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MAGNETOGENESIS IN NATURAL INFLATION MODEL

We study the process of inflationary magnetogenesis in the natural single-field inflation model, whose parameters are chosen in accordance with the recent observations by the Planck collaboration [1]. The conformal invariance of the Maxwell action is broken by a kinetic coupling with the inflaton field by means of the coupling function as a power of the scale factor, $I(\phi) \propto a^{\alpha}$, and $\alpha < 0$ is used in order to avoid the strong coupling problem. For such α, the electric component of the energy density dominates over the magnetic one and, for $\alpha \lesssim -2.2$, it causes a strong back-reaction, which can spoil inflation and terminate the enhancement of the magnetic field. It is found that the magnetic fields generated without back-reaction problem cannot exceed $\sim 10^{-20}$ G at the present epoch, and their spectrum has a blue tilt.

K e y w o r d s: natural inflation, magnetogenesis, kinetic coupling, large-scale magnetic fields.

1. Introduction

Magnetic fields are present on various scales in the Universe including stars, galaxies, and clusters of galaxies. There is also some evidence that they exist in galaxies at cosmological distances. The typical magnetic field strengths range from a few μG in galaxies and galaxy clusters up to 10^{15} G in magnetars (see, e.g., Refs. [2–8]. Observations of the cosmic microwave background (CMB) [9, 10] and the gamma rays from distant blazars [11–14] yield the following upper and lower bounds on the strength of the present large-scale magnetic fields: $10^{-17} \lesssim B_0 \lesssim 10^{-9}$ G. The remarkable feature of the magnetic fields observed in voids is the fact that they have a very large coherence scale. Therefore, in order to understand the origin of these fields, one needs to be able to explain how they could be generated in the vacuum and could have such a large coherence scale.

There are generally two groups of theories trying to explain the origin of these magnetic fields (see Refs. [3, 6–8]). One possibility is that the magnetic fields were generated during the process of structure formation due to astrophysical mechanisms such as Biermann battery [15] and then amplified by means of different types of dynamo [16–19] and spread into the intergalactic medium by outflows from galaxies [20–23]. Another possibility is that the observed magnetic fields have primordial origin and were produced in the early Universe [24–28]. Although cosmological phase transitions could produce primordial magnetic fields of the necessary strength [26–31], the comoving coherence length of such magnetic fields cannot be larger than the Hubble horizon at the phase transition that leads to the coherence scale much smaller than Mpc today. Consequently, the most natural mechanism for the generation of the large-coherence-scale magnetic fields is the inflation [24], which is a period of rapid expansion in the early Universe.

It is well known that quantum fluctuations of massless scalar and tensor fields are very strongly amplified in the inflationary stage and create considerable density inhomogeneities evolving later into the large-scale structure of the observed Universe [32–36] or relic gravitational waves [37–39]. However, the large scale fluctuations of electromagnetic fields usually are not generated, because the Maxwell action is conformally invariant [40]. If the conformal invariance is broken, then the magnetic fields would be generated from quantum perturbations and may become very large due to the expansion of the Universe. Obviously, this could explain the presence of magnetic fields with large coherence scale.

One can break the conformal invariance of the electromagnetic sector in various ways, e.g., by the coupling of the electromagnetic field to a scalar or pseudo-scalar field or to the curvature invariant. Although many models are described in the literature [25, 41–44], we adopt, in our study, the kinetic coupling model $I^2(\phi)F_{\mu\nu}F^{\mu\nu}$ firstly introduced by Ratra [25], where $I(\phi)$ is a coupling function of the inflation field ϕ, and $F_{\mu\nu}$ is the electromagnetic field tensor (see [45–52] for later works).

There is a great variety of inflationary models with different numbers of free parameters (for a review, see Ref. [53]). Their suitability can be judged by the spectral index of scalar primordial perturbations n_s and the tensor-to-scalar ratio r in the corresponding models and could be measured from the cosmic microwave background observations. According to the most recent data by the Planck Collaboration [1], only a few inflationary models are in a satisfactory accordance with the observations. For example, the most popular chaotic inflationary models like the large field inflation with power-like potentials are disfavored due to their high tensor-to-scalar ratio. Among the favorable ones are the R^2 Starobinsky model, the quartic hill-top model, and the natural inflation model with periodic inflaton potential. An advantage of the natural inflation model is that its potential is protected from radiative corrections by a nonperturbative shift symmetry preserving the slow-roll dynamics [54, 55]. In this paper, we will investigate the magnetogenesis process in the natural inflation model. Magnetogenesis in the Starobinsky model was studied in Ref. [52].

This paper is organized as follows. We determine the numerical values of the model parameters and find a solution of the background equations in the natural inflation model in Sec. 2. In Sec. 3, we consider the kinetic coupling of the inflaton field ϕ to the electromagnetic field, calculate the power spectra of generated electric and magnetic fields, determine the range of parameters, for which the back-reaction problem does not occur, and estimate the value of generated magnetic fields at the present epoch. The summary of the obtained results is given in Sec. 4.

2. Natural Inflation

Natural inflation was proposed in Refs. [54, 55] in order to resolve the problem of radiative corrections to the inflaton potential. A successful realization of the slow-roll regime requires the inflaton potential to be sufficiently flat. The radiative corrections can break this flatness and spoil the inflation, unless the potential is protected by some symmetry. The natural inflation model contains a complex scalar field Φ with a "Mexican hat" potential and obeys the global $U(1)$ symmetry. On some energy scale f, this symmetry is spontaneously broken. Then the "absolute value" $|\Phi|$ obtains the finite vacuum expectation value $f/\sqrt{2}$, and the 'phase' degree of freedom becomes a Nambu–Goldstone boson, which plays the role of the inflaton. Since it comes from an angular variable, it acquires the discrete shift symmetry $\phi \to \phi + 2\pi f$, and its effective potential, which arises from the interaction with other fields, must be periodic with a period $2\pi f$. The simplest choice is the well-known cosine potential

$$V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{\phi}{f}\right)\right], \tag{1}$$

where f and Λ are parameters of the model. As we will see, the recent observations of CMB require the super-Planckian values of the decay constant f. In principle, the energy scale of the symmetry breaking could be well below the Planck scale, if there are several identical fields with the same potential, and they will effectively reproduce the picture with a super-Planckian decay constant [56]. However, the presence of many identical fields seems too artificial. Therefore, we will restrict our analysis to the single-field case and will consider f as a free parameter.

The Lagrangian of the inflaton field interacting with the electromagnetic field reads

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] - \\ - \frac{1}{4}\int d^4x\sqrt{-g}\left[g^{\mu\alpha}g^{\nu\beta}I^2(\phi)F_{\alpha\beta}F_{\mu\nu}\right]. \tag{2}$$

The equation of motion for the inflaton is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right] + \frac{dV}{d\phi} = -\frac{I}{2}\frac{dI}{d\phi}F^{\mu\nu}F_{\mu\nu}. \tag{3}$$

In our study, we will neglect the back-reaction of the generated electromagnetic fields on the background evolution and will discuss the applicability of this approximation in the next section. Technically, we omit the right-hand side of Eq. (3) and neglect the electromagnetic energy density on the right-hand side of the Friedmann equation.

We will use the spatially flat Friedmann–Lemaître–Robertson–Walker metric in terms of the cosmic time with $g_{00} = +1$. The inflationary evolution is governed by the following system of equations:

$$H^2 = \frac{1}{3M_p^2}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right], \tag{4}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'_\phi = 0, \tag{5}$$

where $M_p = (8\pi G)^{-1/2} = 2.4 \times 10^{18}\,\text{GeV}$ is the reduced Planck mass. In order to analyze the generation of electromagnetic fields, we need to determine the time dependences of the inflaton and scale factor. However, before numerically integrating Eqs. (4) and (5), it is useful to carry out the slow-roll analysis.

In the slow-roll approximation, the evolutionary equations can be simplified:

$$H^2 \simeq \frac{1}{3M_p^2}V(\phi), \quad \frac{d\phi}{dt} \simeq -\frac{1}{3H}\frac{dV}{d\phi}. \tag{6}$$

The applicability of these equations is determined by the smallness of the slow-roll parameters, which are defined as usual: [57]

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'_\phi}{V}\right)^2, \tag{7}$$

$$\eta = M_p^2\frac{V''_\phi}{V}. \tag{8}$$

Using the explicit expression for the inflaton potential, we obtain

$$\epsilon = \frac{M_p^2}{2f^2}\text{ctg}^2\frac{\phi}{2f}, \quad \eta = \epsilon - \frac{M_p^2}{2f^2}. \tag{9}$$

For $\epsilon \ll 1$ and $\eta \ll 1$, Eqs. (6) approximately determine the scale factor a and inflation field ϕ as functions of time. For the scale factor, we have

$$a(\phi) = \frac{\cos^\beta\left(\frac{\phi}{2f}\right)}{\cos^\beta\left(\frac{\phi_i}{2f}\right)}, \tag{10}$$

where $\beta = 2f^2/M_p^2$. The time evolution of the inflaton field ϕ is determined by the following equation:

$$\sec\left(\frac{\phi}{2f}\right) + \tan\left(\frac{\phi}{2f}\right) = \left[\sec\left(\frac{\phi_i}{2f}\right) + \tan\left(\frac{\phi_i}{2f}\right)\right] \times \times \exp\left(-\sqrt{\frac{2}{3}}\frac{\Lambda^2}{2f^2}M_p t\right). \tag{11}$$

The inflation ends, when $\epsilon = 1$. Using Eq. (9), we find

$$\frac{1}{\beta}\text{ctg}^2\frac{\phi_e}{2f} = 1, \tag{12}$$

that gives the value of the inflaton field at the end of the inflation

$$\phi_e = f\arccos\left(\frac{\beta - 1}{\beta + 1}\right) = 2f\,\text{arcctg}\sqrt{\beta}. \tag{13}$$

It is useful to find out how ϕ depends on the number of e-folds $N = \ln(a_e/a)$ during the inflation. Using Eq. (10), we have

$$N = \beta\ln\frac{\cos(\phi_e/2f)}{\cos(\phi/2f)}. \tag{14}$$

Combining Eqs. (13) and (14), we obtain

$$\phi(N) = 2f\arccos\left(e^{-N/\beta}\sqrt{\frac{\beta}{\beta + 1}}\right). \tag{15}$$

The quasiexponential expansion of the Universe during the inflation stage leads to the amplification of primordial scalar and tensor perturbations. Their amplitudes and the corresponding spectral indices are determined by the model parameters. In particular, the spectral index of scalar perturbations and the tensor-to-scalar ratio can be expressed in terms of the slow-roll parameters as follows [53, 58]:

$$n_s = 1 - 6\epsilon_* + 2\eta_*, \tag{16}$$

$$r = 16\epsilon_*, \tag{17}$$

where the quantities with asterisk should be taken at N_* e-folds before the inflation end, when the pivot scale k_* crosses the horizon. Using Eqs. (9) and (15), we find

$$n_s = 1 - \frac{2}{\beta} - \frac{4}{\beta}\left[e^{2N_*/\beta}\left(1 + \frac{1}{\beta}\right) - 1\right]^{-1}, \tag{18}$$

$$r = \frac{16}{\beta}\left[e^{2N_*/\beta}\left(1 + \frac{1}{\beta}\right) - 1\right]^{-1}, \tag{19}$$

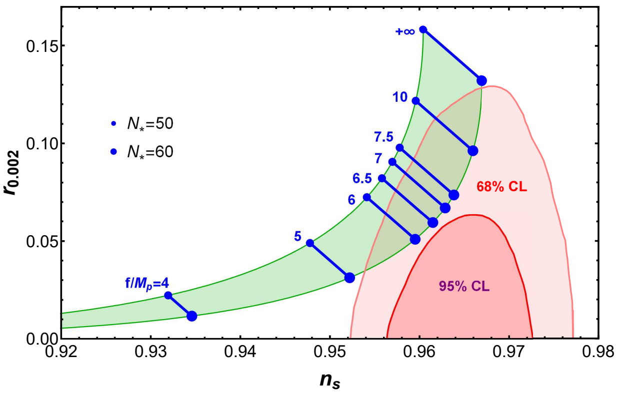

Fig. 1. Theoretical prediction for n_s and r from the natural inflation model (region between two growing curves) for $50 < < N_* < 60$ and $0 < f/M_p < +\infty$ and marginalized joint 68% CL and 95% CL regions for n_s and r at $k_* = 0.002\,\text{Mpc}^{-1}$ from the latest Planck observations [1]

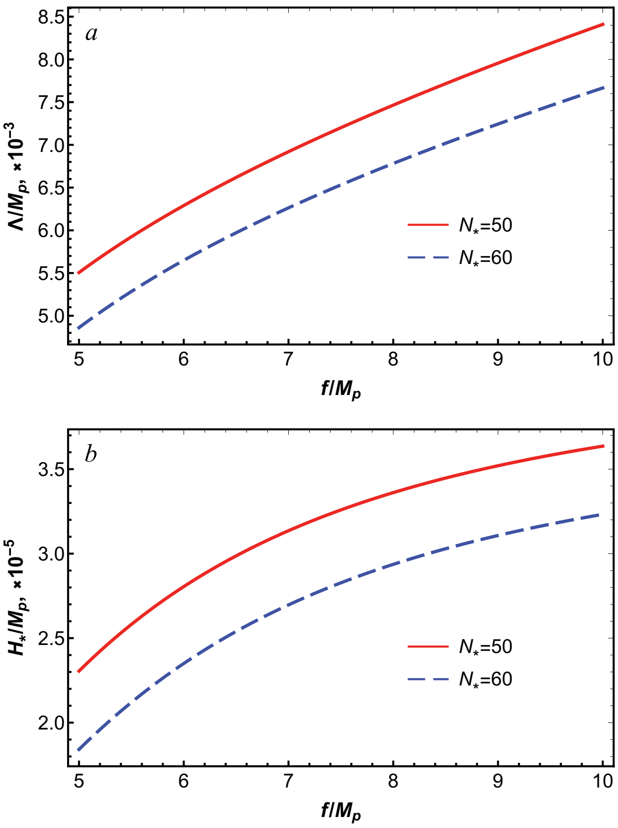

Fig. 2. The parameter Λ (panel *a*) and the Hubble parameter at the moment of the pivot scale horizon crossing H_* (panel *b*) as functions of f for fixed $N_* = 50$ (solid lines) and $N_* = 60$ (dashed lines) obtained from Eqs. (21) and (22)

For given N_* and f, these equations parametrically determine a point on the (n_s, r) diagram, which is often used to constrain inflationary models. Figure 1 shows such a diagram for the natural inflation model. The region between two growing curves is filled with points which can be achieved by a variation of the model parameters: $50 < N_* < 60$ and $0 < f/M_p < +\infty$. Thick lines correspond to fixed values of the parameter f, which are written near the corresponding lines. Smaller and larger points correspond, respectively, to $N_* = 50$ and $N_* = 60$. The lines connecting them correspond to intermediate values. For comparison, we also show the marginalized joint 68% confidence level (CL) and 95% CL regions for n_s and r at $k_* = 0.002\,\text{Mpc}^{-1}$ from the latest Planck observations [1].

According to Fig. 1, the most favorable value corresponds to $f \sim 7M_p$. More precisely, the analysis carried out in Ref. [1] gives at 95% CL that $\log_{10}(f/M_p) > 0.84$, i.e., $f/M_p \gtrsim 6.91$.

The value of Λ in Eq. (1) can be fixed by the requirement that the amplitude of primordial scalar perturbations at N_* e-folds before the inflation end equals [1]

$$\mathcal{P}_{\mathcal{R}} = \left(\frac{H^2}{2\pi|\dot{\phi}|}\right)^2\Bigg|_{N_*} = 2.2 \times 10^{-9}. \tag{20}$$

Using Eqs. (6), we find

$$\left(\frac{\Lambda}{M_p}\right)^2 = \sqrt{\frac{2}{3}}\frac{2\pi\sqrt{\mathcal{P}_{\mathcal{R}}}}{f/M_p}\frac{\cos(\phi/2f)}{\sin^2(\phi/2f)}\Bigg|_{N_*}. \tag{21}$$

Similarly, we calculate the Hubble parameter at the time moment of the pivot mode horizon crossing:

$$\frac{H_*}{M_p} = \frac{2\pi\sqrt{\mathcal{P}_{\mathcal{R}}}}{f/M_p}\text{ctg}\,\frac{\phi_*}{2f}. \tag{22}$$

The scalar field value at N_* e-folds before the end of the inflation is given by Eq. (15). Then, for given N_* and f, we can determine the values of Λ and H_*. The corresponding dependences are shown in Fig. 2.

Without the loss of generality, we can choose the initial moment of time at $N_{tot} = N_*$ e-folds before the end of the inflation, when the physically relevant modes start to cross the horizon. Therefore, we have $\phi_i = \phi(N_*)$. For example, for $N_* = 60$, $f = 7.5\,M_p$, we obtain $\phi_i = 14.2\,M_p$ and $\Lambda = 6.5 \times 10^{-3}\,M_p = 1.56 \times 10^{16}$ GeV. Using these initial conditions, we find numerically the solutions of Eqs. (4), (5) and use them to determine the power spectra of generated electromagnetic fields in the next section.

3. Magnetic Field Generation

In this section, we consider the time evolution of the electromagnetic field in the expanding inflationary background described in Sec. 2. We determine the range of parameters, for which the back-reaction problem can be avoided, and estimate the present value of the observed large-scale magnetic field.

3.1. Basic equations

Maxwell's equations following from action (2) take the form

$$\partial_\mu \left[\sqrt{-g} g^{\mu\alpha} g^{\nu\beta} I^2(\phi) F_{\alpha\beta}\right] = 0. \quad (23)$$

In the Coulomb gauge $A_0 = 0$, $\partial_i A^i = 0$, they yield the following equation for the vector potential:

$$\ddot{A}_j + \dot{A}_j \left(H + 2\frac{\dot{I}}{I}\right) - \partial_i \partial^i A_j = 0. \quad (24)$$

Since the equation of motion (24) is linear, the quantum field $\hat{A}_j$ can be decomposed into the sum over the creation $\hat{b}^\dagger_{\mathbf{k},\lambda}$ and annihilation $\hat{b}_{\mathbf{k},\lambda}$ operators for modes with momentum $\mathbf{k}$ and transverse polarization λ

$$\hat{A}_j(\mathbf{x}, t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \times$$
$$\times \sum_{\lambda=1}^{2} \varepsilon_j(\lambda, \mathbf{k}, t) \hat{b}_{\mathbf{k},\lambda} A_\lambda(\mathbf{k}, t) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.}, \quad (25)$$

where $A_\lambda(\mathbf{k}, t)$ is a mode function, and $\varepsilon_j(\lambda, \mathbf{k}, t)$ is a polarization vector that is also time-dependent, because it satisfies the normalization condition in the expanding Universe $\varepsilon_\mu(\lambda, \mathbf{k}, t)\varepsilon_\nu(\lambda, \mathbf{k}, t)g^{\mu\nu} = -1$ [47]. It is convenient to use a rescaled mode function $\mathcal{A}(t, k) = a(t) I(t) A(t, k)$. Since the coupling between the electromagnetic field and the inflaton is insensitive to the polarization, we will omit the polarization index in the mode function in what follows. Then, substituting decomposition (25) into Eq. (24), we obtain the following equation governing the evolution of the mode function:

$$\ddot{\mathcal{A}}(t, k) + H\dot{\mathcal{A}}(t, k) + \left(\frac{k^2}{a^2} - H\frac{\dot{I}}{I} - \frac{\ddot{I}}{I}\right)\mathcal{A}(t, k) = 0. \quad (26)$$

For modes, which are far inside the horizon, the first term in the brackets gives the leading contribution. Thus, the behavior of the mode function could be described by the Bunch–Davies vacuum initial condition

$$\mathcal{A}(\mathbf{k}, t) = \frac{1}{\sqrt{2k}} e^{-ik\tau(t)}, \quad -k\tau \to \infty, \quad (27)$$

where $\tau(t) = \int^t \frac{dt'}{a(t')}$ is the conformal time.

For an analytical treatment, the simplest case corresponds to the coupling function with power dependence on the scale factor

$$I \propto a^\alpha, \quad (28)$$

where α is a free dimensionless parameter, which must be chosen negative in order to avoid the strong coupling problem [48]. Using Eq. (10) and requiring that the inflaton field lies in the potential minimum $\phi = 0$ at the end of the inflation, the coupling function could be written in the following form:

$$I(\phi) = \cos^{\alpha\beta}\left(\frac{\phi}{2f}\right). \quad (29)$$

The generated electromagnetic fields are characterized by the power spectra

$$\frac{d\rho_B}{d\ln k} = \frac{1}{2\pi^2}\left(\frac{k}{a}\right)^4 k|\mathcal{A}(t, k)|^2, \quad (30)$$

$$\frac{d\rho_E}{d\ln k} = \frac{1}{2\pi^2}\left(\frac{k}{a}\right)^2 k I^2(t) \left|\frac{\partial}{\partial t}\left(\frac{\mathcal{A}(t, k)}{I(t)}\right)\right|^2. \quad (31)$$

3.2. Back-reaction problem

After the inflation in the epoch of reheating, the Universe becomes filled with all sorts of particles, and its conductivity strongly increases. As a result, electric fields quickly dissipate, and magnetic fields survive due to the magnetic flux conservation. Despite the fact that the electric fields disappear in the primordial plasma, one cannot neglect them during the inflation, since their energy density may be large enough to cause the back-reaction and may spoil the inflation.

In order to ensure that the back-reaction problem does not occur, one has to check that the following condition is satisfied for all physically relevant modes [47]:

$$\left.\frac{d\rho_E}{d\ln k}\right|_{\text{inf}} + \left.\frac{d\rho_E}{d\ln k}\right|_{\text{inf}} < \rho_{\text{inf}}. \quad (32)$$

For power-like coupling functions in the de Sitter space-time, the power spectra of generated electromagnetic fields are well-known [47]. For $\alpha < -1/2$, they are

$$\frac{d\rho_B}{d\ln k} = \frac{H^4}{2\pi^2}\mathcal{F}(\alpha)\left(\frac{k}{aH}\right)^{6+2\alpha}, \quad (33)$$

$$\frac{d\rho_E}{d\ln k} = \frac{H^4}{2\pi^2}\mathcal{F}(\alpha)\left(\frac{k}{aH}\right)^{4+2\alpha}, \quad (34)$$

where

$$\mathcal{F}(\alpha) = \frac{\pi}{2^{2\alpha+3}\Gamma^2(\alpha+3/2)\cos^2(\pi\alpha)}. \quad (35)$$

All modes, which crossed the horizon during the inflation, contribute to the energy density. Therefore, condition (32) must be satisfied for all momenta in the range $a_iH < k < a_eH$. Let us analyze the behavior of the power spectra for different values of α.

1. For $\alpha \in (-2; -1/2)$, the main contribution to the energy densities comes from large momenta $k \sim \sim a_eH$. Therefore,

$$\rho_B \sim \rho_E \sim H^4 \ll \rho_{\rm inf} = 3H^2M_p^2 \quad (36)$$

for $H \ll M_p$. Thus, there is no back-reaction.

2. For $\alpha \in (-3; -2)$, the magnetic energy density is determined by short-wavelength modes $k \sim a_eH$ and cannot cause the back-reaction similarly to the previous case. However, the electric power spectrum is dominated by long modes with $k \sim a_iH$

$$\rho_E = \frac{H^4}{2\pi^2}\frac{\mathcal{F}(\alpha)}{2|\alpha|-4}e^{2N_e(|\alpha|-2)}. \quad (37)$$

Therefore, the electric component determines the back-reaction. It can be neglected, if $\rho_E < \rho_{\rm inf}$, which implies that

$$|\alpha| < 2 + \frac{1}{N_e}\left(\ln\frac{M_p}{H} + \frac{1}{2}\ln\frac{12\pi^2(|\alpha|-2)}{\mathcal{F}(\alpha)}\right). \quad (38)$$

3. For $\alpha < -3$, the leading contribution to the electric and magnetic energy densities is given by long-wavelength modes, which cross the horizon at the beginning of the inflation, $k \sim a_iH$. Although both components could attain very large values, the main effect is due to the electric component, because it has two more powers of a_e/a_i. The condition to avoid the back-reaction is, thus, the same as in the previous case.

We would like to mention that, in order to estimate the "critical" value of α, for which the back-reaction problem occurs, one can use the value of the Hubble parameter $H = H_*$, because it varies very slowly during the inflation. On the other hand, the situation with N_e is more delicate. If we assume that the inflation lasts only from the moment, when the pivot scale crosses the horizon, then we may set $N_e = N_*$. In this case for $N_* = 60$ and $f = 7.5\,M_p$, we have $\alpha \gtrsim -2.2$. However, in a more realistic situation, the inflation can last many e-folds before the pivot scale horizon crossing, and a lot of modes which are longer than the pivot one (and, therefore, are not physically relevant at the present epoch) would be also enhanced and contribute to the energy density. In this case, one would not deal with the back-reaction problem only in the case of scale-invariant or blue electric power spectrum, i.e., for $\alpha \geq -2$.

To investigate the most favorable situation, we assume that the enhancement occurs only for the modes, which are shorter than the pivot scale k_*. In this case, we numerically solve the mode equation (26) for all modes, which cross the horizon during the inflation, and plot the corresponding electric and magnetic power spectra in Fig. 3. In numerical simulations, we set $N_* = 60$ and $f = 7.5\,M_p$. The numerical results nicely confirm the theoretical constraint $\alpha > -2.2$ for the absence of the back-reaction.

3.3. Magnetic field at the present epoch

The present-day value of the observed magnetic field is determined by the modes, which can survive the further evolution of the Universe. Assuming the flux conservation, we have

$$B_0 = \left(\frac{a_e}{a_0}\right)^2\sqrt{2\int\limits_{a_iH}^{k_{\rm diff}}\frac{dk}{k}\frac{d\rho_B}{d\ln k}}, \quad (39)$$

where $k_{\rm diff}$ is the momentum, which now corresponds to the cosmic diffusion scale, i.e., the smallest size of a magnetic configuration, which can survive the diffusion in the late stages of the Universe evolution. It could be estimated as $k_{\rm diff}/a_0 \sim 1\,\text{A.U.}^{-1} = = 1.3 \times 10^{-27}\,\text{GeV}$ [3].

Since $6 + 2\alpha > 0$ in the region without back-reaction, the magnetic power spectrum has a blue tilt. Therefore, the main contribution to the magnetic field comes from the upper integration boundary

$$B_0 = \left(\frac{a_e}{a_0}\right)^2\frac{H^2}{2\pi}\sqrt{\frac{2\mathcal{F}(\alpha)}{3+\alpha}}\left(\frac{k_{\rm diff}}{a_eH}\right)^{3+\alpha}. \quad (40)$$

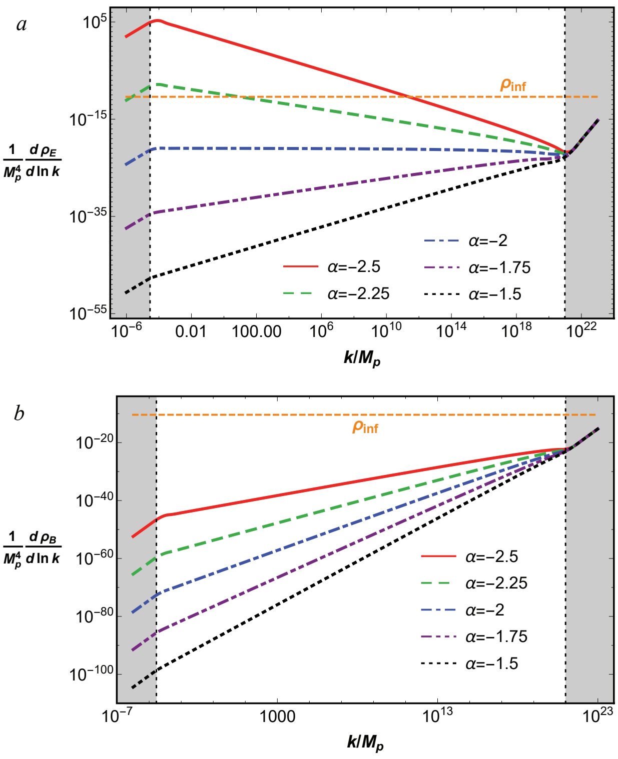

Fig. 3. The magnetic (a) and electric (b) power spectra at the end of the inflation for different values of the parameter α. The left shaded region corresponds to unphysical modes, which are outside the horizon even before the beginning of the inflation. The right shaded region corresponds to very short modes, which are inside the horizon during the inflation stage and undergo no enhancement. The orange horizontal line corresponds to the energy density of the inflaton field at the end of the inflation

The value of $\frac{a_0}{a_e}$ can be found, by using the fact that the pivot scale k_* crosses the horizon N_* e-folds before the end of the inflation

$$\frac{a_0}{a_e} = \frac{a_*}{a_e}\frac{a_0 H_*}{k_*} = e^{-N_*}\frac{a_0 H_*}{k_*}. \qquad (41)$$

Then the present-day strength of the magnetic field equals

$$B_0 = \frac{M_p^2}{2\pi}\sqrt{\frac{2\mathcal{F}(\alpha)}{3+\alpha}}\left[\frac{k_{\rm diff}}{a_0 M_p}\right]^{3+\alpha}\left[\frac{k_* e^{N_*}}{a_0 M_p}\right]^{-1-\alpha} =$$
$$= (1.6\times 10^{-23}\,G)\sqrt{\frac{2\mathcal{F}(\alpha)}{3+\alpha}}\left[\frac{1.03\times 10^{14}}{\exp(N_*)}\right]^{\alpha+2}\times$$
$$\times\left[\frac{k_{\rm diff}/a_0}{1\,\text{A.U.}^{-1}}\right]^{3+\alpha}\left[\frac{k_*/a_0}{0.002\,\text{Mpc}^{-1}}\right]^{-1-\alpha} e^{N_*-60}. \qquad (42)$$

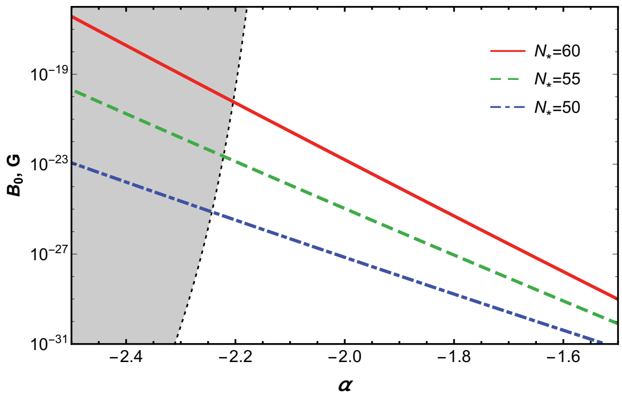

Fig. 4. The present-day value of the magnetic field as a function of α for different numbers of e-folds $N_* = 50, 55, 60$. The shaded region is forbidden in view of the back-reaction problem

Now, setting f and N_*, we calculate the corresponding H_* and determine the allowed region for the parameter α, by using Eq. (38). After that, using Eq. (42), we compute the present-day value of the magnetic field B_0. The corresponding result is shown in Fig. 4 for $f = 7.5\,M_p$ and three different e-fold numbers N_*. In the calculations, we used the standard values of the pivot scale ($k_*/a_0 = 0.002\,\text{Mpc}^{-1}$ used by the Planck Collaboration [1]) and the cosmic diffusion scale ($k_{\rm diff}/a_0 = 1\,\text{A.U.}^{-1}$ from [3]).

Unfortunately, the value of B_0 is considerably smaller than the lower bound required by the observations of distant blazars [11–14]. The power spectrum of the magnetic field has a blue tilt because the spectral index is positive

$$n_B = 6 + 2\alpha > 0. \qquad (43)$$

This means that the coherence length of such magnetic fields is of the order of cosmic diffusion scale, i.e., 1 A.U.

4. Conclusions

In this work, we studied the generation of large-scale magnetic fields in the natural inflation model, which is one of the favored models according to the latest results of the Planck Collaboration [1]. In order to break the conformal invariance of the electromagnetic action, we chose the kinetic coupling $I^2(\phi)FF$ of the inflaton field with the electromagnetic field through the coupling function, which behaves like a negative power of the scale factor $I \propto a^\alpha$, $\alpha < 0$. Since $I = 1$ at the end of the inflation and is a decreasing function, the strong coupling problem does not occur during the inflation [48].

We performed the slow-roll analysis and compared the predictions of the model with observational results of the Planck Collaboration [1]. In our numerical simulations, we used the values of the model parameters, which are in the best accordance with observational data and provide the correct amplitude of scalar primordial perturbations. Using them, we solved the background equations, which govern the evolution of the scale factor and the inflaton field.

The simple choice of the coupling function allowed us to use the well-known analytic expressions for the electromagnetic power spectra from Ref. [47] and to determine the parameter range, for which the back-reaction problem does not occur. Then we determined the power spectra numerically and confirmed the correctness of our analytic estimates.

Finally, we considered the subsequent evolution of the generated magnetic fields up to the present epoch. Considering only the modes, which can survive the cosmic diffusion during the Universe lifetime, we calculated the present value of the large-scale magnetic field.

It is well known [47, 48, 52] that the back-reaction may spoil the inflation and the process of magnetogenesis. However, it is not necessarily the case. This problem only signals that we cannot solve the background equations and the equations for the electromagnetic field separately, and the electromagnetic fields also influence the evolution of the Universe. It is possible that the electromagnetic field will continue to grow but in other regime. We plan to address this question in future studies.

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МАГНЕТОГЕНЕЗИС У МОДЕЛІ ПРИРОДНОЇ ІНФЛЯЦІЇ

Р е з ю м е

Досліджено процес інфляційного магнетогенезису у моделі природної інфляції з одним полем, параметри якої вибрано згідно з даними нещодавніх спостережень Колаборації Planck [1]. Конформну інваріантність максвеллівської дії порушено кінетичним зв'язком з інфлатонним полем через функцію, яка поводить себе як степінь масштабного фактора, $I(\phi) \propto a^{\alpha}$, і значення $\alpha < 0$ використано для того, щоб уникнути проблеми сильного зв'язку. Для таких значень α електрична компонента густини енергії домінує над магнітною і для $\alpha \lesssim -2{,}2$ вона призводить до сильної зворотної реакції, що може порушити інфляційну динаміку і припинити підсилення магнітного поля. Показано, що магнітні поля, згенеровані без спричинення зворотної реакції, не можуть перевищувати в теперішню епоху $\sim 10^{-20}$ Гс, а їхній спектр має блакитний ухил.