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Magneto-optical interaction of light with a periodic bi-gyrotropic structure

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Abstract. Features of the interaction of light with a periodic bi-gyrotropic medium are investigated. The dispersion law for eigen circularly polarized waves propagating at an angle with respect to the axis of periodicity of the medium is found. The characteristics of reflected and transmitted light waves at oblique incidence in resonant and non-resonant cases are obtained.

1. Introduction

Modern techniques for the preparation of monocrystalline structures on the basis of various materials (semiconductors, magnetic substances, liquid crystals) with periodic distributions of refractive index allow one to form one-dimensional structures with a period of about the wavelength of light, which are of considerable interest for their optical properties [1, 2]. The appearance of new magnetic materials which are transparent in the infrared range enables one to realize a variety of easily controlled magnetic domain structures and has stimulated magneto-optical research of media with periodically distributed magnetization [3, 4].

Special attention is paid to media in which the analysis of electromagnetic wave propagation requires consideration of both the electrical and magnetic gyrotropies described by the permittivity $\hat{\varepsilon}$ and the permeability $\hat{\mu}$ tensors [5]. As is well known, the nature of Faraday rotation in dielectrics can be related to the interactions of the electric component of the electromagnetic wave with the electric dipole moment of a substance (gyroelectric mechanism) and the magnetic component of a wave with a magnetic dipole moment (gyromagnetic mechanism). Both properties are characteristic of monocrystalline yttrium iron garnet and its various derivatives with a substitution of some of the yttrium ions for ions of rare-earth metals. Separate measurements of electric and magnetic gyrotropies were carried out in epitaxial ferrite-garnet films such as $Y_3Fe_{3.88}Sc_{0.68}Ga_{0.44}O_{12}$ by waveguide methods at the wavelength $\lambda = 1.15$ μm [6]. The specific Faraday rotation was obtained to be about 40 deg cm^{-1} and 15 deg cm^{-1} for electric and magnetic gyrotropies, respectively.

Propagation of electromagnetic waves in bi-gyrotropic media was already investigated in homogeneously magnetized [7, 8] and layered [9, 10] structures. In the present work features of light propagation at an angle with the axis of periodicity in an inhomogeneously magnetized medium with a continuous periodic distribution of magnetization are analysed. The magnetic structure of the medium is a 'static spin wave'. In this structure the projection of spins on the axis of periodicity changes harmonically, and the spin components perpendicular to the axis are directed chaotically or are equal to zero [11].

2. Equations for circular waves

We choose a coordinate system with the z-axis coincident with the axis of periodicity and the direction of the magnetization in a bi-gyrotropic cubic crystal. An electromagnetic wave is incident on a slab of the magneto-optic medium in the xz plane at an angle α with respect to z (figure 1). This medium has a periodic distribution of magnetization $\boldsymbol{M}(z)$ and therefore an anisotropy within the slab [0, L]. At first we consider propagation of light simply in the anisotropic medium at an angle β with respect to the axis of periodicity. Taking circular birefringence into consideration and neglecting linear birefringence, the permittivity and the permeability tensors in this case can be written in the form [12]

$$\hat{\varepsilon} = \varepsilon \begin{pmatrix} 1 & \mathrm{i}f(z) & 0 \\ -\mathrm{i}f(z) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \hat{\mu} = \mu \begin{pmatrix} 1 & \mathrm{i}g(z) & 0 \\ -\mathrm{i}g(z) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (1)$$

The dependences of magneto-optical parameters on the coordinates in the given structure are

$$f(z) = f_0 + f_1 \cos qz \qquad g(z) = g_0 + g_1 \cos qz \qquad (2)$$

where $q = 2\pi/D$, D is the period of the magnetic structure, f_0 and g_0 are the parameters determined by the constant component of the magnetization in the structure, f_1 and g_1 are their amplitudes of modulation.

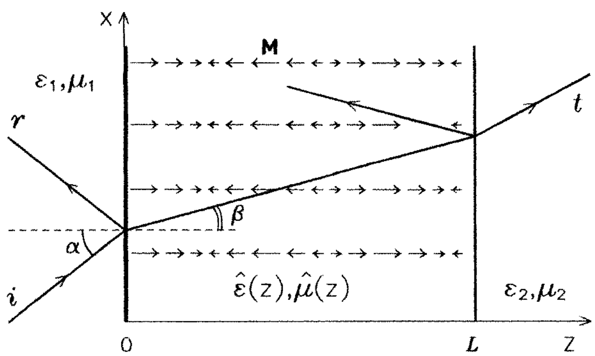

Figure 1. Diagram of the propagation of light waves in a periodic bi-gyrotropic structure.

Propagation of light in the crystal is described by the following equations for circularly polarized eigenwaves having components of fields $E_\pm = E_x \pm \mathrm{i}\sigma E_y$, $H_\pm = H_x \pm \mathrm{i}\sigma H_y$:

$$\begin{aligned} &E''_\pm \mp \frac{g'}{\sigma + g} E'_\pm + k_0^2 \varepsilon\mu(\sigma \pm f)(\sigma \pm g) E_\pm = 0 \\ &H''_\pm \mp \frac{f'}{\sigma + f} H'_\pm + k_0^2 \varepsilon\mu(\sigma \pm f)(\sigma \pm g) H_\pm = 0 \end{aligned} \qquad (3)$$

where a prime denotes the derivative with respect to $z, \sigma = \cos\beta, k_0 = \omega/c, \omega$ is the frequency and c the speed of light in vacuum. In equation (3) we make new designations $f_1^\pm = \pm f_1$, $g_1^\pm = \pm g_1$ and

$$\varkappa_\pm = k_0 \sqrt{\varepsilon\mu[\sigma^2 \pm \sigma(f_0 + g_0)]} \qquad \gamma_\pm = \pm \frac{f_1 + g_1}{\sigma \pm (f_0 + g_0)}.$$

Since magneto-optical parameters are small (for iron garnet in the available region of bi-gyrotropy $f \sim g \sim 10^{-4}$–10^{-5}), one can write the approximation of equations (3) as

$$\begin{aligned} &E''_{\pm} + qg_1^{\pm}/\sigma \sin qz\, E'_{\pm} + \varkappa_{\pm}^2(1+\gamma_{\pm}\cos qz)E_{\pm} = 0 \\ &H''_{\pm} + qf_1^{\pm}/\sigma \sin qz\, H'_{\pm} + \varkappa_{\pm}^2(1+\gamma_{\pm}\cos qz)H_{\pm} = 0\,. \end{aligned} \tag{4}$$

It follows from these equations that circularly polarized waves in the considered case as well as in a homogeneously magnetized medium are eigenwaves. The presence of bi-gyrotropy in a periodic medium results in the appearance of the members with one derivative in wave equations for electric and magnetic fields. Previously, when analysing the features of propagation of waves in mono-gyrotropic media, the simplest of the wave equations not containing terms with E' or H' was solved, i.e. in the case of electric (magnetic) gyrotropy it was the equation for the electric (magnetic) field.

The form of the solution of equations (4) depends critically on the range of its parameters. Two main cases can be marked out: resonant, when an approximately integer number of half-wavelengths of light are present along the period of the magnetic structure, and non-resonant, when this condition is not satisfied. Below we consider both these cases.

3. Non-resonant interaction of light with the medium

In view of the identity of wave equations for both circular components of fields, we shall omit, where it is optional, sign indices '$\pm$'. Each of equations (4) can be written as

$$G'' + qp/\sigma \sin(qz)G' + \varkappa^2(1+\gamma\cos qz)G = 0 \tag{5}$$

where $p = g_1$ for $G = E$ and $p = f_1$ for $G = H$. The presence of small parameters p and γ of the same order allows us to obtain an approximate solution of equation (5) in the form $G = G_0 + G_1 + \cdots$, where G_i is proportional to the ith degree of the specified small parameters. Substitution of such a form in (5) leads to the following equation for G_0 and its solution:

$$G''_0 + \varkappa^2 G_0 = 0 \qquad G_0 = A_1\exp(-\mathrm{i}\varkappa z) + A_2\exp(\mathrm{i}\varkappa z)\,. \tag{6}$$

After substitution of (6) into (5) we have the first correction:

$$\begin{aligned} G_1 = {} & \frac{\varkappa(\varkappa\gamma + pq/\sigma)}{2q(q+2\varkappa)}(A_1\exp[-\mathrm{i}(\varkappa+q)z] + A_2\exp[\mathrm{i}(\varkappa+q)z]) \\ & + \frac{\varkappa(\varkappa\gamma - pq/\sigma)}{2q(q-2\varkappa)}(A_1\exp[-\mathrm{i}(\varkappa-q)z] + A_2\exp[\mathrm{i}(\varkappa-q)z])\,. \end{aligned} \tag{7}$$

The amendment (7) caused by the periodic inhomogeneity is not valid near resonant space frequencies $\varkappa = \pm q/2$, where it strongly perturbs the homogeneous solution G_0. Taking into account the smallness of the bi-gyrotropy parameters we obtain expressions for the ellipticity χ and rotation ϑ of the light wave which was originally linearly polarized:

$$\begin{aligned} \chi &= \frac{\varkappa^2}{\sigma}\frac{f_1 - g_1}{4\varkappa^2 - q^2}(1-\cos qz) \\ \vartheta &= \vartheta_0 z + \frac{\varkappa}{\sigma q}\frac{2\varkappa^2(f_1+g_1) - g_1 q^2}{4\varkappa^2 - q^2}\sin qz \end{aligned} \tag{8}$$

where $\vartheta_0 = \varkappa(f_0+g_0)/2\sigma$ is the specific Faraday rotation of a homogeneously magnetized structure and $\varkappa = k_0\sigma\sqrt{\varepsilon\mu}$. It follows from expressions (8) that at the points $z = mD$ the wave keeps its linear polarization and the maximal deviation of the ellipticity takes place at the points $z = (m+\frac{1}{2})D$. Oscillation imposed on the linear dependence on z

is characteristic of the rotation. At the points $mD/2$ the rotation is determined only by the constant component of the magnetization. Approaching the resonant region ($q \simeq \pm 2\varkappa$) results in an increase of the efficiency of interaction of light with the medium and, therefore, a large oscillation amplitude of the ellipticity and rotation of the polarization.

Consideration of the following amendments G_n in the solution of equation (5) brings new harmonics with higher space frequencies $\varkappa \pm nq$. Their amplitudes are proportional to the nth degree of the specified small parameters, hence their contributions to the solution in the non-resonant case ($\varkappa \ncong nq/2$) can be neglected.

4. Resonant interaction of light with the medium

We consider the character of the solution of equation (5) in the resonant regions. If a plane wave is incident on a weakly anisotropic and weakly absorbing medium then its amplitude undergoes small changes while propagating over the distance of a wavelength, i.e. this amplitude is a slowly varying function of the coordinate. We represent this solution in the vicinity of one of the main resonances $\varkappa \simeq q/2$ as a superposition of two counterpropagating waves:

$$G(z) = A(z)\exp(-\mathrm{i}\varkappa z) + B(z)\exp(\mathrm{i}\varkappa z)\,. \tag{9}$$

Under the approximation that the amplitudes of the counterpropagating waves $A(z)$ and $B(z)$ are changing slowly, after substitution of (9) into (5) we reject terms with two derivatives and those of a small order of magnitude. By designating a small deviation from the resonant space frequency as $\Delta = \varkappa - q/2$ we can obtain from (5) two equations for the coupling waves:

$$A' = -\mathrm{i}\eta B\exp(2\mathrm{i}\Delta z) \qquad B' = \mathrm{i}\eta A\exp(-2\mathrm{i}\Delta z) \tag{10}$$

where $\eta = \varkappa(f_1 - g_1)/4\sigma$ is the coupling constant of the direct and the counterpropagating waves. Thus interaction of counterpropagating waves in the resonant region is determined not by the sum of the modulation parameters of electric and magnetic gyrotropies as in the case of Faraday rotation but by their difference. Media in which f and g have the same sign display their gyrotropic property more actively in Faraday rotation and less actively in resonant interaction. Conversely, the resonant interaction in periodic bi-gyrotropic media is displayed more actively if the parameters f and g have opposite signs.

We find the amplitudes of the direct and opposite waves by solving the system (10):

$$\begin{aligned} A &= \exp(\mathrm{i}\Delta z)[C_1\exp(-\nu z) + C_2\exp(\nu z)] \\ B &= -\frac{1}{\eta}\exp(-\mathrm{i}\Delta z)[(\Delta + \mathrm{i}\nu)C_1\exp(-\nu z) + (\Delta - \mathrm{i}\nu)C_2\exp(\nu z)] \end{aligned} \tag{11}$$

where $\nu = \sqrt{\eta^2 - \Delta^2}$ and the factors $C_{1,2}$ are determined from boundary conditions. The obtained solutions are valid inside the resonant zone where $|\eta| > |\Delta|$ and ν is a real quantity as well as outside the zone where $|\eta| < |\Delta|$ and ν is purely imaginary. The system of equations (10) has an invariant corresponding to the conservation of the total energy of the coupling waves propagating in opposite directions:

$$|A|^2 - |B|^2 = \text{constant}\,. \tag{12}$$

For a half-infinite medium ($z > 0$) the amplitudes of the counterpropagating waves are exponentially decreasing functions. If a light wave with amplitude $A(0)$ is incident on such a medium, the factors $C_1 = A(0)$ and $C_2 = 0$. Resonant interaction of light with a periodic bi-gyrotropic. medium accounts for the strong attenuation of the light wave, when the direct

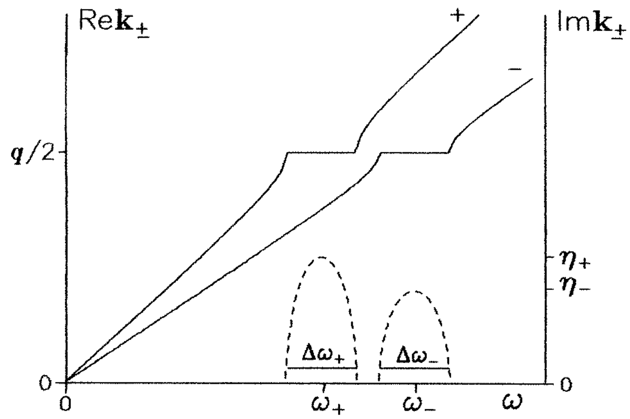

Figure 2. Propagation constants of circularly polarized light waves versus frequency in the case of separate resonant zones: real (full curves) and imaginary (broken curves) parts.

wave gives its energy to the opposite wave and attenuates. The amplitude of the opposite wave increases from zero at $z \to \infty$ up to a maximum value at the output of the medium at $z = 0$:

$$\begin{aligned} A(z) &= A(0)\exp[(\mathrm{i}\Delta - \nu)z] \\ B(z) &= -\sqrt{\frac{\mathrm{i}\Delta - \nu}{\mathrm{i}\Delta + \nu}}A(0)\exp[-(\mathrm{i}\Delta - \nu)z]\,. \end{aligned} \tag{13}$$

It follows from (12) that the amplitudes of the direct and opposite waves in a half-infinite medium satisfy the identity $|A(z)| = |B(z)|$. Thus the resonant interaction of the light with the medium is a Bragg reflection. Outside of the resonant zone, where ν is a purely imaginary quantity, the amplitude oscillates.

Proceeding from the solution of the equations of the coupling waves (10), we have propagation constants of circularly polarized waves:

$$k = q/2 \pm \mathrm{i}\sqrt{\eta^2 - \Delta^2} \tag{14}$$

where the sign on the square root corresponds not to the type of circular polarization but to the choice of branch of the dispersion curve at the point $k = q/2$. In figure 2 the real and imaginary parts of the propagation constants are presented as functions of frequency. The propagation constants of waves with right $k_+(\omega)$ and left $k_-(\omega)$ circular polarizations have a gap at $k_\pm = q/2$. At the specified points there are resonant zones centred on the frequencies $\omega_\pm$ and widths $\Delta\omega_\pm$:

$$\omega_\pm = \frac{cq}{2\sqrt{\varepsilon\mu}}[\sigma^2 \pm \sigma(f_0 + g_0)]^{-1/2} \qquad \Delta\omega_\pm = \frac{1}{2\sigma}|f_1 - g_1|\omega_\pm \tag{15}$$

within which the imaginary part of the propagation constants differs from zero. Propagating in the medium, a wave of the corresponding polarization with its frequency lying within the zone $\Delta\omega_\pm$ quickly attenuates; thus a wave of right polarization attenuates in $\Delta\omega_+$ and left polarization attenuates in $\Delta\omega_-$. One can see from (15) that in the absence of constant magnetization ($f_0 = g_0 = 0$) or if magnetic gyrotropy neutralizes the electrical gyrotropy ($f_0 = -g_0$), the positions and the widths of the zones do not depend on the type of circular polarization of waves interacting with the structure ($\omega_+ = \omega_-$, $\Delta\omega_+ = \Delta\omega_-$). The

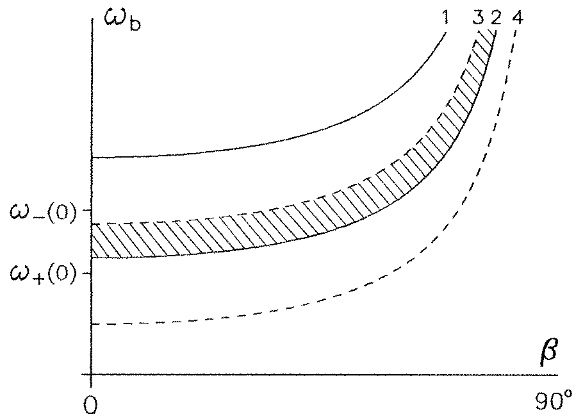

Figure 3. Frequencies of resonant zone boundaries of right (broken curves) and left (full curves) circularly polarized waves versus the direction of light propagation. $\omega_- \pm \Delta\omega_-/2$ (curves 1, 2), $\omega_+ \pm \Delta\omega_+/2$ (curves 3, 4). The area of zone overlap is hatched.

widths of the resonant zones are directly proportional to the differences of the modulation parameters of the electric and magnetic gyrotropies.

Two variants of the arrangement of the resonant zones are possible: with an overlap, when the constant component of the magnetization is reasonably small $2|f_0+g_0| < |f_1-g_1|$ and separate zones without overlap $2|f_0+g_0| > |f_1-g_1|$.

Angular dependences of the resonant zone boundary frequencies for right (broken curves) and left (full curves) circularly polarized light are presented in figure 3. Both the distance between the centres of the resonant zones and their widths increase with the increase of the angle. Thus zones separate (overlapping) at $\beta = 0$ are separate (overlapping) at an arbitrary angle. In the case of coincident resonant zones they remain coincident independent of the angle.

5. Transmission and reflection of light

We consider resonant interaction of a light wave with a bi-gyrotropic slab of periodic magnetic structure and thickness L, when the slab is located inside a homogeneous isotropic medium with permittivity ε and permeability μ (the cover, slab and substrate are the same medium: $\varepsilon_1 = \varepsilon = \varepsilon_2, \mu_1 = \mu = \mu_2$). Fields of waves propagating inside each of the media can be written in the form

$$\begin{aligned} E &= A_i \exp(-\mathrm{i}\varkappa z) + B_i \exp(\mathrm{i}\varkappa z) \\ H &\simeq \pm\mathrm{i}\sqrt{\varepsilon/\mu}(A_i \exp(-\mathrm{i}\varkappa z) - B_i \exp(\mathrm{i}\varkappa z)) \end{aligned} \tag{16}$$

where $i = 1, 2, 3$ indicates the number of the medium. The amplitudes A_i and B_i in the bi-gyrotropic layer ($i = 2$) are determined by relation (11) and they are constant outside of it. The solution of the boundary conditions gives the following expressions for the constants C_1 and C_2 in (11):

$$C_1 = \frac{(\nu + \mathrm{i}\Delta)\exp(\nu L)/2}{\nu\cosh(\nu L) + \mathrm{i}\Delta\sinh(\nu L)} \qquad C_2 = \frac{(\nu - \mathrm{i}\Delta)\exp(-\nu L)/2}{\nu\cosh(\nu L) + \mathrm{i}\Delta\sinh(\nu L)} \tag{17}$$

the reflectance and the transmittance of circularly polarized waves:

$$r_\pm = \frac{-\mathrm{i}\eta_\pm\sinh(\nu_\pm L)}{\nu_\pm\cosh(\nu_\pm L) + \mathrm{i}\Delta_\pm\sinh(\nu_\pm L)} \qquad t_\pm = \frac{\nu_\pm\exp(-\mathrm{i}qL/2)}{\nu_\pm\cosh(\nu_\pm L) + \mathrm{i}\Delta_\pm\sinh(\nu_\pm L)}\,. \tag{18}$$

Reflection and transmission of radiation of arbitrary polarization for a stratified structure can be described by the following matrix relations:

$$\begin{bmatrix} E_{\mathrm{p}}^{\mathrm{r}} \\ E_{\mathrm{s}}^{\mathrm{r}} \end{bmatrix} = \begin{bmatrix} r_{\mathrm{pp}} & r_{\mathrm{ps}} \\ r_{\mathrm{sp}} & r_{\mathrm{ss}} \end{bmatrix} \begin{bmatrix} E_{\mathrm{p}}^{\mathrm{i}} \\ E_{\mathrm{s}}^{\mathrm{i}} \end{bmatrix} \qquad \begin{bmatrix} E_{\mathrm{p}}^{\mathrm{t}} \\ E_{\mathrm{s}}^{\mathrm{t}} \end{bmatrix} = \begin{bmatrix} t_{\mathrm{pp}} & t_{\mathrm{ps}} \\ t_{\mathrm{sp}} & t_{\mathrm{ss}} \end{bmatrix} \begin{bmatrix} E_{\mathrm{p}}^{\mathrm{i}} \\ E_{\mathrm{s}}^{\mathrm{i}} \end{bmatrix} \tag{19}$$

where the indices i, r, t indicate incident, reflected and transmitted light waves, respectively; p or s is the type of polarization. The components of the matrices can be determined from the reflectance and the transmittance of circularly polarized waves:

$$\begin{aligned} -r_{\mathrm{pp}} &= r_{\mathrm{ss}} = (r_+ + r_-)/2 \qquad & r_{\mathrm{sp}} &= r_{\mathrm{ps}} = (r_+ - r_-)/2\mathrm{i} \\ t_{\mathrm{pp}} &= t_{\mathrm{ss}} = (t_+ + t_-)/2 \qquad & t_{\mathrm{sp}} &= -t_{\mathrm{ps}} = -(t_+ - t_-)/2\mathrm{i}\,. \end{aligned} \tag{20}$$

Using the following relations it is possible to find the power reflectance R, rotation ϑ and ellipticity χ [13]:

$$\begin{aligned} &R = |r_{\mathrm{p}k}|^2 + |r_{\mathrm{s}k}|^2 \\ &\tan 2\vartheta = \tan 2\alpha \cos(\varphi - \psi) \\ &\sin 2\chi = \sin 2\alpha \sin(\varphi - \psi) \end{aligned} \tag{21}$$

where $\tan\alpha = |r_{\mathrm{s}k}|/|r_{\mathrm{p}k}|$, $\psi = \arg(r_{\mathrm{p}k})$, $\varphi = \arg(r_{\mathrm{s}k})$, k denotes the polarization of the incident light (p or s).

In the case where the magnetic gyrotropy neutralizes the electrical gyrotropy $(f_0 + g_0 = 0)$, we have $r_+ = -r_-$, $t_+ = t_-$, then

$$\begin{aligned} r_{\mathrm{pp}} &= r_{\mathrm{ss}} = 0 \qquad & t_{\mathrm{ps}} &= t_{\mathrm{sp}} = 0 \\ r_{\mathrm{ps}} &= r_{\mathrm{sp}} = -\mathrm{i}r_+ \qquad & t_{\mathrm{pp}} &= t_{\mathrm{ss}} = t_+\,. \end{aligned}$$

Thus, if the incident light is linearly polarized, the reflected light is also linearly polarized with its polarization rotated by 90° and the transmitted light does not change its polarization.

Power reflectance and transmittance in the considered case are written as

$$R = \frac{\eta^2 \sinh^2 \nu L}{\eta^2 \sinh^2 \nu L + \nu^2} \qquad T = \frac{\nu^2}{\eta^2 \sinh^2 \nu L + \nu^2}\,. \tag{22}$$

For a slab with a large number of periods of magnetization oscillations ($\eta L > 1$) the reflectance in the resonant zone is close to unity and the transmittance is close to zero, which is a consequence of the Bragg reflection of light from the periodic structure. Outside of the resonant zone R and T are obtained by the replacement ν for $\mathrm{i}\nu$ in expressions (22). They have the form

$$R = \frac{\eta^2 \sin^2 |\nu| L}{\eta^2 \sin^2 |\nu| L + |\nu|^2} \qquad T = \frac{|\nu|^2}{\eta^2 \sin^2 |\nu| L + |\nu|^2} \tag{23}$$

and their dependence on the thickness oscillates, which is caused by the resonant interaction of the light with the structure. For a slab with a small number of periods ($\eta L \leqslant 1$), a large part of the energy of the light wave passes through even if the frequency lies in the resonant zone.

On the basis of the relations for the components of the coefficient matrix (20) and the reflectance (18) we obtain the phase shift of the reflected light:

$$\varphi = \arctan\left(\frac{\Delta}{\nu} \tanh \nu L\right). \tag{24}$$

In the presence of absorption ($\varepsilon = \varepsilon' + \mathrm{i}\varepsilon''$) it is possible to obtain the reflectance for a semi-infinite bi-gyrotropic structure. If the absorption is reasonably small ($\varepsilon'' \ll \varepsilon' |f_1 - g_1|$)

and the thickness of the layer is large enough for a wave to attenuate at this thickness ($k_0\varepsilon'' L \gg 1$), then the reflectance from (18) transforms to

$$r_{\pm} = \frac{-\mathrm{i}\eta_{\pm}}{\nu_{\pm}\,\mathrm{sign}\,\Delta_{\pm} + \mathrm{i}\Delta_{\pm}}\,. \tag{25}$$

Now we proceed to a more complex structure in which the permittivity and permeability of the upper medium do not coincide with those of the bi-gyrotropic slab and substrate ($\varepsilon_1 \neq \varepsilon = \varepsilon_2, \mu_1 \neq \mu = \mu_2$). In this case two effects are added together: reflection from the dielectric boundary and reflection from the periodic structure. From the boundary

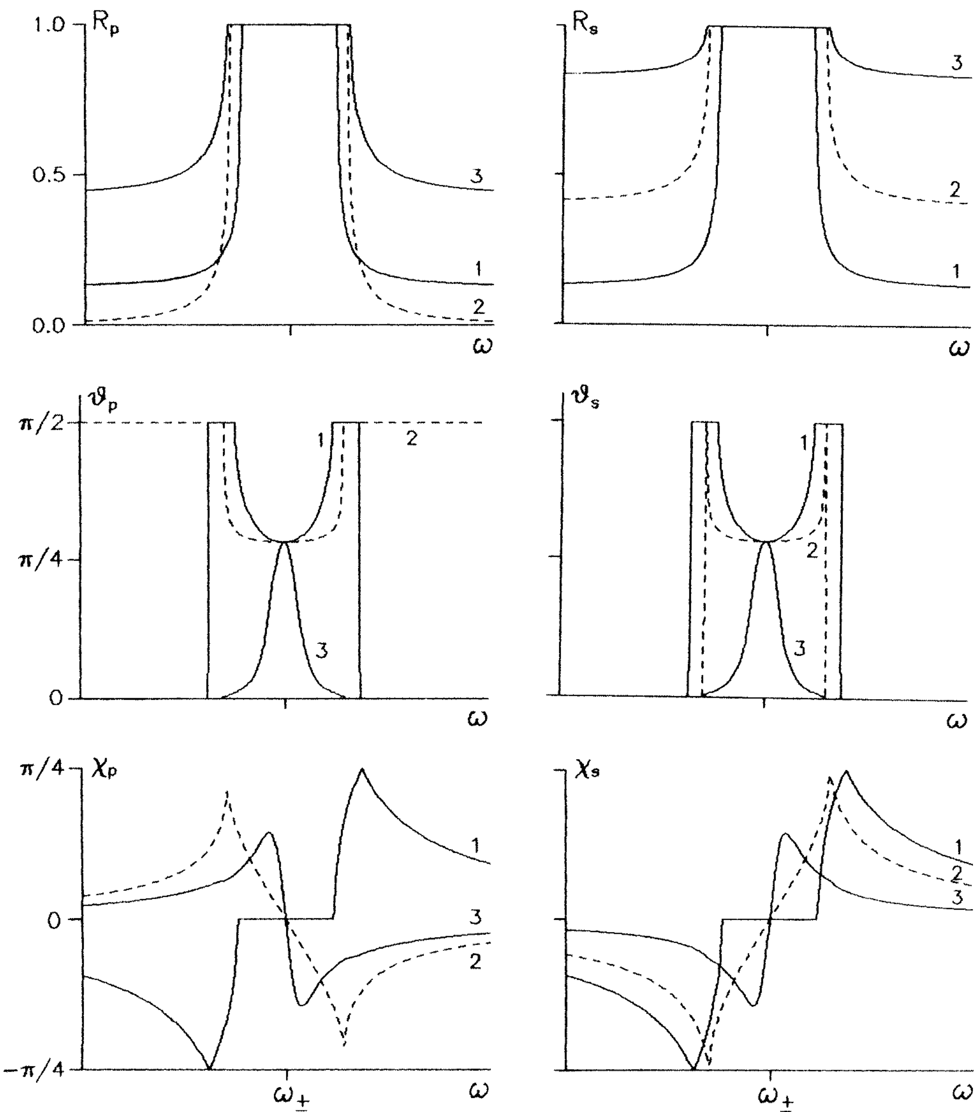

Figure 4. Power reflectance R, rotation ϑ and ellipticity χ of p- and s-polarized light reflected from a semi-infinite iron garnet structure versus frequency in the case of coincident resonant zones ($1, 0^\circ$; $2, 65^\circ$; $3, 85^\circ$).

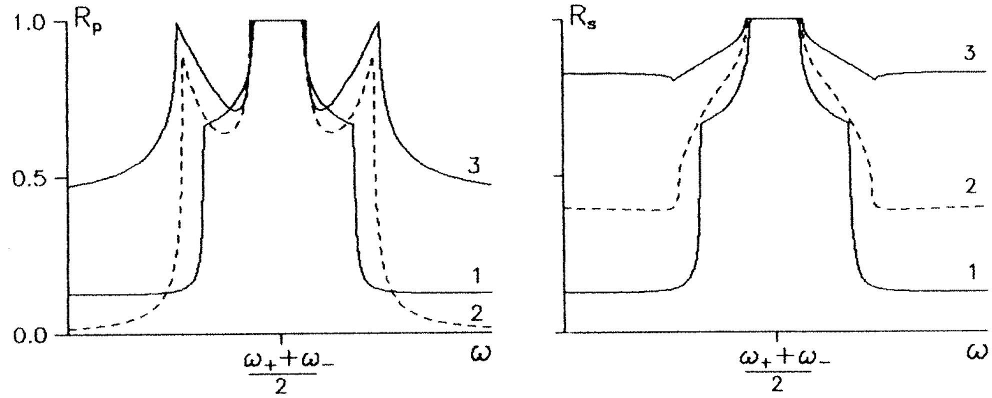

Figure 5. Power reflectance R of p- and s-polarized light reflected from a semi-infinite iron garnet structure versus frequency in the case of overlapping resonant zones $f_0 + g_0 = (f_1 - g_1)/4$ (1, 0°; 2, 65°; 3, 85°).

problem for electromagnetic fields we obtain the components of the reflection coefficients matrix (19):

$$
\begin{aligned}
r_{\mathrm{pp}} &= \frac{1}{D}[\rho_{\mathrm{p}} - r_+ r_- \rho_{\mathrm{s}} - (r_+ + r_-)(1 - \rho_{\mathrm{p}}\rho_{\mathrm{s}})/2] \\
r_{\mathrm{ss}} &= \frac{1}{D}[\rho_{\mathrm{s}} - r_+ r_- \rho_{\mathrm{p}} - (r_+ + r_-)(1 - \rho_{\mathrm{p}}\rho_{\mathrm{s}})/2] \\
r_{\mathrm{ps}} &= \frac{\mathrm{i}}{2D}\frac{\tau_{\mathrm{p}}}{\tau_{\mathrm{s}}}(r_+ - r_-)(1 - \rho_{\mathrm{s}}^2) \\
r_{\mathrm{sp}} &= \frac{\mathrm{i}}{2D}\frac{\tau_{\mathrm{s}}}{\tau_{\mathrm{p}}}(r_+ - r_-)(1 - \rho_{\mathrm{p}}^2)
\end{aligned}
\tag{26}
$$

where $D = 1 - r_+ r_- \rho_{\mathrm{p}}\rho_{\mathrm{s}} - (r_+ + r_-)(\rho_{\mathrm{s}} - \rho_{\mathrm{p}})/2$, $\rho_{\mathrm{p,s}}$, $\tau_{\mathrm{p,s}}$ are the Fresnel reflectance and transmittance of the interface between the cover and the isotropic substrate (ε, μ). In figures 4 and 5 optical characteristics (power reflectance R, rotation ϑ, ellipticity χ) of p- and s-polarized light reflected from iron garnet with magnetic structure ($\varepsilon_1 = 1, \varepsilon = 4.45$) versus frequency are presented. Resonant zones of right and left polarized light waves coincide in figure 4 ($f_0 + g_0 = 0$) and overlap in figure 5 ($f_0 + g_0 = (f_1 - g_1)/4$). Curves 1, 2, 3 correspond to the angles of incidence 0°, 65° (Brewster angle), 85°, respectively.

The following features of the presented characteristics can be seen: (i) in the area where resonant zones overlap $R_{\mathrm{p,s}} = 1$, $\vartheta_{\mathrm{p}} = \vartheta_{\mathrm{s}}$, $\chi_{\mathrm{p}} = -\chi_{\mathrm{s}}$; (ii) in the centre between the resonant zones $(\omega_+ + \omega_-)/2$ we have $\vartheta_{\mathrm{p}} \approx \vartheta_{\mathrm{s}} \approx$ constant, $\chi_{\mathrm{p,s}} = 0$; (iii) outside the resonant zones $\vartheta = 0$ or $\pi/2$ (at Brewster's angle); (iv) when the incidence of light is close to normal ($\alpha \to 90°$) $R_{\mathrm{p,s}} \to 1$, $\chi_{\mathrm{p}} = -\chi_{\mathrm{s}} \to 0$, $\vartheta_{\mathrm{p,s}} \to 0$; (v) R, ϑ are symmetrical with respect to the centre, χ is antisymmetric. Far from the resonant regions the characteristics of light are like those of light reflected from an isotropic dielectric.

6. Conclusion

The analysis showed that resonant interaction of light with a periodic bi-gyrotropic structure depends on the sum of the constant components of the electric and magnetic gyrotropies and the difference of their modulation amplitudes. Therefore, for effective resonant interaction f

and g must be opposite in sign, in contrast with the case of Faraday rotation in homogeneous media. Thus we can conclude that the presence of bi-gyrotropy in periodic structures leads to the appearance of new effects which were not available in mono-gyrotropic media. These effects can be used in the development of various optical devices.

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