Majority Decision Functions of up to Six Variables

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1. Introduction. Recently logical elements based essentially on the majority decision principle have been widely used in electronic computers. Among these elements are parametrons, magnetic cores, transistor-resistor logic, et cetera.

The logical behavior of such elements can be expressed by a model called a "majority decision element" with n Boolean inputs and one Boolean output, whose operation can be described in the form of a logical function called a "majority decision function".

This paper defines the canonical representative of each equivalence class in the classification of the majority decision functions by complementing and permuting variables and by complementing the output. Also, a method is proposed to obtain all the representatives with their optimum structures, and a table of the representatives of the majority decision functions of up to six variables is provided.

The reader should be familiar with the content of a previous paper by the authors, included as reference [1].

2. Majority Decision Functions. A "majority decision element" of n variables is a logical element with n Boolean inputs, $x_1, x_2, \cdots, x_n$ and one Boolean output. The output value of the element is

$$(1)^* \qquad \begin{aligned} &\text{one for} \quad \sum_{i=1}^{n} w_i x_i \geqq T \\ &\text{zero for} \quad \sum_{i=1}^{n} w_i x_i \leqq T - 1 \end{aligned}$$

where w_i is a prescribed constant real number called a "coupling weight" associated with the input x_i and T is also a prescribed constant real number called a "threshold."

In the case of parametrons or magnetic cores, the coupling weight w_i corresponds to the number of turns of the winding of the input x_i. The threshold T is related to the number of turns w_c for the constant input by the relation,

$$(2) \qquad w_c = \sum_{i=1}^{n} w_i + 1 - 2T$$

where $w_c > 0$ means the constant of one is coupled to the element and $w_c < 0$ means the constant of zero.

A set of $(n + 1)$ real numbers $(w_1, w_2, \cdots, w_n ; T)$, which specifies the behavior of a majority decision element, will be called a "structure" of the element.

A logical function represented by a single majority decision element will be called a "majority decision function."

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* The term -1 on the right hand side is introduced as a normalizing factor of w_i's and T.

For example, a majority decision element with the structure (2, 1, 1; 2) represents a function $x_1 + x_2x_3$; hence, this function is a majority decision function. In contrast, the function $x_1x_2 + x_3x_4$ is not a majority decision function since it can not be realized by any single majority decision element.

3. Classification of Majority Decision Functions. Logical functions obtained from a given logical function f by the following operations are defined as equivalent functions with f:

(1) Complementation of one or more input variables,

(2) Permutation among input variables,

(3) Complementation of f.

It is a well known fact that the logical functions can be classified into equivalent classes by this equivalent relation. Once a structure of a majority decision function is given, its equivalent functions can be easily realized in the same element by complementing and/or permuting input variables and/or by complementing the output. Thus, it is not necessary to determine the whole of the majority decision functions; it is sufficient to know the representatives of their equivalence classes. It should be noted that this limits the study to a much smaller number of functions.

In the case of general logical functions, it is difficult to extract systematically one representative from each equivalence class, but in the case of majority decision functions there is a way to define a canonical representative of each equivalence class from the intrinsic nature of majority decision functions.

The method of determining the canonical representative is described below. Hereafter in this section the majority decision function is assumed to have n nonvacuous variables.

Any majority decision function can be expressed by a polynomial without any complemented variable by appropriately complementing one or more variables (refer to [1], Section 3). Such a polynomial will be called a "positive polynomial." The way to complement the variables to obtain a positive polynomial from a given function is unique if complementing one variable more than once is prohibited. Hence we can restrict the possible representatives within positive polynomials. This is equivalent to the condition in which the representative should be realized by a majority decision element with positive coupling weights.

All the variables of a majority decision function can be ordered by a relation $\gtrsim$ (refer to [1], Definition 3 and Theorem 1). Therefore, it is always possible for variables to be permuted and relabelled so that $x_1 \gtrsim x_2 \gtrsim \cdots \gtrsim x_m$ holds. This permutation can be uniquely determined except in the case of arbitrary permutations among some variables such as $x_1, x_2, \cdots, x_m$ for which $x_1 \sim x_2 \sim \cdots \sim x_m$ holds. But $x_1 \sim x_2 \sim \cdots \sim x_m$ means that the given function is symmetric with respect to these variables, and therefore the function is invariant under the permutations among $x_1, x_2, \cdots, x_m$. Thus, the function for which $x_1 \gtrsim x_2 \gtrsim \cdots \gtrsim x_n$ holds is unique and can well be adopted as a possible representative. Of course, this is equivalent to the condition in which $w_1 \geqq w_2 \geqq \cdots \geqq w_n$ holds for the representative majority decision element. Note that as a conclusion from the above requirements, we have $w_1 \geqq w_2 \geqq \cdots \geqq w_n > 0$ except $w_c \leqq 0$.

Only two functions left in each class satisfy both of the conditions just described.

If we denote one of them by f, the other is the dual function f^* of f. But for a majority decision function, either $f^* \supseteq f$, or $f \supseteq f^*$ holds (refer to [1], Corollary 2). A unique representative of the equivalent class can be determined by requiring either of the two inequalities. If we adopt f such that $f \subseteq f^*$, this implies $w_c \leqq 0$.

Thus, it is shown that there is a unique canonical representative in each equivalent class of majority decision functions which satisfies the following three conditions:

Conditions I.

(1) A positive polynomial,

(2) $x_1 \gtrsim x_2 \gtrsim \cdots \gtrsim x_n$,

(3) f such that $f \subseteq f^*$.

Given a majority decision function, we can now effectively obtain the representative of the equivalent class to which the given function belongs.

4. A Method to Obtain the Totality of the Representatives of the Majority Decision Functions. From Section 5 of [1] it can be determined by linear programming whether a given function is a majority decision function or not. Therefore, it is possible, at least in principle, to obtain the totality of majority decision functions by applying the criterion to all of 2^{2^n} logical functions of n variables. It will, however, take an impractically long time to solve 2^{2^n} linear programming problems for large values of n, but the length of time to perform computation will be greatly reduced if we can confine the scope of the functions to be tested.

Accordingly, a method is developed here to obtain a set of logical functions which includes all the representatives of majority decision functions and to apply the criterion only to those functions in the set. The functions in the set will be called "candidates" of the representatives.

Any positive majority decision function can be expressed in the form of $Mx_1 + N$, where M and N are both positive majority decision functions of $(n - 1)$ variables, $x_2, x_3, \cdots, x_n$. Therefore, without loss of generality, we can restrict the candidates within such functions. This assumes that we have already obtained all the majority decision functions of $(n - 1)$ variables; hence the method described here is one of the recursive constructions of majority decision functions with respect to the number of variables.

Moreover, if we choose as the candidates those functions for which Conditions I can be defined, then the set of the candidates will certainly contain the totality of the representatives of the majority decision functions of n variables.

Then the restrictions imposed upon combinations of M and N will be examined.

Condition (1) will be trivially satisfied, for $Mx_1 + N$ is positive from its construction.

Condition (2) requires that the relation

$$x_2 \gtrsim x_3 \gtrsim \cdots \gtrsim x_n \tag{3}$$

must hold for both M and N. Moreover, in order that $x_1 \gtrsim x_2$ may hold in $Mx_1 + N$, it is necessary (Corollary 1 of Reference [1]), that

$$m_2 \supseteq n_1, \tag{4}$$

where

$$m_2 = M(0, x_3, \cdots, x_n)$$

$$n_1 = N(1, x_3, \cdots, x_n).$$

As the relation $\gtrsim$ is an ordering relation (Theorem 1 of [1]), the relation

$$x_1 \gtrsim x_2 \gtrsim \cdots \gtrsim x_n \tag{5}$$

follows from (3) and (4).

M and N are majority decision functions satisfying (3), hence the relations

$$m_1 \supseteq m_2 \quad \text{and} \quad n_1 \supseteq n_2 \tag{6}$$

where

$$m_1 = M(1, x_3, \cdots, x_n)$$

$$n_2 = N(0, x_3, \cdots, x_n)$$

hold (Corollary 1 of Reference [1]). From (4) and (6) we have

$$M \supset N. \tag{7}$$

From (3) in Conditions I, it is necessary that

$$f^* = N^*x_1 + M^*N^* \supseteq f = Mx_1 + N. \tag{8}$$

But as $M^*N^* = M^*$ from (7), (8) reduces to

$$M^* \supseteq N. \tag{9}$$

Thus, we choose as candidates those functions which satisfy the following conditions:

Conditions II

(1) Both M and N are positive majority decision functions of $(n - 1)$ variables, $x_2, x_3, \cdots, x_n$.
(2) For both M and N, $x_2 \gtrsim x_3 \gtrsim \cdots \gtrsim x_n$.
(3) $m_2 \supseteq n_1$.
(4) $M^* \supseteq N$.

By taking all the combinations of M and N which satisfy Conditions II, we can obtain the set of candidates of the representatives of majority decision functions of n variables.

M and N must satisfy (1) and (2) of Conditions II. Such functions are either canonical representatives of majority decision functions or their dual functions. Therefore, once the totality of representatives of majority decision functions of $(n - 1)$ variables are obtained, the scope within which functions M and N must be taken can be easily determined. In this way we can obtain the totality of the representatives of majority decision functions of n variables recursively.

The next problem is to examine each candidate to determine whether or not it is a majority decision function. If so, it is clearly a canonical representative of an equivalent class defined in the preceding section. The discrimination of majority decision functions from other functions can be accomplished by linear programming. The details will be found in Section 5 of [1].

5. Majority Decision Functions of up to Six Variables. Following the procedure described in Section 4, a program was written for the parametron digital computer MUSASINO-I, and all the canonical representatives of the functions of up to six variables were obtained.

The canonical representatives of up to five variables had been obtained by S. Muroga [3] at that time, using a combinatorial method. Both results agreed completely.

The canonical representatives of the functions of up to six variables are shown in Table 1. The functions are numbered according to the magnitude of $V = \sum_{i=1}^{n} w_i$, which is expected to denote the complexity of functions to some extent. Functions are expressed by denoting the variables by means of their subscripts. For instance, $12 + 13 + 23$ stands for the function $x_1x_2 + x_1x_3 + x_2x_3$.

In the same entry of the table an optimum structure of the function is shown. The optimum structure is one with a minimum number of total turns of windings, namely, a structure which minimizes $(w_1 + w_2 + \cdots + w_n + |w_c|)$ (Section 5 in [1]).

To establish the threshold T, the constant input of zero must be coupled to the element with a winding of $2T - V - 1$ turns. Dual functions can be realized by merely reversing the polarity of the constant input, that is, by coupling the constant of one to the same winding.

The numbers in this table are somewhat different from those shown in [1]. This is because f and f^* are considered to belong to the same equivalence class in this paper and that in Table 1 the numbers of functions of n (nonvacuous) variables are shown, while the numbers for up to n variables are shown in [1].

By computing the number of the members of each equivalent class, the total numbers of majority decision functions are obtained and shown in Table 2.

6. Remarks on the Results. Some remarks are added here concerning the representatives of majority decision functions of up to six variables.

First, it is remarkable that all the candidates proved to be true representatives, that is, Conditions II are sufficient for a function of up to six variables to be realized by a single majority decision element.

Second, it is interesting to note that the optimum structures $(w_1, w_2, \cdots, w_n)$ are all integer-valued in spite of the fact that the optimum structure is obtained as a solution of a system of inequalities of the form of equation (1).

A structure of a majority decision function is a solution of a system of 2^n linear inequalities (Section 5 of Reference [1]).

$$Ax \geqq b \qquad A = \begin{Bmatrix} a_{ij}\, i \downarrow 1, 2, \cdots, 2^n \\ i \rightarrow 1, 2, \cdots, n \end{Bmatrix} \tag{10}$$

$$x = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ T \end{bmatrix}$$

The third remark concerns the structure of the solution space of these inequalities. It has been noted that for a majority decision function of up to five

TABLE 1

Representative Functions of Majority Decision Functions of up to Six Variables

No.	V	w_i	T	Representative Function
		$n = 2$		
1	2	11	2	12
		$n = 3$		
1	3	111	2	12 + 13 + 23
2	3	111	3	123
3	4	211	3	12 + 13
		$n = 4$		
1	4	1111	3	123 + 124 + 134 + 234
2	4	1111	4	1234
3	5	2111	3	12 + 13 + 14 + 234
4	5	2111	4	123 + 124 + 134
5	6	2211	4	12 + 134 + 234
6	6	2211	5	123 + 124
7	6	3111	4	12 + 13 + 14
8	7	3211	5	12 + 134
9	8	3221	5	12 + 13 + 234
		$n = 5$		
1	5	11111	3	123 + 124 + 134 + 234 + 125 + 135 + 235 + 145 + 245 + 345
2	5	11111	4	1234 + 1235 + 1245 + 1345 + 2345
3	5	11111	5	12345
4	6	21111	4	123 + 124 + 134 + 125 + 135 + 145 + 2345
5	6	21111	5	1234 + 1235 + 1245 + 1345
6	7	22111	4	12 + 134 + 234 + 135 + 235 + 145 + 245
7	7	22111	5	123 + 124 + 125 + 1345 + 2345
8	7	22111	6	1234 + 1235 + 1245
9	7	31111	4	12 + 13 + 14 + 15 + 2345
10	7	31111	5	123 + 124 + 134 + 125 + 135 + 145
11	8	22211	5	123 + 124 + 134 + 234 + 125 + 135 + 235
12	8	22211	6	123 + 1245 + 1345 + 2345
13	8	22211	7	1234 + 1235
14	8	32111	5	12 + 134 + 135 + 145 + 2345
15	8	32111	6	123 + 124 + 125 + 1345
16	8	41111	5	12 + 13 + 14 + 15
17	9	32211	5	12 + 13 + 234 + 235 + 145
18	9	32211	6	123 + 124 + 134 + 125 + 135 + 2345
		$n = 5$		
19	9	32211	7	123 + 1245 + 1345
20	9	33111	6	12 + 1345 + 2345
21	9	33111	7	123 + 124 + 125
22	9	42111	6	12 + 134 + 135 + 145
23	10	32221	6	123 + 124 + 134 + 234 + 125 + 135 + 145
24	10	32221	7	123 + 124 + 134 + 2345
25	10	33211	6	12 + 134 + 234 + 135 + 235
26	10	33211	8	123 + 1245
27	10	42211	6	12 + 13 + 145 + 2345
28	10	42211	7	123 + 124 + 134 + 125 + 135
29	10	43111	7	12 + 1345
30	11	33221	7	123 + 124 + 134 + 234 + 125
31	11	33221	8	123 + 124 + 1345 + 2345
32	11	43211	7	12 + 134 + 135 + 2345
33	11	52211	7	12 + 13 + 145
34	12	33222	7	123 + 124 + 134 + 234 + 125 + 135 + 235 + 145 + 245
35	12	43221	7	12 + 134 + 234 + 135 + 145
36	12	43221	8	123 + 124 + 134 + 125 + 2345
37	12	43221	9	123 + 124 + 1345
38	12	43311	7	12 + 13 + 234 + 235
39	12	52221	7	12 + 13 + 14 + 2345
40	12	53211	8	12 + 134 + 135
41	13	43321	8	123 + 124 + 125 + 135 + 134 + 234
42	13	53221	9	123 + 124 + 125 + 134
43	13	53311	8	12 + 13 + 2345
44	14	43322	8	123 + 124 + 125 + 134 + 135 + 145 + 234 + 235
45	14	53321	8	12 + 13 + 145 + 234
46	14	54221	9	12 + 134 + 2345
47	15	54321	9	12 + 134 + 135 + 234
48	16	54322	9	12 + 134 + 145 + 135 + 234 + 235

No.	V	$w_1 \sim w_6$	T	Representative Function
		$n = 6$		
1	6	111111	4	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
2	6	111111	5	12345 + 12346 + 12356 + 12456 + 13456 + 23456
3	6	111111	6	123456
4	7	211111	4	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156 + 2345 + 2346 + 2356 + 2456 + 3456

No.	V	$w_1 \sim w_6$	T	Representative Function
		$n = 6$		
5	7	211111	5	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 1456 + 23456
6	7	211111	6	12345 + 12346 + 12356 + 12456 + 13456
7	8	221111	5	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
8	8	221111	6	1234 + 1235 + 1245 + 1236 + 1246 + 1256 + 13456 + 23456
9	8	221111	7	12345 + 12346 + 12356 + 12456
10	8	311111	5	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156 +23456
11	8	311111	6	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 1456
12	9	222111	5	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 235 + 236 + 2456 + 3456
13	9	222111	6	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 2345 + 2346 + 2356
14	9	222111	7	1234 + 1235 + 1236 + 12456 + 13456 + 23456
15	9	222111	8	12345 + 12346 + 12356
16	9	321111	5	12 + 134 + 135 + 145 + 136 + 146 + 156 + 2345 + 2346 + 2356 + 2456
17	9	321111	6	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456 + 23456
18	9	321111	7	1234 + 1235 + 1245 + 1236 + 1246 + 1256 + 13456
19	9	411111	5	12 + 13 + 14 + 15 + 16 + 23456
20	9	411111	6	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156
21	10	222211	6	123 + 124 + 134 + 1256 + 1356 + 1456 + 234 + 2356 + 2456 + 3456
22	10	222211	7	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 2345 + 2346
23	10	222211	8	1234 + 12356 + 12456 + 13456 + 23456
24	10	222211	9	12345 + 12346
25	10	322111	6	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 2345 + 2346 + 2356
26	10	322111	7	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 23456
27	10	322111	8	1234 + 1235 + 1236 + 12456 + 13456
28	10	331111	6	12 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
29	10	331111	7	123 + 124 + 125 + 126 + 13456 + 23456
30	10	331111	8	1234 + 1235 + 1245 + 1236 + 1246 + 1256
31	10	421111	6	12 + 134 + 135 + 145 + 136 + 146 + 156 + 23456
32	10	421111	7	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456
33	10	511111	6	12 + 13 + 14 + 15 + 16
34	11	322211	6	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 234 + 2356 + 2456 + 3456
35	11	322211	7	123 + 124 + 134 + 1256 + 1356 + 1456 + 2345 + 2346
36	11	322211	8	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 23456
37	11	322211	9	1234 + 12356 + 12456 + 13456
38	11	332111	6	12 + 134 + 135 + 136 + 1456 + 234 + 235 + 236 + 2456
39	11	332111	7	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 2345 + 2346 + 2356
40	11	332111	8	123 + 1245 + 1246 + 1256 + 13456 + 23456
41	11	332111	9	1234 + 1235 + 1236 + 12456
42	11	422111	6	12 + 13 + 145 + 146 + 156 + 2345 + 2346 + 2356
43	11	422111	7	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 23456
44	11	422111	8	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356
45	11	431111	7	12 + 1345 + 1346 + 1356 + 1456 + 23456
46	11	431111	8	123 + 124 + 125 + 126 + 13456
47	11	521111	7	12 + 134 + 135 + 145 + 136 + 146 + 156
48	12	322221	7	123 + 124 + 134 + 125 + 145 + 135 + 2345 + 2346 + 2356 + 2456 + 3456
49	12	322221	8	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345
50	12	322221	9	1234 + 1235 + 1245 + 1345 + 23456
51	12	332211	7	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 234 + 2356 + 2456
52	12	332211	8	123 + 124 + 1345 + 1346 + 1256 + 2345 + 2346
53	12	332211	9	1234 + 1235 + 1245 + 1236 + 1246 + 13456 + 23456
54	12	332211	10	1234 + 12356 + 12456
55	12	333111	7	123 + 124 + 134 + 125 + 135 + 126 + 136 + 234 + 235 + 236
56	12	333111	9	123 + 12456 + 13456 + 23456
57	12	333111	10	1234 + 1235 + 1236
58	12	422211	7	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 2345 + 2346
59	12	422211	8	123 + 124 + 134 + 1256 + 1356 + 1456 + 23456
60	12	422211	9	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346
61	12	462111	7	12 + 134 + 135 + 136 + 1456 + 2345 + 2346 + 2356
62	12	432111	8	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 23456
63	12	432111	9	123 + 1245 + 1246 + 1256 + 13456
64	12	441111	8	12 + 13456 + 23456
65	12	441111	9	123 + 124 + 125 + 126
66	12	522111	7	12 + 13 + 145 + 146 + 156 + 23456
67	12	522111	8	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456
68	12	531111	8	12 + 1345 + 1346 + 1356 + 1456
69	13	332221	7	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 235 + 245 + 3456
70	13	332221	8	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
71	13	332221	9	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1256 + 2345
72	13	332221	10	1234 + 1235 + 1245 + 13456 + 23456
73	13	333211	8	123 + 124 + 134 + 1256 + 1356 + 234 + 2356
74	13	333211	9	123 + 1245 + 1345 + 1246 + 1346 + 2345 + 2346
75	13	333211	11	1234 + 12356
76	13	432211	7	12 + 134 + 135 + 145 + 136 + 146 + 234 + 2356 + 2456
77	13	432211	8	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 2345 + 2346
78	13	432211	9	123 + 124 + 1345 + 1346 + 1256 + 23456
79	13	432211	10	1234 + 1235 + 1245 + 1236 + 1246 + 13456
80	13	433111	7	12 + 13 + 1456 + 234 + 235 + 236
81	13	433111	8	123 + 124 + 134 + 125 + 135 + 126 + 136 + 2345 + 2346 + 2356
82	13	433111	10	123 + 12456 + 13456
83	13	442111	8	12 + 1345 + 1346 + 1356 + 2345 + 2346 + 2356
84	13	442111	10	123 + 1245 + 1246 + 1256
85	13	522211	7	12 + 13 + 14 + 156 + 2345 + 2346
86	13	522211	8	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 23456
87	13	522211	9	123 + 124 + 134 + 1256 + 1356 + 1456
88	13	532111	8	12 + 134 + 135 + 136 + 1456 + 23456
89	13	532111	9	123 + 124 + 125 + 1345 + 126 + 1346 + 1356
90	13	541111	9	12 + 13456
91	13	622111	8	12 + 13 + 145 + 146 + 156

TABLE 1—*Continued*

No.	V	$w_1 \sim w_6$	T	Representative Function
		$n = 6$		
92	14	332222	8	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
93	14	332222	9	1234 + 1235 + 1236 + 1245 + 1246 + 1256 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
94	14	333221	8	123 + 124 + 134 + 125 + 135 + 1456 + 234 + 235 + 2456 + 3456
95	14	333221	10	1234 + 1235 + 1245 + 1345 + 1236 + 2345
96	14	333221	11	1234 + 1235 + 12456 + 13456 + 23456
97	14	432221	8	123 + 124 + 134 + 125 + 135 + 145 + 126 + 2345 + 2346 + 2356 + 2456
98	14	432221	9	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456 + 2345
99	14	432221	10	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1256 + 23456
100	14	432221	11	1234 + 1235 + 1245 + 13456
101	14	433211	8	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 2356
102	14	433211	9	123 + 124 + 134 + 1256 + 1356 + 2345 + 2346
103	14	433211	10	123 + 1245 + 1345 + 1246 + 1346 + 23456
104	14	442211	8	12 + 134 + 1356 + 1456 + 234 + 2356 + 2456
105	14	442211	9	123 + 124 + 125 + 1345 + 126 + 1346 + 2345 + 2346
106	14	442211	10	123 + 124 + 1256 + 13456 + 23456
107	14	442211	11	1234 + 1235 + 1245 + 1236 + 1246
108	14	443111	8	12 + 134 + 135 + 136 + 234 + 235 + 236
109	14	443111	11	123 + 12456
110	14	522221	8	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156 + 2345
111	14	522221	9	123 + 124 + 134 + 125 + 145 + 135 + 23456
112	14	532211	8	12 + 134 + 135 + 145 + 136 + 146 + 2345 + 2346
113	14	532211	9	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 23456
114	14	532211	10	123 + 124 + 1345 + 1346 + 1256
115	14	533111	8	12 + 13 + 1456 + 2345 + 2346 + 2356
116	14	533111	9	123 + 124 + 134 + 125 + 135 + 126 + 136 + 23456
117	14	542111	9	12 + 1345 + 1346 + 1356 + 23456
118	14	622211	8	12 + 13 + 14 + 156 + 23456
119	14	622211	9	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146
120	14	632111	9	12 + 134 + 135 + 136 + 1456
121	15	333222	9	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
122	15	333222	10	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 2345 + 2346 + 2356
123	15	433221	8	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234 + 235 + 2456 + 3456
124	15	433221	9	123 + 124 + 134 + 125 + 135 + 1456 + 2345 + 2346 + 2356
125	15	433221	10	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 2345
126	15	433221	11	1234 + 1235 + 1245 + 1345 + 1236 + 23456
127	15	433221	12	1234 + 1235 + 12456 + 13456
128	15	433311	9	123 + 124 + 134 + 1256 + 1356 + 1456 + 234
129	15	433311	10	123 + 124 + 134 + 2345 + 2346
130	15	443211	9	123 + 124 + 134 + 125 + 126 + 1356 + 234 + 2356
131	15	443211	11	123 + 1245 + 1246 + 13456 + 23456
132	15	532221	9	123 + 124 + 134 + 125 + 135 + 145 + 126 + 2345
133	15	532221	10	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456 + 23456
134	15	532221	11	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1256
135	15	533211	8	12 + 13 + 145 + 146 + 234 + 2356
136	15	533211	9	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 2345 + 2346
137	15	533211	10	123 + 124 + 134 + 1256 + 1356 + 23456
138	15	563211	11	123 + 1245 + 1345 + 1246 + 1346
139	15	542211	9	12 + 134 + 1356 + 1456 + 2345 + 2346
140	15	542211	10	123 + 124 + 125 + 1345 + 126 + 1346 + 23456
141	15	542211	11	123 + 124 + 1256 + 13456
142	15	543111	9	12 + 134 + 135 + 136 + 2345 + 2346 + 2356
143	15	632211	9	12 + 134 + 135 + 145 + 136 + 146 + 23456
144	15	632211	10	123 + 124 + 134 + 125 + 126 + 1356 + 1456
145	15	633111	9	12 + 13 + 1456 + 23456
146	15	633111	10	123 + 124 + 134 + 125 + 135 + 126 + 136
147	15	642111	10	12 + 1345 + 1346 + 1356
148	15	722211	9	12 + 13 + 14 + 156
149	16	433222	9	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
150	16	433222	10	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356
151*	16	433222	11	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 23456
152	16	433321	9	123 + 124 + 134 + 125 + 145 + 135 + 234 + 2356 + 2456 + 3456
153	16	433321	11	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 2345
154	16	443221	9	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234 + 235 + 2456
155	16	443221	10	123 + 124 + 125 + 1345 + 1346 + 1356 + 2345 + 2346 + 2356
156	16	443221	11	123 + 1245 + 1345 + 1246 + 1256 + 2345
157	16	443221	12	1234 + 1235 + 1245 + 1236 + 13456 + 23456
158	16	443221	13	1234 + 1235 + 12456
159	16	443311	10	123 + 124 + 134 + 1256 + 234
160	16	443311	11	123 + 124 + 1345 + 1346 + 2345 + 2346
161	16	532222	9	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156 + 2345 + 2346 + 2356 + 2456
162	16	533221	9	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 2345 + 2346 + 2356
163	16	533221	10	123 + 124 + 134 + 125 + 135 + 1456 + 2345
164	16	533221	12	1234 + 1235 + 1245 + 1345 + 1236
165	16	533311	9	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 234
166	16	533311	11	123 + 124 + 134 + 23456
167	16	542221	9	12 + 134 + 135 + 145 + 2345 + 2346 + 2356 + 2456
168	16	542221	10	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456 + 2345
169	16	542221	11	123 + 124 + 125 + 1345 + 23456
170	16	543211	9	12 + 134 + 135 + 136 + 1456 + 234 + 2356
171	16	543211	10	123 + 124 + 134 + 125 + 126 + 1356 + 2345 + 2346
172	16	543211	12	123 + 1245 + 1246 + 13456
173	16	544111	9	12 + 13 + 234 + 235 + 236

No.	V	$w_1 \sim w_6$	T	Representative Function
		$n = 6$		
174	16	552211	10	12 + 1345 + 1346 + 2345 + 2346
175	16	552211	12	123 + 124 + 1256
176	16	632221	9	12 + 134 + 135 + 145 + 136 + 146 + 156 + 2345
177	16	632221	10	123 + 124 + 134 + 125 + 135 + 145 + 126 + 23456
178	16	632221	11	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456
179	16	633211	9	12 + 13 + 145 + 146 + 2345 + 2346
180	16	633211	11	123 + 124 + 134 + 1256 + 1356
181	16	642211	10	12 + 134 + 1356 + 1456 + 23456
182	16	642211	11	123 + 124 + 125 + 1345 + 126 + 1346
183	16	643111	10	12 + 134 + 135 + 136 + 23456
184	16	722221	9	12 + 13 + 14 + 15 + 23456
185	16	732211	10	12 + 134 + 135 + 145 + 136 + 146
186	16	733111	10	12 + 13 + 1456
187	17	433322	10	123 + 124 + 134 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
188	17	433322	11	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346
189	17	443321	10	123 + 124 + 125 + 134 + 1356 + 1456 + 234 + 2356 + 2456
190	17	443321	12	1234 + 1235 + 1236 + 1246 + 1245 + 1345 + 2345
191*	17	533222	11	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456 + 23456
192	17	533321	10	123 + 124 + 134 + 125 + 145 + 135 + 2345 + 2346
193	17	533321	11	123 + 124 + 134 + 1256 + 1356 + 1456 + 2345
194	17	543221	9	12 + 134 + 135 + 145 + 136 + 234 + 235 + 2456
195	17	543221	10	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 2345 + 2346 + 2356
196	17	543221	11	123 + 124 + 125 + 1345 + 1346 + 1356 + 2345
197	17	543221	12	123 + 1245 + 1345 + 1246 + 1256 + 23456
198	17	543221	13	1234 + 1235 + 1245 + 1236 + 13456
199	17	543311	10	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 234
200	17	543311	11	123 + 124 + 134 + 1256 + 2345 + 2346
201	17	543311	12	123 + 124 + 1345 + 1346 + 23456
202	17	544211	10	123 + 124 + 134 + 125 + 135 + 126 + 136 + 234 + 2356
203	17	552221	11	123 + 124 + 125 + 1345 + 126 + 2345
204	17	552221	12	123 + 124 + 125 + 13456 + 23456
205	17	553211	10	12 + 134 + 1356 + 234 + 2356
206	17	553211	13	123 + 1245 + 1246
207	17	633221	10	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 2345
208	17	633221	11	123 + 124 + 134 + 125 + 135 + 1456 + 23456
209	17	643211	10	12 + 134 + 135 + 136 + 1456 + 2345 + 2346
210	17	643211	11	123 + 124 + 134 + 125 + 126 + 1356 + 23456
211	17	644111	10	12 + 13 + 2345 + 2346 + 2356
212	17	652211	11	12 + 1345 + 1346 + 23456
213	17	732221	11	123 + 124 + 134 + 125 + 135 + 145 + 126
214	17	733211	10	12 + 13 + 145 + 146 + 23456
215	17	742211	11	12 + 134 + 1356 + 1456
216	17	743111	11	12 + 134 + 135 + 136
217	18	443322	10	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 234 + 2356 + 2456 + 3456
218	18	443322	11	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
219	18	443322	12	1234 + 1235 + 1236 + 1245 + 1246 + 1256 + 1345 + 1346 + 2345 + 2346
220	18	443331	10	123 + 124 + 134 + 125 + 145 + 135 + 234 + 235 + 245 + 3456
221	18	533322	10	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 2345 + 2346 + 2356 + 2456 + 3456
222	18	543222	10	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 2345 + 2346 + 2356 + 2456
223*	18	543222	11	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356
224	18	543321	10	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 2356 + 2456
225	18	543321	11	123 + 124 + 125 + 134 + 1356 + 1456 + 2345 + 2346
226	18	543321	12	123 + 124 + 1345 + 1346 + 1256 + 2345
227	18	543321	13	1234 + 1235 + 1236 + 1246 + 1245 + 1345 + 23456
228	18	544221	10	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 235
229	18	544221	11	123 + 124 + 134 + 125 + 135 + 2345 + 2346 + 2356
230	18	544221	13	123 + 1245 + 1345 + 23456
231	18	544311	11	123 + 124 + 134 + 1256 + 1356 + 234
232	18	553221	10	12 + 134 + 135 + 1456 + 234 + 235 + 2456
233	18	553221	14	1234 + 1235 + 1236 + 1245
234	18	553311	11	123 + 124 + 125 + 126 + 134 + 234
235	18	553311	13	123 + 124 + 13456 + 23456
236	18	633222	10	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156 + 2345 + 2346 + 2356
237	18	633222	13	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356
238	18	643221	10	12 + 134 + 135 + 145 + 136 + 2345 + 2346 + 2356
239	18	643221	11	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 2345
240	18	643221	12	123 + 124 + 125 + 1345 + 1346 + 1356 + 23456
241	18	643221	13	123 + 1245 + 1345 + 1246 + 1256
242	18	643311	10	12 + 134 + 135 + 145 + 136 + 146 + 234
243	18	643311	12	123 + 124 + 134 + 1256 + 23456
244	18	643311	13	123 + 124 + 1345 + 1346
245	18	644211	10	12 + 13 + 1456 + 234 + 2356
246	18	644211	11	123 + 124 + 134 + 125 + 135 + 126 + 136 + 2345 + 2346
247	18	652221	11	12 + 1345 + 1346 + 1356 + 1456 + 2345
248	18	652221	12	123 + 124 + 125 + 1345 + 126 + 23456
249	18	652221	13	123 + 124 + 125 + 13456
250	18	653211	11	12 + 134 + 1356 + 2345 + 2346
251	18	733221	10	12 + 13 + 145 + 146 + 156 + 2345
252	18	733221	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 23456
253	18	733221	12	123 + 124 + 134 + 125 + 135 + 1456
254	18	733311	10	12 + 13 + 14 + 2345 + 2346
255	18	742221	11	12 + 134 + 135 + 145 + 23456
256	18	743211	12	123 + 124 + 134 + 125 + 126 + 1356
257	18	744111	11	12 + 13 + 23456
258	18	752211	12	12 + 1345 + 1346

TABLE 1—*Continued*

No.	V	$w_1 \sim w_6$	T	Representative Function
		$n = 6$		
259	18	833211	11	12 + 13 + 145 + 146
260	19	443332	11	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
261*	19	543322	11	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
262	19	543322	12	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346
263*	19	543322	13	1234 + 1235 + 1236 + 1245 + 1246 + 1256 + 1345 + 1346 + 23456
264	19	543331	11	123 + 124 + 134 + 125 + 145 + 135 + 2345 + 2346 + 2356 + 2456
265	19	544321	11	123 + 124 + 134 + 125 + 135 + 1456 + 234 + 2356
266	19	544321	13	123 + 1245 + 1345 + 1246 + 1346 + 2345
267	19	553321	12	123 + 124 + 125 + 1345 + 1346 + 2345 + 2346
268	19	553321	13	123 + 124 + 1256 + 1345 + 2345
269	19	554221	11	123 + 124 + 125 + 126 + 134 + 135 + 234 + 235
270	19	554221	14	123 + 1245 + 13456 + 23456
271	19	643321	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 2345 + 2346
272	19	643321	12	123 + 124 + 125 + 134 + 1356 + 1456 + 2345
273	19	643321	14	1234 + 1235 + 1236 + 1246 + 1245 + 1345
274	19	644311	11	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234
275	19	653221	11	12 + 134 + 135 + 1456 + 2345 + 2346 + 2356
276	19	653221	12	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 2345
277	19	653311	11	12 + 134 + 1356 + 1456 + 234
278	19	653311	12	123 + 124 + 125 + 126 + 134 + 2345 + 2346
279	19	653311	14	123 + 124 + 13456
280	19	654211	11	12 + 134 + 135 + 136 + 234 + 2356
281	19	733222	13	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456
282	19	733321	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 2345
283	19	743221	11	12 + 134 + 135 + 145 + 136 + 2345
284	19	743221	12	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 23456
285	19	743221	13	123 + 124 + 125 + 1345 + 1346 + 1356
286	19	743311	13	123 + 124 + 134 + 1256
287	19	744211	11	12 + 13 + 1456 + 2345 + 2346
288	19	752221	13	123 + 124 + 125 + 1345 + 126
289	19	753211	12	12 + 134 + 1356 + 23456
290	19	833221	12	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136
291	19	833311	11	12 + 13 + 14 + 23456
292	20	543332	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 2345 + 2346 + 2356 + 2456 + 3456
293	20	544322	11	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 2356 + 2456 + 3456
294	20	544322	12	123 + 124 + 134 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356
295	20	544322	13	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 2345 + 2346
296	20	544331	11	123 + 124 + 134 + 125 + 145 + 135 + 234 + 235 + 2456 + 3456
297	20	553331	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 235 + 245
298	20	554222	11	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 235 + 236 + 2456
299	20	554321	12	123 + 124 + 125 + 134 + 1356 + 234 + 2356
300	20	554321	14	123 + 1245 + 1246 + 1345 + 2345
301	20	643322	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 2345 + 2346 + 2356 + 2456
302*	20	643322	13	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456 + 23456
303	20	644321	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234 + 2356
304	20	644321	12	123 + 124 + 134 + 125 + 135 + 1456 + 2345 + 2346
305	20	644321	13	123 + 124 + 134 + 1256 + 1356 + 2345
306	20	653222	11	12 + 134 + 135 + 136 + 1456 + 2345 + 2346 + 2356 + 2456
307	20	653321	11	12 + 134 + 135 + 145 + 234 + 2356 + 2456
308	20	653321	13	123 + 124 + 125 + 1345 + 1346 + 2345
309	20	653321	14	123 + 124 + 1256 + 1345 + 23456
310	20	654221	11	12 + 134 + 135 + 136 + 1456 + 234 + 235
311	20	654221	12	123 + 124 + 125 + 126 + 134 + 135 + 2345 + 2346 + 2356
312	20	654221	15	123 + 1245 + 13456
313	20	654311	12	123 + 124 + 134 + 125 + 126 + 1356 + 234
314	20	733322	11	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 156 + 2345 + 2346
315	20	743222	11	12 + 134 + 135 + 145 + 136 + 146 + 156 + 2345 + 2346 + 2356
316	20	743321	13	123 + 124 + 125 + 134 + 1356 + 1456 + 23456
317	20	744221	11	12 + 13 + 145 + 2345 + 2346 + 2356
318	20	744221	12	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 2345
319	20	744221	13	123 + 124 + 134 + 125 + 135 + 23456
320	20	744311	11	12 + 13 + 145 + 146 + 234
321	20	753221	12	12 + 134 + 135 + 1456 + 2345
322	20	753311	13	123 + 124 + 125 + 126 + 134 + 23456
323	20	754211	12	12 + 134 + 135 + 136 + 2345 + 2346
324	20	762221	13	12 + 1345 + 23456
325	20	833321	11	12 + 13 + 14 + 156 + 2345
326	20	843221	12	12 + 134 + 135 + 145 + 136 + 23456
327	20	843221	13	123 + 124 + 134 + 125 + 135 + 126 + 1456
328	20	853211	13	12 + 134 + 1356
329	21	544332	12	123 + 124 + 134 + 125 + 135 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
330	21	544332	14	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 2345
331	21	554322	13	123 + 124 + 1256 + 1345 + 1356 + 1346 + 2345 + 2346 + 2356
332	21	554322	14	123 + 1245 + 1256 + 1246 + 1345 + 1346 + 2345 + 2346
333	21	554331	12	123 + 124 + 134 + 125 + 135 + 1456 + 234 + 235 + 2456
334	21	644331	12	123 + 124 + 134 + 125 + 145 + 135 + 2345 + 2346 + 2356
335*	21	653322	13	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 1456 + 2345 + 2346
336	21	654321	12	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234 + 2356
337	21	654321	13	123 + 124 + 125 + 134 + 1356 + 2345 + 2346
338	21	654321	15	123 + 1245 + 1246 + 1345 + 23456
339	21	655221	12	123 + 124 + 134 + 125 + 135 + 126 + 136 + 234 + 235
340	21	743322	15	1234 + 1235 + 1236 + 1245 + 1246 + 1256 + 1345 + 1346
341	21	744321	12	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 2345 + 2346
342	21	753321	12	12 + 134 + 135 + 145 + 2345 + 2346
343	21	753321	13	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 2345
344	21	753321	14	123 + 124 + 125 + 1345 + 1346 + 23456
345	21	753321	15	123 + 124 + 1256 + 1345
346	21	754221	13	123 + 124 + 125 + 126 + 134 + 135 + 2345

No.	V	$w_1 \sim w_6$	T	Representative Function
		$n = 6$		
347	21	754311	12	12 + 134 + 135 + 136 + 1456 + 234
348	21	755211	12	12 + 13 + 234 + 2356
349	21	763221	13	12 + 1345 + 1346 + 1356 + 2345
350	21	763311	13	12 + 134 + 2345 + 2346
351	21	843321	12	12 + 134 + 135 + 145 + 136 + 146 + 2345
352	21	843321	14	123 + 124 + 125 + 134 + 1356 + 1456
353	21	853221	13	12 + 134 + 135 + 1456 + 23456
354	21	853311	14	123 + 124 + 125 + 126 + 134
355	21	943221	13	12 + 134 + 135 + 145 + 136
356	22	544333	14	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356
357	22	554332	12	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234 + 235 + 2456 + 3456
358	22	554422	13	123 + 124 + 134 + 1256 + 1356 + 1456 + 234 + 2356 + 2456
359	22	644332	12	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 2345 + 2346 + 2356 + 2456 + 3456
360	22	644332	14	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345
361	22	654322	12	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 2356 + 2456
362*	22	654322	13	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 2345 + 2346 + 2356
363	22	654322	14	123 + 124 + 1256 + 1345 + 1356 + 1346 + 2345 + 2346
364*	22	654322	15	123 + 1245 + 1256 + 1246 + 1345 + 1346 + 23456
365	22	654331	12	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 235 + 2456
366	22	654421	13	123 + 124 + 125 + 134 + 1356 + 1456 + 234
367	22	654421	15	123 + 124 + 1345 + 1346 + 2345
368	22	655222	12	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 235 + 236
369	22	655321	13	123 + 124 + 134 + 125 + 135 + 234 + 2356
370	22	744322	12	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 2345 + 2346 + 2356
371	22	753322	12	12 + 134 + 135 + 145 + 136 + 146 + 2345 + 2346 + 2356 + 2456
372	22	754321	12	12 + 134 + 135 + 145 + 136 + 234 + 2356
373	22	754321	13	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 2345 + 2346
374	22	754321	14	123 + 124 + 125 + 134 + 1356 + 2345
375	22	754321	16	123 + 1245 + 1246 + 1345
376	22	755221	12	12 + 13 + 1456 + 234 + 235
377	22	755311	13	123 + 124 + 134 + 125 + 135 + 126 + 136 + 234
378	22	763222	13	12 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356
379	22	763321	14	123 + 124 + 125 + 1345 + 126 + 1346 + 2345
380	22	764221	13	12 + 134 + 135 + 2345 + 2346 + 2356
381	22	764311	13	12 + 134 + 1356 + 234
382	22	843322	12	12 + 134 + 135 + 145 + 136 + 146 + 156 + 2345 + 2346
383	22	843322	15	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456
384	22	853321	15	123 + 124 + 125 + 1345 + 1346
385	22	854221	13	12 + 134 + 135 + 136 + 1456 + 2345
386	22	854221	14	123 + 124 + 125 + 126 + 134 + 135 + 23456
387	22	855211	13	12 + 13 + 2345 + 2346
388	22	863311	14	12 + 134 + 23456
389	22	944221	13	12 + 13 + 145 + 23456
390	22	953221	14	12 + 134 + 135 + 1456
391	23	554333	14	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
392	23	554432	13	123 + 124 + 125 + 134 + 1356 + 1456 + 234 + 2356 + 2456 + 3456
393*	23	654332	13	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 2345 + 2346 + 2356 + 2456
394	23	654332	14	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356
395	23	654431	13	123 + 124 + 134 + 125 + 145 + 135 + 234 + 2356 + 2456
396	23	655322	14	123 + 124 + 134 + 1256 + 1356 + 2345 + 2346 + 2356
397	23	655331	13	123 + 124 + 134 + 125 + 135 + 1456 + 234 + 235
398*	23	754322	15	123 + 124 + 1256 + 1345 + 1356 + 1346 + 23456
399	23	754331	13	123 + 124 + 134 + 125 + 135 + 145 + 126 + 2345 + 2346 + 2356
400	23	754421	13	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234
401	23	754421	15	123 + 124 + 134 + 1256 + 2345
402	23	755321	14	123 + 124 + 134 + 125 + 135 + 2345 + 2346
403	23	763322	13	12 + 134 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
404	23	764321	13	12 + 134 + 135 + 1456 + 234 + 2356
405	23	765221	13	12 + 134 + 135 + 136 + 234 + 235
406	23	854321	13	12 + 134 + 135 + 145 + 136 + 2345 + 2346
407	23	854321	15	123 + 124 + 125 + 134 + 1356 + 23456
408	23	855221	14	123 + 124 + 134 + 125 + 135 + 126 + 136 + 2345
409	23	855311	13	12 + 13 + 1456 + 234
410	23	863321	14	12 + 134 + 1356 + 1456 + 2345
411	23	944321	13	12 + 13 + 145 + 146 + 2345
412	23	954221	15	123 + 124 + 125 + 126 + 134 + 135
413	24	554433	14	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
414	24	654432	13	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 2356 + 2456 + 3456
415	24	654432	14	123 + 124 + 125 + 134 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
416	24	654432	16	1234 + 1235 + 1236 + 1245 + 1246 + 1256 + 1345 + 1346 + 2345
417	24	655332	13	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 235 + 2456 + 3456
418	24	655422	14	123 + 124 + 134 + 1256 + 1356 + 1456 + 234 + 2356
419	24	754332	13	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 2345 + 2346 + 2356 + 2456
420	24	754422	13	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 234 + 2356 + 2456
421	24	755331	13	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234 + 235
422	24	755421	14	123 + 124 + 134 + 125 + 135 + 1456 + 234
423	24	764322	13	12 + 134 + 135 + 136 + 1456 + 234 + 2356 + 2456
424*	24	764322	15	123 + 124 + 125 + 1345 + 126 + 1346 + 1356 + 2345 + 2346
425	24	764331	13	12 + 134 + 135 + 145 + 234 + 235 + 2456
426	24	764421	15	123 + 124 + 134 + 125 + 2345 + 2346
427	24	764421	17	123 + 124 + 1345 + 23456
428	24	765222	13	12 + 134 + 135 + 136 + 1456 + 234 + 235 + 236
429	24	765321	14	123 + 124 + 125 + 126 + 134 + 135 + 234 + 2356
430	24	854322	13	12 + 134 + 135 + 145 + 136 + 146 + 2345 + 2346 + 2356
431	24	854322	17	123 + 1245 + 1256 + 1246 + 1345 + 1346
432	24	855321	13	12 + 13 + 145 + 234 + 2356

TABLE 1—*Continued*

No.	V	$w_1 \sim w_6$	T	Representative Function
				$n = 6$
433	24	864321	14	12 + 134 + 135 + 1456 + 2345 + 2346
434	24	864321	15	123 + 124 + 134 + 125 + 126 + 1356 + 2345
435	24	865311	14	12 + 134 + 135 + 136 + 234
436	24	873321	15	12 + 1345 + 1346 + 2345
437	24	944322	13	12 + 13 + 145 + 146 + 156 + 2345 + 2346
438	24	954321	16	123 + 124 + 125 + 134 + 1356
439	24	955221	14	12 + 13 + 1456 + 2345
440	24	964221	15	12 + 134 + 135 + 23456
441	25	654433	14	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
442	25	655432	14	123 + 124 + 134 + 125 + 135 + 1456 + 234 + 2356 + 2456 + 3456
443	25	754432	16	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456 + 2345
444	25	755431	14	123 + 124 + 134 + 125 + 145 + 135 + 234 + 2356
445*	25	765322	15	123 + 124 + 134 + 125 + 126 + 1356 + 2345 + 2346 + 2356
446	25	765331	14	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234 + 235
447	25	765421	15	123 + 124 + 125 + 134 + 1356 + 234
448	25	855421	14	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234
449	25	864331	14	12 + 134 + 135 + 145 + 2345 + 2346 + 2356
450	25	865321	15	123 + 124 + 125 + 126 + 134 + 135 + 2345 + 2346
451	25	873322	15	12 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346
452	25	954322	17	123 + 124 + 1256 + 1345 + 1356 + 1346
453	25	955321	14	12 + 13 + 145 + 2345 + 2346
454	25	965221	15	12 + 134 + 135 + 136 + 2345
455	26	655433	17	1234 + 1235 + 1245 + 1345 + 1236 + 1246 + 1346 + 1256 + 1356 + 2345 + 2346
456	26	755432	14	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234 + 2356 + 2456 + 3456
457	26	764432	16	123 + 124 + 125 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346
458	26	765332	14	123 + 124 + 134 + 125 + 135 + 126 + 136 + 1456 + 234 + 235 + 2456
459*	26	765422	15	123 + 124 + 134 + 125 + 126 + 1356 + 1456 + 234 + 2356
460	26	855422	14	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 146 + 234 + 2356
461	26	864332	14	12 + 134 + 135 + 145 + 136 + 2345 + 2346 + 2356 + 2456
462	26	865331	14	12 + 134 + 135 + 145 + 136 + 234 + 235
463	26	865421	15	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234
464	26	874322	15	12 + 134 + 1356 + 1456 + 2345 + 2346 + 2356
465	26	875321	15	12 + 134 + 135 + 234 + 2356
466	26	955322	14	12 + 13 + 145 + 146 + 2345 + 2346 + 2356
467	26	964421	17	123 + 124 + 134 + 125 + 23456
468	26	974321	16	12 + 134 + 1356 + 2345
469	27	755433	17	123 + 1245 + 1345 + 1246 + 1346 + 1256 + 1356 + 1456 + 2345 + 2346
470*	27	765432	15	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234 + 2356 + 2456
471	27	765432	16	123 + 124 + 125 + 134 + 1356 + 1456 + 2345 + 2346 + 2356
472	27	765432	18	123 + 1245 + 1256 + 1246 + 1345 + 1346 + 2345
473	27	765441	15	123 + 124 + 134 + 125 + 145 + 135 + 234 + 235 + 2456
474	27	865431	15	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 2356
475	27	874332	15	12 + 134 + 135 + 1456 + 2345 + 2346 + 2356 + 2456
476	27	875331	15	12 + 134 + 135 + 1456 + 234 + 235
477	27	965421	15	12 + 134 + 135 + 145 + 136 + 234
478	27	974421	17	123 + 124 + 125 + 126 + 134 + 2345
479	27	975321	16	12 + 134 + 135 + 2345 + 2346
480	28	765433	17	123 + 124 + 1256 + 1345 + 1346 + 1356 + 1456 + 2345 + 2346 + 2356
481	28	765442	15	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 235 + 2456 + 3456
482	28	765442	16	123 + 124 + 134 + 125 + 135 + 1456 + 2345 + 2346 + 2356 + 2456
483	28	865432	15	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234 + 2356 + 2456
484	28	865432	18	123 + 124 + 1256 + 1345 + 1356 + 1346 + 2345
485	28	875332	15	12 + 134 + 135 + 136 + 1456 + 234 + 235 + 2456
486	28	965422	15	12 + 134 + 135 + 145 + 136 + 146 + 234 + 2356
487	28	975421	16	12 + 134 + 135 + 1456 + 234
488	28	984322	17	12 + 1345 + 1346 + 1356 + 2345 + 2346
489	29	765443	16	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
490	29	765533	17	123 + 124 + 134 + 1256 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456
491	29	875432	18	123 + 124 + 125 + 1345 + 1346 + 1356 + 2345 + 2346
492	29	975431	16	12 + 134 + 135 + 145 + 234 + 2356
493	29	976421	17	123 + 124 + 125 + 126 + 134 + 135 + 234
494	29	985322	17	12 + 134 + 1356 + 2345 + 2346 + 2356
495	30	765543	17	123 + 124 + 125 + 134 + 1356 + 1456 + 2345 + 2346 + 2356 + 2456 + 3456
496	30	876432	18	123 + 124 + 125 + 134 + 1356 + 2345 + 2346 + 2356
497	30	975432	16	12 + 134 + 135 + 145 + 136 + 234 + 2356 + 2456
498	30	985422	17	12 + 134 + 1356 + 1456 + 234 + 2356
499	31	876532	18	123 + 124 + 125 + 134 + 1356 + 1456 + 234 + 2356
500	31	976441	17	123 + 124 + 134 + 125 + 135 + 145 + 126 + 234 + 235
501	31	985432	17	12 + 134 + 135 + 1456 + 234 + 2356 + 2456
502	32	876542	18	123 + 124 + 134 + 125 + 135 + 1456 + 234 + 2356 + 2456
503	32	976442	17	123 + 124 + 134 + 125 + 135 + 145 + 126 + 136 + 234 + 235 + 2456
504	33	876543	18	123 + 124 + 134 + 125 + 135 + 126 + 1456 + 234 + 2356 + 2456 + 3456

TABLE 2
The Number of Majority Decision Functions

n	Number of Logical Functions of up to n Variables	Number of Types of Logical Functions of n Variables*	Number of Types of Majority Decision Functions of n Variables	Number of Majority Decision Functions of n Variables	Number of Types of Self-Dual Majority Decision Functions of n Variables
1	4	1	1	2	1
2	16	2	1	8	0
3	256	10	3	72	1
4	65, 536	208	9	1, 536	1
5	4, 294, 967, 296	615, 904	48	86, 080	4
6	18, 446, 774, 073, 709, 551, 616	—	504	14, 487, 040	14

* These values are obtained from the results in References [4] and [5].

TABLE 3
The Maximum Values of Optimum Parameters of Majority Decision Functions

n	w	$V = \sum_{i=1}^{n} w_i$	T	K
2	1	2	2	3
3	2	4	3	5
4	3	8	5	9
5	5	16	9	17
6	9	33	18	35

variables the solution space of (10) is a pointed cone. That is, there is a certain point x_0 such that

$$Ax_0 \geqq b \tag{11}$$

and any solution x of (10) can be written as

$$x = x_0 + x' \qquad Ax' \geqq 0. \tag{12}$$

This means the solution space of (10) is a cone with x_0 as a sole vertex. These structures for majority decision functions of six variables were examined and it was found that almost all the majority decision functions have solution space of a pointed cone but that 15 out of 504 representatives have spaces of non-cone structure. These functions are marked with * in Table 1.

Fourth, some maximum values of the optimum parameters are shown in Table 3, where V is the sum of coupling weights associated with input variables and K is the total number of turns of windings including the constant winding and the relation $K = 2T - 1$ holds. In Table 3, 26 functions have the maximum value 9 for a weight w and only one function attains the maximum value 33 of V; there are 7 functions with maximum K of 35.

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