Magnetopiezoelastic energy harvesting driven by random excitations

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This letter considers a nonlinear piezomagnetoelastic energy harvester driven by stationary Gaussian white noise. The increase in the energy generated by this device has been demonstrated for harmonic excitation with slowly varying frequency in simulation and validated by experiment. This paper considers the simulated response of this validated model to random base excitation and shows that the system exhibits a stochastic resonance. If the variance of the excitation were known then the device may be optimized to maximize the power harvested, even under random excitation. © *2010 American Institute of Physics*. [doi:10.1063/1.3436553]

Energy harvesting of ambient vibration is important for remote devices, for example, in structural health monitoring.[1–6] Completely wireless sensor systems are desirable and this can only be accomplished by using batteries and/or harvested energy. Harvesting is attractive because the energy generated can be used directly or used to recharge batteries or other storage devices, which enhances battery life. Most of the results using the piezoelectric effect as the transduction method have used cantilever beams and single frequency excitation, i.e., resonance based energy harvesting. The design of an energy harvesting device must be tailored to the ambient energy available. For single frequency ambient excitation the resonant harvesting device is optimum, provided it is tuned to the excitation frequency. Several authors[7–10] have proposed methods to optimize the parameters of the system to maximize the harvested energy. Shu *et al.*[11–13] conducted detailed analysis of the power output for piezoelectric energy harvesting systems.

Most linear energy harvesting devices are designed on the assumption that the (base) excitation has some known form, typically harmonic excitation. However, there are many situations where energy harvesting devices are operating under unknown or random excitations. In such situations the ambient vibration should be described using the theory of random processes[14] and the analysis of harvested power should be performed using the framework of probability theory. Lefeuvre *et al.*[15] considered random vibration in the context of energy harvesting due to random vibrations. Halvorsen[16] used linear random vibration theory to obtain closed-form expressions for the harvested energy. He also derived the Fokker–Planck equation governing the probability density function of the harvested power. Soliman *et al.*[17] considered energy harvesting under wide band excitation. Adhikari *et al.*[18] derived expressions for mean normalized harvested power of a linear system subjected to Gaussian white noise base acceleration.

An alternative approach uses nonlinear structural systems to maximize the harvested energy. McInnes *et al.*[19] investigated using the stochastic resonance to enhance vibrational energy harvesting of a nonlinear mechanism. An alternative system is the piezomagnetoelastic structure shown in Fig. 1. This system (without the piezoelectric patches) was used by Moon and Holmes[20] to demonstrate chaotic motion. Erturk *et al.*[21] investigated the potential of this device for energy harvesting when the excitation is harmonic and demonstrated significant performance improvements over the linear system (without magnets) for nonresonant excitation. This paper considers the performance of the piezomagnetoelastic system to random excitations, using the model of Erturk *et al.*,[21] which was validated against experimental results in their paper. A similar device was investigated by Stanton *et al.*[22]

The device consists of a ferromagnetic cantilever beam that is excited at the support. Two permanent magnets are located symmetrically on the base near the free end, and the static system can have five, three, or one equilibrium positions depending on the geometry of the system,[20] and, in particular, the distance between the beam and the magnets. Here we are interested in the case when the system has three equilibrium positions, two of which are stable, and the mechanical system is characterized by the classical double well potential. The nondimensional equations of motion for this system[21] are

$$\ddot{x} + 2\zeta\dot{x} - \tfrac{1}{2}x(1-x^2) - \chi v = f(t), \qquad (1)$$

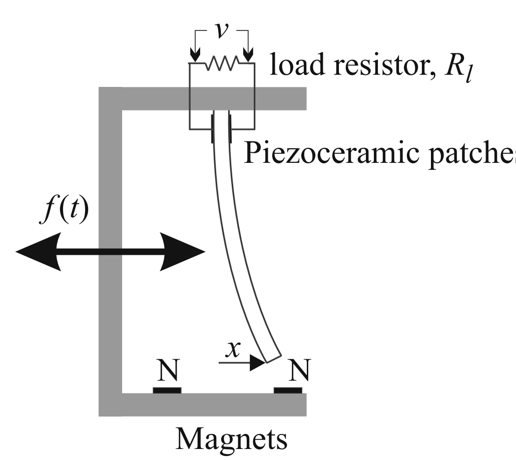

FIG. 1. Schematic of the piezomagnetoelastic device (Ref. 21).

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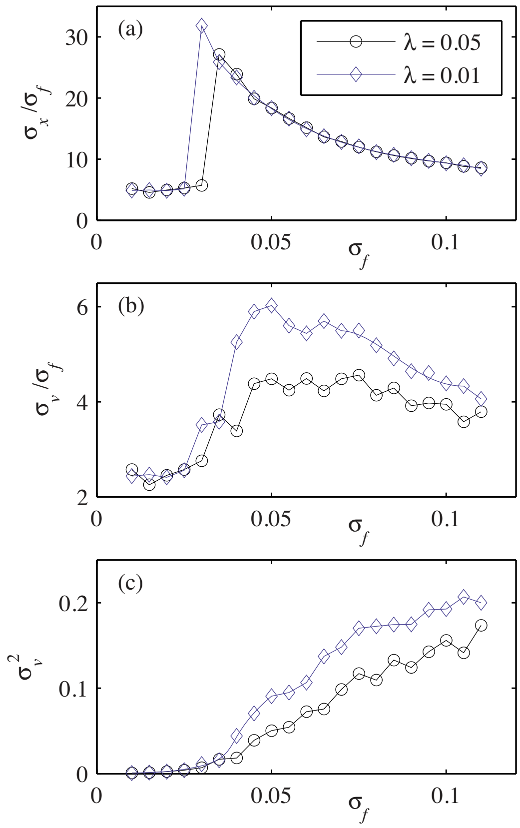

FIG. 2. (Color online) Simulated responses of the piezomagnetoelastic oscillator in terms of the standard deviations of displacement and voltage (σ_x and σ_v) as the standard deviation of the random excitation σ_f varies. (a) gives the ratio of the displacement and excitation; (b) gives the ratio of the voltage and excitation; and (c) shows the variance of the voltage.

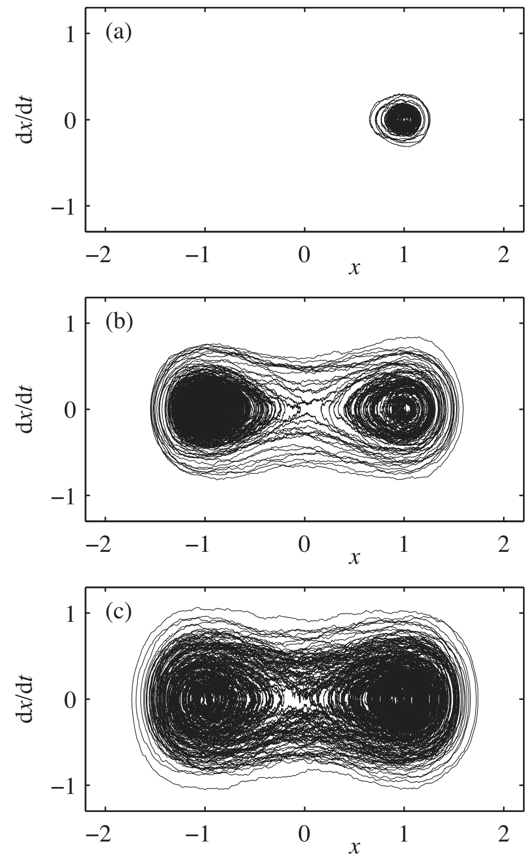

FIG. 3. Phase portraits for $\lambda=0.05$, and the stochastic force for (a) $\sigma_f=0.025$, (b) $\sigma_f=0.045$, and (c) $\sigma_f=0.065$. Note that the increasing noise level overcomes the potential barrier resulting in a significant increase in the displacement x.

$$\dot{v} + \lambda v + \kappa \dot{x} = 0, \tag{2}$$

where x is the dimensionless transverse displacement of the beam tip, v is the dimensionless voltage across the load resistor, χ is the dimensionless piezoelectric coupling term in the mechanical equation, κ is the dimensionless piezoelectric coupling term in the electrical equation, $\lambda \propto 1/R_l C_p$ is the reciprocal of the dimensionless time constant of the electrical circuit, R_l is the load resistance, and C_p is the capacitance of the piezoelectric material. The force $f(t)$ is proportional to the base acceleration on the device, and is assumed to be Gaussian white noise, with zero mean and specified variance.

The system parameters have been taken as follows:[21] $\zeta=0.01$, $\chi=0.05$, and $\kappa=0.5$, while λ was varied between 0.01 and 0.05. The excitation $f(t)$ is stationary Gaussian white noise with standard deviation σ_f. Equations (1) and (2) are integrated using the fourth order Runge–Kutta–Maruyama algorithm.[23] The standard deviations of the displacement x and the voltage v are calculated for a range of excitation noise amplitudes σ_f.

Figure 2(a) shows the standard deviation of the displacement relative to that of the excitation, and the pronounced peaks correspond to the stochastic resonance phenomenon.[24] This is a nonlinear effect; for the linear system the output is proportional to the random excitation and the corresponding plot would be a horizontal line. The sudden increase in the displacement amplitude x is associated with the escape from the single well potential in the presence of stochastic excitation. This may be highlighted by considering the phase plane for three values of σ_f shown in Fig. 3. Figure 2(a) shows the displacement response for two values of λ, which is related to the electrical time constant; the results are only significantly different near the stochastic resonance, showing that the mechanical and electrical systems are only weakly coupled.

Figure 2(b) shows the ratio of the standard deviations of voltage and excitation as the excitation amplitude varies. As the mechanical system is coupled to the electrical circuit through the piezoelectric transducers, the increase in mechanical displacement amplitude causes the voltage amplitude to rise. This also causes the harvested power to increase; Fig. 2(c) shows the variance of the voltage, which is proportional to the mean harvested power since the mean voltage is negligible. Note that the power remains very low for noise intensities below the stochastic resonance. The electrical constant λ significantly affects the voltage produced and hence the power generated. Decreasing λ increases the harvested power, as shown in Fig. 2(c). Note that this increase in power above the stochastic resonance is due to the changes in the electrical system, since the mechanical responses shown in Fig. 2(a) are almost identical. Decreasing λ increases the electrical time constant, and this allows the load to capture energy from the slow scale dynamics, where the mechanical states hop between the two potential wells. For large enough noise intensity the difference disappears. The practical implementation of a reduction in λ requires a

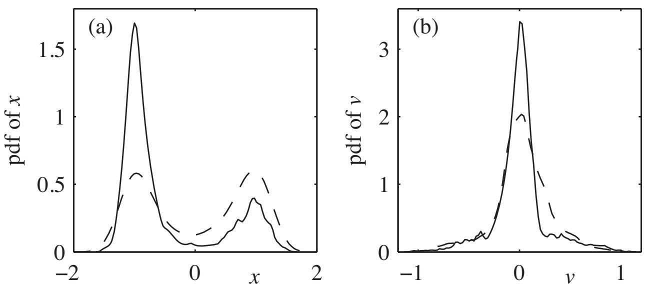

FIG. 4. Probability density functions of the mechanical displacement x and the voltage across the piezoelectric patch, v, for σ_f=0.045 and λ=0.05 (solid) and λ=0.01 (dashed).

reduction in either electrical resistance or capacitance; these requirements should be included in the trade-offs required for the design of a real system.

Figure 4 shows the probability density functions for the displacement x and voltage v for the response shown in Fig. 3(b) for λ=0.05, and the corresponding results for λ=0.01. The bimodal distributions in Fig. 4(a) clearly show that the mechanical system spends most of its time close to the two equilibrium positions. The bi-modal distribution is not visible in the voltage, Fig. 4(b), because the electrical circuit acts as a first order filter.

Our analysis indicates that the energy harvested from a piezomagnetoelastic device is most efficient for a certain range of the noise intensity. In practice, the noise intensity or noise variance, would be known, and the device designed so that it operates at the stochastic resonance condition. Essentially this design process would tailor the height of the energy barrier in the double well potential. The understanding of the slow dynamics of this bistable system can be used to improve the coupling to the electrical system and hence the performance of the energy harvester.

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