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Making Sense of the Soul's Numbers. Middle Platonist Readings of Plato's *Divisio Animae*

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Abstract: The aim of this paper is to show that a new approach to Middle Platonist technical exegesis is both necessary and profitable, for it can shed light on the deep philosophical and methodological background of Middle Platonist exegesis as a whole. Through the exegesis of Plato's *divisio animae*, the Middle Platonists wanted to establish specific ways of both demonstrating and conceiving Plato's authority also in the field of harmonics. In particular, I shall take into account Platonists such as Plutarch, Theon, Nicomachus, and Severus, in order to show that: a) the Middle Platonist exegesis of Plato's *divisio animae* is chiefly based on a literalist approach to Plato's text, which is exploited in such a way as to make good technical sense of Plato's *divisio animae*; b) in this way, Middle Platonists sought to establish Plato's authority in the field of harmonics; c) this conception of authority is however controversial, for some Platonists (e.g., Theon) regarded him as the founder of very specific technical notions, while others (such as Plutarch, Nicomachus, and Severus) considered him the first to have established the general framework of Greek harmonics.

Keywords: Middle Platonism, Plato's Timaeus, Greek Harmonics, Platonist Exegesis

The Issue

The musical exegesis of Plato's *divisio animae* represents quite a widespread philosophical activity from the Early Academy to late Neoplatonism, and from the point of view of modern interpreters it encompasses several intriguing features. On the one hand, these pieces of exegesis represent interesting parallels to the technical divisions of the monochord, and highlight the way in which Platonists approached Plato's 'harmonics' after the technical developments of

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the Hellenistic age. On the other hand, they are puzzling in themselves from a mathematical point of view, since it is difficult to gain even a rough understanding of them – that is, to simply grasp what scales Platonists really had in mind.[1] All in all, then, the *divisiones* which can be found in Platonist texts have been examined from a technical perspective, with the aim of evaluating their technical correctness, or of discovering unusual technical positions. With this paper, however, I would like to go a step further and to apply a fresh approach to the texts at issue by uncovering the ideological and methodological reasons that led Platonists to opt for a specific technical interpretation, to introduce specific coefficients for the unit of Plato's *divisio*, and more generally to read the whole harmonic structure of Plato's soul according to different models. Such a new approach will be better exploited by referring to Middle Platonist texts, for in this period it is possible to observe an intriguing concentration of exegeses of Plato's *divisio*. This will lead us to consider Middle Platonist exegeses of Plato's *divisio animae* within the framework of a strictly philosophical debate, focused on the problem of how one should *consistently* and *effectively* read Plato's text in order to establish the Master's authority.

First of all, it is profitable to take a look at the well-known Platonic text at issue (*Tim*. 35b4–36b5, transl. Barker 1989):

> ἤρχετο δὲ διαιρεῖν ὧδε. μίαν ἀφεῖλεν τὸ πρῶτον ἀπὸ παντὸς μοῖραν, μετὰ δὲ ταύτην ἀφῄρει διπλασίαν ταύτης, τὴν δ' αὖ τρίτην ἡμιολίαν μὲν τῆς δευτέρας, τριπλασίαν δὲ τῆς πρώτης, τετάρτην δὲ τῆς δευτέρας διπλῆν, πέμπτην δὲ τριπλῆν τῆς τρίτης, τὴν δ' ἕκτην τῆς πρώτης ὀκταπλασίαν, ἑβδόμην δ' ἑπτακαιεικοσιπλασίαν τῆς πρώτης· μετὰ δὲ ταῦτα συνεπληροῦτο τά τε διπλάσια καὶ τριπλάσια διαστήματα, μοίρας ἔτι ἐκεῖθεν ἀποτέμνων καὶ τιθεὶς εἰς τὸ μεταξὺ τούτων, ὥστε ἐν ἑκάστῳ διαστήματι δύο εἶναι μεσότητας, τὴν μὲν ταὐτῷ μέρει τῶν ἄκρων αὐτῶν ὑπερέχουσαν καὶ ὑπερεχομένην, τὴν δὲ ἴσῳ μὲν κατ' ἀριθμὸν ὑπερέχουσαν, ἴσῳ δὲ ὑπερεχομένην. ἡμιολίων δὲ διαστάσεων καὶ ἐπιτρίτων καὶ ἐπογδόων γενομένων ἐκ τούτων τῶν δεσμῶν ἐν ταῖς πρόσθεν διαστάσεσιν, τῷ τοῦ ἐπογδόου διαστήματι τὰ ἐπίτριτα πάντα συνεπληροῦτο, λείπων αὐτῶν ἑκάστου μόριον, τῆς τοῦ μορίου ταύτης διαστάσεως λειφθείσης ἀριθμοῦ πρὸς ἀριθμὸν ἐχούσης τοὺς ὅρους ἓξ καὶ πεντήκοντα καὶ διακοσίων πρὸς τρία καὶ τετταράκοντα καὶ διακόσια.
>
> This is how he began to divide. First he took away one part from the whole (1), then another, double the size of the first (2), then a third, hemiolic with respect to the second and triple the first (3), then a fourth, double the second (4), then a fifth, three times the

1 Although valuable studies have been produced, in general, this is a much understudied topic. A pioneering and comprehensive account was provided by Brisson (1974). Analyses of Platonist scalar divisions were then provided by Barker (1989), *passim* (esp. on Thrasyllus/Theon and Nicomachus). In his outstanding book, Creese (2010) takes into account Adrastus, Thrasyllus, Theon, Eratosthenes, Nicomachus, and Timaeus Lokrus. On the philosophical import of these pieces of exegesis see also Ferrari (2000), Petrucci (2012a), *passim*, and 2018a. On Middle Platonist astronomical exegesis see Petrucci (2016).

> third (9), then a sixth, eight times the first (8), then a seventh, twenty-seven times the first (27). Next he filled out the double and triple intervals, once again cutting off parts from the material and placing them in the intervening gaps, so that in each interval there were two means, the one exceeding [one extreme] and exceeded [by the other extreme] by the same part of the extremes themselves, the other exceeding [one extreme] and exceeded [by the other] by an equal number. From these links within the previous intervals there arose hemiolic (3/2), epitritic (4/3) and epogdoic (9/8) intervals; and he filled up all the epitritics with the epogdoic kind of interval, leaving a part of each of them, where the interval of the remaining part had as its boundaries, number to number, 256 to 243 (*leimma*).

I have suggested elsewhere[2] that this passage gave rise to a series of technical problems (ζητήματα), which were systematically dealt with by Middle Platonists in order to better define the text from a technical perspective. However, all these problems somehow converged towards a fundamental issue, that of detecting the exact values of the harmonic structure of Plato's soul. This concern and its dependence on the way in which Plato wrote his text are clearly highlighted by Plutarch in the second part of the *De animae procreatione in Timaeo* (16.1019f8–1020b10, transl. Cherniss, slightly modified)[3]:

> This is the way the means are found; but one must insert them in that designated position and fill up the double and the triple intervals. Of the numbers set out, however, some do not have any room at all between them, and others do not have enough; so by increasing them with the same ratios preserved people produce sufficient accommodations for the aforesaid means. First, for one they substituted as the smallest number six, since it is the first that has both a half and a third; and all those ranged underneath, as drawn below, they made six times as large with room to admit both the means to the double intervals and triple too. Plato said, however, From these links within the previous intervals there arose hemiolic (3/2), epitritic (4/3) and epogdoic (9/8) intervals; and he filled up all the epitritics with the epogdoic kind of interval, leaving a part of each of them, where the interval of the remaining part had as its boundaries, number to number, 256 to 243'; and because of this passage they were compelled again to raise the numbers and make them larger.

Plutarch has some good reasons to emphasise the need for focused exegesis, for Plato's text is quite ambiguous especially with respect to the issue of the numbers involved in the *divisio*. Such obscurity, moreover, applies to a number of different levels. While, on the one hand, it is clear that Plato takes the series 1, 2, 3, 4, 8, 9, 27 (the so-called Platonic *tetraktys*) as the basis for the production of the harmonic structure, on the other such a series is not immediately suitable for achieving the next steps of the *divisio*. First, while Plato just states that arithmetic and harmonic means must be applied, there is no room between the numbers of the series to insert these mean terms in integers. This issue,

2 See Petrucci (2012a), 48–56.

3 See also Calc. *In Tim.* XXXIV 83, 20–27 and Procl. *In Tim.* II 175, 22–32.

however, was easily – and commonly[4] – solved by multiplying the unit and all subsequent numbers of the *tetraktys* by six, as indicted by Plutarch in the text quoted above, and as shown in Figure 1:

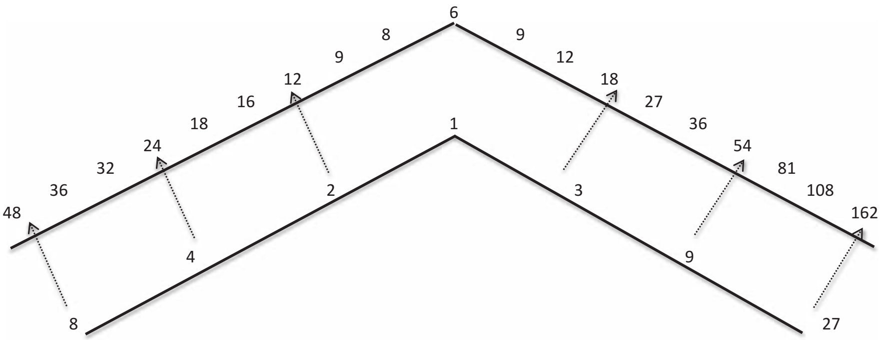

Figure 1: The Platonic *tetraktys* and the mean terms in integers.

There is, however, a further and more puzzling step to be clarified, for once mean terms are inserted between the numbers of the *tetraktys*, Plato inserts other numbers in order to fill the spaces produced, obtaining intervals of 9/8, that is the tone, the remainder being for each fourth 256/243, that is the *leimma*. After this last operation the whole harmonic structure of the soul emerges, and here our narrative starts, since the way in which the final series is to be represented is the core question in the Middle Platonist musical exegesis of Plato's *Timaeus*.

Plutarch's Interpretation as an Argument for Literalism

The narrative starts with Plutarch, who sketches out two methods in order to represent the whole series (*De an. procr*. 16, 1020C3–D9, transl. Cherniss, with modifications):

> **a**) Eudorus, then, following Crantor, took as the first of the numbers 384, which is the product of six multiplied by 64; and they were attracted by the number 64 because it has 72 as epogdoic. **b**) It is more in accord with Plato's words, however, to assume the half of this number, for the *leimma* that is left after the epogdoics are taken will have its ratio expressed in the numbers that Plato has given, 256 to 243, if 192 is made the first number. If the double

4 See e. g. Proc. *In Tim*. II 177, 25–179, 8.

> of this be made the first number, the leimma will be the same in ratio, to be sure, but double in number, being as 512 is to 486, for four thirds of 192 come to 256 but of 384 to 512.

The first model (**a**) is ascribed to Eudorus and, before him, to Crantor, and consists in identifying the unit of Plato's *divisio* with the number 384. If the unit is then multiplied by 384, the first fourth of the system[5] will be reproduced by the numbers 384, 432 (9/8 of 384), 486 (9/8 of 432), and 512, which is in the ratio of a *leimma* (256/243) with 486 and forms a fourth (in the ratio 4/3) with the first number, 384. Plutarch, however, explicitly prefers another model (**b**), the fundamental feature of which is that it preserves for the *leimma* the Platonic value of 256/243. The models can be observed and compared in the following Figure 2[6]:

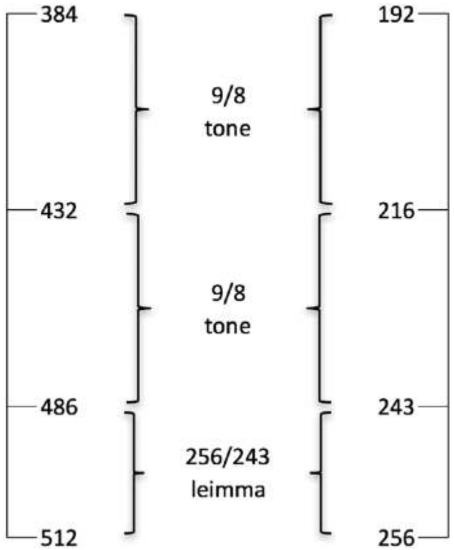

Figure 2: Plutarch's models: first fourth.

5 The fourth is one of the most important concords of Greek harmonics, and coincides with a tetrachord; it corresponds to the ratio 4/3 and, in the case of a Doric mode, it is a sequence of tone-tone-*leimma*. For an overview of Greek harmonic theory see Barker 1989, 1–27; West 1992, 160–6; Barker 2007, 12–18; Hagel 2010, 1–9.

6 It might seem as though *De an. procr.* 1022C5–1028A4 contradicts the fact that Plutarch has his own position on the issue, since here he dismisses the whole operation of filling the scale with intervals by referring to the methods developed by others, namely Crantor, Clearchus and Theodorus. However, it is likely that Plutarch is only resorting to this rhetorical strategy in order to move on to the following step in his exegesis, concerning the disposition of the numbers, since his polemic against Crantor and Eudorus' model, which we are discussing here, is quite sharp and

It is noteworthy that the preservation of Plato's absolute value for the *leimma* is the sole requirement which Plutarch feels to be pressing and discriminating. In fact, in what follows he shows to be even prepared to admit another model (*De an. procr*. 1022A7–B9), identifying the tones as 243/216 and 288/256, *provided that* the *leimma* remains 256/243. In this way Plutarch admits either that two tones are detected before the *leimma*, or that one tone is put before the *leimma* and another after it. This is telling, since it would imply a huge technical shortcoming, for it would not preserve the standard structure of the diatonic fourth (encompassing two tones and, *after them*, the *leimma*). Accordingly, the concern for technical precision would appear to be much less important in Plutarch's eyes than the need to strictly adhere to Plato's λεγόμενα. Of course, one could still absolve Plutarch of the paradoxical endorsement of the series 216–288 by taking it as a sort of provocative point. However, also the preference for the series 192, 216, 243, 256, leads to substantial technical problems, as is clearly indicated by Proclus (*In Tim*. II 177, 16–20):

> If it were possible to detect two epogdoics taking 288 as a starting point, we could have filled up also this epitritic interval with the epogdoics and the *leimma*; this is not the case, however: for the epogdoic of 288, that is 324, is not divisible by eight.

Proclus' point is that, in order for 192 to be a correct coefficient for the unit, it is necessary to detect *three* tones after 256, that is three consecutive intervals of 9/8. This, however, is impossible: 256 is in the ratio of 9/8 with 288, 288 is in the same ratio with 324, but 324 has no ἐπόγδοος, that is no integer with which it can form a tone. This is revealing with respect to the model Proclus has in mind, one coinciding with that first presented by Plutarch (**a**) and first developed by Crantor, which is based on the application of the coefficient 384 and allows the detection of three consecutive tones after the first *leimma*. Indeed, if the first *leimma* must be followed by three consecutive tones, the system consists of a series of single disjoint octaves, that is a sequence made up of a fourth, a tone, and a fourth, where in each fourth the *leimma* is at the bottom. In turn, this implies that Proclus assumes that it is necessary to represent in integers the whole Platonic series, which is much more extended than a fourth; accordingly, the coefficient applied to the unit *must* ensure that it is possible to find integers throughout the whole series, and not only for the first fourth. But, as it emerges from the table below (Figure 3), this is impossible according to Plutarch's model, which effectively represents only the first fourth of Plato's system:

focused. Moreover, in this way Plutarch represents his opponents as supporting a single strategy, which would be consistent with the 'historiographical' strategy which Plutarch often adopts (e. g., at *De an. procr*. 1013E, on the supporters of a sempiternalistic cosmogony).

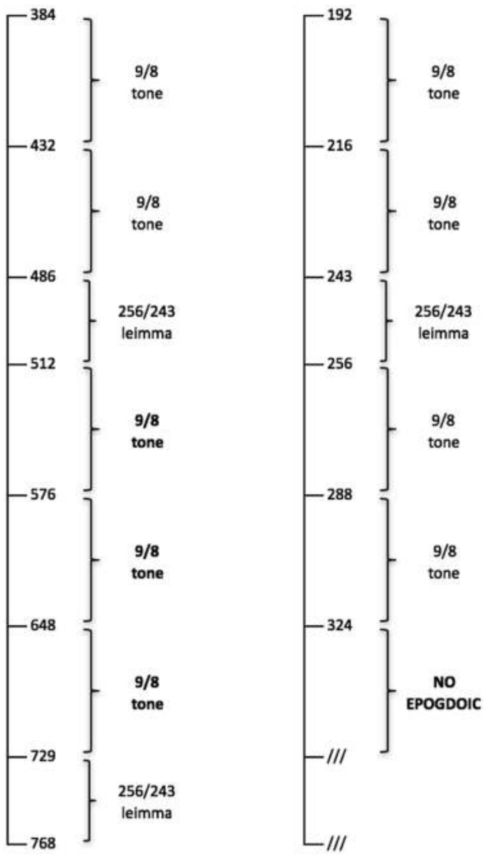

Figure 3: Plutarch's models: development.

Now, considering that Eudorus, Plutarch's source, was inclined to adopt Crantor's model, Plutarch must have been aware of the shortcomings implied by the adoption of the coefficient 192 for the unit. If this is the case, however, why does Plutarch *in any case* opt for a model which presents such problems? On the one hand, it is reasonable to assume that Plutarch would have downplayed his (potential) opponents' objection by claiming that it would be incorrect to say that his model makes it *impossible* to reconstruct the whole series; rather, it makes it impossible to reconstruct it in integers. Indeed, if one disregards the need to map the whole system and accept that the concords after the first fourth are expressed in ratios and not in integers, then Plutarch's model too *can* work, although it is scarcely explanatory. Still, this confirms and further illustrates that Plutarch must have had some important reason to opt for a puzzling model and (possibly) to produce some reasonable reply to quite blatant objections. But, as we have seen, Plutarch is quite explicit on this point, for he states that the values identifying the *leimma* with 256/243 are preferred *simply because they better comply with what Plato says*: the basis for Plutarch's exegesis of Plato's *divisio animae* is not compliance to technical standards, but strict adherence to Plato's account. Strange as it might seem, this line of reasoning is also referred to by Theon of Smyrna (*Expositio* 68, 12–69, 9), confirming that it was used to support the coefficient 192 (transl. Barker 1989):

> Some people, however, take 384 as the first term. For in order to take two epogdoics, they multiply the first term, 6, by 8, making 48, and take this again 69 eight times, making 384, whose epitritic is 512, and between them two epogdoics, that of 384, which is 432, and that of 432, which is 486; and from this to 512 is the ratio of a *leimma*. But other people say that these numbers are not correctly taken. For the excess of the fourth term over the third is not 13, which Plato said the leimma must contain. Yet nothing prevents us from finding in different numbers too the same ratio as 256 has to 243, for Plato did not take a determinate number, but a determinate ratio of number.

By emphasising that by 256/243 Plato is indicating a ratio rather than absolute numbers, Theon admits that the supporters of a model maintaining these values insisted on the fact that Plato used *these very numbers*.

All in all, then, Plutarch's approach is characterised by a clear and consistent exegetical standard, namely a literalist approach to Plato's text, which is a much more pressing matter for him than the search for technical precision. Now, Plutarch's positions and arguments cannot merely be regarded as pedestrian treatments of harmonics.[7] If Plutarch's technical exegesis is framed within the wider picture of his Platonism, it acquires a new and noteworthy import: for it is likely that Plutarch did not merely produce a poor piece of technical exegesis, but rather regarded this exegetical passage as part of his wider polemic against the sempiternalistic interpretation of Plato's psychogony. Indeed, Plutarch emphasises that the model applying a higher coefficient was developed for the very first time in the Old Academy, and was then adopted by Eudorus and the supporters of the sempiternalistic interpretation, whom Plutarch constantly regards as his opponents when it comes to the interpretation of Plato's psychogony and cosmogony. Interestingly enough, the argument which Plutarch employs against a sempiternalistic cosmogony and psychogony is based on a principle of literalist exegesis, leading him to ask why one should depart from what Plato has explicitly declared to his readers (*De an. procr.* 9, 1016C3–4: Τίς

7 This is what would emerge from a superficial and technically oriented reading, and it is not by chance that neither Barker (1989) nor Creese (2010) take Plutarch into account. An excellent commentary on the second part of Plutarch's *De animae procreatione* is to be found in Ferrari (2002).

οὖν τούτων ἐπανόρθωσις ἑτέρα πλὴν ἧς αὐτὸς δίδωσι τοῖς δέχεσθαι βουλομένοις;).[8] It can hardly be a coincidence that this principle is very similar to that which grounds Plutarch's criticism of Crantor's musical exegesis: in the case of both the interpretation of Plato's psychogony and cosmogony and the correct understanding of the *divisio animae*, Plutarch insists on the fact that supporters of the sempiternalistic interpretation in principle apply a mistaken approach to Plato's text, namely one wrongly going beyond Plato's wording. Therefore, Plutarch is not so much directly committing himself to a specific technical point, as considering Plato's *divisio* methodologically crucial: had he accepted his opponents' view, he would probably have left room for the idea that Plato, in his most clear and explicit discussion of the generation of the soul – that is, the *Timaeus* – wanted the reader to supplement his wording. But this is precisely what Plutarch cannot accept, for this would make his 'methodological' criticism of the supporters of a sempiternalistic cosmogony entirely inconsistent. If this reading is correct, what we discover in Plutarch's technical exegesis is a definitely philosophical point, which confirms how radical and accurate Plutarch's reaction against previous exegeses was and provides further support for his overall philosophical strategy.

At the same time, however, a further and more general conclusion can be drawn. Indeed, it seems quite clear that Plutarch's main concern is not to establish the idea that Plato was a technical authority in strict terms: he is even prepared to ascribe to him vague – albeit correct – technical statements (that is, a scarcely explanatory account of the musical system of the soul) *provided that* the philosophical correctness of the overall content of the *Timaeus* is preserved. This suggests that Plutarch rather wishes to depict Plato as the one who established harmonics as a cosmological discipline based on certain epistemological assumptions, regardless of the fact that Plato's statements directly coincide with standard technical views. Interestingly, this is perfectly consistent with Plutarch's account of harmonics in the third *Platonic Question* (esp. 1001e–1002a), where he defines this discipline as dealing with the 'sounds' produced by heavenly motions. The actual import of Plutarch's description is not so clear, but one can fairly exclude that he is referring here to sounds in strict terms (that is, that he is just reproducing the Pythagorean account which Aristotle refers to in *De Caelo* II 9, 290b12–29). Indeed, this would completely overturn the definition of harmonics as a purely theoretical discipline that Plato provides in *Republic* VII, which is after all Plutarch's main point of reference; moreover, it would be inconsistent with many other Plutarchean passages, for instance his criticism of the idea that

8 On Plutarch's literalism, see Opsomer (2004) and Petrucci (2018b), 57-58.

understanding cosmic harmony consists in associating planets with notes (*De an. procr.* 1029A1–C4). So, Plutarch's idea must be that harmonics detect quantitative patterns and values in the heavenly motions (in turn corresponding to concords, of course), which makes harmonics look like a purely mathematical discipline, *at least* in its strictest form. If this is the case, however, we can also understand in what sense Plutarch regarded Plato as having provided a certain foundation for harmonics: Plato must be the founder of this very theoretical kind of harmonics in the sense that he established it as a discipline having a proper epistemological status and object. If this reading is correct, we are in a position to understand why Plutarch does not care about reworking the numerical series of Plato's soul in order to obtain a technically satisfying system: Plato was not interested in these aspects of harmonics, which rather pertain to practice, but was keen to establish in principle the nature and foundation of a harmonic structure. In other words, Plato is an authority in the field of harmonics *because* he did not enter a technical debate, but furnished the discipline with its epistemological bases.

Philosophical Authority vs Technical Authority: A Middle Platonist Debate

All in all, Plutarch illustrates a first important episode in the narrative of the Middle Platonist exegesis of Plato's *divisio*, one establishing with specific methods and for specific reasons the priority of the coefficient 192. His attempt to safeguard Plato's values against interpretations which programmatically altered them, however, was destined to fail, for the model applying the coefficient 384, formulated by Crantor, survived throughout the Platonist tradition and was taken up, e. g., by Pseudo-Timaeus,[9] Adrastus,[10] and then the Neoplatonists, especially Proclus. Its huge diffusion indicates that the need to increase Plato's

9 The details of Ps.-Timaeus' *divisio* are puzzling, above all because it is likely that its arithmetic development (chapt. 22–23) derives from an interpolation (see Baltes 1972, 79–85). Nonetheless, in the textual section which can for sure be taken to have been originally included in the treatise (chapt. 21), Timaeus accepts 384 as coefficient. The only substantial divergence with respect to Crantor's *divisio* consists in the fact that Timaeus detects thirty-six values, two more than Crantor, consisting in two *apotomai* (an *apotome* being the interval completing a tone with a *leimma*): see again Baltes (1972, 77–80).

10 See Procl. *In Tim.* II 187, 15–24, and Calcidius' (*In Tim.* XLIX 98, 3–99, 9) argument against the application of the coefficient 192 – on Calcidus' dependence on Adrastus, in comparison with Theon's *Expositio*, see Petrucci (2012b).

values was widely acknowledged, allowing an exegete to represent Plato's scale in integers. This point established, it should be noted that under this veneer of uniformity a silent quarrel took place, one based on intriguing methodological polemics.

As we have already seen, the traditional representation of Crantor's model consists in applying the coefficient 384 to Plato's unit and all other values which the first steps of the *divisio* detected, and then in filling the intervals obtained with tones and *leimmata*. We obtain a series of *four independent octaves*, which are typically diatonic, since the *leimma* is at the bottom of each tetrachord; to them a fifth and a tone are added. Although the operation of filling up the intervals obtained leaves room for some ambiguity, the most reasonable arrangement of the final part of the system according to this model would be constituted by an anomalous fifth, since the three tones which it encompasses are not consecutive.[11] Be that as it may, two principles were applied for sure: first, the application of 384 as a coefficient for Plato's unit; second, the association of higher values and lower notes.[12] Indeed, in a diatonic Doric system the *leimma* must always be at the bottom of each tetrachord, and the first fourth of the system is always indicated as encompassing the lowest numbers (384, 432, 486, 512). All in all, by associating numbers and notes, Crantor's system can be represented as indicated in Figure 4 below.

Now, from a technical point of view, what is striking in this model is that, inasmuch as it encompasses a series of single octaves, it makes it impossible to regard Plato's system as anticipating the standard harmonic structure which was used from the Hellenistic age onwards, that is the well known Greater Perfect System, encompassing two octaves formed by two fourths, a tone of disjunction, two fourths, and a tone at the bottom. In other words, although Crantor's model is consistent from an arithmetical point of view, and although it correctly applies general laws of harmonics, it implies taking Plato to be in contrast with the whole Hellenistic and post-Hellenistic technical tradition, which essentially relied on Aristoxenus. For obvious reasons this was not a problem for Crantor, but Middle Platonists had at least to take into account this implication, and possibly to undermine the potential polemics which it gave rise to.

11 According to Proclus' testimony (*In Tim.* II 186, 2–187, 9), this anomaly can be amended through the substitution of two numbers (4374 for 4096 and 8748 for 8192), which would produce a regular fifth after the first three octaves, and a supplementary tone at the bottom of the system. The idea proposed by Brisson (1974, 323), that with these modifications Proclus is just seeking to produce as many consecutive tones as possible is neither consistent with the whole system nor harmonically grounded.

12 See Barker (2007, 322), with reference to Plato's system in itself.

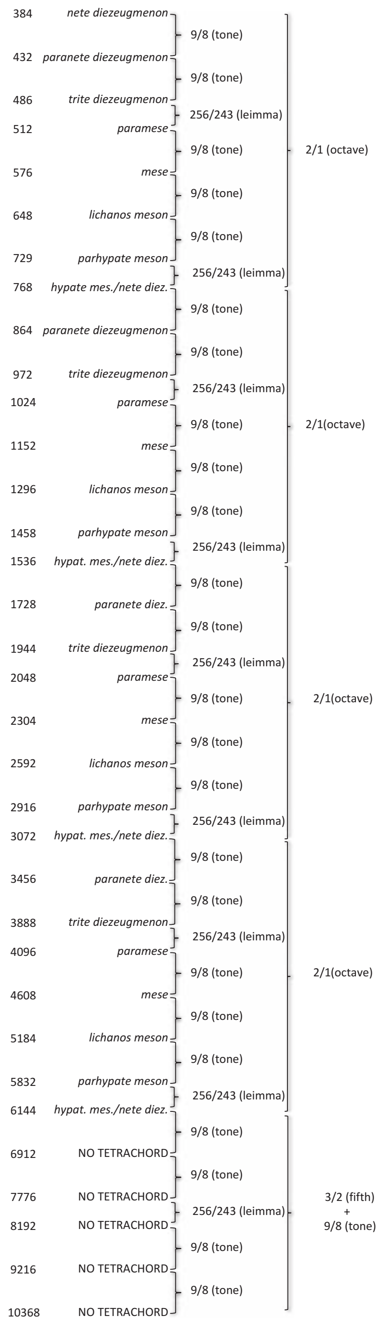

Figure 4: Crantor's model.

This very concern must have served as an inspiration for another reading of Plato's model, briefly mentioned by Theon of Smyrna at the end of his quotation of Thrasyllus' *sectio canonis* (*Exp*. 93, 2–4):

> We will be able to discover the very same things in numerical terms also by starting from the *nete hyperbolaion*, namely assuming it to be associated with 10368.[13]

At face value, Theon is just hinting at Crantor's model, given that 10,368 is its highest number. This is not the case for two reasons, however. The *nete hyperbolaion* is the highest note of the system at issue and is here associated with the highest value, while in Crantor's model, as we have seen, higher values are associated with lower notes. So, while maintaining the standard numerical values for the scale based on the application of 384 as coefficient, Theon reads this scale in the opposite way by associating higher values with higher notes. Second, in order to ensure a consistent diatonic form, the system arranged in this way cannot consist in a series of four separated octaves; rather, it can *only* be consistently read as encompassing two Greater Perfect Systems, each made up of two octaves, plus a fifth and a tone.[14] This is not only required by a consistent association of values and notes, given the relation between higher notes and higher numbers, but also clearly shown by the mention of the *nete hyperbolaion*, a note which can be taken into account only if the Greater Perfect System is at stake. The following Figure (5, next page) offers a comparison between Crantor's model (on the right) and Theon's (on the left).

Now, it is quite clear that Theon's version heavily depends on Crantor's values and model, of which it is a re-thinking: Crantor's model was indeed very well-known and it effectively detected suitable numeric values.[15] If this is the case, however, why did Theon (along with Thrasyllus, his source) wish to alter this well-established reading, and why did he do so exactly the way he did? The answer to this question cannot lie in the mere association of higher values and higher notes, for this aspect is not telling in itself; rather, it is to be sought in its most substantial consequence, namely the fact that in the modified version Plato's soul is no longer made up of single octaves, but encompasses two

13 On this passage see Petrucci (2012a), *ad loc*.

14 The fact that Theon – and Thrasyllus before him – had enough leeway to invert the association of values and pitches, namely by associating higher values and higher pitches, is explained by the traces of an exegetical debate on this issue, on which see Petrucci (2012a), 372-374. This is interestingly testified to by Theon himself in *Expositio* 65, 10–66, 18, a passage which however derives from Adrastus (see Petrucci 2012b) and – not by chance – agrees with Adrastus' reading of Crantor's system, associating higher values with lower notes.

15 It is not by chance that it is quite similar to the readings of Plato's system provided by Barker (2007) and Brisson (1974).

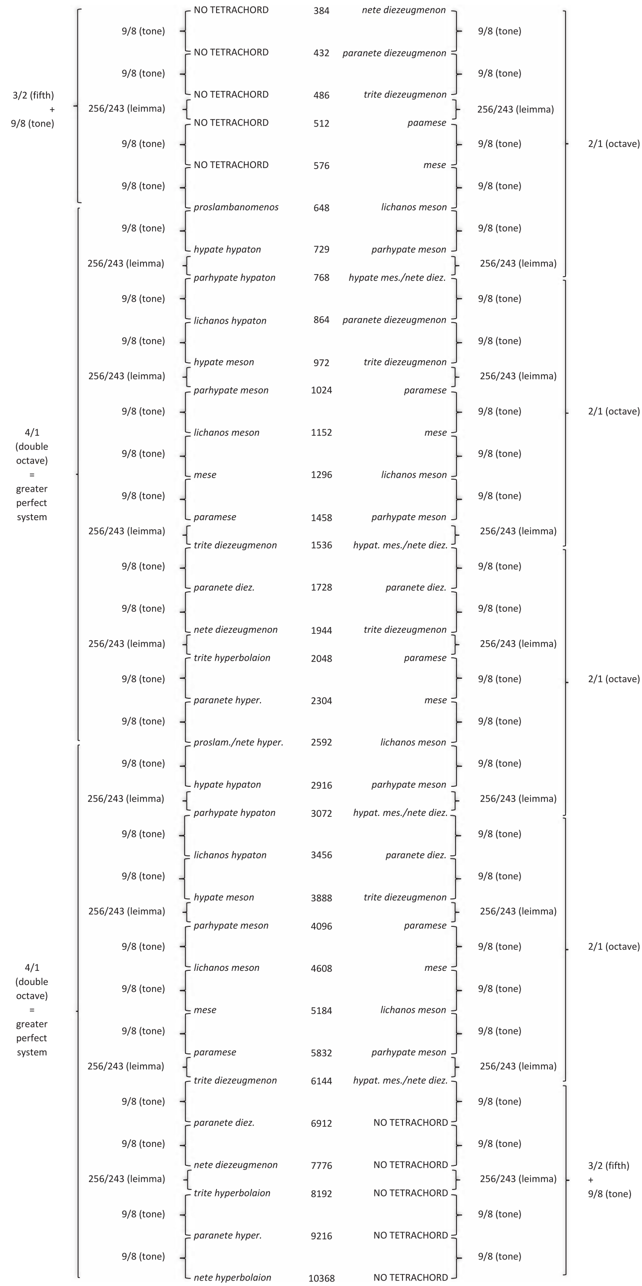

Figure 5: Theon’s model (on the left left) vs Crantor’s model (on the right).

Greater Perfect Systems.[16] Far from being just a pedantic point, this shift has a very strong impact on the exegetical tradition: the new arrangement suggests that Plato was the first to *discover* and *apply* the Greater Perfect System, so that Theon can claim for him a leading role in the history of harmonics in quite a technical sense. More specifically, even admitting that Pythagoras was the first discoverer of the values of concords,[17] it turns out that Plato substantially contributed to harmonics by arranging these concords into the complex system which was employed in the Hellenistic and post-Hellenistic ages.

This is confirmed by the whole process of division of the monochord which immediately precedes the reference to Crantor's system in Theon's *Expositio* – a division which Theon takes again from Thrasyllus (*Exp*. 87, 9–93, 7). This text is too long to be analysed in its entirety,[18] but a reference to some general features will be enough to make my point. Theon follows Plato (and other exegetes) in producing the division in two steps,[19] first detecting fourths and fifths (through the application of arithmetic and harmonic means to limited fundamental values), then filling these with tones and *leimmata*. The first step detects the following notes: *proslambanomenos, hyperhypate, hypate meson, mese, nete diezeugmenon, nete hyperbolaion*. To these notes, specific values from 3 to 12 are associated once the 'monochord' is divided into twelve parts. The selection of both the notes and the values is far from random. The selected notes are the highest of each tetrachord of the Greater Perfect System, plus the lowest note of the system and the *hyperhypate*, which is often considered a fixed note in ancient harmonic theory.[20] This can be regarded, in other words, as a way to represent the fundamental notes determining the structure of the Greater Perfect System. But what about the values? Let us go back to Plato's system, starting from the *tetraktys*. As we have seen, the standard ancient approach allowing one to represent it in integers once the *tetraktys* is filled up with the arithmetic and harmonic means is to multiply each of these values by 6[21]; we will then have the series 6, 8, 9, 12, 16, 18, 24, etc. Now, if we consider only the values forming the

16 If Proclus' explanation of Crantor's system were correct (see footnote 11 above), another positive consequence could be detected, that is the collocation of the supplementary fifth plus tone at the bottom of the system, while in the rival model it brakes up the continuity of the octaves.

17 This was quite a widespread belief, also attested in Theon's *Expositio* (56, 9–57, 10): see Meriani (1995), and Petrucci (2012a), *ad loc*.

18 For a focused analysis see Petrucci (2012a), *ad loc*.

19 A multi-step division is suggested, e. g., in Calc. *In Tim*. XXXIV 83, 20–27, and Procl. *In Tim*. II 175, 22–32.

20 See Eucl., *Sect. can*. 164, 18–165, 3, and Barker (1989, 206 n.65).

21 See Figure 1 above.

first double octave, i. e. up to 24 starting from 6, it is clear that they coincide with the double of those which are employed by Theon (Figure 6):

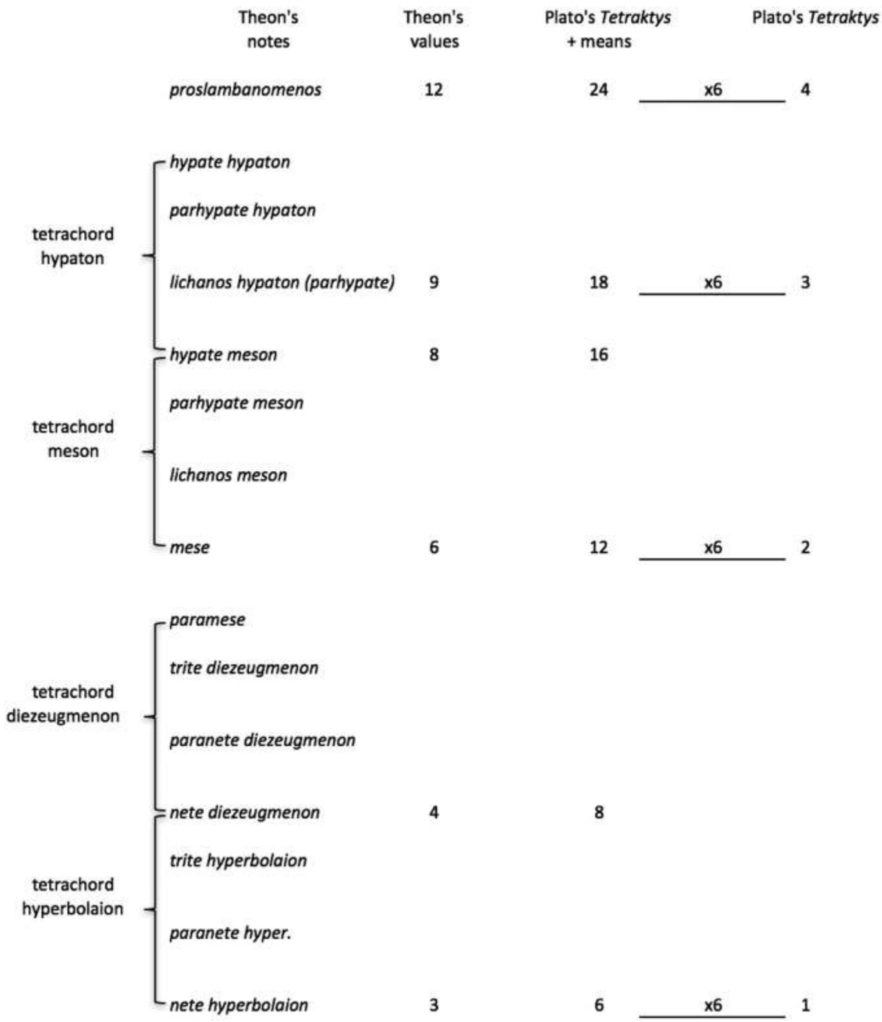

Figure 6: Theon's division of the monochord.

Accordingly, Theon's 12–3 arrangement reproduces Plato's values for the fundamental notes of the first double octave – i. e., for the first Greater Perfect System – after their halving. Also, the only aspect of divergence between the models, namely the absence of any corresponding number in Plato's series for Theon's 9, is not casual, and does not simply depends on the fact that 9 cannot be halved into whole numbers. The point is rather that 9 would correspond, in Plato's scale, to the *paranete diezeugmenon*, which is a fundamental note only in the Lesser Perfect System, which is not taken into account by Theon at this stage. Therefore, the only way to explain Theon's representation of the *divisio* is to regard him as aiming to introduce a strict consistency between Plato's *divisio*, its values and methods, and the most usual harmonic notions. However, this has an important consequence in turn, which goes far beyond the mere technical reading of Plato's *divisio*, for in the light of Theon's exegesis Plato can be regarded again as already employing the Greater Perfect System. If read in this way, then, the same ideological perspective may be seen to lie at the basis of both Theon's re-thinking of Crantor's model and of his way of representing the first two octaves of Plato's 'soul': the projection of the Greater Perfect System onto Plato's *divisio* is the strategic core of the attempt to make of Plato a technical authority in the sense of ascribing to him the shaping of a specific harmonic pattern.

Intriguing as this approach might be, it ultimately represented a failure inasmuch as no-one else apparently adopted it. We have not only the *e silentio* confirmation consisting in the substantial indifference of Proclus, who never refers to either of Thrasyllus/Theon's models in his extensive commentary on Plato's *divisio*, but also specific criticism from Nicomachus. Indeed, in a puzzling passage of the *Encheiridion* (260, 12–17),[22] Nicomachus explicitly criticises Thrasyllus' exegesis[23] and cites that of Timaeus Lokrus as a better model. This reference to the Pseudo-Pythagorean text – which essentially reproduces Crantor's system, with slight modifications[24] – is the only element which might allow us to grasp the meaning of Nicomachus' criticism, since the following scales are probably corrupted. First, Nicomachus' criticism must be directed against Thrasyllus' full scale (i. e., the one associating the value 10,368 with the *nete hyperbolaion*), which Theon just hints at, for it must have displayed the system in its entirety. Moreover, this scale provides Nicomachus with a very effective argumentative tool, for a major peculiarity of Thrasyllus/Theon's model consists in the criterion of association of numbers and notes. Indeed, as we have seen, the strategic device allowing Thrasyllus/Theon to re-think Crantor's model is the association of higher values and higher notes. Passages of the *Encheiridion*

22 For a different interpretation of Nicomachus' criticism, see Creese (2010, 264–81).

23 Along with that of Eratosthenes, on which see Ptol. *Harm*. II 13–14, with Barker (2000, 129–31) and Creese (2010, 178–209).

24 See footnote 9 above.

(e. g. 254, 19–22)[25] show that Nicomachus did not agree with this assumption, preferring to keep closer to the traditional association of higher values and lower notes. If, however, one rejects this aspect of Thrasyllus/Theon's *divisio* and associates higher values and lower notes – just as Timaeus, Adrastus, and Proclus did – the series deriving from the application of the coefficient 384 *unavoidably* produces a series of single octaves. What Nicomachus does is invoke the unusual association of higher values and higher notes, which is necessary to Theon in order to achieve his goal, so as to undermine the overall construction deriving from this assumption, namely the attempt to represent Plato's system in terms of the Greater Perfect System (an attempt which is also reflected in the smaller 3–12 system). This is confirmed by another passage of the *Encheiridion* (250, 3–251, 13), where Nicomachus explicitly refers to a correct exegesis of Plato's *divisio*. Unfortunately this is even less than a hint: Nicomachus just says that in Plato's system, after the application of the means, the number 9 corresponds to the *paramese*. Even more regrettably, all possible ways of considering this association as part of a full arithmetic representation of Plato's *divisio* (i. e. from 1 to 27 considering Plato's *tetraktys*) would be inconsistent. The only way to make sense of this association is rather to consider the series 6, 8, 9, 12 as a prototypic octave, a sort of model for each part of Plato's system – the number 9 corresponding to the arithmetic mean between 6 and 12, that is between 1 and 2 after their usual multiplication by 6. If read this way, the model is consistent: 6, 8, 9, 12 would be an octave-model made of Platonic basic values and applied to the central octave, where 12 is the *nete diezeugmenon*, 9 the *paramese*, 8 the *mese* and 6 the *hypate meson*. If this is the case, it is also clear that no lower octave can be detected after the *hypate meson* (that is, in integers lower than 6), which will then be regarded by Nicomachus as the lowest note of Plato's 'prototypic scale'. This leads us to state that Nicomachus sees Plato's model as consisting of a series of single octaves and not of Greater Perfect Systems, which in turn confirms that Nicomachus' criticism is meant, in general, to undermine Theon's representation of Plato's *divisio* and its ideological background.[26]

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25 See also *Exc. Nicom.* 267, 1–271, 15, and Creese (2010, 274).

26 Nicomachus' reading implies here the association of higher values and higher notes (see Barker 1989, 258 n.53), which would seem to contradict Nicomachus' preferences. However, my interpretation could also solve this difficulty, for it can explain such preference as being directed *ad hominem* against a Theon-like position and indicates, once again, that Nicomachus' concern is not the mere issue of the values to be associate with notes, but the overall representation of Plato's 'soul' as a series of octaves and not of Greater Perfect Systems.

Now, if we assume that Theon's overall goal is to affirm Plato's authority in the field of harmonics, my conclusion could at first sight lead us to ascribe to Nicomachus the very idea of Plato's musical authority. This cannot be the case, however, for after all Nicomachus himself is committed to the idea that Plato did play a key role in the development of a system of mathematics, including for sure arithmetic and harmonics. This clearly emerges, for instance, from the first pages of Nicomachus' *Introductio*, where the definitions of arithmetic, geometry, astronomy, and harmonics are proposed as depending on the Platonic ontology of the *Timaeus* (*Intr. Arithm*. I 2, 1, 9-I 2, 5, 15).[27] So, conversely, Nicomachus *did* agree that Plato played a crucial role in the establishment of harmonics, but must have regarded Theon's strategy to affirm Plato's technical authority as misleading. In this sense, the only reason why one can strongly deny that in the *Timaeus* the Greater Perfect System is detected is to abstract Plato from overly-technical debates and elevate his role 'above' all specific applications of harmonic theory: after all, just as it is not necessary to state that Plato applied specific arithmetic tools in order to affirm his understanding of the deep nature of numbers,[28] it is not necessary – or, rather, it is somewhat dangerous – to bind too strictly Plato's conception of harmonics to overly-specific technical applications. In other words, in Nicomachus' view Plato's authority can better be preserved *only* if the Master is credited with providing the comprehensive epistemological background for harmonic science and its foundations, and in this respect one should avoid ascribing to him any stricter commitment to specific technical aspects – a commitment which, on the contrary, Theon's reading would establish for the very same reasons, namely in order to credit Plato with a certain authority in the field of harmonics.

Severus, or Literalism and Authority Again

This narrative, however, would not be complete without referring to its last step, embodied by Severus. According to Proclus, Severus peculiarly adopted the coefficient 768 for the number one of Plato's series (18T Gioè): this just amounts to doubling Crantor's coefficient. Proclus also indicates the alleged reason why Severus opted for this number, namely that he wanted the system to end with a *leimma* (16T Gioè):

27 See Petrucci 2018a.
28 See Helmig (2007) on Nicomachus' mathematical ontology.

> Or rather as Severus did, for he as well produced the system without excluding the tone, but made it end with a *leimma* and not with a tone.

Consistently with the traditional approach to our texts, most scholars have just accepted Proclus' report and have not searched for any ulterior reason: it has simply been assumed that Severus chose to close the system with a *leimma*, and in order to do so was compelled to double Crantor's values.[29] But can we really assume that Severus had such a peculiar – and, all in all, philosophically pointless – commitment? Now, from a technical perspective it is correct to state that Severus' doubling of Crantor's values and his desire to make the system end with a *leimma* are strictly connected: in order for the highest number of the system to be the numerator of the final *leimma*, it must be divisible by 256; 10,368 divided by 256 makes 40.5; so, Severus' highest value will be its double, 20,736, which will form a *leimma* with 19,683. This point granted, however, the precise reconstruction of Severus' model is a different matter. We have, of course, Proclus' testimony (16T Gioè), listing all numbers encompassed by Severus' model (Figure 7, next page). In fact, Proclus' understanding of Severus' model displays a strong technical shortcoming, which Proclus himself highlights: it encompasses a series of four consecutive fifths, corresponding to the ratio 3/2, which breaks up the sequence of octaves characterising (and giving consistency to) Crantor's model, of which Severus one would only be a pedestrian misrepresentation. The absurd consequence of this would be that Severus, in order to satisfy the (quite pointless) requirement of making the system end with a *leimma*, ultimately undermined the technical consistency of Crantor's model. So, the traditional understanding of Severus' model is not only superficial, but also leads to a very poor representation.

However, it is entirely possible that Proclus' account of Severus' model is misleading. This does not depend only on the application of a principle of charity, but also on the fact that several studies have indicated that Proclus did not have direct access to Middle Platonist texts, and that his reports are sometimes inaccurate and autoschediastic.[30] By taking this as my working hypothesis, I shall attempt to produce a different reading of Severus' model, which is both possible and desirable. Let us focus on the puzzling part of the

29 See e. g. Gioè (2002, 377–433), for an annotated translation of Severus' remarks.

30 See e. g. Tarrant (2004) on Proclus' access to Numenius' writings, and Petrucci (2014) for a comprehensive account. A confirmation of this is that according to Proclus Severus' model just corresponds to the multiplication of the values of Crantor's system according to Proclus' representation of it: but, as we have seen above (footnote 11), this might be an alternative way to read Crantor's system, which probably diverged from the original one.

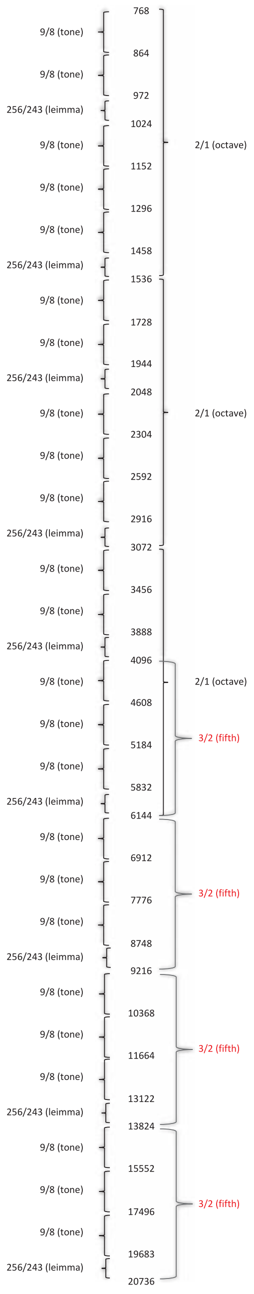

Figure 7: Severus' model according to Proclus.

system, that is the one encompassing the consecutive fifths. The fundamental point is that Severus is bound in this part only to four numbers, that is those corresponding to 9 and 27 of Plato's *tetraktys*, and their arithmetic and harmonic means: the others are detected just by filling the intervals produced with tones and *leimmata*, and the way in which this process is realised depends on the exegete's reasoning. Now, three of these added numbers lead to the production of the consecutive fifths of Proclus' testimony, namely 8748 and 13,122. However, the intervals which these numbers are meant to fill can also be divided by other values. Let us consider, for instance, the interval between 7776 and 9216. The interval formed by these numbers is of a tone and a *leimma*, which can be filled by inserting 8748 – as Proclus suggests in his testimony – or by inserting 8192: the former produces a tone with 7776 and a *leimma* with 9216, the latter a *leimma* with 7776 and a tone with 9216. Similar alternatives are also available in the remaining puzzling case, as shown in Figure 8:

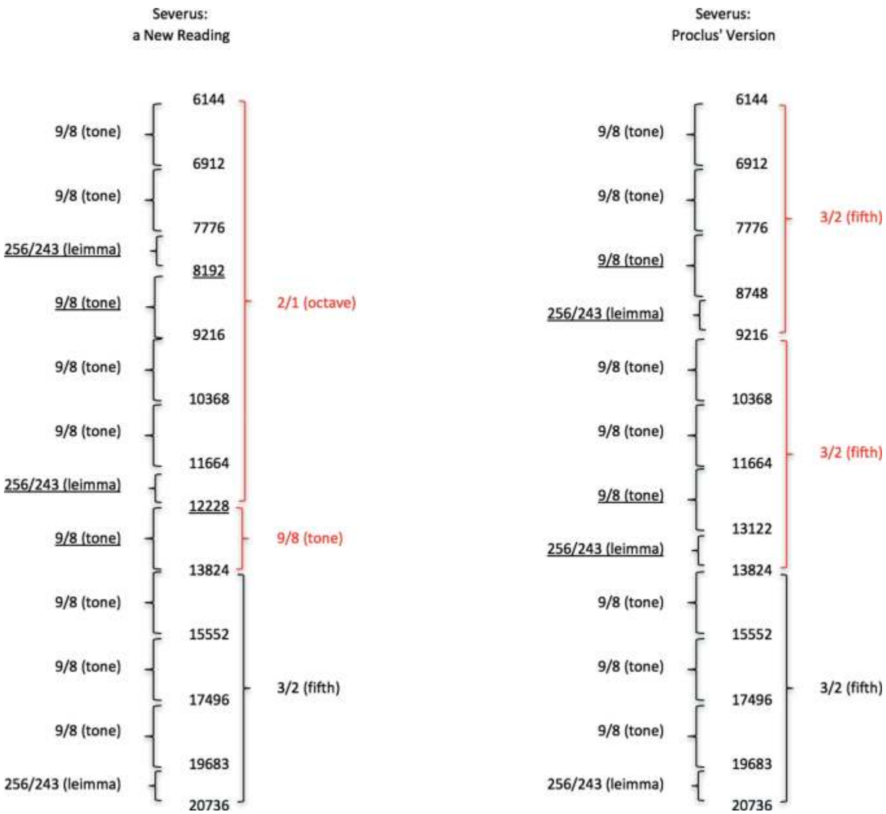

Figure 8: Severus' model: the new reading vs Proclus' version.

Now, if the alternative values with respect to those indicated by Proclus are chosen, the irregularity which Proclus detects ceases to exist: the puzzling part of Severus' system does not encompass three consecutive fifths, but an octave, a tone, and a fifth. Accordingly, the system in its entirety now consistently encompasses a sequence of four octaves, a tone and a fifth, and its overall representation is much more reasonable than is usually believed to be the case on the basis of Proclus' misleading testimony (Figure 9, next page).

On these bases, if we compare Severus' system to Crantor's, we notice only one difference (apart from the values of the chosen numbers), that is the structure of the part out of the octaves (compare Figure 4 with Figure 9): Crantor's system is closed by a fifth and a tone, Severus' by a tone and a fifth, with the result that the latter ends with a *leimma*. And indeed – as we have seen – this would appear to have been Severus' goal according to Proclus. If we limit the enquiry to this point, we obtain a more consistent representation of Severus' system, but at the same time its philosophical grounds are still very poor: after all, Severus seems to be moved by a somewhat fetishistic passion for the *leimma*. Deeper motivations can be discovered, however. As a matter of fact, Severus' system has the strong exegetical advantage of complying very well with Plato's description of the *divisio*, for Plato says that the *leimma* is just what *remains after* the insertion of tones (36b1–6, quoted at the beginning of the paper). So, Severus could have resorted to a strictly literalist interpretation of this passage in order to claim the need to end the system with a *leimma*, a literalist interpretation which was, moreover, much more effective than Plutarch's: while Plutarch's literalism, insisting on the absolute value of the *leimma*, risked proving pretty weak from a technical point of view – for it is obvious that, as Theon pointed out, what counts is the ratio and not the absolute numbers in which it is expressed – Severus' approach ensures a good technical representation which at the same time complies with the harmonic structure that Plato describes literally. This insistence on a specific literal aspect of Plato's text is typical of Middle Platonist exegeses,[31] which are often based on strictly literal interpretations of textual passages. So Severus opted for his slight modification because it better agreed with Plato's text, and in consequence was argumentatively stronger.

We have therefore discovered a better arrangement for Severus' model and its exegetical advantages. But there is also more to this, namely an important philosophical implication for the problem of Plato's authority. Given the position of the final *leimma*, Plato's system, as arranged by Severus, *cannot* be read as Thrasyllus and Theon did, i. e. by associating higher values with higher notes in

31 See Petrucci 2018b, 57-61.

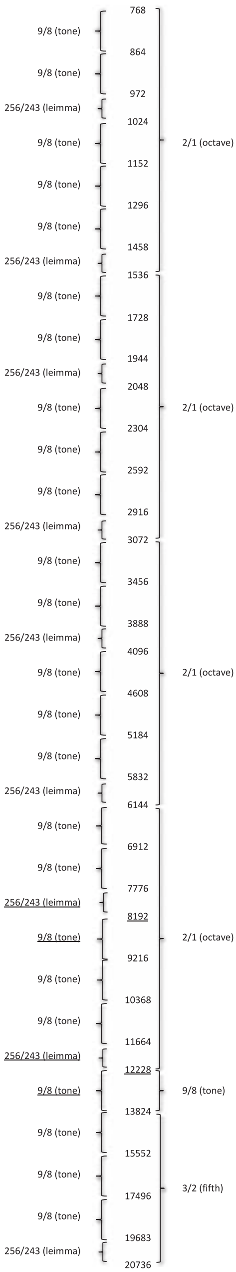

Figure 9: Severus' model according to the new reading.

order to represent Plato's system as a series of two Greater Perfect Systems. Indeed, if one accepts Severus' values and his collocation of the *leimma*, and given that no *leimma* can be produced by the two lowest numbers (i. e. 864/768), Severus' system cannot be read as encompassing Greater Perfect Systems. Therefore, by combining both points, Severus' exegesis emerges as an attempt to dismiss *all* alternative readings of Crantor's system by providing it with a strong exegetical basis, just as Plutarch wanted to do. But this, in turn, leads us to ascribe to Severus quite a definite conception of Plato's authority in the field of harmonics, one that identifies Plato as the founder of harmonics as a very theoretical discipline (as Plutarch and Nicomachus did) rather then as the excellent technician who discovered – and was interested in – the Greater Perfect System (as Thrasyllus and Theon wished to suggest).

Some Conclusions: Platonist Harmonics as a Debate on Methods and Authority

Severus' case is also telling from a more general point of view, for it reveals that the Middle Platonists' musical exegesis, even when it seems to amount to mere calculus, or poor technical speculation, has a much more intriguing core, which can prove important for our general understanding of these philosophers' methods and ideology. First of all, Middle Platonism revolves around an ongoing debate concerning the very bases of all Platonist exegesis: the issue of literalism, which plays a fundamental role in many relevant problems, is also central to this debate, and is often used by Platonists as the key parameter for the correctness of an interpretation. Unless one is aware of such an approach in technical exegesis, this field risks appearing somewhat anomalous, and in turn the general case for the importance of literalism in Middle Platonism exegesis is weakened. My analysis shows, on the contrary, that Middle Platonist technical exegesis rests on the very same ground as more widespread 'philosophical' exegesis: in this sense, methods can be regarded as a unique unifying factor of this strange philosophical 'family'. Second, all technical quarrels conceal a wider and much more important debate concerning the terms in which Plato's role as an authority should be envisaged. Middle Platonists committed themselves to demonstrating that Plato was a technical authority in the field of harmonics, and took his *divisio animae* to be the textual passage which one should focus on. The real question, however, was whether it suited Plato's status more to make him part of a strictly technical history, as the discoverer of notions that everyone could employ, or to place the Master *above* all technical applications, as the philosopher who established the very bases of harmonics in purely philosophical terms. *This* is the intense debate which apparently sterile series of numbers conceal, a debate the investigation of which can significantly contribute to our own understanding of Middle Platonist philosophy.

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