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## **Abstract**

*This paper shows that network effects do not have any implication to the nature of the equilibrium strategic managerial delegation under Cournot type quantity competition, unlike as in the case of Bertrand type price competition a la Hoernig (2012). It also shows that delegation of output/price decision to the manager is optimal for the owner even in the case of monopoly in the product market, due to the existence of network effects. The monopolist offers sales-oriented incentive scheme to her manager in equilibrium, if there is network effect.*

**Keywords:** Strategic delegation, Network effects, Quantity competition, Monopoly

**JEL Code:** D43, L20

# Managerial Delegation under Network Effects<sup>1</sup>

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## Abstract

This paper shows that network effects do not have any implication to the nature of the equilibrium strategic managerial delegation under Cournot type quantity competition, unlike as in the case of Bertrand type price competition *a la* Hoernig (2012). It also shows that delegation of output/price decision to the manager is optimal for the owner even in the case of monopoly in the product market, due to the existence of network effects. The monopolist offers sales-oriented incentive scheme to her manager in equilibrium, if there is network effect.

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## 1 Introduction

In a recent paper Hoernig (2012) demonstrates that it is optimal for the owners to induce the managers to be more aggressive under Bertrand type price competition in the product market and sufficiently strong network effects. This is in contrast to the well known result of Fershtman and Judd (1987) and Sklivas (1987) under price competition in absence of network effects. Based on this finding, Hoernig (2012) argues that the choice of the strategic managerial delegation contract depends on both (a) mode of product market competition - price or quantity, and (b) strength of network effects.

In this paper we show that network effects do not play any role in determining the nature of the equilibrium strategic managerial delegation contract, sales-oriented vis-a-vis

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profit-oriented, under Cournot type quantity competition in the product market. Unlike as under price competition, owners' incentive to induce their managers to be more aggressive in the product market is further reinforced in the presence of network effects and, thus, in equilibrium each owner offers a sales-oriented incentive scheme to manager under quantity competition as in Fershtman and Judd (1987) and Sklivas (1987), irrespective of the strength of network effects. In other words, due to network effects, the possibility of alteration of Fershtman and Judd (1987) and Sklivas (1987)'s results arises only in the case of price competition, but *not* in the case of quantity competition.

Interestingly, it is optimal for a monopolist as well to delegate the pricing/output decision to her manager, if there is network effect. Existence of strategic interaction with other firms in the product market is not necessary for managerial delegation to be profitable for the owner. When there is network effect, the owner offers sales-oriented incentive scheme to the manager in equilibrium in the case of monopoly, as under Cournot type quantity competition.

## 2 The Model

The setup is exactly the same as in Hoernig (2012), except that managers are engaged in Cournot type quantity competition in the product market. There are two firms producing differentiated products using identical technologies. Each firm incurs constant marginal (average) cost of production  $c$ . The utility function of the representative consumer is assumed to be as follows.

$$U(x_1, x_2; y_1, y_2) = m + \frac{\alpha(x_1 + x_2)}{1 - \beta} - \frac{x_1^2 + 2\beta x_1 x_2 + x_2^2}{2(1 - \beta^2)} + n \left[ \frac{(y_1 + \beta y_2)x_1 + (y_2 + \beta y_1)x_2}{1 - \beta^2} - \frac{y_1^2 + 2\beta y_1 y_2 + y_2^2}{2(1 - \beta^2)} \right],$$

where  $x_i$  denotes the quantity of the good produced by firm  $i$  ( $= 1, 2$ ),  $y_i$  denotes the consumers' expectation about firm  $i$ 's equilibrium market share, higher value of the parameter  $n \in [0, 1)$  indicates stronger network effects,  $m$  denotes the consumption of all other goods

measured in terms of money, and  $\alpha > 0$  and  $0 \leq \beta < 1$  are other two preference parameters such that  $0 < c < \frac{\alpha}{1-\beta}$ . Note that, if  $\beta = 0$ , goods are completely differentiated. From the above mentioned utility function, we get the inverse demand functions as follows, where  $p_i$  is the price of good  $i$ .

$$p_i = \frac{\alpha(1+\beta) - x_i - \beta x_j}{1-\beta^2} + \frac{n(y_i + \beta y_j)}{1-\beta^2}; \quad i, j = 1, 2; \quad i \neq j.$$

Owner of each firm delegates the task to take decisions concerning output to her manager, knowing that managers are risk neutral and will engage in Cournot type quantity competition in the product market. Thus, given the incentive structure, which is a linear combination of profits and sales revenue, managers will maximize  $O_i = \lambda_i \pi_i + (1 - \lambda_i) p_i x_i$ ,  $i = 1, 2$ , where  $\pi_i$  is the profit of firm  $i$  and  $\lambda_i$  is the incentive parameter set by firm  $i$ . Note that we can re-write the incentive scheme of firm  $i$  as  $O_i = (p_i - \lambda_i c)x_i$ ,  $i = 1, 2$ . It implies that marginal cost of production of firm  $i$  is perceived to be lower (greater) than  $c$  by the managers, if the value of the incentive parameter  $\lambda_i$  is less (greater) than one. Thus, if  $\lambda_i < 1$ , firm  $i$ 's manager becomes more aggressive in the product market than in the case of  $\lambda_i = 1$ , and the delegation contract is said to be sales-oriented. Alternatively, if  $\lambda_i > 1$ , firm  $i$ 's delegation contract is said to be profit-oriented, which induces its manager to be less aggressive.

Stages of the game involved are as follows. In Stage 1, owners of each firm simultaneously and independently decide the incentive parameter so that the profit is maximized. In Stage 2, each manager simultaneously and independently chooses quantity to maximize the incentive scheme, given the incentive parameter. We solve this game by the backward induction method. We mention here that, as in Hoernig (2012), we also follow Katz and Shapiro (1985) and impose the additional 'rational expectations' conditions  $y_1 = x_1$  and  $y_2 = x_2$  in Stage 2.

It is easy to check that  $\frac{\partial}{\partial x_j} \left( \frac{\partial O_i}{\partial x_i} \right) = -\frac{\beta}{1-\beta^2} < 0$ ,  $\forall n \in [0, 1)$ ;  $i, j = 1, 2$  and  $i \neq j$ . That is, both the managers perceive that  $x_i$  and  $x_j$  are strategic substitutes. Now, in Stage 2, solving the problem of firm  $i$ 's manager,  $\text{Max}_{x_i} O_i(x_i, x_j | \lambda_i, \lambda_j)$ , we get her reaction function

as follows.

$$x_i = \frac{1}{2}[\alpha (1 + \beta) - \lambda_i c (1 - \beta^2) - \beta x_j + n (y_i + \beta y_j)], \quad i, j = 1, 2, i \neq j.$$

As expected, managers' reaction functions are negatively sloped irrespective of whether there is any network effect or not. Above mentioned reaction functions, along with the 'rationality' conditions  $y_1 = x_1$  and  $y_2 = x_2$ , give the Stage 2 equilibrium quantities and prices as follows.

$$x_i = \frac{\alpha \{(2 - \beta) (1 + \beta) - n (1 - \beta^2)\} - c (1 - \beta^2) \{(2 - n) \lambda_i - (1 - n) \beta \lambda_j\}}{(2 - n)^2 - (1 - n)^2 \beta^2},$$

$$p_i = \frac{\alpha \{(2 - \beta) - n (1 - \beta)\} + c (1 - n) (1 - \beta) [\{(2 - \beta^2) - n(1 - \beta^2)\} \lambda_i + \beta \lambda_j]}{(1 - \beta) \{(2 - n)^2 - (1 - n)^2 \beta^2\}},$$

$i, j = 1, 2; i \neq j.$

It is easy to check that both  $x_i$  and  $p_i$  ( $i = 1, 2$ ) are positive for all  $\beta, n \in [0, 1]$ , since  $(2 - \beta) - n (1 - \beta) > 0$ . It is straightforward to check that, as in the case of price competition, owner of firm  $i$  can induce her manager to capture greater market share by decreasing the weight on profit ( $\lambda_i$ ) while designing the incentive scheme, i.e., by offering a more sales-oriented delegation contract to her manager.

Substituting the Stage 2 equilibrium  $(x_i, p_i)$  into the profit expression for firm  $i$ , we get  $\pi_i(\lambda_i, \lambda_j; \alpha, \beta, c, n)$ . Now, in Stage 1, the problem of the owner of firm  $i$  can be written as  $Max_{\lambda_i} \pi_i(\lambda_i, \lambda_j; \alpha, \beta, c, n)$ . From the first order condition of this problem, we get the owner  $i$ 's reaction function as follows.<sup>2</sup>

$$\lambda_i = \frac{\{2 - \beta - n (1 - \beta)\} [c (2 - n) (1 - \beta) \{2 - n + (1 - n) \beta\} - \alpha \{\beta^2 + (2 - n) n (1 - \beta^2)\}]}{2c (1 - n) (2 - n) (1 - \beta) \{2 - n - (1 - n) \beta^2\}} - \frac{\beta \{\beta^2 + (2 - n) n (1 - \beta^2)\}}{2 (2 - n) \{2 - n - (1 - n) \beta^2\}} \lambda_j$$

Clearly,  $\frac{\partial \lambda_j}{\partial \lambda_i} < 0, \forall n \in [0, 1]; i, j = 1, 2, i \neq j$ . That is, unlike as in the case of Bertrand type price competition, incentive parameters  $\lambda_i$  and  $\lambda_j$  are always strategic substitutes

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<sup>2</sup>Second order conditions for maximization and stability conditions are satisfied in each of the two stages of the game.

under Cournot type quantity competition irrespective of whether there is any network effect or not.

Now, solving the owners' reaction functions in Stage 1, we get the equilibrium incentive parameters chosen by the owners in Stage 1 as follows.

$$\lambda_1 = \lambda_2 = \lambda^* = 1 - \frac{\{\alpha - c(1 - \beta)\} \{\beta^2 + (2 - n)n(1 - \beta^2)\}}{c(1 - n)(1 - \beta) \{4 + (2 - \beta)\beta - n(2 - \beta)(1 + \beta)\}}$$

It is easy to observe that  $\lambda^* < 1$  for all  $n \in [0, 1)$  and  $\beta \in (0, 1)$ . That is, the equilibrium incentive parameter is always less than one under Cournot type quantity competition. In other words, in the case of Cournot type quantity competition it is always optimal for the owners to induce the managers to be more aggressive by offering sales-oriented delegation contract, as in Fershtman and Judd (1987) and Sklivas (1987).

**Proposition 1:** *Network effects do not have any implication to the nature of the strategic managerial delegation contract under Cournot type quantity competition in the product market.*

The above proposition is in sharp contrast to the finding of Hoernig (2012) in the case of Bertrand type price competition with strong network effects. Therefore, the nature of the strategic managerial delegation contract need not necessarily depend on the strength of network effects. The strength of network effects becomes important only when managers' strategic variables are complements.

The underlying intuition behind these results is as follows. In the presence of network effects, managerial delegation affects firm's profitability through four channels: (a) *direct effect (DE)* via own output, (b) *strategic effect (SE)* via rival firm's output, (c) *direct network effect (DNE)* via consumers' expectation about own firm's output and (d) *indirect network effect (INE)* via consumers' expectation about rival firm's output. To be more explicit, we can write the change in firm  $i$ 's profit due to change in its incentive parameter

$(\lambda_i)$  as follows.

$$\frac{\partial \pi_i}{\partial \lambda_i} = \underbrace{\frac{\partial \pi_i}{\partial x_i} \frac{\partial x_i}{\partial \lambda_i}}_{\substack{(? \text{ } DE \\ -ve}} + \underbrace{\frac{\partial \pi_i}{\partial x_j} \frac{\partial x_j}{\partial x_i} \frac{\partial x_i}{\partial \lambda_i}}_{(-) SE} + \underbrace{\frac{\partial \pi_i}{\partial y_i} \frac{\partial y_i}{\partial x_i} \frac{\partial x_i}{\partial \lambda_i}}_{(-) DNE} + \underbrace{\frac{\partial \pi_i}{\partial y_j} \frac{\partial y_j}{\partial x_j} \frac{\partial x_j}{\partial x_i} \frac{\partial x_i}{\partial \lambda_i}}_{\substack{(+ \text{ } INE \\ -ve}}; \quad i, j = 1, 2; i \neq j.$$

From the first order conditions in Stage 2 we get  $\frac{\partial \pi_i}{\partial x_i} = -(1-\lambda)c$  and  $\frac{\partial x_i}{\partial \lambda_i} < 0$ . Therefore, the direct effect of a marginal decrease in the value of  $\lambda_i$  from one on  $\pi_i$  is zero. Next, we have  $\frac{\partial \pi_i}{\partial x_j} < 0$  since the two goods are substitutes, and  $\frac{\partial x_j}{\partial x_i} < 0$  since  $x_i$  and  $x_j$  are strategic substitutes. Therefore,  $\pi_i$  increases with a decrease in  $\lambda_i$ . It is easy to observe that the strategic effect always dominates the direct effect. Thus, in absence of network effects, it is optimal for the owners to offer sales-oriented incentive schemes to managers under Cournot type quantity competition. Network effects add an additional dimension to it.

When there are network effects, a decrease in  $\lambda_i$  increases  $x_i$  ( $\frac{\partial x_i}{\partial \lambda_i} < 0$ ), which in turn (i) enhances consumers' expectations about firm  $i$ 's output ( $\frac{\partial y_i}{\partial x_i} > 0$ ) and (ii) dampens consumers' expectations about firm  $j$ 's output by reducing firm  $j$ 's output since  $x_i$  and  $x_j$  are strategic substitutes and  $\frac{\partial y_j}{\partial x_j} > 0$ . Since goods are substitutes, an increase in consumers' expectation about either firm's output increases firm  $i$ 's profit ( $\frac{\partial \pi_i}{\partial y_i} > 0$  and  $\frac{\partial \pi_i}{\partial y_j} > 0$ ). Therefore, the direct (indirect) network effect of a decrease in  $\lambda_i$  increases (decreases) firm  $i$ 's profit. Now, since  $|\frac{\partial x_j}{\partial x_i}| < 1$  and  $\frac{\partial \pi_i}{\partial y_j} < \frac{\partial \pi_i}{\partial y_i}$ , the direct network effect always dominates the indirect network effect. Overall, network effects provide an additional incentive to the owners to reduce incentive parameters (i.e., to reduce weights on profits) while designing incentive contracts for their managers. Clearly, the result of Fershtman and Judd (1987) and Sklivas (1987) under Cournot type quantity competition is further reinforced in the presence of network effects.

In fact, the equilibrium incentive parameter decreases with the strength of network effects ( $n$ ), irrespective of the mode of product market competition - quantity or price. The reason for the negative impact of the strength of the network effects on the equilibrium incentive parameter under Cournot type quantity competition is easy to understand from the above discussions. It is also easy to check that  $\frac{\partial \lambda^*}{\partial n} < 0 \forall n, \beta[0, 1)$ .



Under Bertrand type price competition strategic effect and network effect of managerial delegation work in opposite direction. While due to strategic effect owners find it profitable to induce their managers to be less aggressive by offering profit-oriented incentive scheme, network effects provide negative incentive to the owners to induce their managers to be less aggressive. Borrowing the expression for the equilibrium incentive parameter under Bertrand type price competition from Hoernig (2012), pp. 488, and denoting that by  $\lambda^{*B}$ , we get  $\frac{\partial \lambda^{*B}}{\partial n} = -\frac{\{\alpha-c(1-\beta)\}(1-\beta)\{2(2-n)^2+(2-n)^2\beta-2(2-n)\beta^2-\beta^3\}}{c(1-n)^2\{4-n(2-\beta)-\beta(2+\beta)\}^2} < 0, \forall n, \beta[0, 1)$ . In the absence of network effects  $\lambda^{*B} > 1$ , as the strength of network effects increases  $\lambda^{*B}$  falls. If there is sufficiently strong network effect, network effect of delegation becomes dominant and, thus, we get  $\lambda^{*B} < 1$  as in Hoernig (2012).

**Lemma 1:** *The optimal incentive parameter ( $\lambda$ ) is decreasing in the strength of network effects ( $n$ ), irrespective of the mode of product market competition.*

## 2.1 Delegation under monopoly

It is interesting to note that, if  $\beta = 0$ ,  $\lambda^* = 1 - \frac{n(\alpha-c)}{2c(1-n)} < 1 \forall n \in (0, 1)$ . It implies that, in equilibrium, the incentive parameter is less than one even when products are completely differentiated, unless  $n = 0$ . That is, in the presence of network effect, sales-oriented managerial delegation remains optimal in the case of no strategic interaction among firms as well. In other words, managerial delegation is profitable to the owner in the case of monopoly as well, if there is network effect.

**Proposition 2:** *In the presence of network effect, it is optimal for the monopolist to induce her manager to be more aggressive in the product market by offering sales-oriented delegation contract.*

Existence of strategic interaction among firms, as considered in the literature on strategic managerial delegation following Fershtman and Judd (1987) and Sklivas (1987), or existence of agency problem within firm as considered in the principal-agent literature, is not necessary for managerial delegation through incentive contract to be optimal for the

owner. Delegation of tasks to decide output/price to the manager through sales-oriented incentive scheme can be optimal for the owner solely due to existence of network effects. It is also easy to understand that such delegation by a monopolist is social welfare enhancing.

### 3 Conclusion

In this paper we have shown two results. First, the nature of the equilibrium managerial delegation contract under Cournot type quantity competition in the product market does not depend on the strength of the network effect. This is in contrast to the result of Hoernig (2012) under Bertrand type price competition. In fact, network effects enhance owners' incentives to offer sales-oriented incentive schemes to managers. Thus, optimal weight on profit in the incentive scheme is decreasing in the strength of network effects, irrespective of the mode of product market competition. Second, in the presence of network effects, managerial delegation emerges in the equilibrium under monopoly as well. If there is network effect, it is optimal for the monopolist to induce her manager to be more aggressive in the product market by offering sales-oriented incentive scheme, as in case of Cournot type quantity competition.

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