(IJITR) INTERNATIONAL JOURNAL OF INNOVATIVE TECHNOLOGY AND RESEARCH
Volume No.5, Issue No.3, April – May 2017, 5955-5961.

Magneto-Hydro-Dynamics Boundary Layer Viscous Flow Over A Stretching Sheet: An Approach Based on Homotopy Perturbation Method

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Abstract- **In this paper by means of homotopy perturbation method an approximate solution of the steady magneto-hydrodynamics flow under slip condition over a permeable stretching sheet has been studied. The homotopy perturbation method (HPM) has been used to solve the governing non linear equation to derive an approximate analytical solution. The main feature of the HPM is that it produces analytic expression for the solution to nonlinear differential equations and the obtained analytic solution is in the form of an infinite power series. In this work the analytic solution obtained by using first two terms from HPM solution. The solution is given in a closed form equation and is an approximate solution of the full governing Navier–Stokes equations. The effects of the slip, the magnetic and the mass transfer parameters are discussed. Results show that there is only one physical solution for any combination of the slip, the magnetic, and the mass transfer parameters. The velocity and shear stress profiles are greatly influenced by these parameters.**

Keywords: **Magneto-Hydro-Dynamics; Stretching Sheet; Slip Effect; Homotopy Perturbation Method; Similarity Solution;**

I. INTRODUCTION

During the last decades, flow of incompressible viscous fluid and heat transfer phenomena over stretching sheets have received great attention, owning to the abundance of practical application in the chemical and manufacturing process such as aerodynamics, extrusion of plastic sheets, continuous casting of metals, glass, fibres and paper production. The study of hydrodynamic flow of an electrically conducting fluid caused by the deformation of the wall of a vessel containing a fluid is of considerable interest in a modern metallurgical and metal-working process.

The study of magneto-hydrodynamic has important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field (Ganesan & Palani [1]). At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment (Seddeek [2]).

The viscous flow due to a stretching boundary occurs in expanding or contracting surfaces in a fluid such as extrusion of sheet material form a die and the elongation of pseudo-pods. The flow and heat transfer of a viscous and incompressible fluid over a stretching sheet has attracted the interest of many researchers in view of its applications in many industrial manufacturing process- examples are in the glass blowing, the cooling and/or drying of papers and textiles, the extrusion of a polymer in a melt-spinning process, metals and plastics, continuous casting and spinning of fibers, etc. Crane [3] was the first who studied the two-dimensional steady flow of an incompressible viscous fluid caused by a linearly stretching plate and obtained an exact solution in closed analytical form. Since then, many authors have studied various aspects of this problem, as Ishak et al. [4] have studied the flow behaviors due to a stretching sheet in the presence of magnetic field, considering some other physical features such as power-law velocity and buoyancy effect, with various surface heating conditions.

All the above mentioned studies continued their discussions by assuming the no slip boundary conditions. The no-slip boundary condition that is the assumption that a liquid adheres to a solid boundary, is one of the central tenets of the Navier–Stokes theory. However, there are situations wherein this condition does not hold. Partial velocity slip may occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances. Recently, many researchers investigated the flow problems taking slip flow condition at the boundary. The fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities.

Takher et al. [5] investigated the unsteady magneto hydrodynamic flow due to the impulsive motion of a stretching sheet and reported that the surface heat transfer increase upto a certain portion of time, beyond that it decreases. There are many situations where the flow and heat transfer are unsteady due to sudden stretching of a sheet. Heat transfer of an unsteady boundary layer flow over stretching sheets by Elbashbeshy and Bazid [6] has studied that thermal boundary layer thickness and momentum boundary layer thickness decrease with unsteadiness parameter, on the other hand the effect of thermal radiation and boundary layer flow and heat transfer problem can be quite significant at high operating temperature. In view of this Elbashbeshy et al.[7], [8] and Hossain et al.[9] have studied thermal radiation of gray fluid which is emitting and absorbing radiation in non-scattering medium. The unsteady viscous flow over a continuous shrinking surface with mass transfer has been studied by Fang et al. [10]. Ali et al. [11] have studied the problem of unsteady fluid and heat induced by submerged stretching surface.

The newest analytical methods to solve the mathematical problems are both homotopy and perturbation methods. In recent years, the application of the homotopy perturbation methods in nonlinear problems has been studied by scientist and engineers. Homotopy techniques were applied to find all roots of nonlinear equation first in (Hillermeier, [12]),, (Yildirim, [13]), (Ganji and Rajabi,[14]), (Domairry and Nadim,[15]) and(Rajabi, [16]). Recently, the application of homotopy theory becomes a powerful mathematical tool, when it is successfully coupled with perturbation theory (He, [17], [18]) and (Ganji, [19]).

Motivated by the above investigations and possible applications, the purpose of this article is to studythe nature of the MHD flows over a stretching sheet applying homotopy perturbation method which is useful for finding the approximate analytical solution. In this paper the homotopy perturbation method (HPM) has been used to solve the governing nonlinear equation to derive an approximate analytical solution. HPM is an analytical procedure for finding the solution of problems which is based on the constructing a homotopy with an embedding parameter p that is considered as a small parameter

A. *Abbreviations and Acronyms*

u velocity component in x direction

v velocity component in y direction

P pressure of the fluid

L proportional constant of velocity slip

B magnetic field strength

M magnetic parameter

f dimensionless stream function

υ the kinematic viscosity

σ electrical conductivity of the fluid

$\psi(x, y)$ steam function

η similarity variable

α unknown parameter

γ velocity parameter

ρ density of the fluid

II. FORMULATION OF THE PROBLEM

Two-dimensional laminar flow over a continuously stretching sheet in an electrically conducting quiescent fluid is considered in the influence of transverse magnetic field. The sheet stretching velocity and the wall mass transfer velocity as is taken respectively as $U_w = U_0(x)$ and $v_w = v_w(x)$, which will be determined later. A uniform magnetic field of strength B is applied normal to the stretching surface. The induced magnetic field is being neglected since the magnetic Reynolds number for the flow is considered to be very small. No external electric field is applied so the effect of polarization of fluid is neglected. The x-axis is taken along the continuous surface in the direction of motion with the slot as the origin and y-axis is perpendicular to it as shown in Fig. 1

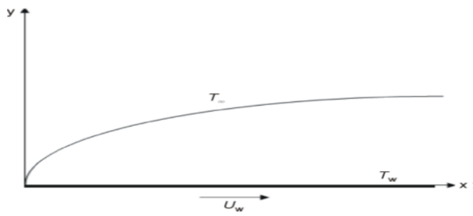

Fig. 1

B. *Equations*

The governing equation of continuity, momentum and energy under above assumptions reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\sigma B^2}{\rho}u \qquad (2)$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \qquad (3)$$

With the boundary conditions:

$$u(x,0)=U_0x+L\frac{\partial u}{\partial y} \quad (4)$$

$$v(x,0)=v_w(x) \quad (5)$$

$$u(x,\infty)=0 \quad (6)$$

Where u and v are velocity component in x and y direction respectively, υ is the kinematic viscosity, p is the pressure, ρ is the density, σ is the electrical conductivity of the fluid, and L is a proportional constant of the velocity slip. The magnetic field with strength B is applied in the vertical direction and the induced magnetic field is neglected considering that this group of N-S equations is valid for small magnetic field strength.

The continuity equation (1) is satisfied by the Cauchy- Riemann equation

$u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ where $\psi(x,y)$ is the steam function.

The following similarity transformation and dimensionless variables are introduced

$$\eta=y\sqrt{\frac{U_0}{\upsilon}} \quad (7)$$

$$\psi(x,y)=\sqrt{\nu U_0}\,xf(\eta) \quad (8)$$

where η is the similarity variable, ψ is the stream function and f is the dimensionless stream function.

The velocity u & v in terms of stream function with similarity variable are expressed as

$$u=u_0xf'(\eta) \quad (9)$$

$$v=-\sqrt{U_0\upsilon}\,f(\eta) \quad (10)$$

The wall-mass-transfer velocity becomes

$$v_w(x)=-\sqrt{U_0\upsilon}\,f(0) \quad (11)$$

The above momentum and energy equations is made free from dimension & reduces to

$$f'''+ff''-f'^2-M^2f'=0 \quad (12)$$

with the boundary conditions

$$f=s,\quad f'=1+\gamma f'' \quad \text{at} \quad \eta=0 \quad (13)$$

$$f'=0 \quad \text{at}\ \eta=\infty \quad (14)$$

where dash represent differentiation with respect toη, s is the wall mass transfer parameter showing the strength of the mass-transfer at the sheet, M is the magnetic parameter with $M^2=\frac{\sigma B^2}{\rho U_0}$ and γ is the velocity parameter with $\gamma=L\sqrt{\frac{U_0}{\nu}}$.

The pressure term can be obtained from equation (3) as $\frac{p}{\rho}=\nu\frac{\partial v}{\partial y}-\frac{v^2}{2}+$ constant

III. HOMOTOPY PERTURBATION METHOD

Equation (12) can be rewritten in the following form

$$u'''+F(u)=0 \quad (15)$$

where

$$F(u)=uu''-u'^2-M^2u' \text{ and } u(\eta)=f(\eta) \quad (16)$$

According to homotopy perturbation method the following has been constructed

$$u'''-\alpha^2u'+p\left[F(u)+\alpha^2u'\right]=0 \quad (17)$$

where α is unknown parameter so that when p=1 perturbation equation (17) reduces to the above equation (15)

The boundary conditions that is equation (13) and equation (14) reduces to

$$u(0)=s,u'(0)=1+\gamma u''(0) \quad (18)$$

$$u'(\infty)=0 \quad (19)$$

The solution of equation (17) is considered as a power series in p as follow:

$$u=u_0+pu_1+p^2u_2+.............. \quad (20)$$

Substituting equation (20) into equation (17) and equating the first two terms of identical power of p, that is

Coefficient of p^0:

$$u_0'''-\alpha^2u_0'=0, \quad (21)$$

with $u_0(0)=s$, $u_0'(0)=1$, $u_0'(\infty)=0$

Coefficient of p^1:

$$u_1'''-\alpha^2u_1'+u_0u_0''-u_0'^2-M^2u_0'+\alpha^2u_0'=0 \quad (22)$$

With

$$u_1(0)=0$$
$$u_1'(0)=\gamma u''_1(0)$$
$$u_1'(\infty)=0$$

The solution of equation (21) is given by,

$$u_0(\eta)=s+\frac{1}{\gamma\alpha^2+\alpha}[1-\exp(-\alpha\eta)] \quad (23)$$

Using the solution of $u_0(\eta)$ equation (22) has been reduced to

$$u_1'''-\alpha^2 u_1'=C_1 e^{-\alpha\eta} \quad (24)$$

Where,

$$C_1=[s\alpha+\frac{1}{\gamma\alpha+1}+M^2-\alpha^2](\frac{1}{\gamma\alpha+1})$$

The solution of $u_1(\eta)$ is obtaining as follow:

$$u_1(\eta)=B\eta e^{-\alpha\eta}, \quad (25)$$

where $B=\frac{C_1}{2\alpha^2}$,

Or

$$B=\frac{1}{2\alpha^2}[s\alpha+\frac{1}{\gamma\alpha+1}+M^2-\alpha^2](\frac{1}{\gamma\alpha+1}) \quad (26)$$

The solution of $u_1(\eta)$ is satisfied by the initial and boundary conditions that is $u_1(0)=0$ and $u_1'(\infty)=0$ respectively. Since $u_1(\eta)=B\eta e^{-\alpha\eta}$ therefore $u_1'(0)=B$ and $u''_1(0)=-2B\alpha$. Henceunder the other initial condition that is $u_1'(0)=\gamma u''_1(0)$, the following result obtain:

$$B(1+2\gamma\alpha)=0$$

Which is true only if either B=0 or $\alpha=-\frac{1}{2\gamma}$

CASE 1: $B\neq 0$ OR $\alpha=-\frac{1}{2\gamma}$

Therefore, the approximate solution under the constraint $B\neq 0$

$$u(\eta)=u_0(\eta)+u_1(\eta)$$

$$u(\eta)=s+\frac{1}{\gamma\alpha^2+\alpha}[1-\exp(-\alpha\eta)] + \frac{1}{2\alpha^2}[s\alpha+\frac{1}{\gamma\alpha+1}+M^2-\alpha^2]\frac{\eta e^{-\alpha\eta}}{\gamma\alpha+1} \quad (27)$$

Where $\alpha=-\frac{1}{2\gamma}$

Differentiating twice with respect to η that is $u''(\eta)$ which is given by

$$u''(\eta)=\frac{-\alpha}{\gamma\alpha+1}[\exp(-\alpha\eta)]+ \frac{1}{2\alpha}[s\alpha+\frac{1}{\gamma\alpha+1}+M^2-\alpha^2][-\frac{2e^{-\alpha\eta}}{\gamma\alpha+1}+\frac{\eta\alpha e^{-\alpha\eta}}{\gamma\alpha+1}] \quad (28)$$

The value of $f''(0)$ becomes

$$f''(0)=u''(0)=-2s-\frac{2(2+M^2)}{\alpha} \quad (29)$$

CASE2: $B=0$

The approximate solution when $B=0$ is given by

$$u(\eta)=u_0(\eta)+u_1(\eta)$$

or

$$u(\eta)=s+\frac{1}{\gamma\alpha^2+\alpha}[1-\exp(-\alpha\eta)],$$

where α is the root of the following third degree algebraic equation:

$$\lambda\alpha^3+(1-s\gamma)\alpha^2-(s+\gamma M^2)\alpha-1-M^2=0 \quad (30)$$

Differentiating twice with respect to η that is $u''(\eta)$ which is given by

$$u''(\eta)=\frac{-\alpha}{\gamma\alpha+1}[\exp(-\alpha\eta)] \quad (31)$$

The value of $f''(0)$ becomes

$$f''(0)=u''(0)=\frac{-\alpha}{\gamma\alpha+1} \quad (32)$$

Equation (30) is a complete cubic algebraic equation.The positive real roots of α is the only physically feasible solution which is based on the flow configuration. Moreover it is interesting to note that the solution by Wang [20] is the special case of our solution when M = 0 and the solution of

Andersson [21] and Wang [22] is also the special case of our general solution when both $M = 0$ and $s = 0$.

IV. RESULT

The physical feasible solution is determined for any combination of three control parameters, namely the mass-suction, the magnetic and the slip parameters considering only the positive root of equation (3.17). Few numerical values of α under different combination of the three parameters are shown in Table-1 and Table- 2 for M =0.5 and M =2.0 respectively. It is observed that the value of α increases with the increase of mass parameter and magnetic parameter. However it decreases with the increase of slip parameter**.**

TABLE.I:The solution of α when $M = 0.5$ under different values of s and γ

ϒ / s	0.0	0.5	1.0	3.0	10.0
3	3.3708	3.1984	3.1555	3.1114	3.0910
2	2.5000	2.3092	2.2481	2.1759	2.1378
1	1.7247	1.5326	1.4527	1.3364	1.2574
0	1.1180	0.9619	0.8837	0.7473	0.6231
-1	0.7247	0.6235	0.5669	0.4594	0.3499
-2	0.5000	0.4388	0.4013	0.3252	0.2420
-3	0.3708	0.3323	0.3070	0.2520	0.1875

TABLE-2: The solution of α when $M = 2.0$ under different values of s and γ

ϒ / s	0.0	0.5	1.0	3.0	10.0
3	4.1926	4.0651	4.0394	4.0153	4.0049
2	3.4495	3.3186	3.2876	3.2567	3.2427
1	2.7913	2.6631	2.6274	2.5890	2.5706
0	2.2361	2.1179	2.0796	2.0349	2.0118
-1	1.7913	1.6891	1.6511	1.6029	1.5760
-2	1.4495	1.3652	1.3301	1.2818	1.2525
-3	1.1926	1.1249	1.0938	1.0478	1.0178

In order to show to study the effect of the three parameters on the flow field and shear stresses, few typical velocity and shear stress profile for different combination of the three parameter are plotted in Figure 1.The effects of the magnetic parameter by keeping the slip and mass suction parameters constant has been shown at Figure 2. (a) and Figure 2(b) respectively as follows:-

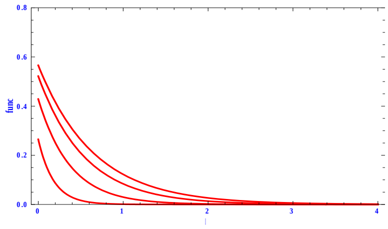

Figure. 2.(a)

Velocity profiles (func = $f'(\eta)$) at s=1 and γ=0.5 under different magnetic parameters M =0.5, 1.0, 2.0, 5.0.

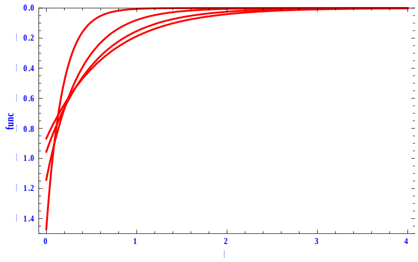

Figure 2.(b)

Shear stress profiles (func = $f''(\eta)$) at s=1 and γ=0.5 under different magneticparameters M =0.5, 1.0, 2.0, 5.0.

The effect of velocity slip parameters are shown in the following Figure3 (a), Figure3 (b), and Figure4 (a), Figure 4(b). A mass suction is applied at the surface with M=1 and s=1 in Figure 3(a) and Figure3(b) and with M=0.1 and s = - 1 in Figure 4(a) & Figure 4(b).

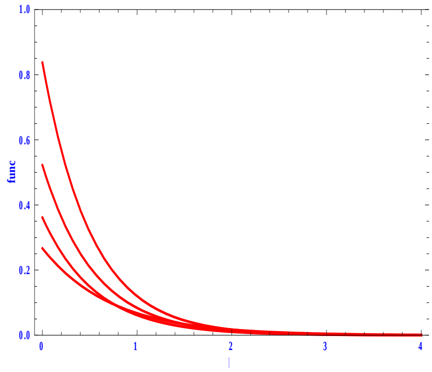

Figure 3(a)

Velocity profiles (func = $f'(\eta)$)at M =1and s = 1 under different velocity slip parameters γ = 0.1, 0.5, 1.0, 2.0.

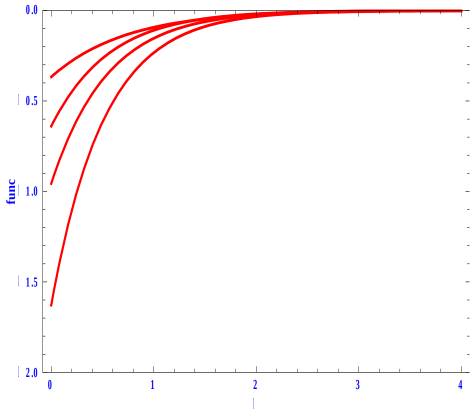

Figure 3 (b)

Shear stress profiles (, func = $f''(\eta)$) at M =1and s = 1 under different velocity slip parameters

γ = 0.1, 0.5, 1.0, 2.0.

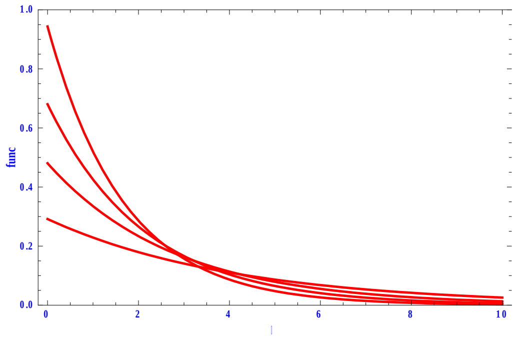

Figure 4(a)

Velocity profiles at s= −1 and M=0.1 under different velocity slip parameters γ = 0.1, 1.0, 3.0, 10.0.

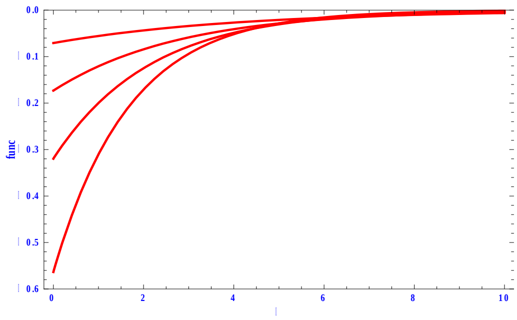

Figure 4(b)

Shear stress profiles at s= −1 and M=0.1 under different velocity slip parameters γ = 0.1, 1.0, 3.0, 10.0)

The effect of mass transfer parameters are studied graphically in figure 5(a) & Figure 5 (b) for M=0.5 and γ= 0.5 to visualize how the mass injection helps the flow to penetrate more into the fluidas follows:

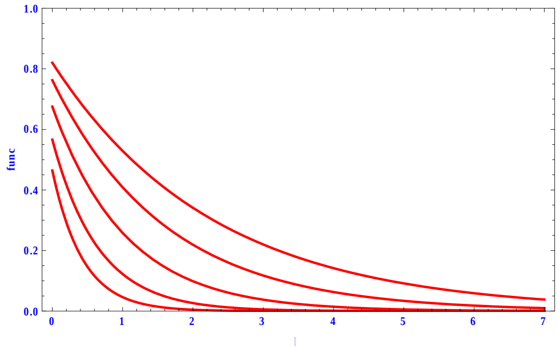

Figure 5(a)

Velocity profiles at$M$=0.5and γ=0.5underdifferent mass suction parameters s = 2.0, 1.0, 0, -1.0, -2.0.

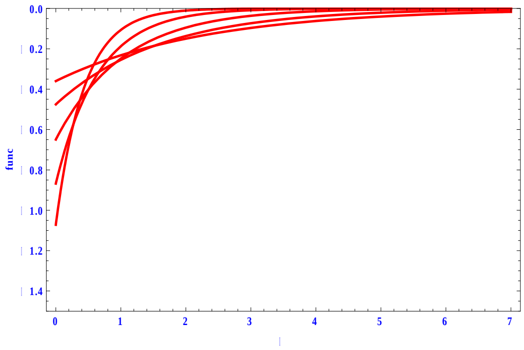

Figure 5(b)

Shear stress profiles at$M$=0.5and γ=0.5under different mass suction parameters s = 2.0, 1.0, 0,-1.0, -2.0.

It is observed from the above Figures 2(a) & 2(b) that the wall slips velocity increases with the increase of M and the wall drag force also increases with the increase of M. The graph as in Figure 3.(a) & 3 (b) shows that higher velocity slip occur for a large value of γ. The wall drag force becomes smaller for a large value of γ. Flow penetrates deeper for a small value of γ. There are no common points between these velocity and shear-stress profiles. Similarly again in Figure 4(a) & 4 (b) the velocity slip increases and the wall drag force decreases with the increase of slip parameter. However for both the velocity and shear stress profile there are interception points. The effect of mass transfer parameters are studied in figure 5(a) & 5 (b) for M=0.5 and γ= 0.5 to visualize that mass injection helps the flow to penetrate more into the fluid.

In the present work , the homotopy perturbation method (HPM) has been applied to compute the solution domains under certain values of control parameters , typical velocity and stress profiles for different combination of three parameter has been illustrated graphically. The combination of the partial slip, the magnetic effect and mass transfer greatly affects the fluid flow and shear stresses on the wall and in the fluid.

V. REFERENCES

[1] Ganesan, P. & Palani, G., Finite Difference Analysis of Unsteady natural Convection MHD past an Inclined Plate with Variable Surface Heat and Mass flux. , Int. J. Heat MassTransfer 47: (2004), pp 4449-4457.

[2] Seddeek, M.A. Effects of Radiation and Variable Viscosity on a MHD free Convection flow Past a Semi-infinite flat Plate with an aligned Magnetic field in the case of unsteady flow. *Int. J. Heat Mass Transfer* 45: (2002), pp 931-935.

[3] Crane, L. J., Flow past a stretching plate. Z. Angew. Math. Phys. 21: (1970), pp 645-655

[4] Ishak, A., Nazar, R., Pop, I., Heat Transfer over an Unsteady Stretching PermeableSurface with Prescribed Wall Temperature, Non-linear Analysis: Real World Applications, 10 5, (2009), pp. 2909-2913

[5] H.S. Takhar, A.J. Chamkha and G. Nath, Unsteady three-dimensional MHD boundary layer due to the Impulsive motion of a Stretching surface,Acta Mechanica, vol. 146, (2001), pp 59-71.

[6] Elbashbeshy EMA, Bazid MAA. , Heat Transfer over an Unsteady Stretching Surface. Heat and Mass Transfer, 41: (2004), pp 1-4.

[7] Elbashbeshy, E. M. A., Dimian, M. F., Effects of Radiation on the Flow and Heat Transfer over a Wedge with Variable Viscosity, Appl. Math. Comp., 132, 2-3, (2002), pp. 445-454

[8] Elbashbeshy, E. M. A, Emam, T.G, Effect of Thermal Radiation and Heat Transfer Over an Stretching surface embedded in a Porous medium in the presence of Heat Source Or Sink, J. Thermal science, 15, 2, (2011), pp. 477-485.

[9] Hossain, M. A., Alim, M. A., Rees, D., The Effect of Radiation on Free Convection from a Porous Vertical Plate, Int. J. Heat Mass Transfer, 42 , 1, (1999),pp. 181-191.

[10] Fang T, Zhang J. Flow between two stretchable disks – an exact solution of the Navier–Stokes equations. Int Commun Heat Mass Transfer; 35:, (2008), 892–5.

[11] Ali et al. Ali, M. E., Magyari, E., Unsteady Fluid and Heat Flow Induced by a Submerge Stretching Surface while its Steady Motion is Slowed Down Gradually, Int. J. HeatMass Transf., 50, 1-2, ((2007), pp. 188-195

[12] Hillermeier C., Generalized homotopy approach to multi-objective optimization. Int Optim Theory Appl, 110: (2001), pp 557–583

[13] Yildirim A.,on the solution of the nonlinear Korteweg-de Vries equation by the homotopy perturbation method. Commun Numer Meth Engng, 25: (2009), pp 1127–1136

[14] Ganji D D, Rajabi A., Assessment of homotopy perturbation and perturbation methods in heat radiation equations. Int Commun Heat Mass Transfer, 33: (2006), pp 391–400

[15] Domairry G, Nadim N., Assessment of homotopy analysis method and homotopy perturbation method in non-linear heat transfer equation. Int Commun Heat MassTransfer, 35: (2008), pp 93–102

[16] Rajabi A., Homotopy perturbation method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity. Phys Lett A, 364: (2007), pp 33–37.

[17] He J H., Homotopy perturbation method for bifurcation of nonlinear problems .Int J NonlinearSic Numer Simul, (2005), pp 207–208

[18] He J H., Homotopy perturbation method for solving boundary value problems. Phys LettA, 350: (2006), pp 87–88

[19] Ganji D., The application of He's homotopy perturbation method to Non-linear equations arising in heat transfer. Phys Lett A, 355: (2006), pp 337–341.

[20] Wang CY. Analysis of viscous flow due to a stretching sheet with surface slip and suction. Nonlinear Anal Real World Appl; 10(1): (2009), 375–80.

[21] Anderson HI, Slip flow past a stretching surface. Acta Mech; 158: (2002), 121–5

[22] Wang CY., Flow due to a stretching boundary with partial slip – an exact solutionof the Navier–Stokes equations. Chem Eng Sci; 57(17): (2002), 3745–7.

