Sādhanā, Vol. 19, Part 2, April 1994, pp. 337–346. © Printed in India.

Magneto-visco-elastic surface waves in stressed conducting media

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MS received 23 November 1992; revised 9 March 1994

Abstract. The present paper is concerned with magneto-visco-elastic surface waves in conducting media involving time rate of strain and stress of first order, the media being under an initial stress of hydrostatic tension or compression. The theory of magneto-visco-elastic surface waves in a conducting medium involving time rate of strain and stress of first order is derived under an initial stress. The above general theory is then employed to characterise Rayleigh, Love and Stoneley waves. Results obtained in the above cases reduce to well-known classical results when viscosity and magnetic field are absent.

Keywords. Magneto-visco-elastic; first order; surface waves; initial stress; hydrostatic tension or compression.

1. Introduction

Surface waves play an important role in the study of earthquakes, seismology, geophysics and geodynamics. The theory of surface waves has been widely investigated and developed by Rayleigh (1885), Voigt (1887), Stoneley (1924), Ewing *et al* (1957, pp. 257–259, 311), Hunter (1960, pp. 1–57), Bland (1960, pp. 30–75), Flugge (1967, pp. 3–21) and Jeffreys (1959, pp. 35–38). As the inner parts of the earth are under considerable stress from the weight of the matter resting on its surface, we may suppose that the initial equilibrium stress is approximately of hydrostatic nature. Moreover, the earth is placed in its own magnetic-field. Therefore, the investigation presented in this paper may be of importance when surface waves propagate under initial stress, magnetic fields and the viscous nature of the medium are involved. It is believed that the considered problem has not been so far investigated.

The interplay of an electromagnetic field with the motion of deformable solids has also been undertaken by many investigators (Knopoff 1955; Banos 1956; Chadwick 1957; Suhubi 1965; Yu & Tang 1966; De & Sengupta 1971). Yu & Tang (1966) thoroughly discussed the dilatational and rotational waves in a magneto-elastic initially stressed conducting medium. De & Sengupta (1971, 1972) investigated magneto-elastic waves and disturbances in initially stressed conducting media. Acharya & Sengupta (1978) investigated the problem of magneto-thermo-elastic surface waves in initially stressed conducting media. More recently, the effect of viscosity on the elastic surface waves is receiving greater attention from many investigators (Das & Sengupta 1990a, 1990b, 1992; Roy & Sengupta 1983a, 1983b).

In the present paper the authors investigate magneto-visco-elastic surface waves in a conducting medium under hydrostatic stress (tension or compression) paying special attention to Rayleigh, Love and Stoneley waves. Dispersion relations are derived for Rayleigh and Love and some comments on Stoneley waves are also included.

2. Basic equations

The equations of motion for a perfectly conducting elastic solid under initial stress (hydrostatic tension or compression) in a uniform magnetic field are (Yu & Tang 1966)

$$\rho \frac{\partial^2 u_i}{\partial t^2} = -p_0 \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \mu_e H_0 \left(\frac{\partial H_i}{\partial x_1} - \frac{\partial H_1}{\partial x_i} \right) + \frac{\partial \tau_{ij}}{\partial x_j}, \tag{1}$$

$$H_i = H_0 \left(\frac{\partial u_i}{\partial x_1} - \frac{\partial u_1}{\partial x_i} \right), \qquad i,j = 1,2,3,$$

where p_0 is the hydrostatic tension or compression (tension when $p_0 < 0$ and compression when $p_0 > 0$), τ_{ij} is the stress tensor over the initial stress, u_i is the displacement vector with respect to coordinates x_1, x_2, x_3 and time t, ρ is the density of the material, H_0 is the intensity of the uniform magnetic field parallel to x_1-axis, μ_e is the magnetic permeability.

3. Formulation of the problem

Let M_1 and M_2 be two electrically conducting charge free isotropic, homogeneous, visco-elastic, semi-infinite solid media in welded contact under an initial hydrostatic

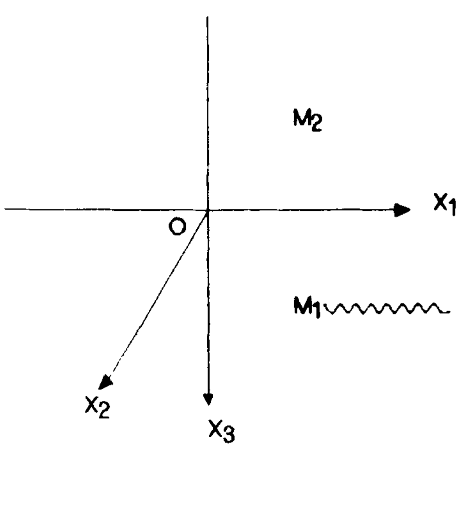

Figure 1. Interface geometry.

tension or compression permeated by uniform magnetic field (figure 1). We further assume that the medium still remains homogeneous and isotropic under the combined influence of magnetic field and initial stress. The magnetic properties of the conducting solids in the domains M_1 and M_2 are assumed to be sensibly the same. We consider a system of orthogonal Cartesian axes $Ox_1x_2x_3$, the origin O being on the interface, and Ox_3 being normal to the interface (figure 1).

We consider the possibility of a type of wave travelling in the positive x_1-direction in such a manner that the disturbance is largely confined to the neighbourhood of the boundary and all the particles at any instant on any line parallel to x_2-axis have equal displacements. Due to the first assumption we assert that the wave is a surface wave and the second assumption concludes that all partial derivatives with respect to the coordinates x_2 are zero. Then using the formulae $u = \mathrm{grad}\,\phi + \mathrm{curl}\,\psi$, the displacement components, u_1 and u_3 at any point may be expressed in the form

$$u_1 = \frac{\partial\phi}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \quad u_3 = \frac{\partial\phi}{\partial x_3} + \frac{\partial\psi}{\partial x_1}, \tag{2}$$

so that

$$\nabla^2\phi = \Delta, \quad \nabla^2\psi = \frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}, \quad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}, \quad \Delta = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3},$$

where ϕ and ψ are functions of co-ordinates x_1, x_3 and time t.

The first order stress-strain relation for an isotropic visco-elastic medium is (Voigt 1887)

$$\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\tau_{ij} = \left(\lambda_1 + \lambda_2\frac{\partial}{\partial t}\right)\Delta\delta_{ij} + 2\left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)e_{ij}, \tag{3}$$

where η_1, λ_1, μ_1 are elastic constants, η_2, λ_2 and μ_2 are constants due to viscosity, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ is the strain tensor and δ_{ij} is the Kronecker symbol.

Using (3) in (1), the displacement equations of motion for a conducting first order visco-elastic medium under hydrostatic stress in an uniform magnetic field as

$$\left[(\lambda_1 + \mu_1) + (\lambda_2 + \mu_2)\frac{\partial}{\partial t}\right]\frac{\partial\Delta}{\partial x_1} + \left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)\nabla^2 u_1 - \left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)p_0\nabla^2 u_1 = \rho\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_1}{\partial t^2},$$

$$\left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)\nabla^2 u_2 - p_0\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\nabla^2 u_2 + K\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_2}{\partial x_1^2} = \rho\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_2}{\partial t^2},$$

$$\left[(\lambda_1 + \mu_1) + (\lambda_2 + \mu_2)\frac{\partial}{\partial t}\right]\frac{\partial\Delta}{\partial x_3} + \left(\mu_1 + \mu_2\frac{\partial}{\partial t}\right)\nabla^2 u_3 - p_0\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\nabla^2 u_3 + K\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_3}{\partial x_1^2} - \frac{\partial^2 u_1}{\partial x_1\partial x_3}\right) = \rho\left(\eta_1 + \eta_2\frac{\partial}{\partial t}\right)\frac{\partial^2 u_3}{\partial t^2}, \tag{4}$$

where $K = \mu_e H_0^2$. The above relations apply to both M_1 and M_2.

Introducing (2) in (4) yields

$$\frac{\partial^2\phi}{\partial t^2}=\left[\frac{(V_{1T}^2+V_{2T}^2(\partial/\partial t))}{L}-\frac{p_0}{\rho}\right]\nabla^2\phi+\frac{K}{\rho}\frac{\partial^2\psi}{\partial x_1\partial x_3},$$

$$\frac{\partial^2\psi}{\partial t^2}=\left[\frac{(V_{1S}^2+V_{2S}^2(\partial/\partial t))}{L}-\frac{p_0}{\rho}\right]\nabla^2\psi+\frac{K}{\rho}\frac{\partial^2\psi}{\partial x_1^2}, \tag{5}$$

$$\frac{\partial^2 u_2}{\partial t^2}=\left[\frac{(V_{1S}^2+V_{2S}^2(\partial/\partial t))}{L}-\frac{p_0}{\rho}\right]\nabla^2 u_2+\frac{K}{\rho}\frac{\partial^2 u_2}{\partial x_1^2},$$

where

$$V_{1T}^2=\frac{\lambda_1+2\mu_1}{\rho},\quad V_{2T}^2=\frac{\lambda_2+2\mu_2}{\rho},\quad V_{1S}^2=\frac{\mu_1}{\rho},\quad V_{2S}^2=\frac{\mu_2}{\rho},$$

$$L=\eta_1+\eta_2\frac{\partial}{\partial t}.$$

Again, (5) apply to both M_1 and M_2. In the sequel, M_2 is identified by $\rho',\eta'_1,\eta'_2,\lambda'_1,\lambda'_2,\mu'_1,\mu'_2$ for its properties.

3.1 *Boundary conditions*

To obtain the frequency equation we apply the following conditions of continuity across the interface

(i) The components of displacement at the interface between M_1 and M_2 must be continuous.
(ii) Stress components τ_{31}, τ_{32}, τ_{33} are continuous across the interface. They are respectively given by

$$L\tau_{31}=\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\left(2\frac{\partial^2\phi}{\partial x_1\partial x_3}+\frac{\partial^2\psi}{\partial x_1^2}-\frac{\partial^2\psi}{\partial x_3^2}\right),$$

$$L\tau_{32}=\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\frac{\partial u_2}{\partial x_3}, \tag{6}$$

$$L\tau_{33}=\left(\lambda_1+\lambda_2\frac{\partial}{\partial t}\right)\nabla^2\phi+2\left(\mu_1+\mu_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2\phi}{\partial x_3^2}+\frac{\partial^2\psi}{\partial x_1\partial x_3}\right).$$

4. Harmonic solutions

To investigate equations (5), let us take harmonic solutions

$$(\phi,\psi,u_2)=[\hat\phi(x_3),\hat\psi(x_3),\hat u_2(x_3)]e^{i(\eta x_1-\omega t)}, \tag{7}$$

for medium M_1. For medium M_2, the dashed functions $\hat\phi',\hat\psi',\hat u'_2$ replace $\hat\phi,\hat\psi,\hat u_2$.

Introducing (7) in (5) we get

$$\frac{d^2\hat\phi}{dx_3^2}-\left[\eta^2-\frac{\omega^2\eta_K^*}{V_{KT}^2-\eta_K^*(p_0/\rho)}\right]\hat\phi=\frac{-i\eta K\eta_K^*}{\rho(V_{KT}^2-(p_0\eta_K^*/\rho))}\frac{d\hat\psi}{dx_3},$$

$$\frac{d^2\hat{\psi}}{dx_3^2}-\left[\eta^2-\frac{\omega^2\eta_K^*}{V_{KS}^2-\eta_K^*(p_0/\rho)}+\frac{K\eta^2\eta_K^*}{\rho(V_{KS}^2-\eta_K^*(p_0/\rho))}\right]\hat{\psi}=0, \tag{8}$$

$$\frac{d^2\hat{u}_2}{dx_3^2}-\left[\eta^2+\frac{((K/\rho)\eta^2-\omega^2)\eta_K^*}{V_{KS}^2-(p_0/\rho)\eta_K^*}\right]\hat{u}_2=0,$$

in which

$$\eta_K^*=(\eta_1-i\omega\eta_2),\quad V_{KT}^2=V_{1T}^2-i\omega V_{2T}^2,\quad V_{KS}^2=V_{1S}^2-i\omega V_{2S}^2.$$

Similar relations for M_2 can be obtained by using dashed variables $\hat{\phi}', \hat{\psi}', \hat{u}_2', \eta_1', \eta_2'$, $V_{1T}', V_{2T}', V_{1S}', V_{2S}', \eta_K^{*\prime}, V_{KT}'^2, V_{KS}'^2, \lambda_1', \mu_1', \lambda_2', \mu_2', \rho'$. According to our assumption K is the same for both the media.

Clearly, (8) must have exponential solutions; and, in order that ϕ, ψ and u_2 describe surface waves, they must become vanishingly small as x_3 tends to infinity. Thus for the medium M_1 the solutions of (5) may be taken in the following form:

$$\phi=[A\exp\{-x_3(\eta^2-\zeta_1^2)^{1/2}\}+\beta\exp\{-x_3(\eta^2-\zeta_2^2)^{1/2}\}]\exp\{i(\eta x_1-\omega t)\},$$

$$\psi=B_1\exp\{-x_3(\eta^2-\zeta_2^2)^{1/2}+i(\eta x_1-\omega t)\},$$

$$u_2=C\exp\left\{-x_3\left[\eta^2+\frac{(K/\rho)\eta^2-\omega^2)\eta_K^*}{V_{KS}^2-(p_0/\rho)\eta_K^*}\right]^{1/2}+i(\eta x_1-\omega t)\right\}. \tag{9}$$

For the medium M_2

$$\phi'=[A'\exp\{x_3(\eta^2-\zeta_1'^2)^{1/2}\}+B'\exp\{x_3(\eta^2-\zeta_2'^2)^{1/2}\}]\exp\{i(\eta x_1-\omega t)\},$$

$$\psi'=B_1'\exp\{x_3(\eta^2-\zeta_2'^2)^{1/2}+i(\eta x_1-\omega t)\},$$

$$u_2'=C'\exp\left\{x_3\left[\eta^2+\frac{((K/\rho')\eta^2-\omega^2)\eta_K^{*\prime}}{V_{KS}'^2-(p_0/\rho')\eta_K^{*\prime}}\right]^{1/2}+i(\eta x_1-\omega t)\right\}, \tag{10}$$

where

$$\zeta_1^2=\frac{\omega^2\eta_K^*}{V_{KT}^2-\eta_K^*(p_0/\rho)},\qquad \zeta_1'^2=\frac{\omega^2\eta_K^{*\prime}}{V_{KT}'^2-\eta_K^{*\prime}(p_0/\rho')},$$

$$\zeta_2^2=\frac{(\omega^2-(K/\rho)\eta^2)\eta_K^*}{V_{KS}^2-\eta_K^*(p_0/\rho)},\qquad \zeta_2'^2=\frac{(\omega^2-(K\eta^2/\rho'))\eta_K^{*\prime}}{V_{KS}'^2-\eta_K^{*\prime}(p_0/\rho')}, \tag{11}$$

and

$$B=\alpha_1 B_1,\quad B'=\alpha_1' B_1',$$

with

$$\alpha_1=\frac{(i\eta K/\rho)(\eta^2-\zeta_2^2)^{1/2}}{\omega^2-\zeta_2^2[(V_{KT}^2/\eta_K^*)-(p_0/\rho)]},\qquad \alpha_1'=\frac{(-i\eta K/\rho')(\eta^2-\zeta_2'^2)^{1/2}}{\omega^2-\zeta_2'^2[(V_{KT}'^2/\eta_K^{*\prime})-(p_0/\rho')]}.$$

In evaluating quantities like $(\eta^2-\zeta^2)^{1/2}$, the root with positive real part will be taken in each case.

Now applying the boundary conditions (i) and (ii) we get

$$A+(\alpha_1-iQ_2)B_1=A'+(\alpha_1'+iQ_2')B_1', \tag{12a}$$

$$C=C', \tag{12b}$$

$$iQ_1A+(iQ_2\alpha_1+1)B_1=-iQ_1'A'-(i\alpha_1'Q_2'-1)B_1', \tag{12c}$$

$$\rho\left(\frac{V_{KS}^2}{\eta_K^*}\right)[2iQ_1A + \{(1+Q_2^2) + 2iQ_2\alpha_1\}B_1]$$

$$= \rho'\left(\frac{V'^2_{KS}}{\eta_K^{*'}}\right)[-2iQ'_1A' + \{-2iQ'_2\alpha'_1 + (1+Q'^2_2)\}B'_1], \tag{12d}$$

$$-\rho\left(\frac{V_{KS}^2}{\eta_K^*}\right)\left\{\eta^2 + \frac{((K/\rho)\eta^2 - \omega^2)\eta_K^*}{V_{KS}^2 - (p_0/\rho)\eta_K^*}\right\}^{1/2} C$$

$$= \rho'\left(\frac{V'^2_{KS}}{\eta_K^{*'}}\right)\left\{\eta^2 + \frac{((K/\rho')\eta^2 - \omega^2)\eta_K^{*'}}{(V'^2_{KS} - (p_0/\rho')\eta_K^{*'})}\right\}^{1/2} C', \tag{12e}$$

$$\left(\frac{\rho}{\eta_K^*}\right)[\{V_{KT}^2(Q_1^2 - 1) + 2V_{KS}^2\}A + \{V_{KT}^2(Q_2^2 - 1)\alpha_1 + 2V_{KS}^2(\alpha_1 - iQ_2)\}B_1]$$

$$= \left(\frac{\rho'}{\eta_K^{*'}}\right)[\{V'^2_{KT}(Q'^2_1 - 1) + 2V'^2_{KS}\}A' +$$

$$+ \{V'^2_{KT}(Q'^2_2 - 1)\alpha'_1 + 2V'^2_{KS}(\alpha'_1 + iQ'_2)\}B'_1] \tag{12f}$$

where

$$Q_k = \left(1 - \frac{\zeta_K^2}{\eta^2}\right)^{1/2}, \quad Q'_K = \left(1 - \frac{\zeta'^2_K}{\eta^2}\right)^{1/2}, \quad k = 1, 2.$$

From (12b) and (12e) we get

$$C = C' = 0.$$

Thus we conclude that there is no propagation of the displacement u_2. The wave velocity equation is, therefore, obtained from (12a), (12c), (12d), (12f) by the elimination of the constants A, B_1, A', B'_1, in the determinant form as

$$|M_{ij}| = 0, \quad (i, j = 1, 2, 3, 4), \tag{13}$$

where

$$M_{11} = 1, \quad M_{12} = (\alpha_1 - iQ_2), \quad M_{13} = -1, \quad M_{14} = -(\alpha'_1 + iQ'_2),$$
$$M_{21} = iQ_1, \quad M_{22} = (iQ_2\alpha_1 + 1), \quad M_{23} = iQ'_1, \quad M_{24} = (i\alpha'_1Q'_2 - 1),$$
$$M_{31} = \rho(V_{KS}^2/\eta_K^*)2iQ_1, \quad M_{32} = \rho(V_{KS}^2/\eta_K^*)\{(1+Q_2^2) + 2iQ_2\alpha_1\},$$
$$M_{33} = \rho'(V'^2_{KS}/\eta_K^{*'})2iQ'_1, \quad M_{34} = -\rho'(V'^2_{KS}/\eta_K^{*'})\{-2iQ'_2\alpha'_1 + (1+Q'^2_2)\},$$
$$M_{41} = (\rho/\eta_K^*)\{V_{KT}^2(Q_1^2 - 1) + 2V_{KS}^2\},$$
$$M_{42} = (\rho/\eta_K^*)\{V_{KT}^2(Q_2^2 - 1)\alpha_1 + 2V_{KS}^2(\alpha_1 - iQ_2)\},$$
$$M_{43} = -(\rho'/\eta_K^{*'})\{V'^2_{KT}(Q'^2_1 - 1) + 2V'^2_{KS}\},$$
$$M_{44} = -(\rho'/\eta_K^{*'})\{V'^2_{KT}(Q'^2_2 - 1)\alpha'_1 + 2V'^2_{KS}(\alpha'_1 + iQ'_2)\}.$$

From (13) we obtain the wave velocity in the common boundary under consideration in the presence of magnetic field, initial stress in the nature of hydrostatic tension or compression, viscosity where viscosity is of first order including strain rate and stress rate.

5. Particular cases

5.1 *Rayleigh waves*

In the particular case of Rayleigh waves the interface becomes a free surface and M_2 is treated as vacuum.

Hence in view of (12d) and (12f) we get

$$2iQ_1 A + \{(1+Q_2^2) + 2iQ_2\alpha_1\} B_1 = 0, \tag{14}$$

$$\{V_{KT}^2(Q_1^2-1) + 2V_{KS}^2\} A + \{V_{KT}^2(Q_2^2-1)\alpha_1 + 2V_{KS}^2(\alpha_1 - iQ_2)\} B_1 = 0. \tag{15}$$

For the indispensable constants A, B_1 from (14) and (15) to assume non zero values we have

$$|M'_{ij}| = 0 \quad (i,j = 1,2), \tag{16}$$

where

$$M'_{11} = 2iQ_1, \quad M'_{12} = (1+Q_2^2) + 2iQ_2\alpha_1,$$

$$M'_{21} = V_{KT}^2(Q_1^2-1) + 2V_{KS}^2, \quad M'_{22} = V_{KT}^2(Q_2^2-1)\alpha_1 + 2V_{KS}^2(\alpha_1 - iQ_2).$$

Equation (16) represents the magneto-visco-elastic Rayleigh wave velocity equation under the initial stress in the nature of hydrostatic tension or compression in a medium, including strain rate and stress rate.

In the absence of viscous effects, (16) reduces to

$$\begin{vmatrix} 2iP_1 & (1+P_2^2) + 2in_1P_2 \\ V_{1T}^2(P_1^2-1) + 2V_{1S}^2 & V_{1T}^2(P_2^2-1)n_1 + 2V_{1S}^2(n_1 - iP_2) \end{vmatrix} = 0, \tag{17}$$

where

$$P_1^2 = 1 - \frac{\omega^2}{(V_{1T}^2 - (p_0/\rho))\eta^2}, \quad P_2^2 = 1 - \frac{\omega^2 - K\eta^2/\rho}{(V_{1S}^2 - (p_0/\rho))\eta^2},$$

$$n_1 = \frac{iK[1 - (\omega^2/\eta^2 V_{1S}^2) + (K/\rho V_{1S}^2)]^{1/2}}{\rho V_{1T}^2[(\omega^2/\eta^2 V_{1T}^2) - (\omega^2/\eta^2 V_{1S}^2) + (KV_{1T}^2/\rho V_{1S}^2)]}.$$

Equation (17) represents the magneto-elastic Rayleigh-wave velocity equation under the initial stress of hydrostatic tension or compression. This agrees with the result obtained by Acharya & Sengupta (1978).

Moreover, in the absence of the magnetic field and initial stress ($p_0 = 0$, $K = 0$) we get from (17) the Rayleigh wave velocity equation for the elastic medium as

$$4[1 - (c^2/V_{1T}^2)]^{1/2}[1 - (c^2/V_{1S}^2)]^{1/2} = (2 - (c^2/V_{1S}^2))^2, \tag{18}$$

where

$$c^2 = \omega^2/\eta^2.$$

Equation (18) is in complete agreement with the classical result of Rayleigh.

5.2 *Love waves*

We know that for such types of waves u_2 is the only component of displacement vector $\mathbf{u}$ to play the role. Let us consider that the medium M_2 is bounded by two

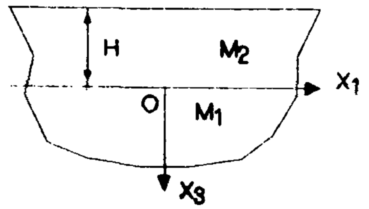

Figure 2. Love wave formulation.

horizontal plane surfaces at a finite distance H apart, the upper plane surface being free while the medium M_1 extends to an infinitely great distance (figure 2).

The notable fact here is that the displacement in M_2 may no longer diminish with distance from the boundary between M_1 and M_2 so that for the medium M_2 we preserve the full solution as

$$u_2' = \left[C_1' \exp\left\{ x_3 \left(\eta^2 + \frac{((K/\rho')\eta^2 - \omega^2)\eta_K^{*\prime}}{V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}} \right)^{1/2} \right\} + C_2' \exp\left\{ -x_3 \left(\eta^2 + \frac{((K\eta^2/\rho') - \omega^2)\eta_K^{*\prime}}{(V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime})} \right)^{1/2} \right\} \right] \times \exp(i(\eta x_1 - \omega t)), \tag{19}$$

where the restriction that the real part of $\{\eta^2 + ((K/\rho')\eta^2 - \omega^2)\eta_K^{*\prime}/[V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}]\}^{1/2}$ be positive is not required.

For the present case the boundary conditions are

(i) u_2 and τ_{32} are continuous at $x_3 = 0$,
(ii) $\tau_{32}' = 0$ at $x_3 = -H$.

Employing the boundary conditions (i) and (ii) we obtain

$$C = C_1' + C_2' \tag{20a}$$

$$-\rho\left(\frac{V_{KS}^2}{\eta_K^*}\right)\left[\eta^2 + \frac{((K\eta^2/\rho) - \omega^2)\eta_K^*}{V_{KS}^2 - (p_0/\rho)\eta_K^*}\right]^{1/2} C = \rho'\left(\frac{V_{KS}'^2}{\eta_K^{*\prime}}\right) \times \left[\eta^2 + \frac{((K\eta^2/\rho') - \omega^2)\eta_K^{*\prime}}{V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}}\right]^{1/2} (C_1' - C_2'), \tag{20b}$$

$$C_1' e^{-H}\left[\eta^2 + \frac{((K/\rho')\eta^2 - \omega^2)\eta_K^{*\prime}}{V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}}\right]^{1/2} - C_2' e^{-H}\left[\eta^2 + \frac{((K\eta^2/\rho') - \omega^2)\eta_K^{*\prime}}{V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}}\right]^{1/2} = 0. \tag{20c}$$

Eliminating C, C_1', C_2' from equations (20a)–(20c) we get

$$\rho\left(\frac{V_{KS}^2}{\eta_K^*}\right)\left[1 - \frac{(c^2 - (K/\rho))\eta_K^*}{V_{KS}^2 - (p_0/\rho)\eta_K^*}\right]^{1/2} - \rho'\left(\frac{V_{KS}'^2}{\eta_K^{*\prime}}\right)\left[\frac{(c^2 - (K/\rho'))\eta_K^{*\prime}}{V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}} - 1\right]^{1/2} \times \tan\left\{\eta H\left[\frac{(c^2 - (K/\rho'))\eta_K^{*\prime}}{V_{KS}'^2 - (p_0/\rho')\eta_K^{*\prime}} - 1\right]^{1/2}\right\} = 0, \tag{21}$$

where $c = \omega/\eta$.

Equation (21) represents the required wave velocity equations for Love waves in a magneto-visco-elastic solid medium including strain and stress rates of first order under an initial hydrostatic tension or compression. It is seen from (21) that Love waves depend upon viscous field as well as magnetic field and also on the initial stress in the nature of hydrostatic tension or compression.

5.3 *Stoneley waves*

In the classical theory, Stoneley waves are the generalised form of Rayleigh waves propagating in the vicinity of interface of two semi-infinite media M_1 and M_2. Hence in our general case Stoneley waves propagating along the common boundary of M_1 and M_2 are determined by the roots of the wave velocity equation (13). In the absence of magnetic field, initial stress, viscosity and strain-stress rates this equation of course reduces to the classical result obtained by Stoneley.

We are grateful to the referees for their valuable comments and suggestions towards improvement of the paper and its appearance in the present form.

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