euler equation (2.1), we have upon substitution

$$\frac{\partial^2 \underline{v}}{\partial t^2} + \frac{a^2}{h}\nabla(\underline{v}\cdot\underline{e}_z) - a^2\nabla(\nabla\cdot\underline{v})$$

$$+\nabla\left[\frac{a^2}{\rho_0}(\underline{v}\cdot\underline{e}_z)\left.\frac{\partial\rho}{\partial s}\right)_p\frac{ds_{eq}}{dz}\right] - \frac{a^2}{h^2}(\underline{v}\cdot\underline{e}_z)\,\underline{e}_z$$

$$+\frac{a^2}{h}(\nabla\cdot\underline{v})\,\underline{e}_z - \frac{a^2}{h\rho_0}(\underline{v}\cdot\underline{e}_z)\left.\frac{\partial\rho}{\partial s}\right)_p\frac{ds_{eq}}{dz}\;\underline{e}_z$$

$$-\frac{(\underline{v}\cdot\underline{e}_z)}{h}\underline{g} + (\nabla\cdot\underline{v})\underline{g}$$

$$+\frac{1}{4\pi\rho_{eq}}\underline{H}_0\times\left\{\nabla\times\left[\nabla\times(\underline{v}\times\underline{H}_0)\right]\right\} = 0 \tag{3.2}$$

where $\underline{e}_z$ is the unit vector in the z-direction and a is the velocity of sound. This vector differential equation (3.2) is the linearized equation of motion and it represents a set of three linear homogeneous differential equations for the unknowns v_x , v_y and v_z.

Using the assumption of a perfect gas, the expression given above for the scale height, and well-known thermodynamic expressions relating the density, pressure, entropy and temperature, one finds

$$a^2 \left.\frac{\partial \rho}{\partial s}\right)_p \frac{ds_{eq}}{dz} = - \rho_0 g(\gamma - 1) \qquad , \qquad (3.3)$$

where γ is the ratio of specific heats.

By the substitution of eq. (3.3) into (3.2), we have the linearized equation of motion

$$\frac{\partial^2 \underline{v}}{\partial t^2} - a^2 \text{ grad div } \underline{v} - \text{grad}(\underline{v} \cdot \underline{g}) - \underline{g}(\gamma - 1) \text{ div } \underline{v}$$

$$+ \frac{1}{4\pi \rho_{eq}} \underline{H}_0 \times \text{curl curl}(\underline{v} \times \underline{H}_0) = 0 \quad . \qquad (3.4)$$

4. LOCAL DISPERSION RELATION

In the local situation, we assume the wavelength to be small compared to the scale height. This assumption allows us to consider ρ_{eq} as a constant in the last term of eq. (3.4). Thus, this equation is now a second-order differential equation with constant coefficients, which yields to a plane-wave solution. For plane waves, we have

$$\underline{v}(\underline{r},t) = \underline{v}\, e^{i(\underline{k}\cdot\underline{r} - \omega t)} \tag{4.1}$$

where $\underline{v}$ is now a constant vector.

Upon substituting eq. (4.1) into eq. (3.4) and performing the mathematical operations, we have

$$\omega^2\underline{v} - a^2(\underline{k}\cdot\underline{v})\,\underline{k} + i\,(\underline{v}\cdot\underline{g})\,\underline{k} + i\,(\gamma - 1)\,(\underline{k}\cdot\underline{v})\,\underline{g}$$

$$+ \frac{1}{4\pi\rho_{eq}}\;\underline{H}_o \times\left\{\underline{k}\times\left[\underline{k}\times(\underline{v}\times\underline{H}_o)\right]\right\} = 0 \tag{4.2}$$

The vector equation (4.2) represents a set of three linear homogeneous equations for the unknowns v_x, v_y and v_z. The condition for a non-trivial solution, the vanishing of the determinant of the coefficients, is properly called the local dispersion relation

$$\omega^6 + \omega^4\left[-a^2k^2 - i\,g\gamma k_z - \frac{H_o^2\,k^2}{4\pi\rho_{eq}} - \frac{\left(\underline{H}_o\cdot\underline{k}\right)^2}{4\pi\rho_{eq}}\right]$$

$$+ \omega^2\left\{g^2(\gamma-1)(k^2-k_z^2) + i\,g\gamma\left[\frac{(\underline{H}_o\cdot\underline{k})^2}{4\pi\rho_{eq}}\,k_z + \frac{H_z(\underline{H}_o\cdot\underline{k})}{4\pi\,\rho_{eq}}\;k^2\right]\right.$$

$$
+ 2 a^2 k^2 \frac{(\underline{H}_o \cdot \underline{k})^2}{4\pi \rho_{eq}} + \frac{H_o^2(\underline{H}_o \cdot \underline{k})^2}{(4\pi \rho_{eq})^2} k^2 \Big\} - ig\gamma \frac{H_z(\underline{H}_o \cdot \underline{k})^3}{(4\pi \rho_{eq})^2} \quad k^2
$$

$$
- g^2(\gamma - 1) \frac{(\underline{H}_o \cdot \underline{k})^2}{4\pi \rho_{eq}} (k^2 - k_z^2) - \frac{a^2 k^2 (\quad \cdot \underline{k})^4}{(4\pi \rho_{eq})} = 0. \tag{4.3}
$$

Eq. (4.3) can be factorized into two parts, where one part turns out to be the dispersion relation for the Alfvén mode and the other part is the dispersion relation for two distinct modes, which involve the intimate coupling of magneto-hydrodynamic and gravity wave motion. By factorization, we obtain

$$
\omega^2 = \frac{(\underline{H}_o \cdot \underline{k})^2}{4\pi \rho_{eq}} \qquad \text{(the Alfvén mode)} \tag{4.4}
$$

and

$$
\omega^4 + \omega^2\Big(- i\, g\, \gamma k_z - a^2 k^2 - \frac{H_o^2}{4\pi \rho_{eq}} k^2\Big) + g^2(\gamma - 1)(k^2 - k_z^2)
$$

$$
+ a^2 \frac{(\underline{H}_o \cdot \underline{k})^2}{4\pi \rho_{eq}} k^2 + ig\gamma \frac{H_z(\underline{H}_o \cdot \underline{k})}{4\pi \rho_{eq}} k^2 = 0 \tag{4.5}
$$

From eq. (4.5), which is quadratic in ω^2, we obtain two other modes which we call the + mode and the − mode.

$$W_{\pm}^2 = \left(\tfrac{1}{2}K^2 + \frac{K^2}{\gamma\beta}\right) + i\left[\left(\frac{\gamma}{2}\right)^{3/2}\beta^{\frac{1}{2}}\; K\cos\theta\right]$$

$$\pm\left\{\left[\tfrac{1}{4}K^4 + \frac{K^4}{\gamma^2\beta^2} + \frac{K^4}{\gamma\beta} - \frac{1}{8}\beta\gamma^3\; K^2\cos^2\theta \;-\frac{1}{2}\gamma\beta(\gamma-1)K^2\sin^2\theta\right.\right.$$

$$\left. - \frac{2}{\gamma\beta}K^4\cos^2\phi\right] + i\left[\left(\frac{\gamma}{2}\right)^{3/2}\beta^{\frac{1}{2}}\cos\theta - \left(\frac{2\gamma}{\beta}\right)^{\frac{1}{2}}\cos\phi\;\cos\eta\right.$$

$$\left.\left. + \left(\frac{\gamma}{2\beta}\right)^{\frac{1}{2}}\cos\theta\right]K^3\right\}^{\frac{1}{2}}$$

(4.6)

Eq. (4.6) is written in a non-dimensional form by the use of a characteristic frequency and a characteristic wave number, defined by

$$\omega_0 = \frac{H_0\,g}{a^2(4\pi\rho_{eq})^{\frac{1}{2}}} \quad \text{and} \quad k_0 = \frac{\omega_c}{a} \qquad (4.7)$$

The non-dimensional frequency and wave number are

$$W = \frac{\omega}{\omega_c} \qquad \text{and} \qquad K = \frac{k}{k_o}$$

and furthermore

$$\beta = \frac{p_{eq}}{H_o^2/8\pi}$$ is the ratio of

hydrostatic pressure to magnetic pressure. θ is the angle between $\underline{k}$ and $\underline{g}$, η is the angle between $\underline{H}_o$ and $\underline{g}$, and φ is the angle between $\underline{k}$ and $\underline{H}_o$.

In general, the dispersion relation is complex, which implies the existence of waves with either exponentially increasing or decreasing wave amplitude, corresponding to rising or falling waves, respectively. This behaviour can be easily understood. A rising wave propagates into a medium of decreasing density. As a result, a decreasing number of fluid elements will participate in the wave motion, so that in order to conserve wave energy, the wave amplitude has to grow. By a similar argument the wave amplitude for falling waves must decrease. In the following analysis we will omit the falling waves with decreasing wave amplitude, since they are of no relevance to the heating of the solar corona. The rising waves with growing wave amplitudes will eventually steepen into shock waves accompanied by a large energy dissipation. However, under certain conditions the waves can propagate with constant amplitude, depending on the magnitude and direction of the wave number vector, if the restoring forces of compressibility, gravity, and magnetic field tend to interfere.

The peculiar dependence of growing wave amplitudes upon the angles θ, η, and φ is absent in treatments which neglect the coupling of hydromagnetic and internal gravity wave motion. For certain angle configurations and ranges of the wave number there is no wave growth and hence no shock waves. In these cases the wave propagates with constant amplitude; therefore the imaginary part of the frequency must be zero. This condition is given in nondimensional form by

$$B = \mp\left[\tfrac{1}{2}(C^2 + D^2)^{\frac{1}{2}} - \frac{C}{2}\right]^{\frac{1}{2}} , \qquad (4.8)$$

where the upper sign represents the + mode and and the lower sign represents the − mode, and where

$$B = \left(\frac{\gamma}{2}\right)^{3/2} \beta^{\frac{1}{2}} K \cos\theta ,$$

$$C = \tfrac{1}{2} K^4 + \frac{K^4}{\gamma^2\beta^2} + \frac{K^4}{\gamma\beta} - \frac{2K^4}{\gamma\beta} \cos^2\varphi$$

$$- 1/8\ \beta\gamma^3 K^2 \cos^2\theta - \tfrac{1}{2}\gamma\beta(\gamma - 1) K^2 \sin^2\theta \quad ,$$

and

$$D = \left[\left(\frac{\gamma}{2}\right)^{3/2} \beta^{\frac{1}{2}} \cos\theta + \left(\frac{\gamma}{2\beta}\right)^{\frac{1}{2}} \cos\theta - \left(\frac{2\gamma}{\beta}\right)^{\frac{1}{2}} \cos\varphi \cos\eta \right] K^3$$

5. W.K.B. APPROXIMATION

We assumed above that ρ_{eq} in the magnetic term in the equation of motion (3.4) could be considered a constant in the lowest approximation if the wavelength was small compared with the scale height. A better approximation is obtained by taking into account the slowly varying change of the equilibrium density in the vertical direction by a generalized W.K.B. method.

Toward this end, we take a Fourier transform of eq. (3.4) for the x − y − t dependence and then introduce the z-dependence by means of an unknown function $f(z)$. We put

$$\underline{v}(x, y, z, t) \Rightarrow \underline{v}_0 \ f(z) \ e^{i(k_x x + k_y y - \omega t)} \quad , \qquad (5.1)$$

where $\underline{v}_0$ is a constant vector and

$$f(z) = A(z)\, e^{\Phi(z)} \qquad (5.2)$$

$A(z)$ is a slowly varying function of z. We collect the terms $\left(\frac{dA}{dz}\right)\frac{d\Phi}{dz}$ and $A\frac{d^2\Phi}{dz^2}$ into one equation and the terms A, $A\frac{d\Phi}{dz}$ and $A\left(\frac{d\Phi}{dz}\right)^2$ into a second equation. In the spirit of the W.K.B. approximation (short wavelength approximation), we neglect terms multiplied by $\frac{dA}{dz}$ and $\frac{d^2A}{dz^2}$. Both of these equations are three-dimensional vector equations for $\underline{v}_c$ and the condition for a non-trivial solution is determined by setting the determinant of the coefficients of each system of equations equal to zero. From the first equation we obtain a first-order differential equation with the solution

$$A = \text{const}\left(\frac{d\Phi}{dz}\right)^{-\frac{1}{2}}, \qquad (5.3)$$

which is the expected relation between the amplitude and phase from employing the W.K.B. method. From the second equation we obtain a first-order differential equation of the sixth degree for the phase $\Phi(z)$. This differential equation can be factorised into two differential equations of the second and fourth degree in a similar manner as the local dispersion relation was factorised into two algebraic equations of the second and fourth order. These two differential equations are

$$\left(\frac{d\Phi}{ds}\right)^2 + 2i\,\frac{(\underline{H}_o\cdot\underline{k}_\perp)}{(\underline{H}_o\cdot\underline{e}_s)}\,\frac{d\Phi}{ds} - \frac{(\underline{H}_o\cdot\underline{k}_\perp)^2}{(\underline{H}_o\cdot\underline{e}_s)^2}$$

$$+ \frac{4\pi\omega^2}{(\underline{H}_o\cdot\underline{e}_s)^2}\,\rho_{eq}(s) = 0 \quad , \tag{5.4}$$

and

$$\left(\frac{d\Phi}{ds}\right)^4 + \left(\frac{d\Phi}{ds}\right)^3\left[2i\,\frac{(\underline{H}_o\cdot\underline{k}_\perp)}{(\underline{H}_o\cdot\underline{e}_s)} - \frac{\varepsilon\gamma}{a^2}\right]$$

$$+\left(\frac{d\Phi}{ds}\right)^2\left[\frac{4\pi\omega^2}{(\underline{H}_o\cdot\underline{e}_s)^2}\,\rho_{eq}(s) - \frac{\omega^2 H_o^2}{a^2(\underline{H}_o\cdot\underline{e}_s)^2} - k_\perp^2\right.$$

$$\left.- \frac{(\underline{H}_o\cdot\underline{k}_\perp)^2}{(\underline{H}_o\cdot\underline{e}_s)^2} - i\,\frac{\varepsilon\gamma}{a^2}\,\frac{(\underline{H}_o\cdot\underline{k}_\perp)}{(\underline{H}_o\cdot\underline{e}_s)}\right] + \frac{d\Phi}{ds}\left[\frac{\varepsilon\gamma k_\perp^2}{a^2}\right.$$

$$\left.- \frac{\varepsilon\gamma\omega^2 4\pi}{a^2(\underline{H}_o\cdot\underline{e}_s)^2}\,\rho_{eq}(s) - 2ik_\perp^2\,\frac{(\underline{H}_o\cdot\underline{k}_\perp)}{(\underline{H}_o\cdot\underline{e}_s)}\right]$$

$$+ \frac{4\pi\omega^4}{a^2(\underline{H}_o \cdot \underline{e}_z)^2} \rho_{eq}(z) - \frac{4\pi\omega^2 k_\perp^2}{(\underline{H}_o \cdot \underline{e}_z)^2} \rho_{eq}(z)$$

$$- \frac{\omega^2}{a^2} \frac{H_o^2 k_\perp^2}{(\underline{H}_o \cdot \underline{e}_z)^2} + \frac{4\pi g^2(\gamma - 1) k_\perp^2}{a^2(\underline{H}_o \cdot \underline{e}_z)^2} \rho_{eq}(z)$$

$$+ k_\perp^2 \frac{(\underline{H}_o \cdot \underline{k}_\perp)^2}{(\underline{H}_o \cdot \underline{e}_z)^2} + i g \gamma \frac{k_\perp^2}{a^2} \frac{(\underline{H}_o \cdot \underline{k}_\perp)}{(\underline{H}_o \cdot \underline{e}_z)} = 0$$

(5.5)

where $\underline{k}_\perp = k_x \underline{e}_x + k_y \underline{e}_y$, and $\underline{e}_x$ and $\underline{e}_y$ are unit vectors in the x- and y-direction, respectively. The solution of the differential eq. (5.4) gives the phase for the W.K.B. solution for the Alfvén mode. The solution of the differential eq. (5.5) gives the phase for the W.K.B. solution of the two remaining modes. Both of these differential equations can be brought into a form that can be solved by the separation of variables, after first solving both equations as algebraic equations of the second and fourth degree in the unknown $\frac{d\phi}{dz}$. This can always be done in closed form for an algebraic equation up to the fourth degree. By putting $\frac{d\phi}{dz} = i k_z$ in eq. (5.5), we obtain the same dispersion relation as eq. (4.5), which therefore justifies calling eq. (4.5) the local dispersion relation.

The condition $\frac{d\Phi}{dz} = 0$ implies that the z-component of the wave number vector $\underline{k}$ is zero. This gives the condition for the termination of the wave propagation in the z-direction, that is, the vertical penetration depth. At this point $\underline{k} = \underline{k}_\perp$. Setting $\frac{d\Phi}{dz} = 0$ in eq. (5.4) and using the definition of ρ_{eq} (the first equation of (2.6)), gives upon solving for the penetration depth, z_{pd},

$$z_{pd} = h \ln\left[\frac{4\pi \rho_o \omega^2}{(\underline{H}_o \cdot \underline{k}_\perp)^2}\right] , \tag{5.6}$$

where h is the scale height. This is the penetration depth for the Alfvén mode. Setting $\frac{d\Phi}{dz} = 0$ in eq. (5.5) gives upon solving for the penetration depth

$$z_{pd} = h \ln\left[\frac{4\pi \rho_o \omega^4 - 4\pi \rho_o \omega^2 a^2 k_\perp^2 + 4\pi \rho_o g^2(\gamma-1) k_\perp^2}{\omega^2 H_o^2 k_\perp^2 - a^2 k_\perp^2(\underline{H}_o \cdot \underline{k}_\perp)^2 - i g\gamma(\underline{H}_o \cdot \underline{k}_\perp)(\underline{H}_o \cdot \underline{e}_z) k_\perp^2}\right] , \tag{5.7}$$

where $\omega = \omega(k)$ has to be substituted from the relation (4.5).

6. CONCLUSION

We have considered small-amplitude low-frequency waves in a plasma of infinite conductivity under the influence of a constant external magnetic and gravitational field. We have shown the existence of three wave modes, which in general have exponentially growing amplitudes. These growth rates may easily be determined from our dispersion relation. One mode, identical with the Alfvén mode, has always constant amplitude and is independent of compressibility and gravity.

From the dispersion relations for the + and - modes, it can be seen that there is a large anisotropy with regard to constant amplitude wave propagation. This behaviour does not occur in a treatment neglecting the wave mode coupling. Therefore, our results suggest that a search be made for this anistropy in gravity wave propagation in the outer solar atmosphere.

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