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Magnetohydrodynamic processes in strongly magnetized young neutron stars

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ABSTRACT

We examine the early evolution of a neutron star with a very strong magnetic field ($B \geq 4 \times 10^{13}$ G) that occupies a significant fraction of the core volume. The electrical conductivity of the core matter is a strong function of the magnetic field, therefore the evolution of magnetized neutron stars (magnetars) may well be different from that of ordinary radiopulsars. We consider magnetohydrodynamic processes in the core for two possible models of nuclear matter, with normal and superfluid neutrons. In the case of the normal matter, an enhancement of the resistivity perpendicular to the magnetic field can result in rapid field decay during the early evolutionary stage. If neutrons are in the superfluid state, we find that the Hall effect can lead to oscillatory behaviour of the magnetic field. This oscillatory behaviour is caused by the generation of large-scale helicoid modes resulting from non-linear coupling between the different field components.

Key words: MHD – stars: magnetic fields – stars: neutron – pulsars: general.

1 INTRODUCTION

The magnetic fields of relatively young pulsars, as inferred from their spin-down rates, lie in a narrow range $4 \times 10^{13} \geq B \geq 5 \times 10^{11}$ G. It cannot be excluded by theoretical reasons, however, that some neutron stars are born with stronger magnetic fields. Several arguments coming from observations hint at the existence of neutron stars with magnetic fields in excess of 4×10^{13} G. Analysing soft gamma-ray repeater sources and their associations with young supernova remnants, Kulkarni & Frail (1993) have pointed out that these sources can be young long-period neutron stars with magnetic fields $\sim 10^{14}$–10^{15} G. The very strong magnetic field can spin down these pulsars to a period ~ 10 s on the time-scale of the order of $\sim 10^3$–10^4 yr. Heyl & Hernquist (1998) have discussed the possibility that recently discovered soft X-ray source RX J0720.4 − 3125 (Haberl et al. 1997) is a nearby neutron star with period $P = 8.39$ s and magnetic field $\sim 10^{14}$ G.

The presence of such a strong magnetic field in some young pulsars may have dramatic consequences on their early evolution. The electric resistivity of plasma becomes anisotropic in a strong magnetic field, and this anisotropy may noticeably alter the behaviour of the magnetic field compared with ordinary radiopulsars. For example, the electric resistivities along and across the magnetic field may differ and as a result of this different components of the electric current may dissipate at different rates. Apart from this, the Hall component of the resistivity tensor leads to a drift of the magnetic field with the velocity perpendicular to both the current density, $\boldsymbol{j}$, and the magnetic field, $\boldsymbol{B}$. In a strongly magnetized plasma the Hall drift can operate on a much shorter time-scale than ohmic dissipation. The Hall current is non-dissipative and does not contribute directly to the rate of dissipation of the magnetic field. However, this current is non-linear, and therefore it couples different magnetic modes and redistributes the magnetic energy among them. The behaviour of the redistribution is oscillatory (Shalybkov & Urpin 1997) and owing to this, the magnetic multipoles outside the star can also oscillate. In strongly magnetized neutron stars, these non-linear magnetohydrodynamic phenomena may play a dominating role.

The influence of non-linear magnetohydrodynamic effects on the evolution of magnetized stars has been considered by a number of authors (see e.g. Muslimov 1994, Muslimov, Van Horn & Wood 1995, Thompson & Duncan 1996 and Heyl & Kulkarni 1998). In their analysis of the transfer of energy among the modes caused by the Hall effect, Muslimov (1994) and Muslimov et al. (1995) have omitted the term that couples the dipole field with the toroidal one. As a result, their calculations cannot reproduce the well-known phenomena resulting from the Hall effect (like helicoid magnetic oscillations). In their analysis of the thermal and magnetic evolution of strongly magnetized young neutron stars (‘magnetars’), Heyl & Kulkarni (1998) modelled the Hall drift in terms of the characteristic time-scale (following the earlier paper by Goldreich & Reisenegger 1992) and included this non-dissipative effect among the mechanisms of the magnetic field decay. By making use of this approximation the authors found that in young neutron stars ‘fields of intermediate strength ($B \sim 10^{12}$–10^{13} G) decay via the Hall drift’.

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In this paper we discuss magnetohydrodynamic phenomena that may influence the early evolution ($t \lesssim 10^6$ yr) of the magnetic field in the core of strongly magnetized neutron stars. We focus on anisotropic properties of the electric resistivity and effects associated with the Hall current that can lead to rather unusual oscillatory behaviour of the magnetic field. In Section 2, we discuss the conductive properties of neutron-star interiors and equations governing the magnetic evolution. The results of the calculations of the magnetic evolution of magnetized stars are represented in Section 3. Finally, a brief summary of results is given in Section 4.

2 THE CONDUCTIVE PROPERTIES OF NEUTRON-STAR INTERIORS

The plasma in neutron-star cores is usually regarded as a mixture of strongly degenerate fermions. At densities of the order of the nuclear density, the main constituents are neutrons with a small admixture of protons and electrons. Both sorts of nucleons in the core can generally undergo phase transitions to the superfluid state. Evidently, the presence of superfluid and/or superconductive phases can dramatically change the transport properties of the nuclear matter. In the present paper we consider magnetohydrodynamic processes only in the case when protons are in the normal state but neutrons can be in normal or superfluid states. At relatively low densities, the $^1\mathrm{S}_0$ interaction governs pairing, thus s-wave superfluidity occurs. At high densities, s pairing is supressed but the $^3\mathrm{P}_2$ interaction becomes efficient and p-wave superfluidity can occur (see e.g. Shapiro & Teukolsky 1983 for details). For some ratios of the nucleon effective masses and nucleon–nucleon potentials, the p-wave gaps are smaller than the s-wave gaps. As neutrons in neutron-star interiors pair through p waves while the less abundant protons interact through s waves, this suggests that the protons are generally superfluid if the neutrons are. However, at a sufficiently high density ($\rho \geq 5 \times 10^{14}$ g cm^{-3}) the proton gap becomes zero, thus protons cannot be superconductive in this region (see e.g. Van Riper 1991). On the contrary, neutrons can be superfluid at such high densities. Besides, both the s- and p-wave gaps are sensitive functions of the nucleon effective masses and in the absence of the precise values of these masses the relative magnitudes of the proton and neutron gaps must be considered as only order-of-magnitude estimates (in particular, when we deal with strongly magnetized neutron stars). Therefore, the model adopted in this paper does not contradict with the current understanding of the properties of superdense matter and may well represent the behaviour of the magnetic field at some evolutionary stages or in some layers of the core.

In the presence of the magnetic field the electrical resistivity is given by the tensor $\boldsymbol{R}$. If the axis z is directed along the magnetic field, this tensor is

$$\boldsymbol{R} = \begin{pmatrix} R_\perp & R_\wedge & 0 \\ -R_\wedge & R_\perp & 0 \\ 0 & 0 & R_\parallel \end{pmatrix}, \tag{1}$$

where the tensor components along and across the field are marked by $\parallel$ and $\perp$, respectively. $R_\wedge$ is the so-called Hall component. In the present paper we use the resistivities of nuclear matter calculated by Yakovlev & Shalybkov (1991). These resistivities do not take into account the influence of chemical reactions and the inhomogeneity of the chemical potential in the npe matter (nuclear matter consisting of neutrons, protons and electrons) on the rate of dissipation of the electrical currents. As shown by Pethick (1991), the effect of build-up of composition changes may generally be comparable to the rate of dissipation resulting from the perpendicular resistivity and may slow down the decay of the magnetic field. However, the simple fitting expressions obtained by Yakovlev & Shalybkov (1991) give a reasonable order-of-magnitude estimate of the resistivity in the neutron-star core.

In the case of npe matter with normal neutrons, the components of the resistivity tensor are

$$R_\parallel = R_0 = 6.06 \times 10^{-29} T_8^2 (\rho_\mathrm{n}/\rho)^3 (1 + \rho/2\rho_\mathrm{n}) \ \ s,$$
$$R_\perp = R_0(1 + B^2/B_0^2), \quad R_\wedge = R_0(B/B_\mathrm{e}), \tag{2}$$

where T and ρ are the temperature and density, respectively; $T_8 = T/10^8$ K; $\rho_\mathrm{n} = 2.8 \times 10^{14}$ g cm^{-3} is the nuclear density. The characteristic magnetic fields are given by

$$B_0 = 7.1 \times 10^{10} T_8^2 (\rho_\mathrm{n}/\rho)^{1/2} \ \ G, \tag{3}$$

$$B_\mathrm{e} = 8.4 \times 10^8 T_8^2 (\rho_\mathrm{n}/\rho + 0.17) \ \ G. \tag{4}$$

In a strong magnetic field, $B \gg B_0$, the transverse resistivity $R_\perp$ is much larger than the parallel one because of the magnetization of the charge carriers. Owing to this, the component of the electrical current perpendicular to the magnetic field dissipates faster than the parallel component.

In the course of evolution, the internal temperature of a neutron star falls down and a phase transition to the superfluid state may occur. If the neutrons are in the superfluid state they hardly influence the kinetic phenomena in the nuclear matter. Owing to this, the transport properties of npe plasma are similar to those of fully ionized two-component plasma. The components of the resistivity tensor in this case are (Yakovlev & Shalybkov 1991)

$$R_\perp = R_\parallel = R_0, \quad R_\wedge = R_0 \frac{B}{B_\mathrm{e}}. \tag{5}$$

As $R_\perp$ is independent of the magnetic field, all the components of the current decay at the same rate. In strongly magnetized stars, the Hall resistivity, $R_\wedge$, may reach a very high value compared with R_0 because the magnetization field B_e is relatively small.

The evolution of the magnetic field is governed by the induction equation, which in the absence of hydrodynamic motions has the form

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c^2}{4\pi} \nabla \times [R_\parallel (\nabla \times \boldsymbol{B})_\parallel + R_\perp (\nabla \times \boldsymbol{B})_\perp + R_\wedge (\nabla \times \boldsymbol{B}) \times \boldsymbol{b}], \tag{6}$$

where $\boldsymbol{b} = \boldsymbol{B}/B$. The neglect of hydrodynamic motions in the core may be a reasonable approximation because these motions have to dissipate on a very short time-scale owing to the high viscosity of the nuclear matter caused by β-processes (Gourgoulhon & Haensel 1993; Urpin & Shalybkov 1996).

Note that sometimes it is convenient to use another representation of the induction equation (6). Taking account of the fact that $\boldsymbol{a}_\perp = \boldsymbol{B} \times (\boldsymbol{a} \times \boldsymbol{B})/B^2$ for any vector $\boldsymbol{a}$, one can re-express equation (6) as

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c^2}{4\pi} \nabla \times [R_\parallel (\nabla \times \boldsymbol{B}) + R_\wedge (\nabla \times \boldsymbol{B}) \times \boldsymbol{b}] + \nabla \times (\boldsymbol{w}_a \times \boldsymbol{B}), \tag{7}$$

where

$$\boldsymbol{w}_a = \frac{c(R_\perp - R_\parallel)}{B^2} \boldsymbol{j} \times \boldsymbol{B} \tag{8}$$

is the velocity associated with the ambipolar drift of the charged fluid relative to the neutral one. Equation (7) does not contain the perpendicular resistivity because the corresponding term is included in the ambipolar drift velocity $\boldsymbol{w}_a$. The representations (6) and (7) obviously describe the same evolution of the magnetic field but using different terms (for more details see Shalybkov & Urpin 1995).

The magnetic evolution of the core is essentially determined by its thermal evolution because the parallel and perpendicular resistivities depend on the temperature. For the range of magnetic fields considered in the present paper ($\leq 4\times10^{14}$ G), ohmic dissipation does not noticeably influence the thermal evolution during the early evolutionary stage, $t \lesssim 10^6$ yr. Therefore, when calculating the magnetic evolution, one can use the time dependence of the internal temperature obtained by Van Riper (1991) for the standard neutron-star model with normal neutrons. Note that Joule heating may be of importance during the later evolution (Urpin & Shalybkov 1995); however, this is beyond the scope of the present paper.

3 CALCULATIONS

The computations have been performed for the 'standard' neutron star with mass $M = 1.4\,\mathrm{M}_\odot$ and radius $a = 10.6$ km. For the sake of simplicity, we assume in the numerical calculations that the core of a neutron star is uniform and isothermal.

If matter in the core is in the normal state then the behaviour of the magnetic field is very sensitive to its initial geometry. For example, a force-free configuration with the electrical current parallel to the magnetic field evolves on a very long time-scale determined by R_0 because $\boldsymbol{B}\times(\nabla\times\boldsymbol{B}) = 0$ for this configuration, and the enhanced perpendicular resistivity as well as the Hall drift do not contribute to the magnetic evolution. An estimate of the magnetic decay time for this case has been obtained by Baym, Pethick & Pines (1969). On the contrary, for relatively simple magnetic configurations the electrical current has a component perpendicular to $\boldsymbol{B}$, and this component can evolve on a short time-scale owing to an enhancement of the perpendicular resistivity. For instance, a pure toroidal magnetic field or the field corresponding to a dipole outside the star are maintained by currents exactly perpendicular to $\boldsymbol{B}$, and the evolution of such fields is determined by $R_\perp$. In the present paper, we consider the case when the initial distribution of currents in the core has a substantial perpendicular component, because there is no evidence suggesting that neutron stars are born with a force-free field.

For configurations with $\boldsymbol{j}\perp\boldsymbol{B}$, the relative importance of the decay rate and the Hall effect is given by the dimensionless parameter, ξ,

$$\xi = \frac{R_\wedge}{R_\perp} = \frac{84.5(\rho_n/\rho)^{1/2}}{(\rho_n/\rho + 0.17)} \times \frac{y}{1+y^2}, \tag{9}$$

where $y = B/B_0$. As a function of B, this quantity has a maximum at the low magnetic field $B = B_0$; however, even the maximum value is $\lesssim 50$. For a stronger magnetic field, $B \gg B_0$, the parameter ξ is much smaller. Direct calculations of the influence of the Hall effect on the decay of the magnetic field (Urpin & Shalybkov 1991; Shalybkov & Urpin 1997) indicate that this influence becomes appreciable only if ξ is relatively large, $\xi \gtrsim 20 - 50$. As such values of ξ can never be reached in strongly magnetized neutron stars we have to conclude that the Hall effect plays an insignificant role in the magnetic evolution if the core matter does not experience the phase transition to superfluidity.

In Fig. 1, we plot the behaviour of the magnetic field in the neutron-star core with normal neutrons. As has been mentioned, the field decay is approximately the same for the toroidal and dipole configurations which are both governed only by the perpendicular and Hall resistivities. We show the results for the toroidal mode. The initial field distribution is assumed to be proportional to the fundamental eigenmode of the toroidal field corresponding to the uniform conducting sphere. The maximum initial field strength inside the core, B_m, is taken to be 4×10^{13} and 4×10^{14} G.

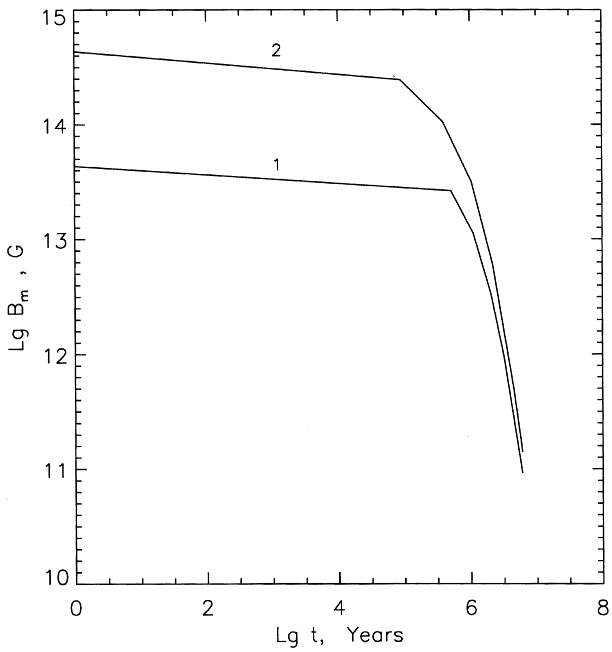

Figure 1. The time dependence of the maximum field strength in the core, B_{max}, for the neutron-star model with normal neutrons and for the maximum initial field 4×10^{13} (curve 1) and 4×10^{14} G (curve 2).

Owing to a specific dependence of $R_\perp$ on B, the evolution of the field inside the core is quite unusual. If $B \gg B_0$, the rate of dissipation is higher in the region with a stronger magnetic field because $R_\perp \propto B^2$. Therefore, the enhancement of the transverse resistivity tends to produce a distribution that becomes with time more and more uniform in the main fraction of the volume. Regions with strong field gradients (or, in other words, with a high density of electrical current) are concentrated only near the surface and the magnetic axis. Obviously, a strong enhancement of the resistivity drastically increases the rate of dissipation. For example, in a weak magnetic field, $B \leq B_0$, the decay time-scale can be estimated as

$$t_d = \frac{4a^2}{\pi R_0 c^2} \sim 5\times10^{11} T_8^{-2}\ \mathrm{yr}. \tag{10}$$

The decay caused by $R_\perp$ is much faster: at the beginning of evolution the decay time-scale is $\approx 10^6$ yr for $B_m = 4\times10^{13}$ G and only $\approx 4\times10^5$ yr for $B_m = 4\times10^{14}$ G. The behaviour of the maximum field in the core is quite remarkable: the difference in the field strength becomes smaller and smaller with time for configurations with different initial B_m. For example, after ≈ 6 Myr the field is only a factor of ~ 1.5 stronger for the initial configuration with $B_m = 4\times10^{14}$ G than with $B_m = 4\times10^{13}$ G. The reason for this behaviour is the dependence of resistivity on B: dissipation is faster in the star with a stronger magnetic field. This effect was found first by Haensel, Urpin & Yakovlev (1991) and rediscovered recently by Heyl & Kulkarni (1998). The decay caused by $R_\perp$ is so efficient that the maximum field strength falls down to below 10^{12} G after ≈ 4 Myr for both considered models.

The behaviour of the field can be qualitatively different if matter inside the neutron star experiences the phase transition to superfluidity at some evolutionary stage. As has been mentioned, the transition temperature is a sensitive function of a rather uncertain nucleon effective mass. Estimates of some authors (see e.g. Shapiro & Teukolsky 1983 and Maxwell 1979) give the value $\sim(5–8)\times10^8$ K for this temperature. If the star actually experiences the phase transition at such a high temperature then the duration of the first stage (when the evolution of the magnetic field is driven by an enhanced transverse resistivity) is probably not longer than $\sim$100 yr for the standard cooling model. It is seen from Fig. 1 that the decay is insignificant on such a short time-scale, thus the neutron star enters the superfluid phase with the magnetic field practically the same as the initial magnetic field.

In a superfluid core, the dissipative components of the resistivity tensor, $R_{\parallel}$ and $R_{\perp}$, are very small but the Hall resistivity can be much greater. Even if the phase transition occurs at $T\sim10^9$ K, the Hall parameter is sufficiently large, $\xi = R_{\wedge}/R_0 = B/B_e \gtrsim 100$ for $B \geq 4\times10^{13}$ G. Unfortunately, numerical solution of equation (6) for very large ξ meets some computational problems (numerical instability). However, a qualitative behaviour of the magnetic field may well be understood from the solution for smaller values of the Hall parameter, $\xi \lesssim 100$ (see also Shalybkov & Urpin 1997) . Using a simple physical interpretation of calculations for $\xi \leq 100$, we can reliably interpolate the results to larger ξ.

If neutrons are in a superfluid state, the induction equation (6) transforms to

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c^2R_0}{4\pi}\nabla\times[\nabla\times\boldsymbol{B}+\xi(\nabla\times\boldsymbol{B})\times\boldsymbol{b}]. \tag{11}$$

The magnetic field can be represented as a sum of the poloidal, $\boldsymbol{B}_{\rm p}$, and toroidal, $\boldsymbol{B}_{\rm t}$, components. Introducing the dimensionless time variable, $d\tau = c^2R_0 dt/4\pi a^2$, which expresses the time in units of a dissipative time, we obtain equations for the components in the form

$$\frac{\partial \boldsymbol{B}_{\rm p}}{\partial \tau} = -a^2\nabla\times\{\nabla\times\boldsymbol{B}_{\rm p}+\beta(\nabla\times\boldsymbol{B}_{\rm t})\times\boldsymbol{B}_{\rm p}\}, \tag{12}$$

$$\frac{\partial \boldsymbol{B}_{\rm t}}{\partial \tau} = -a^2\nabla\times\{\nabla\times\boldsymbol{B}_{\rm t}+\beta[(\nabla\times\boldsymbol{B}_{\rm p})\times\boldsymbol{B}_{\rm p} + (\nabla\times\boldsymbol{B}_{\rm t})\times\boldsymbol{B}_{\rm t}]\}, \tag{13}$$

where $\beta = \xi/B = 1/B_e$. As the Hall current is non-linear, different components are coupled. For instance, a toroidal field will be generated as a result of the Hall current associated with the term proportional to $(\nabla\times\boldsymbol{B}_{\rm p})\times\boldsymbol{B}_{\rm p}$ even if the initial magnetic configuration is pure poloidal. In its turn, the toroidal field contributes to a generation of poloidal modes [see equation (12)]. Owing to this coupling, even the evolution of the simplest initial poloidal configuration may be rather complex and may well illustrate the main qualitative features of the non-linear field decay (Shalybkov & Urpin 1997). Note, however, that the Hall current cannot provide a generation of poloidal modes if the original magnetic configuration is purely toroidal.

We consider the behaviour of the magnetic field assuming that its initial distribution in the core is given by the fundamental poloidal eigenfunction corresponding to the dipole mode outside the star. The boundary condition requires the field to be continuous at the surface, $r = a$. For the sake of simplicity, we neglect changes in the internal temperature. It is seen from equations (12) and (13) that the solution can be characterized only by one dimensionless parameter $\xi_0 = \beta B_0$, where B_0 is the strength of the initial field at the magnetic equator. In Fig. 2, we plot the time dependence of the toroidal component of the magnetic field inside the core for $\xi_0 = 100$. Initially, $\boldsymbol{B}_{\rm t} = 0$, but the Hall current generates the toroidal field from the poloidal one on a very short time-scale. The oscillatory behaviour is especially pronounced for the toroidal mode. During the initial stage of evolution when the dipole component does not decay appreciably, the field oscillates with the period

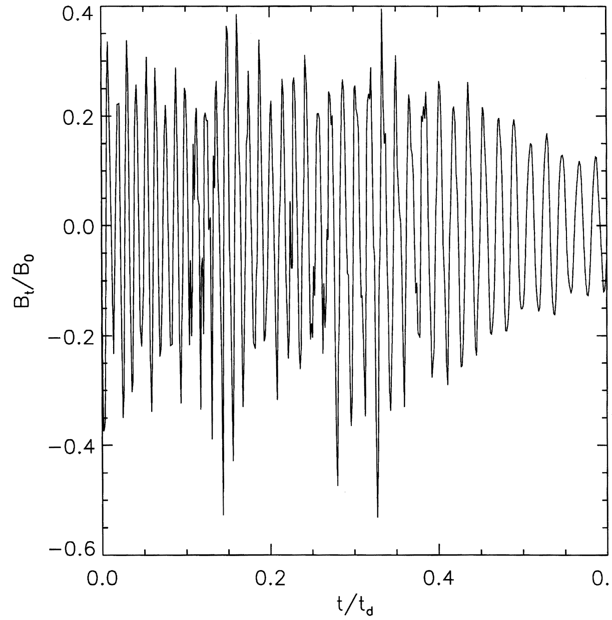

Figure 2. The time dependence of the toroidal magnetic field inside the core for $\xi_0 = 100$. The toroidal field is normalized to the initial surface field strength at the magnetic equator. The curve shows the oscillations of the field at the polar angle $\theta = 0.8\pi$ and at $r = 0.7a$.

$$P \approx t_{\rm d}/\xi_0. \tag{14}$$

Owing to the decay, the period of oscillations increases with time but for a young strongly magnetized star it may be relatively short ($\lesssim$1 Myr). At fixed θ, the amplitude of oscillations reaches its maximum at $r \approx 0.7a$. The generated toroidal field generally may be rather strong reaching $\sim$40 per cent of B_0 at its maximum. Owing to dissipation, the toroidal field, as well as other components, decays on a long time-scale determined by $t_{\rm d}$.

The time dependence of the poloidal field components at the surface is shown in Fig. 3 for the case $\xi_0 = 100$. In the calculations, we expanded the field outside the star into spherical harmonics and followed the behaviour of the multipoles up to the number $\ell = 9$. However, in all the considered cases the multipoles with $\ell > 5$ give a negligible contribution to the external magnetic field. Therefore, only the evolution of multipoles with $\ell = 1, 3$ and 5 is shown in Fig. 3. All poloidal modes exhibit oscillations with the same period P as the toroidal mode. For example, the dipole component experiences oscillations with the amplitude of the order of 10 per cent of the original field strength. Owing to this, the dipole magnetic moment of the star can increase during some periods of evolution. The duration of these periods of growth is $\sim P/2 \sim t_{\rm d}/\xi_0$ at the beginning of the evolution when the decay is insignificant. Of course, the total magnetic energy decreases monotonically despite the increase of the dipole component. For newly generated modes with $\ell = 3$ and 5, the oscillations are much more pronounced. The amplitude of the oscillations for the mode with $\ell = 3$ is of the order of its average strength which in its turn is $\sim$20 per cent of the dipole component. The mode with $\ell = 5$ can even change its sign but this mode contributes only 2–3 per cent to the total magnetic field.

The considered behaviour is quite general for all original poloidal magnetic configurations with a sufficiently large Hall parameter,

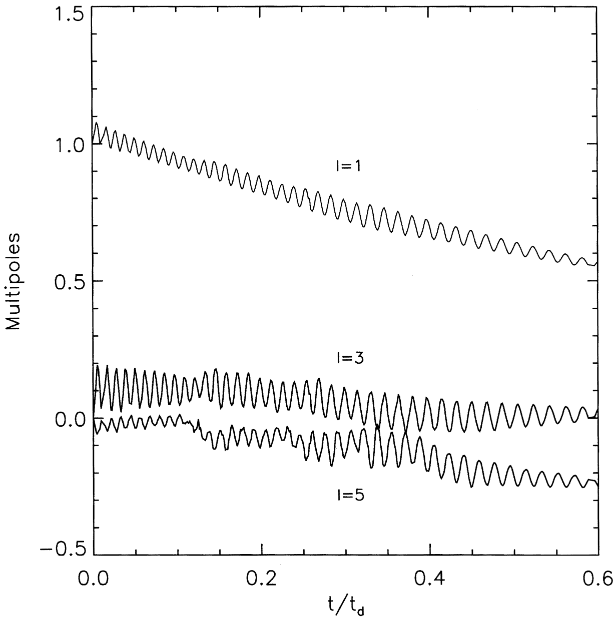

Figure 3. The time dependence of the poloidal field components with $\ell = 1, 3$ and 5 at the surface for $\xi_0 = 100$. The poloidal multipoles are normalized to the initial magnitude of the dipole component.

$\xi_0 \gtrsim 20$–50 (see Shalybkov & Urpin 1997). Amplitudes of newly generated modes reach a quasi-steady state value on a short time-scale associated with the Hall effect, $\sim t_d/\xi_0$. All modes undergo oscillations with a period P. The mechanism of these oscillations is rather simple (see Shalybkov & Urpin 1997). They are caused by the generation of large-scale helicoid modes (helicons) which exist in a strongly magnetized plasma as a result of the Hall effect (see e.g. Lifshitz & Pitaevskii 1981). These modes are maintained by a non-dissipative exchange of energy between different field components. Owing to ohmic dissipation, the helicons decay slowly on the time-scale $\sim t_d$. Most likely the same oscillatory behaviour is also typical for larger values of ξ ($\xi > 100$).

4 CONCLUSIONS

We have considered the behaviour of a strong magnetic field, $B \gtrsim 4 \times 10^{13}$ G, in young neutron stars under the assumption that this field occupies a significant fraction of the core volume. Such a strong field may be typical for some long-period neutron stars (see e.g. Kulkarni & Frail 1993). This field can magnetize the motion of charged particles in the core and drastically change the transport properties of the nuclear matter. The effect of a strong field may be essentially different for the npe matter with normal and superfluid neutrons (see Yakovlev & Shalybkov 1991).

In the case of normal npe matter, the magnetization of the electrons and protons leads to a strong enhancement of the resistivity across the magnetic field. Owing to this, magnetic configurations with the electrical current perpendicular to $\boldsymbol{B}$ decay relatively rapidly. The decay time depends on the strength of the magnetic field and may be as short as $\approx 5 \times 10^5$ yr for a star with an initial magnetic field of 4×10^{14} G. As the perpendicular resistivity and the rate of dissipation are larger for a star with a stronger magnetic field, the evolution proceeds in such a way that the difference in the field strength of stars with different initial fields becomes smaller and smaller with age. For example, if the stars start the evolution with $B = 4 \times 10^{13}$ and 4×10^{14} G, their fields are practically indistinguishable after $\sim 10^6$ yr. Despite a high dissipation rate, heating caused by this dissipation cannot essentially change the early thermal evolution ($t \lesssim 10^6$ yr) of a neutron star for all considered values of the magnetic field. However, Joule heating may be of importance for the later evolution. The Hall effect plays an insignificant role in the evolution of strongly magnetized stars with normal npe matter.

If the neutron star experiences the phase transition to superfluidity then the duration of the initial phase when the neutrons are in the normal state is likely to be very short (~ 100 yr for standard cooling). Even the enhanced transverse resistivity cannot lead to an appreciable dissipation of the magnetic field during such a short time. It is most likely that the neutron star enters the superfluid phase with the magnetic field approximately the same as the initial magnetic field. However, the behaviour of the field in the matter with superfluid neutrons can be qualitatively different compared with the initial phase when the neutrons are normal. Superfluid neutrons do not contribute to collision frequencies, thus the parallel and perpendicular resistivities become equal and very small. On the contrary, the Hall resistivity can reach a very high value as a result of the magnetization of the electrons. The evolution of the magnetic field is very specific under these conditions. Because of its non-linear origin, the Hall current couples the different components of the field and redistributes the magnetic energy among them. Owing to the coupling between the components, there exists a specific type of mode called helicons, and the field decay in neutron-star cores can be accompanied by a generation of large-scale helicons. These modes are oscillatory, and therefore a redistribution of energy caused by the Hall effect has non-monotonic oscillatory behaviour. All field components, including the dipole one, experience oscillations. For example, the amplitude of the dipole oscillations reaches ≈ 10 per cent of the initial field strength during the early evolutionary stage. Oscillations of the toroidal field inside the core may be even stronger: their amplitude is of the order of 40 per cent of the initial dipole field strength at the equator. The period of oscillation is given by a simple expression (14) and may be rather short for a strongly magnetized neutron star, $P \lesssim 10^6$ yr. If the conductivity of the crust surrounding the core is relatively low, the oscillations of the external field are generally observable. The amplitude of oscillation decreases on a long time-scale determined by the ohmic dissipation time in the core. Note that the Hall current is non-dissipative and cannot directly influence the rate of dissipation. However, this current can alter the distribution of the currents and, in this way, it can affect the process of dissipation indirectly. Such indirect influence is not of importance for the initial dipole field because the Hall current generates relatively small deviations from the dipole configuration. This influence can be appreciable, however, for some other initial magnetic configurations (e.g. for the toroidal magnetic field, see Shalybkov & Urpin 1995) where the rate of decay can be increased by several orders of magnitude. Nevertheless, this effect seems to be unimportant for young neutron stars with an age $\lesssim 10^6$ yr.

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