Magnitude and energy of earthquakes

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ABSTRACT

Discrepancies arise among magnitudes as derived from local earthquake data (M_L)*, body waves* (M_B) *and surface waves* (M_S)*. The relation of* M_L *to the others is as yet not definitive; but* $M_S - m_B = a (M_S - b)$*. The latest revision gives* a = 0.37, b = 6.76*. Pending further research it is recommended that* M_L *continue to be used as heretofore, but* M_S *(and ultimately* M_L*) should be referred to* m_B *as a general standard, called the unified magnitude and denoted by* m*. Tentatively* log E = 5.8 + 2.4 m *(*E *in ergs). Revised tables and charts for determining m are given.*

This paper is in continuation of previous investigations [Gutenberg and Richter 1942, 1956].

The earthquake magnitude has statistical and other uses independent of the relation between magnitude and energy. Indeed, it is possible that there is no complete one-to-one correlation between magnitude and energy for large and complex tectonic events. Even so, a mean or representative relation is a legitimate object of inquiry. In attempting to refine the magnitude-energy relation it was found [Gutenberg and Richter 1956] that three imperfectly consistent magnitude scales had been in use:

M_L determined from records of local earthquakes according to the original definition [Richter 1935];

M_S from the amplitudes of surface waves for shallow teleseisms, [Gutenberg and Richter 1936; Gutenberg 1945a];

m_B from the amplitude/period ratio of body waves for teleseisms, shallow and deep-focus [Gutenberg 1945b, 1945c].

The two latter were originally adjusted to coincide near $M = 7$, but were later found to diverge linearly so that

$$M_S - m_B = a\,(M_S - b) \qquad (1)$$

For a number of years reductions were carried out with $a = 1/4$, $b = 7$, converting values of m_B into the corresponding M_S. The result of this reduction may be designated M_B. The final value given for M was a weighted mean between M_B and M_S. This may be taken as defining M without subscript. The adjustment between M_S and M_B can now be performed with considerable accuracy, using the relation (1) with revised parameters $a = 0.37$, $b = 6.76$. This is equivalent to

$$m_B = 0.63\,M_S + 2.5 = M_S - 0.37\,(M_S - 6.76) \qquad (2)$$

The revision is based on a large body of data. Magnitudes have been derived by the senior author, from surface waves and from body waves separately, for a selection of better recorded large shallow earthquakes as listed by Gutenberg and Richter [1954]. Those for which there was suspicion of depth in excess of the normal (believed to be about 25 km) were rejected. Values of m_B were plotted against those of M_S, and (1) derived from the plot. The values $a = 0.37$, $b = 6.76$ are comparable with those found by Bath [1955] as follows:

Station	Body wave used	a	b
Uppsala	PZ	0.45	6.3
	PH	0.46	6.4
	SH	0.23	5.6
Kiruna	PZ	0.59	6.2
	PH	0.50	6.5
	SH	0.30	6.1

At Pasadena, a weighted mean is taken between m_B as found directly from body waves, and m_S, the corresponding value derived from M_S by applying the relation (1), or still better from tables and charts set up to give m_S directly from surface wave data. This weighted mean is designated the *unified magnitude* denoted by m.

In Figure 1 residuals $m_B - m_S$ on the basis $a = 0.37$, $b = 6.76$ are plotted against m, using amplitude and period data from all available station bulletins,

a) for all shocks in Table 13 of Gutenberg and Richter [1954] for which there was no indication of depth exceeding 30 km, excluding all uncertain or doubtful magnitudes;

b) using all similar data for Table 14 of Gutenberg and Richter [1954] for 1936-1939 and 1950-1952 (inclusive). There is little indication of systematic deviation from the axis of zero residuals. The slight apparent excess of positive residuals may be due to the use of a few shocks with depths somewhat greater than supposed, which should result in a

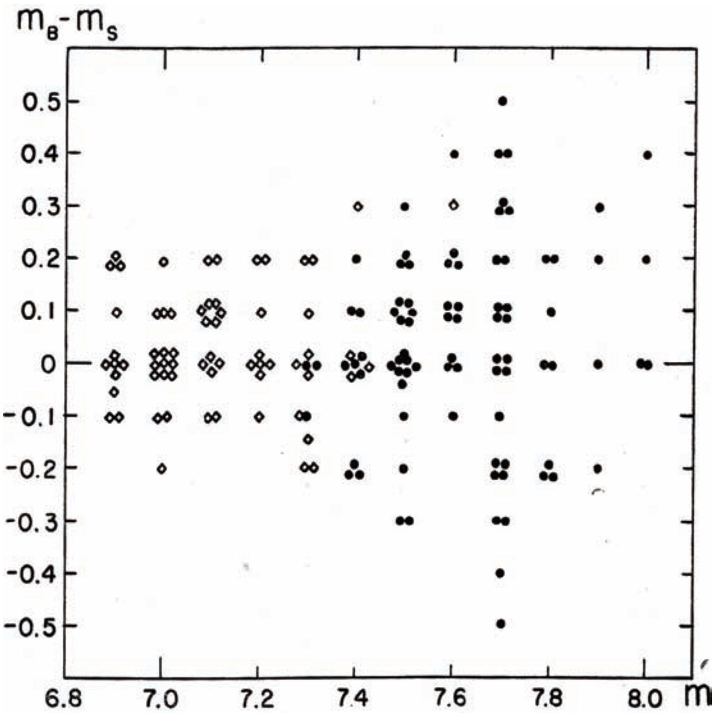


Figure 1. Residuals $m_B - m_S$ on the basis $a = 0.37$, $b = 6.76$ are plotted against *m*, using amplitude and period data from all available station bulletins.

decrease of m_S.

Comparable data for magnitudes below 7 are rare. Ten of eleven shocks in the California region, with magnitudes near 6, give $m_B - m_S$ from + 0.1 to – 0.2; the eleventh gives – 0.4.

Figure 2 is a nomogram prepared by Mr. J.M. Nordquist for the direct determination of m_S from surface wave amplitudes. The corresponding values of M_S and of log *E* from equation (6), are also indicated.

The adjustment of M_L to *m* or *M* cannot yet be determined so closely as that of *m* to *M* but can be stated with an error not likely to exceed 0.5 magnitude unit for those shocks (magnitudes 3 to 6) most often rated in terms of M_L. Representative results are given in Table 1. Values in parenthesis are outside the observable range.

Until 1954, the writers generally reported magnitudes for large deep shocks, and for large shallow teleseisms as determined from body waves, effectively in terms of M_S, first determining m_B and then correcting to M_S by applying equation (1) or an earlier approximation to it. The correction was usually applied only to shocks of magnitude 7 or over.

It now develops [Gutenberg and Richter 1956] that many outstanding difficulties disappear if the linear relation

ML	3	4	5	6	7	8	9
M	(2.4)	(3.6)	4.7	5.8	6.8	7.9	(8.9)
m	(4.0)	(4.7)	5.4	6.1	6.8	7.5	(8.1)
log E	15.4	17.2	18.9	20.5	22.1	23.7	(25.2)

Table 1. Values of *M*, *m* and log *E* for given values of M_L, using $M = 1.27(M_L - 1) - 0.016 M_L^2$, $m = 0.63M + 2.5$, $\log E = 5.8 + 2.4m$ (*E* = energy in ergs).

(2) is consistently extended to magnitudes below 7. Whereas shocks of the largest magnitude record with surface waves relatively large compared with the body waves, shocks of magnitude below 7 show relatively small surface waves when recorded at teleseismic distances. Many long-period instruments do not record such shocks clearly; this makes assignment of magnitude from the data of distant stations difficult. Short-period instruments in such cases may show a measurable *P*; the absence of recorded surface waves is then sometimes misinterpreted as evidence for deep focus.

When equation (2) is used, and data for both body waves and surface waves are available, two different determinations are in effect available for either *m* or *M*. The equation gives $m_B = M_S$ for a value near 6 3/4. When the magnitude does not greatly differ from this figure, problems of adjustment are minor, and reduce to judgement as to the relative reliability of the two groups of data.

Although at present many more stations report amplitudes for surface waves than for body waves, some ten years' experience indicates that m_B provides the better data in practice as shown by fewer systematic errors and more consistent results, as well as being theoretically preferable.

In using station bulletins to determine M_S, the maxima of surface waves can be used for magnitude only when the period is near 20 seconds. If the period is not specified, there is risk that the reported maximum amplitude may refer to much longer or shorter waves, which seriously falsifies M_S. With some exceptions, magnitudes currently being reported in station bulletins are either M_L determined from nearby stations, or M_S. There is less general determination of m_B, and the relation in equation (1) or (2) is often overlooked. Occasionally M_S is even given for deep shocks as found directly from surface waves; if the hypocenter is deeper than about 30 km, calculation on this basis gives too low a value.

Routine station bulletins issued from Pasadena continue to list magnitudes *M* which are either M_L or M_S; but beginning with 1954 the annual list of large shocks also tabulates *m*, which is an intermediate step toward a definitive magnitude-energy relation.

The practical definition of the unified magnitude *m* consists in a system of tables and charts for calculating magnitude from the quotient amplitude/period for the maximum waves of the principal wave groups *P*, *PP*, and *S*. This quotient is used in the form

$$q = \log u/T \text{ or } q = \log w/T \tag{3}$$

where *u* and *w* are respectively the horizontal and vertical components of the ground displacements in microns and *T* the period in seconds. Each table or chart gives for all distances and focal depths a quantity *Q* such that for corresponding distance and depth

$$m = q + Q + s \tag{4}$$

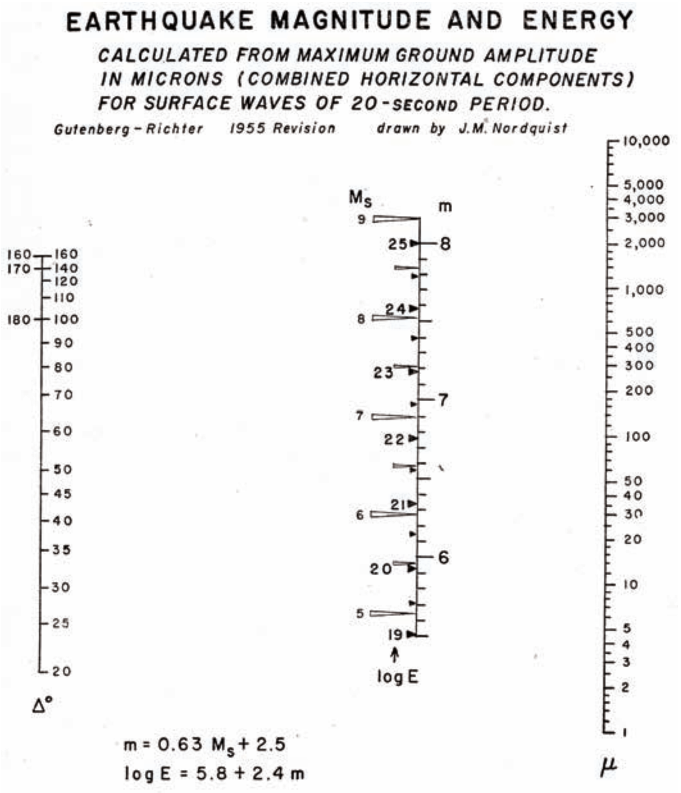


Figure 2. Nomogram for the direct determination of m_S from surface wave amplitudes. The corresponding values of M_S and of log *E* from equation (6) are also indicated.

Δ	PZ	PH	PPZ	PPH	SH	Δ	PZ	PH	PPZ	PPH	SH	Δ	PZ	PH	PPZ	PPH	SH
16	59	60			72	56	68	71	69	70	66	96	73	76	72	74	71
17	59	60			68	57	68	71	69	70	66	97	74	78	72	74	72
18	59	60			62	58	68	71	70	71	66	98	75	78	72	74	73
19	60	61			58	59	68	71	70	72	66	99	75	78	72	74	73
20	60	61			58	60	68	71	71	73	66	100	74	77	72	74	74
21	61	62			60	61	69	72	72	74	67	101	73	76	72	74	74
22	62	63			62	62	70	73	73	74	67	102	74	77	72	74	74
23	63	64			62	63	69	73	73	74	67	103	75	79	72	74	73
24	63	65			62	64	70	73	73	75	68	104	76	79	73	75	73
25	65	66			62	65	70	74	73	75	69	105	77	81	73	75	72
26	64	66			62	66	70	74	73	74	69	106	78	82	74	76	72
27	65	67			63	67	70	74	72	74	69	107	79	83	74	76	72
28	66	67			63	68	70	74	71	73	69	108	79	83	74	76	72
29	66	67			63	69	70	74	70	72	69	109	80	84	74	76	72
30	66	68	67	68	63	70	69	73	70	72	69	110	81	85	74	76	72
31	67	69	67	68	63	71	69	73	71	73	70	112	82	86	74	76	
32	67	69	68	69	64	72	69	73	71	73	70	114	86	90	75	77	
33	67	69	68	69	64	73	69	72	71	73	69	116	88		75	77	
34	67	69	68	69	65	74	68	71	70	72	68	118	90		75	77	
35	67	69	68	69	66	75	68	71	69	71	68	120			75	77	
36	66	68	67	68	66	76	69	72	69	71	68	122			74	76	
37	65	67	67	68	66	77	69	72	69	71	68	124			73	75	
38	65	67	67	68	66	78	69	73	69	71	69	126			72	74	
39	64	66	66	67	67	79	68	72	69	71	68	128			71	74	
40	64	66	66	67	67	80	67	71	69	71	67	130			70	73	
41	65	67	65	66	66	81	68	72	70	72	68	132			70	73	
42	65	67	65	66	65	82	69	72	71	73	69	134			69	72	
43	65	67	66	67	65	83	70	74	72	74	69	136			69	72	
44	65	67	67	68	65	84	70	74	73	75	69	138			70	73	
45	67	69	67	68	65	85	70	74	73	75	68	140			71	74	
46	68	71	67	68	66	86	69	73	73	75	67	142			71	74	
47	69	72	67	68	66	87	70	73	72	74	68	144			70	73	
48	69	72	67	68	67	88	71	75	72	74	68	146			69	72	
49	68	71	67	68	67	89	70	74	72	74	68	148			69	72	
50	67	70	67	68	66	90	70	73	72	74	68	150			69	72	
51	67	70	67	68	65	91	71	75	72	74	69	152			69	72	
52	67	70	67	68	65	92	71	74	72	74	69	154			69	72	
53	67	70	67	68	66	93	72	75	72	74	69	156			69	72	
54	68	71	68	69	66	94	71	74	72	74	70	158			69	72	
55	68	71	69	70	66	95	72	76	72	74	70	160			69	72	
												170			69	72	

Table 2. Values of *Q* for shallow shocks.

where *s* is a ground correction characteristic of the station used.

Charts and tables of this type were first given by Gutenberg [1945b, 1945c], where *Q* was designated *A*. Those accompanying the present paper (Table 2, Figures 3, 4 and 5) represent no change in fundamental concept, but only a revision. The statistical processes by which the tables and charts published in 1945 were derived have now been repeated by the senior author using a much larger body of data, and, it is hoped, with greater precision. One effect has been to remove a persistent discrepancy between magnitudes determined from horizontal and vertical components; this discrepancy was discovered independently by Bath [1955].

This procedure places the unified magnitude *m* on a self-consistent and independent basis as satisfactory for teleseisms as that of M_L for local earthquakes, and with the great advantage of being applicable directly to seismograms recorded on instruments of all types and at all stations. If desired, a formal definition for *m* may be phrased as follows:

$$m - 7.0 = q\,n \qquad (5)$$

at a distance of 90° for normal shallow focal depth, where $q = \log w/T$ refers to *PZ*, and the station constant *s* is taken as zero, representing average station ground conditions.

Since the relation of M_L to *m* is not yet on a definitive basis, the authors suggest that the «Richter scale» as defined in 1935 be retained for determining magnitudes of local shocks. For teleseisms, the use of the unified scale *m* is preferred and strongly recommended. For magnitudes from about 5½ to 7, the departure between the two scales is within the usual limits of error under the now existing conditions of recording and reporting amplitudes. Gutenberg and Richter [1954] have not assigned magnitudes below 6 to shocks outside the California area (within which M_L is reported); such shocks are merely designated by the letter *d*. Above magnitude 7½ the scales diverge significantly; but then determinations from the data of numerous stations scatter increasingly, and it is advisable to distinguish carefully between determinations from body waves and from surface waves. It is urgent that magnitudes determined from seismograms at single stations should not be published unaccompanied by the amplitude and period readings on which they are based.

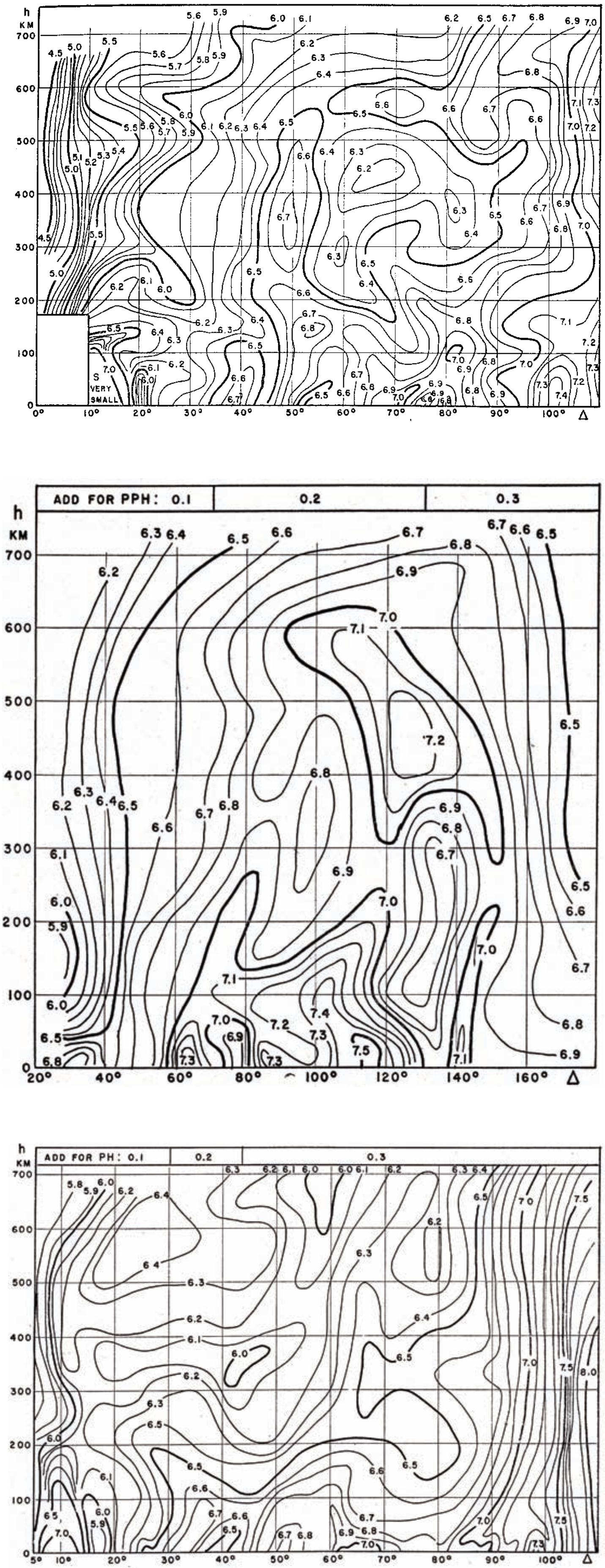
It is hoped that before many years have passed it will be possible to express the entire range of observed magnitudes in terms of the unified magnitude *m*.

Since the provisional use of *m* is especially intended for investigations relating to energy, *m* is being published together with the energy calculated from it by the relation

$$\log E = 5.8 + 2.4\,m \qquad (6)$$

to be established on a later page.

For most types of publication the writers think it preferable to follow a suggestion by Dr. L.B. Slichter, giving the



Figures 3, 4, 5. From top: Revised values of *Q* for *SH*; Revised values for *Q* for *PPZ*; Revised values for *Q* for *PZ*.

value of log E together with equation (6), and so avoiding confusion due to use of numerically different magnitude scales.

Most calculations of the magnitude-energy relation depend directly or indirectly on the equation for a wave group from a point source [Gutenberg and Richter 1956]

$$E = 2\pi^3 h^2 v\rho \ (A/T)^2 t \tag{7}$$

where E is energy, h is linear distance from the source, v is velocity, ρ is density, A and T are amplitude and period of sinusoidal waves, and t is the duration of the wave group (which hence contains $n = t\ /T$ waves).

This applies at the epicenter when h is hypocentral depth, and includes a factor which takes account of the effect of the free surface. Taking $v = 3.4$ km/sec for transverse waves, applying a factor 3/2 to allow for half as much energy in longitudinal waves, and using $h = 16$ km, $\rho = 2.7$ gm/cc, this reduces to

$$\log E = 12.34 + 2\, q_o + \log t_o \tag{8}$$

where $q = \log A/T$ and the subscript zero refers to the epicenter. A fundamental empirical equation is

$$q_o = -\,0.6 + 0.8\, M_L - 0.01\, M_L{}^2 \tag{9}$$

This is a revised result drawn from the plot of q_o as a function of M for California shocks (Figure 3) [Gutenberg and Richter 1956]. Two further important equations are derived from plotted data (Figures 6):

$$\log t_o = -\,1 + 0.4\, q_o \tag{10}$$

and

$$q_o = m - 2.3 \tag{11}$$

For the latter result most of the data cover a relatively small range of m. Combining these

$$\log t_o = 0.4\, m - 1.9 \tag{12}$$

On the other hand, if in (10) we substitute for q_o its expression in terms of M_L from (9), we obtain

$$\log t_o = -\,1.24 + 0.32\, M_L - 0.04\, M_L{}^2 \tag{13}$$

which differs only slightly from the corresponding equation setup empirically in Gutenberg and Richter [1956], showing that the derivation of (9) and (10) has been consistent. Combining (9) and (11)

$$m = 1.7 + 0.8\, M_L - 0.01\, M_L{}^2 \tag{14}$$

This is drawn on Figure 7; it is not inconsistent with the plotted data.

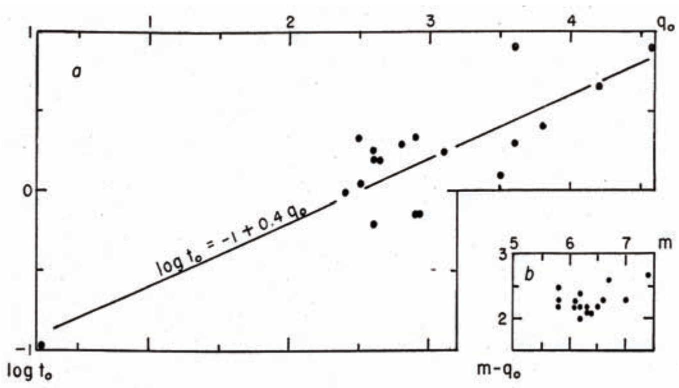


Figure 6. Two important equations are derived from plotted data: $\log t_o = -1 + 0.4\, t_o$ and $q_o = m - 2.3$.

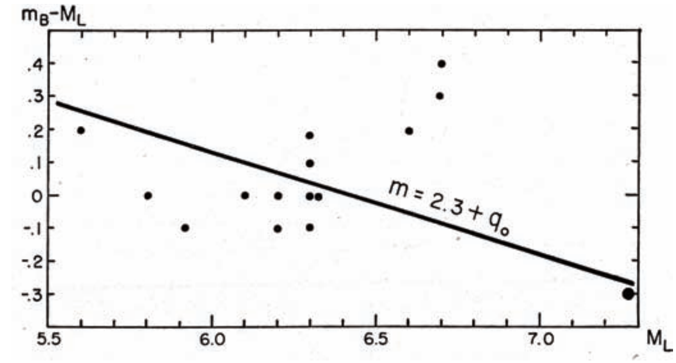


Figure 7. Combining equations (9) and (11), $m = 1.7 + 0.8\, M_L - 0.01\, M_L{}^2$.

If instead of (9) we had used the corresponding equation in Gutenberg and Richter [1956], which has a larger coefficient of the quadratic term, the resulting equation replacing (14) would lead to calculated values of $M_B - M_L$ which for large m are systematically too small to suit the observations. This is the chief reason for revising the empirical relation between q_o and M_L to the form (9).

If we apply the relation (2) to (14) we find

$$M_S = 1.27\,(M_L - 1) - 0.016\, M_L{}^2 \qquad (15)$$

Equation (6) results from substituting in (8) the expressions for q_o and $\log t_o$ from (11) and (12). It has also been verified approximately by the following calculation. For a train of n $(= t/T)$ sinusoidal body waves, emerging to the surface of the earth at arc distance Θ from a surface source with horizontal ground displacement u, the total energy calculated as radiated from the source is

$$E = 8\pi^3 R^2 \rho v t\,(u/T)^2/U^2 L \qquad (16)$$

where $$U^2/f_i^2 = \tan i\, di \;/\; \sin \Theta\, d\,\Theta \qquad (17)$$

Here E = energy, R = radius of the earth, ρ = density, v = velocity, t = duration of wave train, T = period, i = angle of incidence, f_i is a factor expressing the effect of the free surface as a function of i (otherwise it depends only on Poisson's ratio; see Gutenberg 1944), and L is a factor to allow for absorption, scattering, internal friction, effects of discontinuities, etc.

There are several simplifying assumptions: the earth is taken as spherically symmetrical, the effect of hypocentral depth is neglected (it is easily corrected for), energy flux is calculated by the ray method as used in geometrical optics, and the use of (16) to calculate total energy implies radiation equally in all directions from the source. In what follows it is assumed that 1/3 of the original energy is radiated as longitudinal waves, and a factor 3 is accordingly applied.

Analogous equations to (17) and (18) apply to the vertical component of ground displacement, replacing u by w and U^2 by a similar factor W^2.

We next take $\rho = 3$ gm/cc, $v = 6.3$ km/sec (applying to longitudinal waves), $R = 6370$ km; we also take $q = \log u/T$ or $q = \log w/T$, where u and w are expressed in microns. Taking the logarithm of (17) with proper attention to the units used, we arrive at

$$\log E = 18.8 + \log t + 2q - \log U - \log L \qquad (19)$$

We now assume that $t = t_o$; this has been confirmed roughly by Dr. C. Lomnitz from seismograms recorded at Pasadena. Applying (12) with t in place of t_o, and putting $q = m - Q$,

$$\log E = 16.9 + 2.4\, m - 2Q - 2\log U - \log L \qquad (20)$$

Comparing this with (6) we should have

$$2Q + 2\log U + \log L = 11.1 \qquad (21)$$

and a similar equation for the vertical component. Here, $2Q$ may be taken from Table 2, and $\log U$ can be calculated from (17). Working this out for the vertical component of P waves, the following values are found:

Θ	20°	42°	100°
Log L	−2.0	−1.3	−1.5

The calculation cannot be extended reliably to distances less than 20°. The contribution of absorption to $\log L$ should be about 0.4 near 100° and 0.3 at moderate distances. Loss by refraction at the Mohorovicic, Conrad, and other discontinuities in the crust may account for a few tenths in $\log L$. This leaves about one unit in $\log L$ unaccounted for. If all assumptions are correct, energy flux is reduced to roughly one tenth within the first 20° of distance; this must occur within the upper 200 km of the mantle. If this is correct, we should expect a smaller constant term in the energy-magnitude relation corresponding to (6) for shocks at greater depths. This would agree with the relatively low energy calculated by Sagisaka [1954] for a shock at a depth of 360 km. However, the constant term 5.8 in (6) and the coefficient 0.4 in (12) are not accurately fixed, and log E calculated from (6) may be in error by as much as one unit.

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