

# Magnetostrictive actuation of a smart beam with hysteretic material behaviour

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Magnetostrictive materials can be used as actuators in smart structures technology. The relation between induced strain and the applied magnetic field is nonlinear and shows hysteretic behaviour. Thus the magnetomechanical coupling coefficient is not constant and should be defined as a function of strain or magnetic field in computations.

In this study the hysteresis of a mechanically unconstrained actuator is determined using the Michelson interferometry. The hysteretic behaviour is modelled phenomenologically by a Preisach model. Using these experimental data for the modelling of an active structure with embedded magnetostrictive actuators, the actual coupling coefficient can be determined utilising the Preisach model. With this procedure the actuation strain of an embedded actuator, including the physical nonlinearities, can be calculated using the material characteristics obtained with an unconstrained actuator. For the determination of the actual coupling coefficient a strain- and field-dependent approach is used.

For an experimental validation of the method outlined above, a magnetostrictive actuator is characterised experimentally and then applied to a cantilever aluminium beam. Then, the tip displacement of the actuated beam is measured with a laser triangulation sensor and compared with the numerical results.

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## 1 Introduction

In literature, in most cases active shape and vibration control is performed assuming a constant magnetomechanical coupling factor [1], which is only valid for a certain signal range. To abstain from additional closed-loop operations, in some papers the hysteretic behaviour is accounted for directly in the control algorithm. A method for hysteresis compensation in piezoelectric structures, which show a similar hysteretic effect, based on the Preisach model is presented in [2]. Prior to developing the model, the nonlinearity of the active structure has to be determined experimentally as shown by Hughes and Wen [3], also for piezoceramics. Their model for hysteresis compensation can only be developed, if the active structure is already present in the laboratory. With the method presented in this paper it is possible to develop a model for compensation using the hysteresis data obtained from a mechanically unconstrained magnetostrictive actuator. For this purpose a strain- and field-dependent coupling coefficient is used to determine the nonlinear actuation strain in a smart structure, according to [4,5,6]. This is accomplished using a Preisach model.

## 2 Determination of the Magnetomechanical Coupling Coefficient

The magnetomechanical coupling coefficient  $d_{11}$  is determined with a direct measurement method, the Michelson interferometry. With this method the movement of a mirror is measured according to the alternation of the interference pattern. Due to this measurement procedure the complete magnetostrictive patch is taken into consideration for characterisation, while in the case of a strain gauge only a part of it is used to obtain local information about the coupling coefficient. The maximum applied magnetic field  $H_1$  is  $4.1 \cdot 10^4$  A/m and generated by an actuation coil. During the measurement the magnetostrictive actuator can expand freely, i.e. it is mechanically unconstrained, so that the constitutive relation leads to the simplified form  $\varepsilon_{11} = d_{11}H_1$ . For the linear constitutive equation  $d_{11}$  is assumed to be a constant. Anderson [4] introduced a variable  $d_{31}^*$  for piezoelectrics, which indicates that the coefficient can adopt different values depending on the electric field and strain. It is defined as the total strain divided by the total applied electric field, which is the reciprocal slope of a secant between the origin and a point on the electric field-strain curve. The same assumption is taken here for the magnetic field-strain curve measured by the Michelson interferometry (Fig.1 (2)). The displayed results are obtained by altering the voltage applied to the actuation coil with consecutively decreasing amplitude.

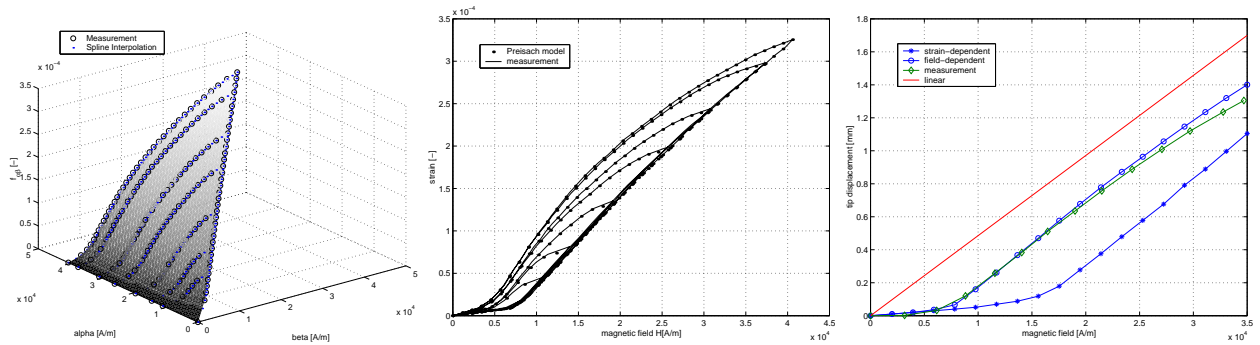
## 3 Preisach Model

The Preisach model is a phenomenological approach to describe the hysteretic behaviour between an input  $u(t)$  and an output  $f(t)$  of various physical systems in a mathematical way. The suitability of the model to describe a system is given with the fulfilment of the wiping-out and congruent minor-loop property. These conditions are fulfilled for the magnetostrictive

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actuator used in this study. Mayergoyz [7] gives a numerical approach of the model to calculate the system output depending on the history path of the input. Therefore a parameter surface has to be derived from the descending curves of a hysteresis measurement (Fig.1 (1)).



**Fig. 1** From left to right: (1) Parameter surface determined from the actuator characterisation measurement; (2) Results of the Preisach model and the characterisation measurement; (3) Comparison of the experimental and theoretical results of the active beam.

## 4 Active Cantilever Beam

The investigated active beam consists of aluminium, which is fixed on one side to a stiff mount. The magnetostrictive actuator is glued with an epoxy resin near the clamping onto the beam. The tip displacement of the beam is measured with a laser triangulation position sensor. The magnetic field is generated with a copper solenoid.

The bending moment  $M$  generated by the actuator can be calculated considering the Bernoulli beam theory as

$$M = H_1 d_{11} E_2 b_2 t_2 \left( t_1 + \frac{t_2}{2} - z_S \right); \quad w'' = -\frac{M}{EI}; \quad \varepsilon_{11} = -w'' z; \quad \varepsilon_{11} = \frac{H_1 d_{11}^*(\varepsilon) E_2 b_2 t_2}{EI} \left( t_1 + \frac{t_2}{2} - z_S \right)^2, \quad (1; 2; 3; 4)$$

where  $H_1$  denotes the actuator driving field,  $E_2$  the Young's modulus of the actuator,  $b$  the width,  $t$  the thickness,  $z_S$  the location of the neutral axis and  $EI$  the bending stiffness of the laminated beam. The index 1 indicates the aluminium beam, index 2 the actuator patch. The nonlinear magnetomechanical coupling coefficient  $d_{11}^*$  can be considered as magnetic field- or strain-dependent. Replacing the linear coupling coefficient  $d_{11}$  in eqn.(1) by the strain-dependent coefficient  $d_{11}^*(\varepsilon)$ , inserting in eqn.(2) and eqn.(3) leads to eqn.(4), which is dependent on the actual strain. This problem can be solved iteratively, starting with an assumed value for  $d_{11}^*$ . The resulting strain is the input for the Preisach model, by means of which the output, a virtual magnetic field, is determined. With the input and output of the Preisach model, i.e. a strain and a magnetic field, the new coupling coefficient  $d_{11}^*$  is determined. Inserting it into eqn.(4) will start a new iteration step, which is performed until the change of strain is unnoticeably small. With this procedure the tip displacement of the actuated beam can be determined for a given actuator driving field path in a strain- and field-dependent sense.

## 5 Results

Fig.1 (2) shows the performance and the suitability of the Preisach model to reproduce the field-strain relation of an unconstrained magnetostrictive actuator. The results of a strain- and field-dependent calculation of an active structure for an increasing driving field up to  $3.5 \cdot 10^4$  A/m are shown in Fig.1 (3). The tip displacement measurement of the active beam is compared to the nonlinear and linear calculation. The field-dependent approach is the best one for the introduced method. Based on this conclusion every arbitrary loading path can be predicted using the Preisach model in a field-dependent sense.

## References

- [1] J. S. Kumar, N. Ganesan, S. Swarnamani and C. Padmanabhan, *Comp. & Struc.* **81**, 1375–1382 (2003).
- [2] J. M. Cruz-Hernández and V. Hayward, Reduction of Major and Minor Hysteresis Loops in a Piezoelectric Actuator, in: *CDC Conference Proc.* 1998
- [3] D. Hughes, J. Wen, *Smrt. Mat. Str.* **6**, 287–300 (1997).
- [4] E. H. Anderson, Piezoceramic Induced Strain Actuation of One- and Two-Dimensional Structures, Master Thesis, MIT (1989).
- [5] E. F. Crawley and E. H. Anderson, *J. Int. Mat. Sys. Str.* **1**, 4–25 (1990).
- [6] E. F. Crawley and K. B. Lazarus, *AIAA J.* **29**, 944–951 (1991).
- [7] I. Mayergoyz, *Mathematical Models of Hysteresis and Their Applications*, Elsevier (2003).