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Magnetohydrodynamic Flow of Viscous Fluid Between Two Parallel Porous Plates

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Abstract: This study deals with the problem of steady laminar flow of an electrically conducting viscous incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of a channel at the same rate. A solution for the case of small R (Suction Reynolds number) and M (Hartmann number) is discussed. Expressions for the velocity components and the pressure are obtained. The governing nonlinear differential equations are solved numerically using R-K Gill's method and the graphs of axial and radial velocity profiles have been drawn.

Key words: Parallel porous plates, transverse magnetic field, suction reynolds number, hartmann number, perturbation technique, pressure drop

INTRODUCTION

The Magnetohydrodynamic flow between two parallel porous plates is a classical problem in fluid dynamics and it is known as the Hartmann flow. The solution of the above problem has many applications in magnetohydrodynamic power generators, magnetohydrodynamic pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays.

The influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite, parallel, stationary and insulated plates was studied (Hartmann, 1937).

The problem of steady flow of an incompressible viscous fluid through a porous channel with rectangular cross section, when the Reynolds number is low was studied and a perturbation solution assuming normal wall velocities to be equal was obtained (Berman, 1953).

A detailed analysis of forced convection heat transfer to an electrically conducting liquid flowing in a channel with transverse magnetic field was studied (Perlmutter and Siegel, 1961).

The Hall effect on the steady motion of electrically conducting and viscous fluids in channels was studied (Tani, 1962).

The effect of the Hall currents on the steady magnetohydrodynamic couette flow with heat transfer was studied (Soundalgekar *et al*., 1979; Soundalgekar and Uplekar, 1986). The temperatures of the two plates were assumed either to be constant (Soundalgekar *et al*., 1979) or to vary linearly along the plates in the direction of the flow (Soundalgekar and Uplekar, 1986).

The effect of Hall current on the steady Hartmann flow subjected to a uniform suction and injection at the boundary plates was studied (Abo-El-Dhat, 1993).

The effect of temperature dependent viscosity on the flow in a channel has been studied in the hydromagnetic case (Attia and Kotb, 1996; Attia, 1999).

Here we consider the steady two dimensional laminar flow of an incompressible viscous fluid between two parallel porous plates in the presence of a transverse magnetic field by assuming the normal wall velocities to be equal. The perturbation solution obtained for this problem reduces to the results of Berman when the Hartmann number is zero (Berman, 1953).

MATHEMATICAL FORMULATION

The steady laminar flow of an incompressible viscous fluid between two parallel porous plates is considered in the presence of a transverse magnetic field of strength Ho applied perpendicular to the walls. The origin is taken at the centre of the channel and let x and y be the coordinate axes parallel and perpendicular to the channel walls.

The length of the channel is assumed to be L and 2 h is the distance between the two plates. Let u and v be the velocity components in the x and y directions, respectively.

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The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

The equations of momentum are

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B^2 u}{\rho} \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \quad (3)$$

where, σ is the electrical conductivity and $B = \mu_e$ Ho, μ_e being the magnetic permeability.

The boundary conditions are u (x, h) = 0, u (x,-h) = 0, v (*x*, h) = V and v (x,-h) = -V where V is the velocity of suction at the walls of the channel. Let η = y/h and the Eqs. 1-3 become

$$\frac{\partial u}{\partial x}+\frac{1}{h}\frac{\partial v}{\partial \eta}=0 \quad (4)$$

$$u\frac{\partial u}{\partial x}+\frac{v}{h}\frac{\partial u}{\partial \eta}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right)-\frac{\sigma B^2 u}{\rho} \quad (5)$$

$$u\frac{\partial v}{\partial x}+\frac{v}{h}\frac{\partial v}{\partial \eta}=-\frac{1}{\rho h}\frac{\partial p}{\partial \eta}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}\right) \quad (6)$$

where ν is the kinematic viscosity, ρ the density of the fluid, μ the coefficient of viscosity and p the pressure. The boundary conditions are converted into

$$u(x, 1) = 0, \quad u(x,-1) = 0 \quad (7)$$

and

$$v(x, 1) = V, \quad v(x,-1) = -V \quad (8)$$

Let Ψ be the stream function such that

$$u=\frac{1}{h}\frac{\partial \psi}{\partial \eta} \quad (9)$$

$$v=\frac{-\partial \psi}{\partial x} \quad (10)$$

The equation of continuity can be satisfied by a stream function of the form

$$\Psi(x, y) = [\, h\,U(0)-Vx\,]f(\eta) \quad (11)$$

where U (0) is the average entrance velocity at x = 0. From Eq. 11, the velocity components (9) and (10) are given by

$$u=\frac{1}{h}[hU(0)-Vx]\,f'(\eta) \quad (12)$$

$$v = V f(\eta) \quad (13)$$

where the prime denotes the differentiation with respect to the dimensionless variable η = y/h. Since the fluid is being withdrawn at constant rate from both the walls, v is independent of x. Using (12) and (13) in (5) and (6), the equation of momentum reduces to

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=\left(U(0)-\frac{Vx}{h}\right)\left(\frac{V}{h}\left(ff''-f'^2\right)-\frac{\nu}{h^2}f'''+\frac{\sigma B^2}{\rho}f'\right) \quad (14)$$

$$-\frac{1}{h\rho}\frac{\partial p}{\partial \eta}=\frac{V^2}{h}ff'-\frac{\nu V}{h^2}f'' \quad (15)$$

Now differentiating (15) w.r.t. x, we get

$$\frac{\partial^2 p}{\partial x \partial \eta}=0 \quad (16)$$

Differentiating (14) w.r.t. 'η', we get

$$\frac{\partial^2 p}{\partial x \partial \eta}=\left(U(0)-\frac{Vx}{h}\right)\frac{d}{d\eta}\left(\frac{V}{h}\left(ff''-f'^2\right)-\frac{\nu}{h^2}f'''+\frac{\sigma B^2 f'}{\rho}\right) \quad (17)$$

From (16), Eq. 17 can be written as

$$\frac{d}{d\eta}\left[\frac{V}{h}\left(ff''-f'^2\right)-\frac{\nu}{h^2}f'''+\frac{\sigma B^2}{\rho}f'\right]=0 \quad (18)$$

which is true for all x.

Let

R = Suction Reynolds number = hV/ν

M = Hartmann number = $Bh\left(\frac{\sigma}{\nu\rho}\right)^{1/2}$

Integrating (18) w.r.t. η and substituting the above expressions we get

$$f''' + R\,(f'^2 - ff'') - aR\,f' = K \quad (19)$$

where $a=\frac{H_0^2\mu_e^2\sigma h}{\rho V}$ and K is an arbitrary constant.

Boundary conditions on f (η) are f (1) = 1, f (-1) =-1, f′ (1) = 0 and f′ (-1) = 0 (20)

Hence the solution of the equations of motion and continuity is given by a nonlinear third order differential Eq. 19 subject to the boundary conditions (20).

RESULTS

Approximate analytic solution: The nonlinear ordinary differential Eq. 19 subject to conditions (20) must in general be integrated numerically. However for the special case when 'R' and 'a' are small, approximate analytic results can be obtained by use of a regular perturbation approach. In this situation f may be expanded in the form.

$$f=\sum_{n=0}^{\infty} R^n f_n(\eta) \quad (21)$$

$$K=\sum_{n=0}^{\infty} K_n R^n \quad (22)$$

where f_n satisfies the conditions

$$f_0(-1) = -1,\ f_0'(-1) = 0,\ \ f_0(1) = 1,\ f_0'(1) = 0 \tag{23}$$

and

$$f_n(0) = f_n'(0) = 0 \text{ and } f_n(1) = f_n'(1) = 0 \text{ when } n>0 \tag{24}$$

Here f_n's and k_n's are independent of R. Substituting (21) and (22) in (19) we get

$$(f_0''' + R f_1''' + R^2 f_2''' \ldots)+ R\,[(f_0' + Rf_1' + R^2f_2' \ldots)^2-((f_0 + R f_1 + R^2f_2 \ldots\ldots).$$
$$(f_0'' + Rf_1'' + R^2f_2'' \ldots\ldots))-a\,(f_0' + Rf_1' + R^2f_2'\ldots)] = K_0 + K_1 R + K_2R^2 + \ldots\ldots$$

Equating the coefficients of R, we get

$$f_0''' = K_0 \tag{25}$$

$$f_1''' + f_0'^2 - f_0f_0'' - af_0' = K_1 \tag{26}$$

$$f_2''' + 2 f_0' f_1' - f_0f_1'' - f_0'' f_1 - af_1' = K_2 \tag{27}$$

Solution of Eq. 25 subject to the boundary conditions (23) is

$$f_0(\eta) = \frac{K_0}{6}\eta^3 + \frac{A}{2}\eta^2 + B\eta + C$$

Applying the boundary conditions (23) to the above equation we get

$$f_0(\eta) = \frac{\eta}{2}\left(3-\eta^2\right) \tag{28}$$

Solution of Eq. 26 and 27 subject to the boundary conditions (24) are

$$f_1(\eta) = \frac{1}{280}\left(-\eta^7 + 3\eta^3 - 2\eta\right) + \frac{a}{40}\left(-\eta^5 + 2\eta^3 - \eta\right) \tag{29}$$

$$\begin{aligned} f_2(\eta) = &\frac{1}{280}\left(\frac{\eta^{11}}{330} - \frac{a\eta^9}{24} - \frac{\eta^9}{12} - \frac{3a\eta^7}{10} - \frac{a^2\eta^7}{6} + \frac{3\eta^7}{70} + \frac{3a\eta^5}{20} + \frac{7a^2\eta^5}{10}\right) \\ &+ \eta^3(0.000671 + 0.0027695a - 3.211667\times10^{-3}a^2) \\ &+ \eta\left(-0.00054 - 0.00209a + 0.0013a^2\right) \end{aligned} \tag{30}$$

$$K_0 = -3 \tag{31}$$

$$K_1 = -\frac{6a}{5} + \frac{81}{35} \tag{32}$$

$$K_2 = -3 + \left(\frac{-6a}{5} + \frac{81}{35}\right)R + R^2\left(-0.0174 - 0.05121a + 0.00573a^2\right) \tag{33}$$

Hence the first order perturbation solutions for f (η) and K are

$$f^{(1)}(\eta) = f_0(\eta) + R\, f_1(\eta) \text{ and}$$
$$K^{(1)} = K_0 + K_1 R$$

i.e.

$$f^{(1)}(\eta) = \frac{\eta}{2}(3-\eta^2) + \frac{R}{280}\left(-\eta^7 + 3\eta^3 - 2\eta\right) + \frac{M^2}{40}\left(-\eta^5 + 2\eta^3 - \eta\right) \tag{34}$$

where $aR = M^2$

$$K^{(1)} = -3 + \left(\frac{-6a}{5} + \frac{81}{35}\right)R \quad (35)$$

The Second order perturbation solutions for f(η) and K are

$$f^{(2)}(\eta) = f_0(\eta) + R\, f_1(\eta) + R^2 f_2(\eta) \text{ and}$$
$$K^{(2)} = K_0 + K_1 R + K_2 R^2$$

i.e.,

$$\begin{aligned} f^{(2)}(\eta) &= \frac{\eta}{2}(3-\eta^2) + \frac{R}{280}(-n^7 + 3\eta^3 - 2\eta) + \frac{M^2}{40}(-\eta^5 + 2\eta^3 - \eta) \\ &+ \frac{R^2\eta^{11}}{92400} - \frac{M^2R\eta^9}{6720} - \frac{R^2\eta^9}{3360} - \frac{3M^2R\eta^7}{2800} - \frac{M^4\eta^7}{1680} + \frac{3R^2\eta^7}{19600} + \frac{3M^2R\eta^5}{5600} + \frac{7M^4\eta^5}{2800} \\ &+ \eta^3(0.000671\,R^2 + 0.0027695\; M^2R - 0.003211667M^4) \\ &+ \eta(-0.00054R^2 - 0.00209M^2R + 0.0013M^4) \end{aligned} \quad (36)$$

$\because$ ($aR = M^2$, $aR^2 = aR.R = M^2R$, $a^2R^2 = M^4$)

$$K^{(2)} = -3 + \left(-\frac{6a}{5} + \frac{81}{35}\right)R - 0.0174R^2 - 0.05121\,M^2R + 0.00573\,M^4 \quad (37)$$

The above results reduces to the results of Berman when M = 0 (Berman, 1953).
Hence the first order expressions for the velocity components are

$$u(x,\eta) = \left(U(0) - \frac{Vx}{h}\right).\, f'(\eta) = \left(U(0) - \frac{Vx}{h}\right).\frac{3}{2}(1-\eta^2)\left(1 - \frac{R}{420}(2 - 7\eta^2 - 7\eta^4) + \frac{M^2}{60}(5\eta^2 - 1)\right) \quad (38)$$

$$v(\eta) = Vf(\eta) = V\left[\frac{1}{2}\eta(3-\eta^2) + \frac{R}{280}(3\eta^3 - 2\eta - \eta^7) + \frac{M^2}{40}(2\eta^3 - \eta - \eta^5)\right] \quad (39)$$

Pressure distribution: From Eq. (14), we have

$$\frac{h^2}{\rho v}\frac{\partial p}{\partial x} = \left(U(0) - \frac{Vx}{h}\right).\left(f'''(\eta) + R\begin{pmatrix} f'^2(\eta) - \\ f(\eta)f''(\eta)\end{pmatrix} - M^2 f'(\eta)\right)$$

and since $f'''(\eta) + R\,(f'^2(\eta) - f(\eta)f''(\eta)) - M^2f'(\eta) = K$ (from 19)

we have

$$\begin{aligned} \frac{\partial p}{\partial x} &= \frac{K\rho v}{h^2}\left(U(0) - \frac{Vx}{h}\right) \\ &= \frac{K\mu}{h^2}\left(U(0) - \frac{Vx}{h}\right) \qquad \because \left(v = \frac{\mu}{\rho}\right) \end{aligned} \quad (40)$$

Now, from Eq. 15, we have

$$\frac{\partial p}{\partial \eta} = \frac{\mu V}{h} f''(\eta) - \rho v^2\, f(\eta)\, f'(\eta) \quad (41)$$

Since

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial y}dy$$

$$\Rightarrow dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial \eta}.d\eta \left(\because \eta = \frac{y}{h} \right)$$

$$\Rightarrow dp = \frac{K\mu}{h^2}\left(U(0) - \frac{Vx}{h} \right)dx + \left(\frac{\mu V}{h} f''(\eta) - \rho v^2 f(\eta).f'(\eta) \right)d\eta \tag{42}$$

Integrating (42) we get

$$p(x,\eta) = p\,(0,0) - \frac{\rho v^2}{2} f^2(\eta) + \frac{K\mu}{h^2}\left[U(0)x - \frac{Vx^2}{2h} \right] + \frac{\mu V}{h}\left(f'(\eta) - f'(0) \right) \tag{43}$$

∴ The pressure drop in the major flow direction is given by

$$p\,(0,\eta) - p\,(x,\eta) = \frac{K\mu}{h^2}\left(\frac{Vx^2}{2h} - xU(0) \right) \tag{44}$$

NUMERICAL SOLUTION

The approximate results of the previous section are not reliable when the Reynolds number is not small. To obtain the detailed information on the nature of the flow for different values of R and M, a numerical solution to the governing equations is necessary. For different ranges of the parameters R and M, the two point boundary value problem expressed by Eq. (19) and (20) has been integrated by using R-K Gill's method and the graphs have been drawn.

DISCUSSION

The axial velocity and the radial velocity profiles have been drawn for values of R in the range of $0 \le R \le 10$ and different values of M. These are shown in Fig. 1-6. Figure 1 represents the axial velocity profiles for M = 0 when R takes the values 0.5, 1.0 and 5.0. These profiles decrease in the central region and increase near the walls with the increase of R.

Figure 2 shows the axial velocity profiles for M = 0.5, 0.707, 1.581 for the values of R = 0.5, 1.0 and 5.0, respectively. As M increases the axial velocity profiles decrease in the central region and increase near the walls.

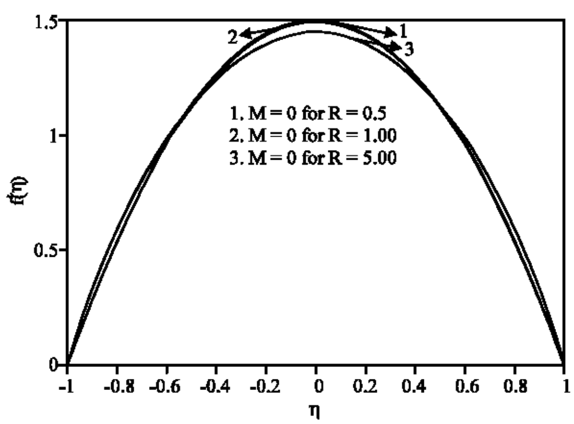

Fig. 1: Axial velocity profiles when M = 0 for different values of R

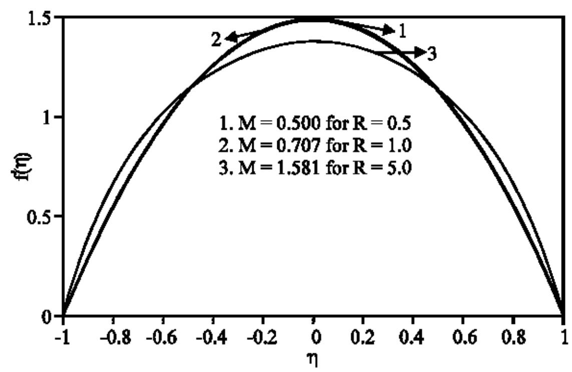

Fig. 2: Axial velocity profiles when M = 0.5, M = 0.707, M = 1.581 for different values of R

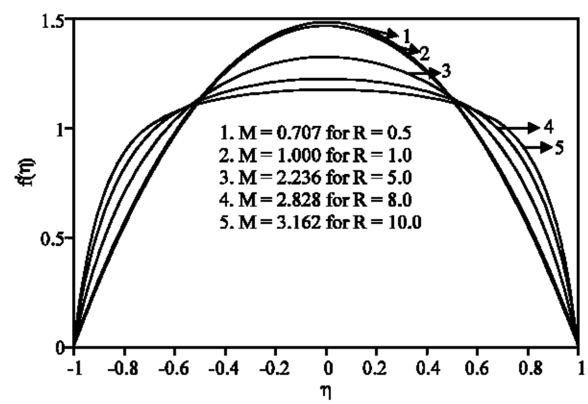

Fig. 3: Axial velocity profiles when M = 0.707, M = 1.0, M = 2.828, M = 3.162 for different values of R

In Fig. 3 and 4 it is seen that as M increases the axial velocity profiles become flat in the central portion and steep near the walls. This gives that for large M, the fluid moves like a block which shows some sort of rigidity. This confirms the idea that in conducting fluids, magnetic field

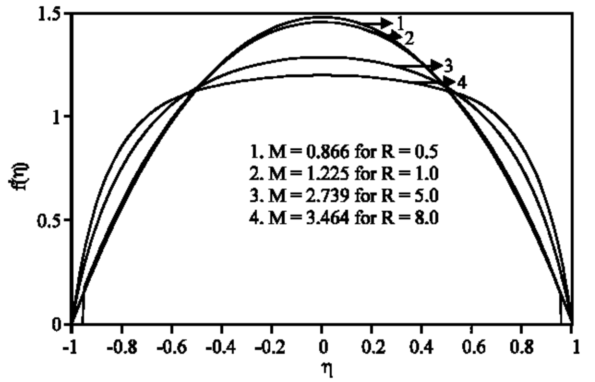

Fig. 4: Axial velocity profiles when M = 0.866, M = 1.225, M = 2.739, M = 3.464 for different values of R

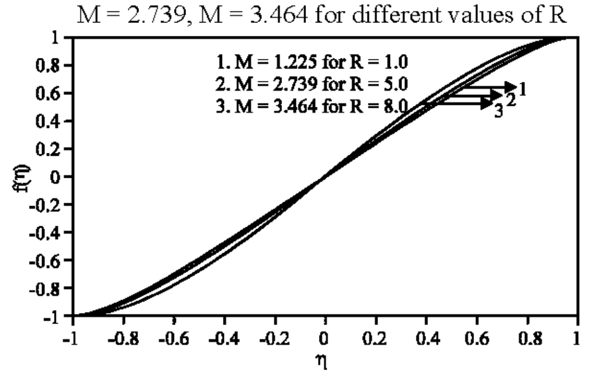

Fig. 5: Radial velocity profiles when M = 1.225, M = 2.739, M = 3.464 for different values of R

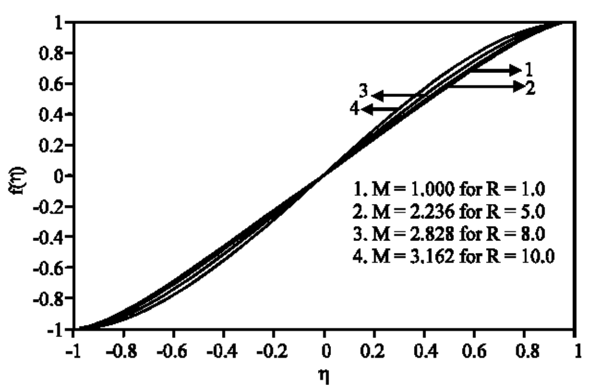

Fig. 6: Radial velocity profiles when M = 1.0, M = 2.236, M = 2.828, M = 3.162 for different values of R

brings rigidity in the fluid. Hence it is observed that $f'(\eta)$ decreases with increase in the values of R and the profile is parabolic. This is in good agreement and correlates well with the results of Berman (1953).

The function $f(\eta)$ (η-velocity profiles) is plotted against η for various values of R in Fig. 5 and 6, respectively. It is observed that for R>0 and for different values of M in the region $-1 \leq \eta \leq 0$, f decreases with increase of R while in the region $0 \leq \eta \leq 1$, f increases with increase of R.

CONCLUSION

In the above analysis a class of solutions of the magnetohydrodynamic flow of viscous fluid between two parallel porous plates is presented, in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of a channel at the same rate. The result obtained for this problem reduces to the result of (Berman, 1953) when the Hartmann number is Zero (i.e., when M = 0).

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REFERENCES

Abo-El-Dhat, E., 1993. Hartmann flow with uniform suction and injection at the bounding plates. MS Thesis, Helwan University, Egypt.

Attia, H.A. and N.A. Kotb, 1996. MHD flow between two parallel plates with heat transfer. Acta Mechanica, 117: 215.

Attia, H.A., 1999. MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Mech. Res. Comm., 26: 115.

Berman, A.S., 1953. Laminar flow in channels with porous walls. J. Applied Phys., 24: 1232.

Hartmann, J., 1937. Hg-Dynamics-I, Theory of the laminar flow of an electrically conducting liquid in a homogeneous magnetic field, Kgl, Danske Videnskab. Selskab Mat.-Fys. Medd. 15, No. 6.

Perlmutter, M. and R. Siegel, 1961. Heat transfer to an electrically conducting fluid flowing in a channel with a transverse magnetic field. NASA, TN-875.

Soundalgekar, V., N. Vighnesam and H. Takhar, 1979. Steady MHD couette flow with heat transfer considering constant temperature. IEEE Trans. Plasma Sci., 7: 178.

Soundalgekar, V. and A. Uplekar, 1986. Steady MHD couette flow with heat transfer considering varying temperature. IEEE Trans. Plasma Sci., 14: 579.

Tani, I., 1962. Steady motion of electrically conducting and viscous fluids in channels. J. Aerospace Sci., 29: 287.