Making and characterizing negative Poisson's ratio materials

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We present an introduction to the use of negative Poisson's ratio materials to illustrate various aspects of mechanics of materials. Poisson's ratio is defined as minus the ratio of transverse strain to longitudinal strain in simple tension. For most materials, Poisson's ratio is close to 1/3. Negative Poisson's ratios are counter-intuitive but permissible according to the theory of elasticity. Such materials can be prepared for classroom demonstrations, or made by students.

Key words: Poisson's ratio, deformation, foam, honeycomb

1. INTRODUCTION

Poisson's ratio is the ratio of lateral (transverse) contraction strain to longitudinal extension strain in a simple tension experiment. The allowable range of Poisson's ratio ν in three dimensional isotropic solids is from –1 to $\frac{1}{2}$ [1]. Common materials usually have a Poisson's ratio close to $\frac{1}{3}$. Rubbery materials, however, have values approaching $\frac{1}{2}$. They readily undergo shear deformations, governed by the shear modulus G, but resist volumetric (bulk) deformation governed by the bulk modulus K; for rubbery materials $G \ll K$. Even though textbooks can still be found [2] which categorically state that Poisson's ratios less than zero are unknown, or even impossible, there are in fact a number of examples of negative Poisson's ratio solids. Although such behaviour is counter-intuitive, negative Poisson's ratio structures and materials can be easily made and used in lecture demonstrations and for student projects. Such solids become fatter in cross section when stretched. A solid with $\nu \approx -1$ would be the opposite of rubber: difficult to shear but easy to deform volumetrically: $G \gg K$.

Negative Poisson's ratio materials offer a new direction for achieving unusual and improved mechanical performance.

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2. DEMONSTRATION OF THE RANGE OF POISSON'S RATIO

We begin with the assumption that students have been exposed to the definitions of stress and strain, and have further been exposed to the relationships between the two through the use of Mohr's circle. The permissible range of Poisson's ratio can then be explained during the introduction of Hooke's law. We suggest proceeding as follows.

From a pedagogical perspective, it is most natural to introduce the fundamental quantities in the isotropic constitutive equations as Young's modulus, E, Poisson's ratio, ν, and shear modulus, G. The dependence of G on E and ν can be illustrated with an exercise suggested, for instance, by McClintock and Argon [3]. The student begins with a state of pure shear, rotates it 45°, and writes the normal stress and strain components in terms of the corresponding shear stress and strain. Employing the Hooke's law relations illustrates that G must be related to E and ν through

$$G = \frac{E}{2(1+\nu)} \tag{1}$$

Each engineering modulus E and G must be positive if a free block of the material is to be in stable equilibrium, therefore $\nu > -1$.

Summing the normal components of stress and defining the bulk modulus as the ratio of pressure to volumetric strain provides a definition for bulk modulus,

$$K = \frac{E}{3(1-2\nu)} \tag{2}$$

Requiring $K > 0$ and $E > 0$ for stability gives $\nu < 0.5$. Combining these gives the range of ν from –1 to $\frac{1}{2}$. If one modulus were to be exactly zero, a condition of neutral stability would obtain, associated with Poisson's ratio exactly attaining one of the limits.

One may alternatively use an energy approach. Observe that the elementary Hooke's law relations are still a bit cumbersome, requiring a set of six equations to represent all components of the stress state. A different representation may be considered desirable, first because a different set of stress measures are more physically meaningful, and second because this alternative set of measures permits the representation of Hooke's law in a more compact and pleasing form. A brief discussion of deviatoric stress components is required, and this permits the instructor to touch on the role of such stress components in material failure as a precursor to later introduction of failure theories. With deviatoric components of stress and strain defined, the student or instructor may be asked to show that the entire set of Hooke's law relations may be reduced to:

$$\sigma'_{ij} = 2G\varepsilon'_{ij} \tag{3}$$

The instructor is now in a position to define the range of ν based on an energy argument, as suggested, for instance, by Malvern [4]. We begin with the definition of a differential element of internal strain energy density,

$$\mathrm{d}W = \sigma_{ij}\mathrm{d}\varepsilon_{ij} \tag{4}$$

This provides an opportunity to revisit the topic from a basic physics course, where the increment is one-dimensional, as expressed for instance through a spring constant, $\mathrm{d}W = F\,\mathrm{d}x$. We remind the student that forces normal to the spring do no work during the deformation. Equation (4) then represents nothing more than a spring-like statement of incremental work for a multi-dimensional spring. It is our experience that students are initially intimidated by indicial notation and the implicit summation over indices. Instructors may be advised to write out the summation in full, emphasizing that only normal components of stress do work through their respective normal strain increments, and only shear components of stress do work through their respective shear strain increments. Writing both σ_{ij} and $d\varepsilon_{ij}$ in terms of their deviatoric components, we then show that the energy density increment can be recast as:

$$dW = -p\, d\varepsilon + \sigma'_{ij}\, d\varepsilon'_{ij} \tag{5}$$

where p and $d\varepsilon$ are pressure and increments of volumetric strain respectively. Using both the definition of bulk modulus and the compact form of Hooke's law, we integrate the expression to obtain

$$W = \tfrac{1}{2} K\varepsilon^2 + G\varepsilon'_{ij}\varepsilon'_{ij} \tag{6}$$

The instructor can then return to the one-dimensional spring, noting the equivalent statement $W = \frac{1}{2}kx^2$ and the implications of negative k. Stability arguments may be used to make the case that the spring constant k must be positive for the spring to have any energy storage capacity ($W > 0$) and to be stable under a small perturbation. Similarly, the instructor can argue that the multi-dimensional statement embodied in equation (6) requires both the bulk and shear modulus to be positive, which in turn requires ν to fall in the range $-1 < \nu < \frac{1}{2}$. Specifically, one may have a purely volumetric strain state, so $K > 0$ for stability; one may have a purely deviatoric strain state, so $G > 0$ for stability. Finally, the instructor may note that many texts, while making this argument, are quick to point out that there are no known materials with negative Poisson's ratio. Beer and Johnston [2] state even more emphatically 'On the other hand, the very definition of Poisson's ratio requires it to be a positive quantity. We thus conclude that, for any engineering material, $0 < \nu < \frac{1}{2}$'.

3. NEGATIVE POISSON'S RATIO HONEYCOMBS

3.1 Honeycomb and its analysis

Structural honeycomb, like that made by the bee, consists of a regular array of hexagonal hollow cells. It may be, for many purposes, considered a two-dimensional cellular solid. A typical structure of convex hexagonal cells is shown in Fig. 1(a).

Following Gibson and Ashby [5], the mechanism which dominates the linear elastic deformation of honeycombs is that of bending of the cell walls. Analysis of stiffness and Poisson's ratio of honeycomb for deformation in its plane is by application of beam theory.

To explain and illuminate these results, we require students to have had some practice finding the deflections of beams under a combination of moments and lateral forces. The honeycomb is permitted to have a general shape, distinguished by the four parameters shown in Fig. 1: a lattice height h, an inclined leg length l with an associated angle of inclination θ, and a lattice thickness t. Before embarking on an analytical treatment, it is advisable to have the students perform a simple exercise with both isotropic ($h/l = 1$, $\theta = 30°$) and anisotropic honeycombs. In the spirit of strain-controlled deformation, have the students compress one of the principal axes of the honeycomb by a given increment, then measure the extension along the other in-plane principal axis. Then repeat the exercise with the compression applied on the second axis while extension is measured along the first. The value of this exercise is three-fold. First, it provides the instructor with the opportunity to make a distinction between n_{12} and n_{21}. Second, it permits the students to observe the

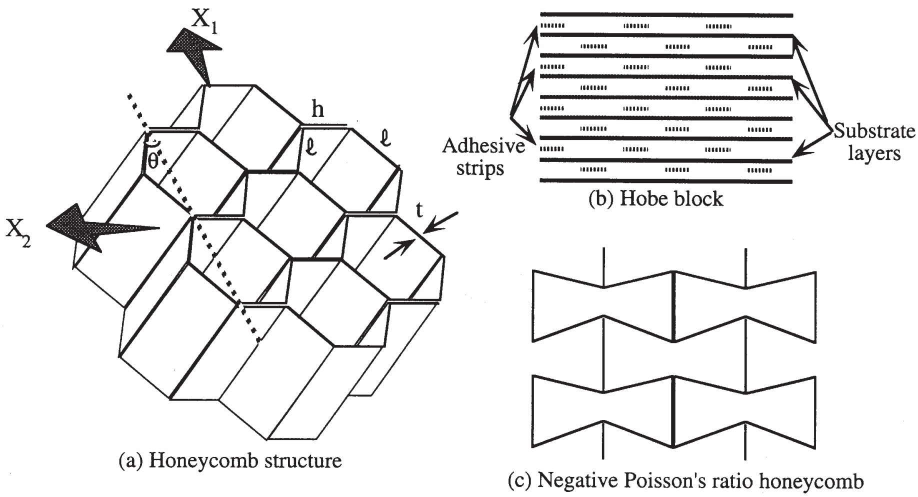

Fig. 1. (a) Honeycomb structure, (b) Hobe block, (c) negative Poisson's ratio honeycomb.

difference between the isotropic ($\nu_{12} = \nu_{21}$) and anisotropic ($\nu_{12} \neq \nu_{21}$) honeycombs. And finally, it provides the students with the opportunity to observe where the deformation is actually occurring under load.

The instructor is then prepared to discuss the analytical model of honeycomb deformation as described in [5]. At the outset, the instructor should state that elastic constants of the honeycomb are *defined* as

$$E_1^* = \sigma_1 / \varepsilon_1, \qquad E_2^* = \sigma_2 / \varepsilon_2 \tag{7}$$

$$\nu_{12}^* = -\varepsilon_2 / \varepsilon_1, \qquad \nu_{21}^* = -\varepsilon_1 / \varepsilon_2 \tag{8}$$

The star refers to the fact that the honeycomb structure, though discrete, is treated as a continuum. The stress σ and strain ε is based on uniaxial tension or compression. The results are obtained from straightforward application of beam theory on the inclined leg of the lattice. The resulting expressions for the four elastic constants defined above are [5]:

$$E_1^* / E_s = \left(\frac{t}{l}\right)^3 \frac{\cos\theta}{(h/l + \sin\theta)\sin^2\theta} \tag{9}$$

$$E_2^* / E_s = \left(\frac{t}{l}\right)^3 \frac{(h/l + \sin\theta)}{\cos^3\theta} \tag{10}$$

$$\nu_{12}^* = \frac{\cos^2\theta}{(h/l + \sin\theta)\sin\theta} \tag{11}$$

$$\nu_{21}^* = \frac{(h/l + \sin\theta)\sin\theta}{\cos^2\theta} \tag{12}$$

With their experimental results in hand from the initial exercise, the students can compare their measured values of ν_{12}^* and ν_{21}^* to those predicted by equations (11) and (12). This also permits them to verify that the theory predicts $\nu_{12}^* = \nu_{21}^* = 1$ for the transversely isotropic honeycomb. Since this development began with the proof that ν must lie in the range $-1 < \nu < +\frac{1}{2}$, some students may question how this result squares with that initial development. This provides a good opportunity to review the assumptions made in that analysis and assure students that there are no inconsistencies between the two results. Specifically, although a honeycomb of regular hexagons is isotropic in the 1–2 plane, any honeycomb is anisotropic in three dimensions. It is much stiffer in the 3 direction, than in the 1 or 2 directions, referring to Fig. 1. Therefore the range of Poisson's ratio for an isotropic material is inapplicable to honeycomb.

Having developed some confidence in these expressions, students may then be asked to calculate values of ν_{12}^* and ν_{21}^* for *negative* values of θ. The instructor is then in a position to make the case that such inverted cell honeycombs comprise one class of materials with negative Poisson's ratios. These honeycombs are manifestly nonlinearly elastic, since modulus and Poisson's ratio depends on angle. If linear behaviour is desired, one can restrict the range of strain to small values as is done in steel or aluminium if one wishes to avoid nonlinearity due to yield.

3.2 Making paper honeycomb

In the hobe (honeycomb before expanding) block method which is used in industry, strips of material are bonded together via bands of adhesive so that the bonded regions of one strip are above the unbonded regions of adjacent strips (Fig. 1(b)). If regular hexagons are desired in the final honeycomb, the width of glued and unglued sections is made equal. The stack of strips is pulled apart so that the web between the bonded strips forms the cell walls. A modified hobe block approach may be used to make paper honeycomb for student projects. Alternatively, paper strips may be formed into zig-zag patterns and attached with glue or double-stick tape. Negative Poisson's ratio honeycomb (Fig. 1(c)) consists of inverted cells. To achieve $\nu_{12}^* = \nu_{21}^* = -1$, one may solve equations (11) and (12) for the appropriate values of θ and h/l.

3.3 Bending: transverse curvature

Transverse curvature occurs in the pure bending of a beam or a plate. If Poisson's ratio is positive, transverse curvature is opposite the principal curvature of bending. This is known as *anticlastic curvature*. Honeycombs with convex hexagonal cells exhibit marked anticlastic curvature and deform into a saddle shape when bent. If, as shown in Fig. 2(a), the hexagons are regular, $\nu = +1$, which is possible in view of the anisotropy of the honeycomb. A material with a negative Poisson's ratio exhibits transverse curvature in the same sense as the principal curvature: synclastic curvature, as is anticipated in the exact, three-dimensional solution for the problem of pure bending [6]. Synclastic (or conclastic) curvature is demonstrated in Fig. 2(b) for a bent re-entrant honeycomb. The convex shape is spherical for $\nu = -1$ (which is possible in a re-entrant honeycomb), it is cylindrical for $\nu = 0$, and ellipsoidal for $-1 < \nu < 0$. Sandwich panels of convex shapes may be found useful for curved sandwich panels in the aircraft industry or in the construction of domes.

3.4 Projects

Several projects offered in a class in composite materials offered the following tasks. Design a honeycomb core of sufficient strength to support your body weight. Use the paper which

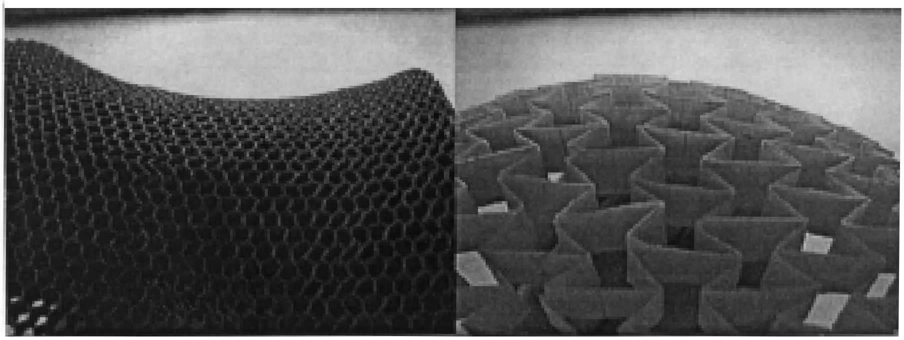

Fig. 2. (a) Anticlastic curvature of honeycomb with convex hexagonal cells. (b) Synclastic (ellipsoidal) curvature of honeycomb with re-entrant cells.

you studied in the previous assignment, and minimize the weight. Make the honeycomb. Measure the Poisson's ratio in the transverse direction. Stand on the honeycomb to demonstrate its strength. How do your results compare with theory? If the honeycomb fails by elastic buckling, the compressive strength [5] in the 3 direction is given, for regular hexagons of material with solid Poisson's ratio of 0.3, by

$$\frac{\sigma_3^*}{E_s} = 5.2\left(\frac{t}{l}\right)^3 \qquad (13)$$

If the honeycomb fails by plastic buckling, the honeycomb strength in terms of the solid strength σ_{ys} is

$$\frac{\sigma_3^*}{\sigma_{ys}} = 5.6\left(\frac{t}{l}\right)^{5/3} \qquad (14)$$

4. NEGATIVE POISSON'S RATIO FOAMS

4.1 Background

Foam materials with a negative Poisson's ratio as small as –0.7 were developed [7–10] by one of the authors and further studied by his co-workers. The negative Poisson's ratio is due to an inverted or re-entrant cell structure achieved by isotropic permanent volumetric compression of a conventional foam. This compression results in microbuckling of the cell ribs. Polymer foams which exhibit a softening point, ductile metallic foams, and thermosetting polymer foams have been prepared with a negative Poisson's ratio.

4.2 Making the foams

Negative Poisson's ratio foam is made by the following procedure involving heat and compression. Each kind of polymer foam has its own softening point and transition temperature. These instructions are for polyester and polyether polyurethane foams. For other foams the transition temperature is to be found empirically. Scott Industrial Foam with 10 to 20 pores per inch (cell size 2.5–1.2 mm) is appropriate. If one prefers less obvious microstructure, this foam is available with a cell size 0.25 mm. It is a reticulated, open cell foam used for air filters. Open-cell polymeric packing foams can be used; they may be more sensitive to processing temperature and humidity. Initial foam density should be low. Foam of 0.043 g/cm^3 (2.7 lb/ft^3) is suitable. If the initial solid volume fraction is too high, there may be insufficient space in the structure to achieve the required permanent compression. Aluminium square tube, 1 in square, has been successfully used for a mould, however choice of a specific type of mould is not critical [11]. Preheat the furnace to about 160–170°C. A kitchen oven is adequate. Cut a piece of foam which is oversize in comparison with the mould, in each of three orthogonal directions. The best results are obtained if the final permanent volumetric compression is between a factor of two and a factor of four. Either measure or mark foam for later determination of strains. Mark foam in all three orthogonal directions, i.e. two adjacent corners and down one side. Stuff the foam in the tube. Use a tongue depressor if necessary to remove wrinkles. Pull the foam a little on both ends to get rid of creases created by stuffing the material. This procedure will result in a pre-stretched sample in the tube. The actual original length of the sample must be used when determining the amount of pre-compression to apply. Restrain the length of the sample either by wrapping the mould with copper wire or by clamping the outside of the mould. Place the assembly in the centre of the furnace, and heat for the predetermined time, typically 15 to 20 min, depending on the foam. Remove and cool the specimen completely. Taking the specimen out of the mould before complete cooling may result in premature release of the pre-compression. It may be helpful to release foam ribs which have stuck together: stretch the specimen gently in each of three directions. Measure the amount of permanent compression retained by the specimen by either measuring the new distance between the marks or by measuring the size of the transformed sample.

Observe the shape of the cells in the foams. Most foams can be readily observed with a magnifying lens or low-power microscope. The conventional foam has convex cells which may be idealized as tetrakaidecahedra (Fig. 3). The physical origin of the positive Poisson's ratio in the normal foam is the elongation under stress of the foam cells into ovoid shapes which become narrower in cross section. The physical origin of the negative Poisson's ratio in re-entrant foam is the unfolding of the cells.

4.3 Testing the foams

The Poisson's ratio of foam may be measured with either a micrometer or a travelling microscope. Measurement of Poisson's ratio is limited by the resolution of displacement measurement. Therefore error bars will be larger at smaller strain than at larger strain. Polymer foams of conventional structure exhibit reasonably linear stress–strain curves up to about 5% strain, followed by a 'plateau' region, due to cell rib buckling. Negative Poisson's ratio foams exhibit a more progressive stress–strain behaviour since the cell ribs are already collapsed. It is instructive to plot Poisson's ratio versus strain. The strain-dependence of Poisson's ratio is due to strain-induced reorientation of the foam ribs, which changes the cell geometry as the foam is deformed.

4.4 Projects

If equipment is available for the measurement of force, one can compare conventional and re-entrant foams in the mitigation of impact, as in a buffer or a knee pad. One may also explore the effectiveness of the transformed foams as filters [7] or as seat cushions.

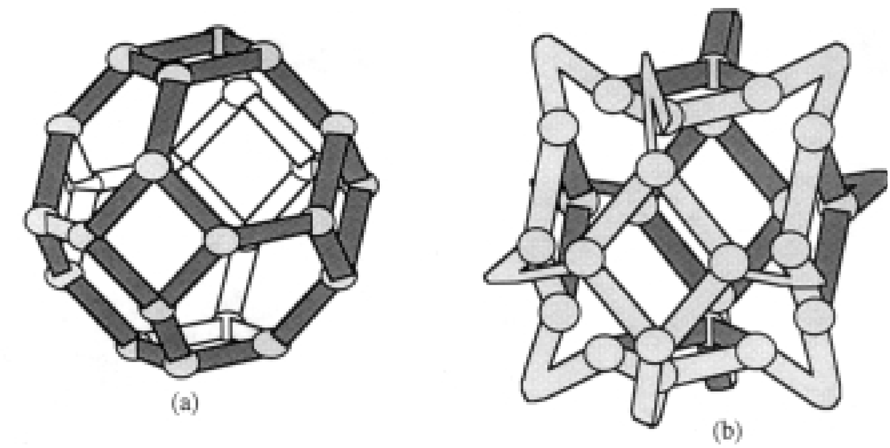

Fig. 3. Tetrakaidecahedron models for foam cells. (a) Conventional cell. (b) Re-entrant cell.

5. THE CONTINUUM HYPOTHESIS

For many purposes it is expedient to make use of continuum representations of materials, all of which actually have structure. No real material is a continuum. The foams and honeycombs presented here have large scale structure which is apparent to the unaided eye. This structure is not the cause of the negative Poisson's ratio, since the theory of elasticity has no characteristic length scale [12]. Indeed, it has been known for a long time, based on continuum concepts, that negative Poisson's ratios were energetically permissible: for isotropic solids ν is from –1 to $\frac{1}{2}$. An early elementary model [13] of atomic structure assuming central forces and affine deformation predicted $\nu = \frac{1}{4}$, but most common materials have $\nu \approx \frac{1}{3}$, therefore either interatomic forces are usually non-central or deformation is non-affine or both. Negative Poisson's ratios can be obtained by a non-affine deformation geometry alone or from a combination of non-central forces and pre-strain [14]. Moreover, following our original disclosure, many other negative Poisson's ratio materials have been created or discovered. The following negative Poisson's ratio materials have structure too small to see with the unaided eye: microporous polyethylene [15], α-cristobalite [16] which is silicon dioxide with a negative Poisson's ratio, certain cubic metals [17] in single crystal form, ordered plasmas and other extreme matter [18]. Some of these are anisotropic. Poisson's ratios for anisotropic materials are well known and defined for both composite materials [19] and for crystals. As we have indicated earlier, anisotropy is not a necessary condition for negative Poisson's ratio; both our foams and polycrystalline arrays of α-cristobalite can be isotropic. If desired, negative Poisson's ratio foams can be made using foams with cell sizes at or below the threshold of visibility. Such a material is not a continuum, but neither is steel. The visible size of the structure in the examples presented here is beneficial for the development of physical insight, since the underlying cause of the Poisson effect is the geometrical angulation of structural elements whether they be visible or not. The materials are sufficiently compliant that recoverable nonlinear effects can readily be observed. These can be handled within the framework of engineering strain, or via more advanced concepts in the theory of elasticity [20].

6. CONCLUSION

Negative Poisson's ratio honeycombs and foams are interesting and counter-intuitive. Their existence illustrates the bounding theorems of the theory of elasticity. They may be readily fabricated for demonstrations or projects.

REFERENCES

[1] Fung, Y. C., *Foundation of Solid Mechanics*, Prentice-Hall, Englewood, NJ, p. 353, 1968.
[2] Beer, F. P. and Johnston, E. R., *Mechanics of Materials*, McGraw Hill, New York, 1981, 2nd ed., 1992.
[3] McClintock, F. A. and Argon, A. S., *Mechanical Behavior of Materials*, Addison-Wesley, p. 79, 1966.
[4] Malvern, L. E., *Introduction to the Mechanics of a Continuous Medium*, Prentice-Hall, p. 292, 1969.
[5] Gibson, L. J. and Ashby, M. F., *Cellular Solids*, Pergamon, 1988.
[6] Timoshenko, S. P. and Goodier, J. N., *Theory of Elasticity*, McGraw-Hill, 3rd edition, 1970.
[7] Lakes, R. S., 'Foam structures with a negative Poisson's ratio', *Science*, **235** , 1038–1040, (1987).
[8] Friis, E. A., Lakes, R. S. and Park, J. B., 'Negative Poisson's ratio polymeric and metallic materials', *Journal of Materials Science*, **23** , 4406–4414, (1988).
[9] Choi, J. B. and Lakes, R. S., 'Nonlinear properties of metallic cellular materials with a negative Poisson's ratio', *J. Materials Science*, **27** , 5373–5381, (1992).
[10] Lakes, R. S., 'Advances in negative Poisson's ratio materials', *Advanced Materials* (Weinheim, Germany), **5**, 293–296, (1993).
[11] Ellis, A. B., Geselbracht, M. J., Johnson, B. J., Lisensky, G. C. and Robinson, W. R., *Teaching general Chemistry, A Materials Science Companion*, American Chemical Society, Washington, DC, 1993.
[12] Lakes, R. S., 'Negative Poisson's ratio materials', *Science*, **238** , 551, (1987).
[13] Weiner, J. H., *Statistical Mechanics of Elasticity*, J. Wiley, NY, 1983.
[14] Lakes, R. S., 'Deformation mechanisms of negative Poisson's ratio materials: structural aspects', *J. Materials Science*, **26** , 2287–2292 (1991).
[15] Caddock, B. D. and Evans, K. E., 'Microporous materials with negative Poisson's ratio: I. Microstructure and mechanical properties', *J. Phys. D. Appl. Phys*., **22** , 1877–1882, (1989).
[16] Haeri, A. Y., Weidner, D. J. and Parise, J. B., 'Elasticity of α-cristobalite: a silicon dioxide with a negative Poisson's ratio', *Science*, **257** , 650–652, (1992).
[17] Baughman, R. H., Shacklette, J. M., Zakhidov, A. A. and Stafstrom, S., 'Negative Poisson's ratios as a common feature of cubic metals', *Nature*, **392** (6674), 362–365, Mar. 26, (1998).
[18] Baughman, R. H., Dantas, S. O., Stafstrom, S., Zakhidov, A. A., Mitchell, T. B. and Dubin, D. H. E., 'Negative Poisson's ratios for extreme states of matter', *Science*, **288** , 2018–2022, (2000).
[19] Agarwal, B. D. and Broutman, L. J., *Analysis and Performance of Fiber Composites*, J. Wiley, 2nd ed., (1990).
[20] Beatty, M. F. and Stalnaker, D. O., 'The Poisson function of finite elasticity', *J. Appl. Mech. Trans. ASME*, **53** (4), 807-813, (1986).