



ELSEVIER

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Managing network mobility with tradable credits

Hai Yang*, Xiaolei Wang

Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, PR China

ARTICLE INFO

Article history:

Received 27 July 2010

Received in revised form 4 October 2010

Accepted 4 October 2010

Available online xxxx

Keywords:

Mobility management

Traffic equilibrium

Tradable credits

Networks

Congestion pricing

ABSTRACT

A system of tradable travel credits is explored in a general network with homogeneous travelers. A social planner is assumed to initially distribute a certain number of travel credits to all eligible travelers, and then there are link-specific charges to travelers using that link. Free trading of credits among travelers is assumed. For a given credit distribution and credit charging scheme, the existence of a unique equilibrium link flow pattern is demonstrated with either fixed or elastic demand. It can be obtained by solving a standard traffic equilibrium model subject to a total credit consumption constraint. The credit price at equilibrium in the trading market is also conditionally unique. The appropriate distribution of credits among travelers and correct selection of link-specific rates is shown to lead to the most desirable network flow patterns in a revenue-neutral manner. Social optimum, Pareto-improving and revenue-neutral, and side-constrained traffic flow patterns are investigated.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Problems of sustainability in transportation are already quite noticeable in almost all large cities worldwide, and it is clear that the current unrestricted use of private cars is at the center of this problem. Due to congestion, more travel time is wasted on the roads, drivers are more likely to get stressed and frustrated, which in turn causes accidents. Forecasts of traffic condition are less reliable, so punctual arrival becomes difficult and travelers suffer losses from early departure or late arrival. Traffic congestion also contributes substantially to environmental problems. In the United States, transportation sources accounted for 29% of total greenhouse gas (GHG) emissions in 2006, and have been the fastest-growing source of GHGs, accounting for 47% of the net increase in total US emissions since 1990 (US Environmental Protection Agency, 2006). Indeed, vehicle emissions are substantially higher in congested conditions than under conditions of freely flowing traffic (Transek, 2002). It is reasonable to believe that the world will soon have to confront high levels of air pollution and congestion problems caused principally by the unrestricted use of private cars, and have to deploy practical instruments to achieve transportation sustainability efficiently, effectively and in a politically feasible manner.

There are in general two ways to alleviate traffic congestion: increasing road capacity (supply) or reducing traffic (demand). As the cliché ‘you cannot pave your way out of traffic congestion’ says, providing more road space has been proven to be self-defeating in congested areas because the increased capacity will soon be absorbed by induced travel demand (Goodwin, 1996; Hansen and Huang, 1997). Therefore, the focus is now turning to demand management as a tool to address congestion problems. Treating road space as a common commodity, travel demand can be controlled either by price instruments or quantity instruments.

Historically, road pricing as a demand management instrument has received far more attention than quantity control in both theory and practice. Since the influential work by Pigou (1920) who suggested that vehicles using congested roads

* Corresponding author. Tel.: +852 2358 7178; fax: +852 2358 1534.

E-mail address: cehyang@ust.hk (H. Yang).

should bear a tax equal to the difference between marginal social and marginal private costs involved, a large body of research has been developed on congestion pricing. In the context of a congested network, various mathematical models and algorithms have been proposed to determine the optimal link tolls for system performance optimization under given physical and economic pricing constraints. Comprehensive reviews have been published by Yang and Huang (2005), Lawphongpanich et al. (2006), Lindsey (2006, 2010), and recently Tsekeris and Voss (2009).

Economists advocate congestion pricing as an efficient pricing strategy that requires the users to pay more for a public good, thus increasing the welfare gain or net benefit for society. However, one of the major concerns with road pricing is that it is perceived as unfair or just another flat tax. Indeed, equity debates play prominent roles in road pricing and explain why its application on urban roads is still limited to a small number of cities worldwide.

To develop more equitable and acceptable pricing schemes, Lawphongpanich and Yin (2010) and Song et al. (2009) studied a class of Pareto-improving pricing schemes without revenue rebate as they apply to networks. They formulated the problem of finding Pareto-improving tolls as a mathematical program with complementary constraints, and proposed a solution algorithm via manifold sub-optimization. However, such Pareto-improving toll schemes exist only conditionally.¹ In parallel, Liu et al. (2009) and then Nie and Liu (2010) adopted a continuous value of time (VOT) distribution and examined the existence of Pareto-improving and revenue-neutral road pricing and transit fare schemes in a simple bi-modal network consisting of a single road and a parallel transit line. Guo and Yang (2010) investigated Pareto-improving congestion pricing *cum* revenue refunding schemes in general networks and established a sufficient condition for the existence of origin–destination-specific but class-anonymous Pareto-improving refunding schemes. However, even with revenue redistribution and the appealing Pareto-improvement, the government plays a role as an objectionable toll collector, and its claim of revenue-neutrality is not always easy to verify and thus perhaps hard to believe. Compensation (to those who are priced off the roads due to road pricing) is insufficient and procedures play an essential role (Frey et al., 1996).

Given the general political resistance to congestion charges, some researchers and planners have turned to quantity instruments to curtail the unrestricted use of private vehicles. In quantity control, the government determines the travel demand to be served, and then assigns fixed mobility rights equally to all individual travelers or inhabitants so that fairness is explicitly demonstrated. The simplest quantity control method—plate-number-based road space rationing, has been in place in many Latin American cities like Mexico City and Sao Paulo, and has recently been tried in a few large cities in China including Beijing and Guangzhou. Modeling and efficiency analysis of plate-number-based rationing schemes in general networks have been carried out recently by Wang et al. (in press) and Han et al. (2010). Under such a rationing policy, every private car's access to the road is restricted on the same fraction of days according to its plate number, with enforcement carried out through an automated traffic surveillance network. Observable congestion reduction and air quality improvement have been reported under short-term rationing, but it is also known that the pure plate-number-based rationing strategy tends to promote undesirable second-car ownership in order to circumvent the restriction. Indeed, there is evidence that driving restrictions in Mexico City (*Hoy No Circula*) led to an increase in the total number of vehicles in circulation as well as a change toward old, cheap and polluting vehicles (Davis, 2008; Mahendra, 2008). So long-term number plate rationing may lose its effectiveness over time as car ownership increases.

Daganzo (1995) has suggested a hybrid strategy between pricing and rationing for a single bottleneck, and proved the possibility of achieving Pareto-optimum without toll redistribution. He further specified a certain time-dependent congestion reduction scheme involving tolls and rationing which can benefit every driver when each driver is assumed to pass the bottleneck at a given time (Daganzo, 2000). Nakamura and Kockelman (2002) applied Daganzo's idea to the San Francisco-Oakland bay bridge and showed the practical difficulty or impossibility of Pareto-improvement through pricing and rationing.

A more sophisticated quantity control method—termed the tradable driving permit (right or credit) or the emission cap-and-trade scheme—has been proposed in the transportation literature. A variant of this method was originated earlier by Dales (1968) for the purpose of attaining water quality targets in a cost-effective manner, and discussion of similar schemes now abounds in the realm of environmental policy (including, for example, in the Kyoto Protocol and the European Union Emission Trading Scheme (Perrels, 2010)). In a tradable credit scheme, a policy target is defined in terms of quantity and the associated consistent equilibrium price of credits is determined by the market through free trading.

Chin and Smith (1997) and Seik (1998) studied the vehicle ownership quota system implemented in Singapore in 1995. The government designed that system to control and limit the growth in supply of private vehicles. Under the system, the number of new vehicle registrations allowed each year is determined by the Land Transport Authority (LTA) while the market determines the price of owning a vehicle.² Raux (2004) looked at two existing permit systems, the Ecopoint program in Austria designed to limit pollution and noise from truck traffic, and the Zero Emission Vehicle (ZEV) program in California which aims to reduce vehicle emissions. Raux and Mariot (2005) described a potential application of a domestic market for car fuel consumption permits and outlined an evaluation of the socioeconomic consequences of market operation and the surpluses that

¹ The existence of the Pareto-improving tolls without revenue redistribution discussed by Lawphongpanich and Yin (2010) requires that the untolled equilibrium flow pattern be dominated by an alternative feasible flow pattern, under which some users are better off and no user is worse off than at the untolled equilibrium. Such a dominating flow distribution and the Pareto-improving tolls exist only for certain special networks that exhibit the generalized Braess paradox defined and characterized by Hagstrom and Abrams (2001).

² Following the strategy of Singapore, the city of Shanghai has implemented policies to restrain both car use and ownership. Since 1998, the number of new car registrations per year has been limited and the rights are sold in a public auction.

would result from it. Goddard (1997) proposed using tradable vehicle use permits as a cost-effective complement to technological abatement for mobile emissions control. Verhoef et al. (1997) explored the potential of a tradable permit scheme in the regulation of road transport externalities and discussed many practical applications on both the demand side (user-oriented) and the supply side (the automobile and fuel industries). They mentioned a tradable road-pricing smart cards (TRPS) scheme and appraised it as 'combining the advantages of electronic road pricing in terms of efficiency and of tradable permits in terms of social feasibility'. A similar idea was addressed by Viegas (2001), who suggested a monthly distribution of mobility rights to all taxpayers. These mobility rights can be used either for private car driving in a tolled area or for riding public transport, and are freely tradable among citizens. Kockelman and Kalmanje (2005) and Raux (2007) further evaluated these concepts and proposals, and made valuable case studies appraising policy impacts such as wealth redistribution between motorists and the community.

Apart from these conceptual studies, a few relevant mathematical analyses are worthy of mention. Nagurney and Dhand (1996) and later Nagurney (2000) modeled systems of marketable pollution permits using a variational inequality approach. Friesz et al. (2008) introduced a European-type congestion call option to value commuting to work along a given path for a given departure time selected by the driver when a telecommuting alternative exists. Teodorović et al. (2008) put forward an auction-based congestion pricing scheme, the basic idea of which is that all drivers who want to enter a cordoned downtown area in a specific time period have to participate in an auction. The operator or traffic authority is the auctioneer who makes the decision whether to accept or reject particular bids by the drivers. Akamatsu (2007) examined a tradable bottleneck permits scheme under which the road manager initially issues link- and time-specific permits that allow a permit holder to pass through a certain link or bottleneck in a pre-specified time period, and permit holders can either use these permits themselves or sell them in a competitive market. By fixing the number of link-specific permits at any time, queuing delay can be eliminated. However, such a tradable bottleneck permit scheme entails a number of problematic issues. First, a traveler has to acquire all the link permits along his or her chosen path, and the trading prices are link-specific, requiring as many trading markets as the number of links. These properties make the system complicated for a general network unless it has only a very limited number of bottlenecks, or unless it is simplified into an area-based single permit system like Singapore's, which sells permits for entering the controlled area in tandem with its vehicle ownership quota system. A second consideration is that if the link-specific permits are sold to individual travelers through auctions organized by the traffic authority, the 'market selling scheme' considered by the author becomes just a variant of traditional road pricing with endogenous formation of individual link prices, thereby losing the revenue-neutral property. Akamatsu proposed a 'free distribution scheme' for revenue-neutrality, but it is difficult or complicated for the traffic authority to freely distribute the permits for each individual link to each individual traveler in an equitable and transparent manner.

This study examines an alternative simple but forceful tradable credit distribution and charging scheme, where credits are universal for all links but link-specific in the amount of credit charge. Such a scheme would have three parts. (1) The government initially issues credits to all eligible travelers. (2) The government predetermines the charge or the consumption rate for each roadway link. (3) Credits can then be traded freely in a competitive market without government intervention. If there is an initial uniform distribution of credits to all registered travelers, the scheme involves at least the same equity as a strict rationing policy, so it would avoid the serious political resistance that would result if some group were favored over another.³ Link-specific (or cordon-based) charges regulate traffic flow as pricing does in both static and dynamic control settings by allowing for unrestricted trading of credits. And the scheme leaves everyone free to choose what to do with their initial endowment of credits. In particular, with the free credit allocation and trading travelers have a supplementary incentive to limit their vehicle use (by, for example, using public transport or carpooling), because they can sell their extra credits and then get a tangible reward for the inconvenience. Most importantly, in contrast with conventional congestion charging that involves a financial transfer from motorists to the government, the scheme confines transfers to within a predefined group of travelers throughout the whole exercise. The scheme is thus clearly revenue-neutral.⁴

We will now proceed to discuss this tradable credit scheme in more detail. We will investigate how to obtain the equilibrium traffic flow pattern and the credit market price in the fixed demand case for a given credit scheme, and then show that tradable credits can lead to certain desirable network flow patterns. The analysis will then be extended to the elastic demand case.

2. A tradable credit distribution and charging scheme

2.1. Initial distribution of credits

A well-formulated initial distribution of credits to travelers is important for political acceptance of the policy. The government must first define the eligible receivers of free credits. Presumably, the government can assume that all taxpayers

³ Clearly, the equity impacts of the tradable credit scheme in question here also depend on how the link-specific credits are charged, which should be addressed in both social and spatial contexts as in the network road pricing case (Yang and Zhang, 2002).

⁴ The scheme does offer the flexibility for the government to collect a certain amount of revenue periodically that can be earmarked for system implementation by charging an appropriately low initial price for each credit according to the total number of credits to be issued.

(or all qualifying travelers/workers) are eligible to receive free credits for every period of the credit scheme, which could be a month.^{5,6} For simplicity, assume that in the fixed demand case, credits are distributed freely to all travelers; in the elastic demand case, credits are distributed to both revealed and latent travelers in order to avoid any distortion of the travelers' incentive to drive.

The government then needs to set the rules for the distribution of free credits to registered travelers. Again for simplicity, consider two simple initial credit distribution schemes: uniform credit distribution, and origin–destination (O–D) specific credit distribution. The former is simple and fair, while the latter depends on the travelers' home and job locations but has the potential to make everyone better off (Pareto-improving).⁷ The total amount of credits is predetermined and the credits are month specific, so no one can gain by banking or stocking credits for the future.

2.2. Link-specific credit charges

The credits that travelers hold can be used on any link but the amount of credit charge is link-specific. This link-specific charge is similar to the tradable road-pricing smart card discussed by Verhoef et al. (1997). With today's electronic road pricing facilities and GPS technologies, such a credit charging system is conceivable. Without making the system unnecessarily complicated, we will require that all link-specific credit charges be nonnegative.^{8,9}

2.3. Trading credits

Travelers' requirements for credits will in general differ from their allocations depending on their origin, destination and route choices. We suppose there is a free market for credit trading where travelers can buy or sell credits according to their individual travel needs. The government does not interfere in the market as a buyer or seller, but solely acts as a manager to monitor the system. Assume the market is competitive (reasonable in view of the large number of agents involved), thus a uniform and constant credit price prevails, and the transaction cost is low enough that it can be ignored in the context of this study (Robert, 1995).¹⁰

3. Tradable credits with fixed demand

Consider a general network $G = (N, A)$ with a set N of nodes and a set A of directed links. Let W denote the set of O–D pairs and R_w the set of all routes between an O–D pair $w \in W$. The travel demand for each O–D pair $w \in W$ is denoted by d_w , $d_w > 0$. To minimize complexity, consider a separable link travel time function, $t_a(v_a)$, that is assumed to be nonnegative, differentiable, convex and monotonically increasing with the amount of flow v_a on link $a \in A$. Assume also that travelers are homogeneous and, without loss of generality, that they each have a value of time (VOT) equal to unity.

3.1. Traveler equilibrium

This tradable credit scheme is characterized by its initial distribution and the charging scheme. The initial distribution schemes considered here will be either uniform or O–D specific. Let $k = \{k_w, w \in W\}$ denote a general O–D based credit distribution scheme, where k_w is the amount of credit distributed to each traveler over the O–D pair $w \in W$. For a uniform distribution scheme, $k_w \equiv k_{w'}$ for any $w' \in W$ and $w' \in W$. For a given and fixed demand d_w and a given distribution scheme, the total amount of credits, denoted by K , is given by $K = \sum_{w \in W} k_w d_w$. As already mentioned, this analysis is restricted to link-specific credit charging. Let $\kappa = \{\kappa_a, a \in A\}$ denote the charging scheme, where κ_a is the credit charge for using link $a \in A$. In what follows, we will use (K, κ) to represent a credit charging scheme κ with a total number of credits K issued for all links, or simply a credit scheme in short.

Let $f_{r,w}$ denote the traffic flow on route $r \in R_w$ between O–D pair $w \in W$. f is a path flow vector $f = (f_{r,w}, r \in R_w, w \in W)$, and Ω_f represents the set of feasible path flow patterns defined by

⁵ A decision about the boundaries of the system is also required. In cities with well-defined geographical limits (Hong Kong for example), this could be straightforward, but it will be less so in a megalopolis.

⁶ The system must also deal with individuals who live outside the jurisdiction but occasionally drive in. Letting them travel without incurring charges would be unpalatable to residents. Visitors could be required to purchase credits on the market, but the transaction costs per trip might be high. Otherwise, some alternative form of payment would be necessary which would add to the overall system costs.

⁷ Distribution based on the traveler's origin (home location) and destination (job location) would cover commuting trips but it would miss a large number of other trips such as for shopping which are in reality fairly common. In the long run, the distribution scheme may induce people to change their home and job locations to become eligible for more credits.

⁸ This assumption is not essential. One can assume an initial nil distribution of credits but allow for both positive and negative credit charges on links, thereby leading to various revenue-neutral and Pareto-improving traffic flow patterns examined in Liu et al. (2009), Nie and Liu (2010) and Xiao and Zhang (2010).

⁹ Negative charges could create a negative full cycle through which a traveler may have an incentive to travel and accumulate credits.

¹⁰ This is similar to the case of electronic road pricing and has been addressed in the literature thanks to the electronic technology which is affordable today (Raux, 2007).

$$\Omega_f = \left\{ f \mid f_{r,w} \geq 0, \sum_{r \in R_w} f_{r,w} = d_w, r \in R_w, w \in W \right\} \quad (1)$$

Let $v = (v_a, a \in A)$ be the link flow vector, and Ω_v represent the set of feasible link flow patterns defined by

$$\Omega_v = \left\{ v \mid v_a = \sum_{w \in W} \sum_{r \in R_w} f_{r,w} \delta_{a,r}, f \in \Omega_f, a \in A \right\} \quad (2)$$

where $\delta_{a,r} = 1$ if route r uses link a and 0 otherwise.

With given and fixed O–D demand, it is obvious that not all credit distribution and charging schemes can guarantee the existence of feasible network flow patterns. The total amount of credits might be too small to meet the needs of all travelers going through the network even if all of them use least-credit paths. So for a given O–D demand, define the corresponding feasible set of credit schemes that ensures the existence of feasible network flow patterns. Let Ψ denote the feasible set of credit schemes where

$$\Psi = \left\{ (K, \kappa) \mid \exists f \in \Omega_f \text{ such that } \sum_{a \in A} \kappa_a v_a \leq K, v \in \Omega_v \right\} \quad (3)$$

Ω_f and Ω_v have been defined in (1) and (2) respectively. Hereafter, assume that Ψ is nonempty.

Let p denote the unit credit price in the credit market. Given a feasible scheme with tradable credits, $(K, \kappa) \in \Psi$, the user equilibrium (UE) conditions can be written as:

$$\sum_{a \in A} (t_a(v_a) + p\kappa_a) \delta_{a,r} = \mu_w, \quad \text{if } f_{r,w} > 0, \quad r \in R_w, \quad w \in W \quad (4)$$

$$\sum_{a \in A} (t_a(v_a) + p\kappa_a) \delta_{a,r} \geq \mu_w, \quad \text{if } f_{r,w} = 0, \quad r \in R_w, \quad w \in W \quad (5)$$

and the credit market equilibrium conditions are given by:

$$\sum_{a \in A} \kappa_a v_a = K, \quad \text{if } p > 0 \quad (6)$$

$$\sum_{a \in A} \kappa_a v_a \leq K, \quad \text{if } p = 0 \quad (7)$$

where $f \in \Omega_f$ and $v \in \Omega_v$. Eqs. (4) and (5) state that, at traffic equilibrium, all utilized paths between the same O–D pair $w \in W$ have equal and minimal generalized travel cost μ_w . The generalized travel cost consists of two parts: (1) the usual travel time cost; and (2) the credit cost. The credit cost of using link $a \in A$ is given by the number of credits consumed, κ_a , multiplied by the unit credit price, p . At the same time, Eqs. (6) and (7) represent the credit market clearing conditions, which stipulate that the equilibrium credit price is positive only when all the issued credits are consumed.

Consider now the following equivalent mathematical formulation for the UE and the credit market equilibrium:

$$\min_{v \in \Omega_v} \sum_a \int_0^{v_a} t_a(\omega) d\omega \quad (8)$$

subject to:

$$\sum_{a \in A} \kappa_a v_a \leq K \quad (9)$$

This is a simple variant of the standard UE model (Beckmann et al., 1956) with the network-wide credit feasibility condition (9) added. Since the problem is a convex nonlinear programming problem with linear constraints, for an optimum solution $f^* = (f_{r,w}^*, r \in R_w, w \in W) \in \Omega_f$ and $v^* = (v_a^*, a \in A) \in \Omega_v$, there exist the Lagrange multipliers p and $\mu = (\mu_w, w \in W)$ associated with the credit feasibility constraint (9) and the path flow conservation constraint (1) respectively, such that the following first-order optimality conditions hold:

$$\left(\sum_{a \in A} (t_a(v_a^*) + p\kappa_a) \delta_{a,r} - \mu_w \right) f_{r,w}^* = 0, \quad r \in R_w, \quad w \in W \quad (10)$$

$$\sum_{a \in A} (t_a(v_a^*) + p\kappa_a) \delta_{a,r} - \mu_w \geq 0, \quad f_{r,w}^* \geq 0, \quad r \in R_w, \quad w \in W \quad (11)$$

$$\left(K - \sum_a \kappa_a v_a^* \right) p = 0 \quad (12)$$

$$K - \sum_{a \in A} \kappa_a v_a^* \geq 0, \quad p \geq 0 \quad (13)$$

It is evident that the first-order optimality conditions (10)–(13) are equivalent to the UE conditions of (4) and (5), and the credit market equilibrium conditions of (6) and (7). The Lagrange multipliers p and μ correspond to the unit credit price at market equilibrium p^* and the minimal generalized travel cost at equilibrium. It is worth mentioning here that the first-order optimality conditions or the equivalent UE conditions and thus the resulting UE flow pattern are scale-invariant with the credit scheme. To put it differently, the UE conditions and the market equilibrium conditions remain unaltered if the total amount of credits issued, K , and the link credit charge, κ , are uniformly scaled up or down, because the market price of credits will be driven down or up by the same factor.

Since both the objective function and the constraints of problem (8) and (9) are convex in v , the link flow pattern at equilibrium, v^* , is unique. However, as in the standard UE model, the path flow pattern, f^* , at equilibrium is usually not unique. The uniqueness of the credit price, p^* , at market equilibrium is described in the following proposition.

Proposition 1. *Given a tradable credit scheme $(K, \kappa) \in \Psi$, the equilibrium credit price p^* is unique if there exists at least one O–D pair whose equilibrium path set always contains the same two (or more) paths with different credit charges.*

Proof. The equilibrium credit price is zero and hence unique when constraint (9) is inactive or if the amount of credits issued is excessive. We only need to consider the case with binding credit constraint (9). Suppose two paths $r_1, r_2 \in R_w$ with different path credit charges are always equilibrium paths between an O–D pair $w \in W$ for any equilibrium path flow patterns and credit prices. That is,

$$t_{r_1,w}^* + p\kappa_{r_1,w} = \mu_w \tag{14}$$

$$t_{r_2,w}^* + p\kappa_{r_2,w} = \mu_w \tag{15}$$

where $\kappa_{r,w} = \sum_{a \in A} \kappa_a \delta_{a,r}$ and $t_{r,w}^* = \sum_{a \in A} t_a(v_a^*) \delta_{a,r}$ are the path credit charge and path travel time that are defined uniquely at UE. Then the only equilibrium credit price that exists can be expressed explicitly by

$$p^* = \frac{t_{r_1,w}^* - t_{r_2,w}^*}{\kappa_{r_2,w} - \kappa_{r_1,w}}, \quad \kappa_{r_2,w} \neq \kappa_{r_1,w} \tag{16}$$

Therefore, the credit price at market equilibrium is uniquely determined if all equilibrium path flow patterns contain at least the two paths with different credit charges connecting the same O–D pair.¹¹ □

To provide an alternative and general expression of the equilibrium credit price, sum Eq. (10) over all $r \in R_w$ and $w \in W$ and use the definitions $\sum_{r \in R_w} f_{r,w}^* = d_w$ and $\sum_{w \in W} \sum_{r \in R_w} f_{r,w}^* \delta_{a,r} = v_a^*$. This leads to

$$\sum_{a \in A} (t_a(v_a^*) + p\kappa_a) v_a^* = \sum_{w \in W} \mu_w d_w \tag{17}$$

The equilibrium credit price is then given by

$$p^* = \frac{\sum_{w \in W} \mu_w d_w - \sum_{a \in A} t_a(v_a^*) v_a^*}{\sum_{a \in A} \kappa_a v_a^*} = \frac{\sum_{w \in W} \mu_w d_w - T^*}{K^*} = \frac{\sum_{w \in W} \mu_w d_w - T^*}{K} \tag{18}$$

where $T^* = \sum_{a \in A} t_a(v_a^*) v_a^*$ is the total travel time at UE under the credit scheme and $K^* = \sum_{a \in A} \kappa_a v_a^*$ is the total amount of credits actually consumed at equilibrium. Note that Eq. (18) is valid for both $K^* < K$ and $K^* = K$. If $K^* < K$ we have $p^* = 0$ from (12) and, as a result, $T^* = \sum_{w \in W} \mu_w d_w$. Thus $p^* = 0$ as well from (18) with either K or K^* .

If the sufficient condition in Proposition 1 is met and hence the equilibrium credit price is unique, then the generalized cost inclusive of travel time cost and credit cost for any path is unique, since the link flow pattern is unique at equilibrium. Thus the set of least-cost equilibrium paths is uniquely defined or invariant. But the converse is not true. That the set of least-cost equilibrium paths is invariant does not ensure the uniqueness of the credit price. This can be seen from Examples 2 and 3 which will be presented below, where a set of all-or-nothing equilibrium paths is invariant but the equilibrium credit price is indeterminate.

The condition in Proposition 1 is generally satisfied for real networks. Nonetheless, there are rare cases where the condition is not satisfied and hence the credit price may not be unique.

Example 1. This simple example shows that the sufficient condition in Proposition 1 is satisfied and the credit price at equilibrium is unique. The network, shown in Fig. 1, consists of two links or paths connecting a single O–D pair. Let

$$t_1(v_1) = 8 + 2v_1, \quad t_2(v_2) = 16 + v_2, \quad d_{12} = 10, \quad K = 30, \quad \kappa_1 = 4, \quad \kappa_2 = 2.$$

¹¹ As an alternative proof, regard path flow f as the basic variable and the link flow v as a function of f in (8) and (9). Then, for any given $(K, \kappa) \in \Psi$, if the condition in Proposition 1 is satisfied, one can show without difficulty that any optimal path flow solution f^* to the problem (8), (9) satisfies the linear independence constraint qualification (LICQ) (Bazaraa et al., 1993). Namely, the gradients of the active inequality constraints (nonnegative condition of path flows in (1) and the credit feasibility condition (9)) and the gradients of the equality constraints (O–D flow conservation conditions in (1)) are linearly independent at f^* . This implies that the Lagrange multipliers $\mu_w, w \in W$ (O–D generalized cost) and p (credit price) are unique.

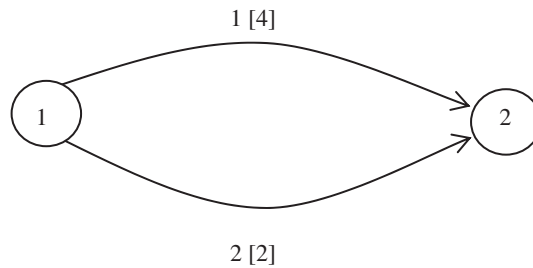


Fig. 1. A simple network example with two paths connecting a single O-D pair.

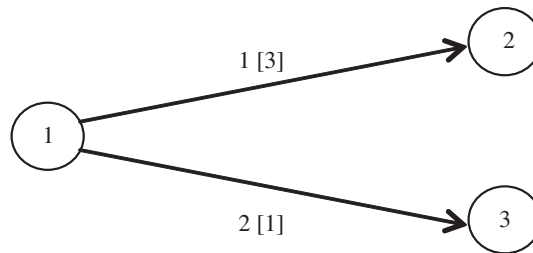


Fig. 2. A simple network example without path choices.

For each link, the number outside the bracket is the link number and the number inside the bracket is that link’s credit charge. Suppose both paths and all credits are used (as in the resultant equilibrium case). Then the user equilibrium and credit market equilibrium conditions are given by

$$\begin{aligned} 8 + 2v_1 + 4p &= 16 + v_2 + 2p \\ v_1 + v_2 &= 10 \\ 4v_1 + 2v_2 &= 30 \end{aligned}$$

The unique solution is $v_1^* = v_2^* = 5$, $p^* = 1.5$. The credit price at equilibrium is unique, because the number of equilibrium paths $|R_{12}^*| = 2$ and the condition of Proposition 1 is met.

Example 2. This example is shown in Fig. 2, where the credit price is indeterminate when an all-or-nothing traffic assignment is the only feasible flow pattern due to network topology. We simply let

$$d_{12} = d_{13} = 5, \quad \kappa_1 = 3, \quad \kappa_2 = 1, \quad K = 20, \quad k_{12} = k_{13} = 2$$

This uniform credit distribution but link-based credit charge scheme is feasible, leading to $v_1^* = v_2^* = 5$ and all credits are used up. Clearly, any $p^* \geq 0$ is an equilibrium solution for the credit price.

In this example, there is one and only one path available for each O-D pair, so the condition for unique credit price in Proposition 1 is obviously violated. The credit price is indeterminate in this case, because the necessity of traveling from node 1 to node 2 forces each traveler to purchase one additional credit from those traveling from node 1 to node 3 at any positive price. The example also illustrates that in special cases the credit system may create an unhealthy market; so in a credit system it is essential to offer travelers freedom of travel choices.

Example 3. In this example the condition in Proposition 1 is not satisfied and the credit price is not unique, even with alternative paths available. Consider the network in Fig. 3 that contains the two O-D pairs 1–2 and 1–3 with travel demands $d_{12} = 4$ and $d_{13} = 6$, respectively. The total amount of credits is set at $K = 22$ (through either a uniform or an O-D based credit distribution). The link travel time functions are given by

$$t_1 = v_1, \quad t_2 = 2v_2 + 15, \quad t_3 = v_3, \quad t_4 = v_4 + 10$$

It is easy to verify that the unique link flows are: $v_1^* = 10$, $v_2^* = 0$, $v_3^* = 6$, $v_4^* = 0$, and that travelers between 1 and 2 use only the path through link 1, and travelers between 1 and 3 only use the path through links 1 and 3 at equilibrium. So this is in fact an all-or-nothing flow pattern. This all-or-nothing UE flow pattern is sustained by the following credit prices:

$$10 + 2p \leq 2 \times 0 + 15 + p; \quad 6 + 2p \leq 0 + 10 + p$$

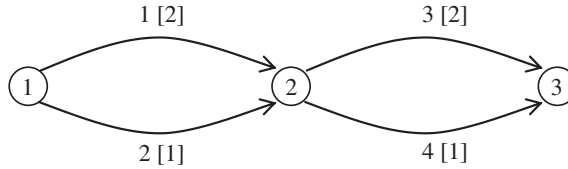


Fig. 3. A simple network example with path choices.

The two conditions, together with the non-negativity of p , imply that any $p^* \in [0, 4]$ is an equilibrium credit price. The non-uniqueness of the credit price in this case is due to the large travel time differences between the different paths, such that a small increase or decrease in the credit price is insufficient to induce travelers to switch paths.

These examples show that the credit price need not be unique when an all-or-nothing flow pattern occurs in the network. Such a pattern could appear when alternative paths do not exist due to network topology, or when alternative paths exist but (1) both the travel times and credit charges of alternative paths are higher than those of the utilized path or (2) the credit charges for the alternative paths are less than that for the utilized path but not enough less to offset a much longer travel time. The examples also demonstrate that non-uniqueness in the credit prices has nothing to do with the initial credit distribution, which nevertheless has great potential to affect individual travelers' wealth if their initial endowments differ.

3.2. A credit scheme for a socially optimum flow pattern

Hearn and Ramana (1998) have shown that a unique socially optimum (SO) flow pattern v^{so} can be sustained by the positive link toll scheme¹² $\tau^{so} = (\tau_a^{so}, a \in A) \geq 0$ if and only if there exists a vector $\mu = (\mu_w, w \in W)$ such that the following conditions are satisfied:

$$\sum_{a \in A} (t_a(v_a^{so}) + \tau_a^{so}) \delta_{a,r} \geq \mu_w, \quad r \in R_w, \quad w \in W \tag{19}$$

$$\sum_{a \in A} (t_a(v_a^{so}) + \tau_a^{so}) v_a^{so} = \sum_{w \in W} \mu_w d_w \tag{20}$$

In our tradable credit scheme, if the planner sets the credit charges: $\kappa^{so} = \tau^{so}$ or $\kappa_a^{so} = \tau_a^{so}, a \in A$ ¹³ and an initial allocation of $K^{so} = \sum_{a \in A} \kappa_a^{so} v_a^{so}$, then the necessary and sufficient optimality conditions for the convex program (8) with linear constraints (9) are

$$\left(\sum_{a \in A} (t_a(v_a) + p \kappa_a^{so}) \delta_{a,r} - \mu_w \right) f_{r,w} = 0, \quad r \in R_w, \quad w \in W \tag{21}$$

$$\sum_{a \in A} (t_a(v_a) + p \kappa_a^{so}) \delta_{a,r} - \mu_w \geq 0, \quad f_{r,w} \geq 0, \quad r \in R_w, \quad w \in W \tag{22}$$

$$\left(K^{so} - \sum_{a \in A} v_a \kappa_a^{so} \right) p = 0 \tag{23}$$

$$K^{so} - \sum_{a \in A} v_a \kappa_a^{so} \geq 0, \quad p \geq 0 \tag{24}$$

Obviously, a path flow pattern $f = f^{so}$ which gives $v = v^{so}$ satisfies the above conditions with $p^* = 1$ under the credit scheme $(K^{so}, \kappa^{so}) \in \Psi$, with either a uniform or O–D based distribution of the total amount of credits, K^{so} .¹⁴

This leads to

Proposition 2. *If all SO path flow patterns, f^{so} , satisfy the condition in Proposition 1, then any tradable credit scheme (K, κ) contained in the following nonempty polyhedron given by the (K, κ) part of the linear system (K, κ, μ) can decentralize a given SO flow pattern (v^{so}, d^{so})*

$$\sum_{a \in A} [t_a(v_a^{so}) + \kappa_a] \delta_{a,r} \geq \mu_w, \quad r \in R_w, \quad w \in W \tag{25}$$

$$\sum_{a \in A} (t_a(v_a^{so}) + \kappa_a) v_a^{so} = \sum_w \mu_w d_w^{so} \tag{26}$$

$$\sum_{a \in A} \kappa_a v_a^{so} = K \tag{27}$$

¹² Hearn and Ramana (1998) considered a SO toll set that allows for both positive and negative link tolls. Here we are restricted to nonnegative link tolls.

¹³ Doing so implies that we have implicitly presumed that the endogenous market price of credits at equilibrium is unity. Indeed, as shown later by using the general Eq. (18), the equilibrium credit price under the given credit setting is constantly equal to 1.

¹⁴ As mentioned before, multiplying both K^{so} and κ^{so} by a positive constant does not affect the equilibrium flow pattern but merely drives down the credit price p^* by the same factor. The current credit settings with an resulting equilibrium credit price $p^* = 1$ in fact define and fix their scale in a range that is analytically and computationally convenient.

If the condition for Proposition 1 is satisfied and thus the credit price at equilibrium is unique, then the unique equilibrium credit price can be determined from (18) as follows:

$$p^* = \frac{\sum_{w \in W} \mu_w^{so} d_w - T^{so}}{K} = \frac{\sum_{a \in A} \kappa_a v_a^{so}}{K} = \frac{K}{K} \equiv 1.0 \quad (28)$$

where the second equality stems from Eq. (26).

The SO credit scheme is independent of the details of the initial credit distribution and only involves the charging scheme and the total amount of credits distributed.¹⁵ However, as has been mentioned, the initial distribution of credits will undoubtedly influence individual travelers' wealth. If, for equity reason, a uniform credit distribution scheme is adopted together with a SO credit charging scheme, the net loss to travelers between O–D pair $w \in W$ before and after the scheme is introduced can be measured by the change in generalized cost $\mu_w^{so} - p^* k^{so} - \bar{\mu}_w^{ue}$, $w \in W$, where $\mu_w^{so} = (\mu_w^{so}, w \in W)$ denotes travelers' generalized cost under the SO credit charging scheme; $\bar{\mu}_w^{ue} = (\bar{\mu}_w^{ue}, w \in W)$ is the generalized travel cost under UE without any policy intervention; and $k^{so} = K^{so} / \sum_{w \in W} d_w$ is the initial uniform allocation of credits. The term $p^* k^{so}$ can be regarded as a uniform 'pre-refunding' to all travelers. Unfortunately, such a uniform credit allocation scheme does not necessarily make every traveler better off.

3.3. A credit scheme for a Pareto-improving SO flow pattern

In this section we are interested in defining an O–D-specific credit distribution scheme that can make everyone better off. Guo and Yang (2010) found Pareto-improving congestion pricing *cum* revenue refunding schemes which made every road user better off compared with the untolled case. For a transportation network with homogeneous travelers, they showed that a Pareto-improving O–D-specific scheme for refunding total toll revenue to all travelers exists if the pricing scheme reduces the total system travel time. Correspondingly, in a tradable credit scheme such a Pareto-improvement can be achieved by an appropriate O–D-specific distribution of free credits, as shown in the following proposition.¹⁶

Proposition 3. *Given a SO credit charging scheme κ^{so} and a total amount of credits issued $K^{so} = \sum_{a \in A} \kappa_a^{so} v_a^{so}$, if the credit price p^* at market equilibrium is unique and the total system travel time at SO is less than that at UE without any policy intervention, then there exists an O–D based credit distribution scheme that can make every traveler better off.*

Proof. Note first that the credit price p^* given by (28) for a SO credit scheme (K^{so} , κ^{so}) is strictly positive if the total system travel time at SO is less than that at UE before introducing the credit scheme (otherwise, the original UE is intact with $p^* = 0$). Now consider the following O–D-specific free distribution of credits $k^{pi} = (k_w^{pi}, w \in W)$, where 'pi' stands for Pareto-improving:

$$k_w^{pi} = \frac{(\mu_w^{so} - \bar{\mu}_w^{ue})}{p} + \frac{\alpha_w}{pd_w} \left(\sum_{w \in W} \bar{\mu}_w^{ue} d_w - \sum_{w \in W} \mu_w^{so} d_w + pK^{so} \right), \quad w \in W \quad (29)$$

where α_w is a positive number satisfying $\alpha_w > 0$, $w \in W$ and $\sum_{w \in W} \alpha_w = 1$. Multiplying both sides of Eq. (29) by d_w and adding them together over all O–D pairs $w \in W$, we have

$$\sum_{w \in W} k_w^{pi} d_w = \frac{\sum_{w \in W} (\mu_w^{so} - \bar{\mu}_w^{ue}) d_w}{p} + \sum_{w \in W} \frac{\alpha_w}{p} \left(\sum_{w \in W} \bar{\mu}_w^{ue} d_w - \sum_{w \in W} \mu_w^{so} d_w + pK^{so} \right) = K^{so} \quad (30)$$

so all credits are distributed to travelers by $k^{pi} = (k_w^{pi}, w \in W)$. Furthermore, from Eqs. (29) and (28) we have

$$\mu_w^{so} - pk_w^{pi} = \bar{\mu}_w^{ue} - \frac{\alpha_w}{d_w} \left(\sum_{w \in W} \bar{\mu}_w^{ue} d_w - \sum_{w \in W} \mu_w^{so} d_w + pK^{so} \right) = \bar{\mu}_w^{ue} - \frac{\alpha_w}{d_w} (\bar{T}^{ue} - T^{so}), \quad w \in W \quad (31)$$

where T^{so} is the total system travel time at SO, and \bar{T}^{ue} is the total travel time at UE without policy intervention. Eq. (31) then implies that as long as $T^{so} < \bar{T}^{ue}$, we have $\mu_w^{so} - pk_w^{pi} < \bar{\mu}_w^{ue}$, $w \in W$. That is, every traveler is made strictly better off under the credit distribution scheme k^{pi} defined by (29).¹⁷ □

¹⁵ As can be observed easily from the linear system (25)–(27), the total market value of all SO credits is not constant. In the spirit of the minimal-revenue congestion pricing (Dial, 1999, 2000), one may seek a SO credit scheme to minimize the market value of all credits (or the total number of credits issued) and thus potentially reduce the financial transfer (or credit transactions) among travelers. This argument, however, does not apply for the elastic demand case, where the total market value of all SO credits is constant, as shown in the subsequent section.

¹⁶ Although such a Pareto-improving credit distribution scheme exists in theory, the following distribution Eq. (29) suggests that the calculation would be complicated for large networks in view of the amount of information required to compute the credit for each O–D pair.

¹⁷ It is clear from the proof that Proposition 3 is also valid for a second-best network flow pattern under a tradable credit scheme. Namely, if a tradable credit scheme reduces the total system travel time (not necessarily to a minimal SO total travel time), then there exists a Pareto-improving, O–D based credit distribution scheme that can make every traveler better off.

4. Tradable credits with elastic demand

In the case of elastic demand, we assume that all revealed and latent travelers are eligible credit receivers. Latent travelers who, for example, telecommute or take public transport can sell their credits to the revealed car travelers. For simplicity, assume no income effects on demand; i.e., the demand curve for trips is invariant with respect to the value of the credits (the number of credits allocated to each traveler multiplied by their market price). This assumption is reasonable inasmuch as driving accounts for a relatively small fraction of most people's expenditures.¹⁸

4.1. Traffic equilibrium under a given tradable credit scheme

Let $\Omega_{(v,d)}$ represent the feasible set of O–D demand and link flows defined by

$$\Omega_{(v,d)} = \left\{ (v, d) \left| v_a = \sum_{w \in W} \sum_{r \in R_w} f_{r,w} \delta_{a,r}, d_w = \sum_{r \in R_w} f_{r,w}, f_{r,w} \geq 0, d_w \geq 0, r \in R_w, w \in W, a \in A \right. \right\} \quad (32)$$

Let $B_w(d)$ be the inverse demand function (the marginal benefit function), which is assumed to be monotonically decreasing, and let $\bar{d}_w, \bar{d}_w > 0$, be the potential or maximum demand (from qualifying credit receivers) over an O–D pair $w \in W$. Then, given a tradable credit scheme (K, κ) , the UE problem with elastic demand can be given by the following convex formulation:

$$\min_{(v,d) \in \Omega_{(v,d)}} \sum_{a \in A} \int_0^{v_a} t_a(\omega) d\omega - \sum_{w \in W} \int_0^{d_w} B_w(\omega) d\omega \quad (33)$$

subject to:

$$\sum_{a \in A} \kappa_a v_a \leq K \quad (34)$$

$$d_w \leq \bar{d}_w, \quad w \in W \quad (35)$$

Again, this is the standard elastic-demand UE model (Beckmann et al., 1956) with an added network-wide credit feasibility condition (34) and a demand upper-bound condition (35). Note that condition (35) is added into the above problem merely for the analysis of the initial credit distribution (to all $d_w, w \in W$ travelers) and the uniqueness of the credit price at market equilibrium. Different from the fixed demand case, here any given tradable credit scheme (K, κ) can guarantee the existence of a feasible solution to problem (33)–(35), because $(0, 0) \in \Omega_{(v,d)}$ always satisfies constraints (34) and (35).

Let $\mu = (\mu_w, w \in W)$ be the Lagrange multipliers associated with the O–D flow conservation conditions in (32), let $\rho = (\rho_w, w \in W)$ and p be the Lagrange multipliers associated with constraints (35) and (34). The following first-order Karush–Kuhn–Tucker conditions for the optimality of the above convex program with linear constraints then hold (Bazaraa et al., 1993):

$$\left(\sum_{a \in A} (t_a(v_a) + p\kappa_a) \delta_{a,r} - \mu_w \right) f_{r,w} = 0, \quad r \in R_w, \quad w \in W \quad (36)$$

$$\sum_{a \in A} (t_a(v_a) + p\kappa_a) \delta_{a,r} - \mu_w \geq 0, \quad f_{r,w} \geq 0, \quad r \in R_w, \quad w \in W \quad (37)$$

$$(\mu_w + \rho_w - B_w(d_w)) d_w = 0, \quad w \in W \quad (38)$$

$$\mu_w + \rho_w - B_w(d_w) \geq 0, \quad d_w \geq 0, \quad w \in W \quad (39)$$

$$(\bar{d}_w - d_w) \rho_w = 0, \quad w \in W \quad (40)$$

$$\rho_w \geq 0, \quad \bar{d}_w - d_w \geq 0, \quad w \in W \quad (41)$$

$$\left(K - \sum_{a \in A} \kappa_a v_a \right) p = 0 \quad (42)$$

$$K - \sum_{a \in A} \kappa_a v_a \geq 0, \quad p \geq 0 \quad (43)$$

Note that the multipliers (μ, ρ, p) exist, since the problem (33)–(35) forms a convex nonlinear programming problem with linear constraints. The first-order conditions coincide with traffic equilibrium and market equilibrium conditions with elastic demand. The Lagrange multiplier p can be regarded as the credit price, and the Lagrange multiplier μ can be regarded as the minimal generalized travel cost. Like the fixed demand case, the first-order optimality or UE conditions and hence the resulting UE flow pattern is scale-invariant with the credit schemes.

¹⁸ A simple and loose way to incorporate income effects is to assume that each traveler spends a certain portion of the initial endowment of credits for traveling. A general and rigorous way is to take the initial endowment as an input in the production of car trips within the framework of consumer behavior including both time and money budget constraints (Jara-Diaz, 2007).

The objective function (33) is strictly convex with respect to v and d , and thus the equilibrium link flow, v^* and travel demand d^* under a given credit scheme (K, κ) are unique. In addition, as long as there exists one O–D pair $w \in W$ whose realized demand d_w^* is strictly positive but does not reach its upper-bound \bar{d}_w and the realized demand goes through at least one path with nonzero credit charge, we have $\rho_w = 0$, $\mu_w = B_w(d_w^*)$ and $\sum_{a \in A} (t_a(v_a^*) + p\kappa_a)\delta_{a,r} - \mu_w = 0$ for at least one path $f_{r,w}^* > 0$, $r \in R_w$, $w \in W$. Hence, the equilibrium credit price that exists, is uniquely determined from the equilibrium conditions and is given by

$$p^* = \frac{B_w(d_w^*) - t_{r,w}^*}{\kappa_{r,w}}, \quad \kappa_{r,w} \neq 0 \tag{44}$$

where $r \in R_w$ is any utilized path with $f_{r,w}^* > 0$ and $\kappa_{r,w} > 0$. Otherwise, if the realized travel demands for all O–D pairs reach their upper-bound, $d_w = \bar{d}_w, \forall w \in W$, then the problem reduces to the UE problem with fixed demand examined in the preceding section. In this case the uniqueness of the credit price at equilibrium can be ascertained using Proposition 1.

For simplicity and for consistency with the standard elastic-demand network equilibrium models in the literature, hereafter, without loss of generality, we will discuss only the case when $d_w < \bar{d}_w$ for all $w \in W$. In this case, the UE conditions (36)–(41) can be simplified and rewritten as:

$$\sum_{a \in A} (t_a(v_a^*) + p^* \kappa_a) \delta_{a,r} = B_w(d_w^*), \quad \text{if } f_{r,w}^* > 0, \quad r \in R_w, \quad w \in W \tag{45}$$

$$\sum_{a \in A} (t_a(v_a^*) + p^* \kappa_a) \delta_{a,r} \geq B_w(d_w^*), \quad \text{if } f_{r,w}^* = 0, \quad r \in R_w, \quad w \in W \tag{46}$$

where $(v^*, d^*) \in \Omega_{(v,d)}$. Multiplying both sides of Eq. (45) by $f_{r,w}^*$ and adding them up over all $r \in R_w, w \in W$ yields

$$\sum_{w \in W} \sum_{r \in R_w} f_{r,w}^* \left[\sum_a (t_a(v_a^*) + p^* \kappa_a) \delta_{a,r} \right] = \sum_{w \in W} d_w^* B_w(d_w^*) \tag{47}$$

The left-hand side is equal to

$$\sum_{w \in W} \sum_{r \in R_w} f_{r,w}^* \left[\sum_a (t_a(v_a^*) + p^* \kappa_a) \delta_{a,r} \right] = \sum_{a \in A} t_a(v_a^*) v_a^* + p^* \sum_{a \in A} \kappa_a v_a^* = T^* + p^* K \tag{48}$$

where T^* is the total travel time at equilibrium. The second equality in (48) stems from the fact that, at equilibrium, $\sum_{a \in A} \kappa_a v_a^* \leq K$ if $p^* = 0$ and $\sum_{a \in A} \kappa_a v_a^* = K$ if $p^* > 0$. Therefore, from Eqs. (47) and (48), the equilibrium credit price is uniquely given by

$$p^* = \frac{\sum_{w \in W} d_w^* B_w(d_w^*) - T^*}{K} \tag{49}$$

In summary, we have the following proposition.

Proposition 4. *Given a tradable congestion credit scheme (K, κ) , the equilibrium credit price p^* is unique and given by (49) as long as there exists at least one O–D pair whose realized positive demand does not reach its upper-bound.*

The condition for uniqueness of the credit price set out in Proposition 4 is much milder than that in Proposition 1 with fixed demand. It can be considered as always being satisfied.

4.2. A credit scheme for the SO flow pattern

The standard SO problem with elastic demand can be formulated as:

$$\min_{(v,d) \in \Omega_{(v,d)}} \sum_{a \in A} v_a t_a(v_a) - \sum_{w \in W} \int_0^{d_w} B_w(\omega) d\omega \tag{50}$$

Let d^{so} and v^{so} be the unique O–D demand and link flow solutions of (50). Based on the work of Hearn and Yildirim (2002) and Yildirim and Hearn (2005), we can characterize the positive link toll pattern $\tau = (\tau_a, a \in A) \geq 0$ that can decentralize the given SO flow pattern (v^{so}, d^{so}) as the following polyhedron given by the linear system defined in τ :

$$\sum_{a \in A} (t_a(v_a^{so}) + \tau_a) \delta_{a,r} \geq B_w(d_w^{so}), \quad r \in R_w, \quad w \in W \tag{51}$$

$$\sum_{a \in A} (t_a(v_a^{so}) + \tau_a) v_a^{so} = \sum_{w \in W} B_w(d_w^{so}) d_w^{so} \tag{52}$$

If we replace τ_a in (51) and (52) by κ_a , we can easily arrive at the following proposition.

Proposition 5. *Any tradable credit scheme (K, κ) contained in the following nonempty polyhedron given by the linear system defined in (K, κ) can decentralize a given SO flow pattern (v^{so}, d^{so}) :*

$$\sum_{a \in A} (t_a(v_a^{so}) + \kappa_a) \delta_{a,r} \geq B_w(d_w^{so}), \quad r \in R_w, \quad W \tag{53}$$

$$\sum_{a \in A} (t_a(v_a^{so}) + \kappa_a) v_a^{so} = \sum_w B_w(d_w^{so}) d_w^{so} \tag{54}$$

$$\sum_{a \in A} \kappa_a v_a^{so} = K \tag{55}$$

Note that from Eqs. (49), (54), and (55) we have

$$p^* = \frac{\sum_{w \in W} d_w^{so} B_w(d_w^{so}) - T^{so}}{K} = \frac{\sum_{a \in A} \kappa_a v_a^{so}}{K} = \frac{K}{K} \equiv 1.0 \tag{56}$$

Therefore, for any SO credit scheme (K^{so}, κ^{so}) contained in the above polyhedron (53)–(55) the equilibrium credit price remains constantly equal to unity. In addition, the total market value of all credits is constant at

$$\Pi^{so} = p^* K^{so} = \sum_{w \in W} d_w^{so} B_w(d_w^{so}) - T^{so} = \text{constant} \tag{57}$$

Namely, the credit market value Π^{so} given by (57) is invariant across the set of SO credit scheme (K^{so}, κ^{so}) contained in (53)–(55) for a given unique SO solution $(d_w^{so}, w \in W)$ and $(v_a^{so}, a \in A)$.

4.3. A credit scheme for traffic restraint

The proposed credit scheme can accomplish the same task as the tolling schemes for traffic restraint in the road pricing problems examined by Yang and Bell (1997), Larsson and Patriksson (1999), Nie et al. (2004) and recently by Yang et al. (2010). The problem is to seek a link toll pattern or a link-based credit scheme on a road network so as to hold traffic demand within a given level such as the network’s environmental or physical capacity.

The standard elastic-demand UE problem with link capacity constraints is described by

$$\min_{(v,d) \in \Omega_{(v,d)}} \sum_{a \in A} \int_0^{v_a} t_a(\omega) d\omega - \sum_{w \in W} \int_0^{d_w} B_w(\omega) d\omega \tag{58}$$

subject to

$$v_a \leq C_a, \quad a \in A \tag{59}$$

where $\Omega_{(v,d)}$ is defined in (32) and $C_a, C_a > 0$ is either the physical or environmental capacity of link $a \in A$ (Yang and Bell, 1997).¹⁹

Let (v^{tr}, d^{tr}) be the unique equilibrium link flow and O–D demand solutions to the convex system described by (58) and (59), where ‘tr’ stands for traffic restraint. The Lagrange multiplier $\tau = (\tau_a, a \in A)$ associated with capacity constraint (59) provides an effective solution for link tolls that can be used to hold network traffic flow within the predetermined physical or environmental capacity. In addition, $\tau_a = 0$ if $v_a^{tr} < C_a$, and $\tau_a \geq 0$ if $v_a^{tr} = C_a$. So a positive link toll is needed only when the flow through that link reaches its capacity. The link tolls that can accomplish the capacity restraint task or support (v^{tr}, d^{tr}) as an equilibrium are not unique, but can be characterized by the following polyhedron given by the linear system defined in τ :

$$\sum_{a \in A} [t_a(v_a^{tr}) + \tau_a] \delta_{a,r} \geq B_w(d_w^{tr}), \quad r \in R_w, \quad w \in W \tag{60}$$

$$\sum_{a \in A} (t_a(v_a^{tr}) + \tau_a) v_a^{tr} = \sum_{w \in W} B_w(d_w^{tr}) d_w^{tr} \tag{61}$$

$$\sum_{a \in A} (C_a - v_a^{tr}) \tau_a = 0 \tag{62}$$

$$C_a - v_a^{tr} \geq 0, \quad \tau_a \geq 0, \quad a \in A. \tag{63}$$

If we replace τ_a in (60)–(63) by κ_a , we readily reach the following proposition.

Proposition 6. Any tradable credit scheme (K, κ) contained in the following nonempty polyhedron given by the linear system defined in (K, κ) can hold network flow pattern within a given desirable target level $v_a^{tr} \leq C_a, a \in A$:

¹⁹ The environmental capacity of a link is assumed to be less than its physical capacity. If physical capacity is adopted for $C_a, a \in A$, wasteful queuing delay will be removed. If, however, lower environmental capacity is adopted, traffic demand is held within a desirable level without physical vehicle queues.

$$\sum_{a \in A} (t_a(v_a^{\text{tr}}) + \kappa_a) \delta_{a,r} \geq B_w(d_w^{\text{tr}}), \quad r \in R_w, \quad w \in W \quad (64)$$

$$\sum_{a \in A} (t_a(v_a^{\text{tr}}) + \kappa_a) v_a^{\text{tr}} = \sum_{w \in W} B_w(d_w^{\text{tr}}) d_w^{\text{tr}} \quad (65)$$

$$\sum_{a \in A} (C_a - v_a^{\text{tr}}) \kappa_a = 0 \quad (66)$$

$$C_a - v_a^{\text{tr}} \geq 0, \quad \kappa_a \geq 0, \quad a \in A \quad (67)$$

$$\sum_{a \in A} \kappa_a v_a^{\text{tr}} = K \quad (68)$$

Like the SO credit scheme, we have the unique equilibrium credit price $p^* = 1.0$ from (49) and (65), and the total market value of all credits is unique as well, because

$$\Pi^{\text{tr}} = p^* K = \sum_{a \in A} \kappa_a v_a^{\text{tr}} = \sum_{a \in A} \kappa_a C_a = \sum_{w \in W} d_w^{\text{tr}} B_w(d_w^{\text{tr}}) - \sum_{a \in A} t_a(v_a^{\text{tr}}) v_a^{\text{tr}} = \text{constant} \quad (69)$$

where $(v^{\text{tr}}, d^{\text{tr}})$ is the unique equilibrium solution of link flows and O–D demands to problem (58) and (59).

5. Discussion of results

This study has investigated a tradable credit scheme in a general network with homogeneous travelers. Under such a scheme, the government initially distributes a certain amount of credits to all eligible receivers and charges a link-specific amount of credits to travelers using each link. Travelers can sell or buy additional credits among themselves in a competitive credit trading market. For a given credit scheme, we have shown that a unique user equilibrium flow pattern exists under either fixed or elastic demand, and it can be obtained by solving simple convex optimization problems. We also have shown that the price of credits at market equilibrium is unique subject only to very mild assumptions. Encouragingly, a properly designed tradable credit scheme with a suitably chosen total number of credits and link charges can emulate a system of congestion toll charges and support various desirable traffic flow optima.

The advantages of the tradable credit scheme are clear. It can guarantee that a predefined quantitative objective will be attained, whether this involves congestion abatement or not exceeding a certain vehicle emissions threshold.²⁰ Secondly, unlike road pricing (or emissions charges or gasoline taxes) which generate large financial transfers from the public to the government and thus usually meet with considerable skepticism, the tradable credit scheme offers revenue-neutral incentives for mobility and environmental quality. Unlike plate-number-based traffic rationing, which would be spatially and temporally less inefficient and ineffective, the tradable credit scheme plays essentially the same role as road tolling in the regulation of location-dependent and time-dependent congestion and environmental externalities, while avoiding the incentive to buy more cars. Thirdly, the fact that the initial credit distribution is free greatly enhances the political acceptability of the scheme. The scheme would actually improve the distribution of income among travelers, as the higher-income groups would need to acquire more credits from the lower-income groups to fulfill their travel plans diverting money from the wealthy to the less so.²¹ By adding a price component to the initial credit distribution, the tradable credit scheme can be made a blend of quantity and price-based demand management with self-financing system implementation. The tradable credit scheme thus offers the best combination of cost-effectiveness, administrative flexibility and distributional fairness.

Beyond this preliminary investigation, there are many open and potentially valuable avenues for further study. Among them are user heterogeneity, implementation with limited information, multiple objectives and hybrid credit schemes, each with either a static or a dynamic setting.

5.1. Tradable credits with heterogeneous travelers

The current modeling of credit schemes applies only to homogeneous user cases but it could be extended to the heterogeneous case (Yang and Huang, 2004; Guo and Yang, 2009,2010). This becomes more complicated but more interesting when credits are freely traded not only among travelers on different paths and O–D pairs, but also among travelers with different incomes or values of time. The extension would offer more dimensions for Pareto-improvement with financial neutrality.

²⁰ One should be careful in making such a statement. Apart from the fact that emissions depend heavily on vehicle maintenance and other things, because travelers have a choice of route, there is no guarantee that a given quantitative objective is always met unless credit charges on the links are set appropriately. Fixing a total number of credits (K) does not constrain total travel as tightly as, say, an emissions cap constrains CO₂ emissions, or a fishing quota limits the number of whales caught. These are one-dimensional control problems whereas the problem here is complicated by network considerations.

²¹ In this regard, the credit scheme can be contrasted with a congestion pricing system. If higher-income groups need to acquire more credits to fulfill their travel plans then they would also pay more road tolls. Both systems would be absolutely progressive if not also progressive on average in terms of income. To the extent that higher-income workers have greater work-hours flexibility than lower-income workers they may be better able to avoid peak-period tolls. If so, it is conceivable that they would pay less in tolls and correspondingly require fewer credits under a tradable credit scheme. The credit scheme would then be regressive.

5.2. Scheme design with limited information

The demand functions, link travel times and travelers' valuations of travel time savings called for in this analysis are not usually readily available in practice. To circumvent this difficulty, the tradable credit scheme could be implemented in the spirit of the trial-and-error road pricing method developed by Yang et al. (2004, 2010) and Meng et al. (2005). In this case, one more piece of market information—the revealed credit price—becomes available, signaling the need for an upward or downward adjustment of the total amount of credits to be issued in a subsequent implementation period.²²

5.3. Hybrid schemes for multiobjective optimization

Apart from the system travel time or cost envisaged here, one can naturally consider total network emissions as a secondary objective function (Yin and Lawphongpanich, 2006). Or one may introduce a scheme based on, for instance, fuel consumption. Such a hybrid form with dual tradable credits together with an initial allotment fee offers a more flexible and perhaps more acceptable way to achieve multiobjective network optimization (Yang and Huang, 2004; Guo and Yang, 2009). Also, credit charges can take alternative forms, such as a distance-based or time-based charge. Together with a proper total amount of credits issued, such a credit charging scheme would allow for effective control of the total number of vehicle-kilometers driven and the total number of vehicle-hours spent on the network (the two standard engineering measures).

All in all, the tradable credit scheme offers a powerful tool to manage network mobility, traffic congestion and environmental quality in an effective, efficient and equitable manner. Its various potential applications remain largely unexplored.

Acknowledgements

We wish to express our sincere thanks to an enthusiastic anonymous reviewer whose quick but thorough and constructive comments have improved the exposition of the paper. We gratefully acknowledge stimulating discussions with Ziyu Gao, Haijun Huang, Zhijia Tan, Qiong Tian, and Yafeng Yin. The work described in this paper was supported by a grant from Hong Kong's Research Grants Council under Project HKUST620910E.

References

- Akamatsu, T., 2007. Tradable Network Permits: A New Scheme for the Most Efficient Use of Network Capacity, Working Paper, Tohoku University, Japan. <[http://www.plan.civil.tohoku.ac.jp/~akamatsu/Publications/PDF/TS-TNP-P1\(070311\).pdf](http://www.plan.civil.tohoku.ac.jp/~akamatsu/Publications/PDF/TS-TNP-P1(070311).pdf)>.
- Bazaraa, M.S., Sherali, H.D., Shetty, C.M., 1993. *Nonlinear Programming: Theory and Algorithms*. John Wiley & Sons, Inc.
- Beckmann, M.J., McGuire, C.B., Winsten, C.B., 1956. *Studies in the Economics of Transportation*. Yale University Press, New Haven, CT.
- Chin, A.T., Smith, P., 1997. Automobile ownership and government policy: the economics of Singapore's vehicle quota scheme. *Transportation Research Part A* 31 (2), 129–140.
- Dales, J.H., 1968. *Pollution, Property, and Prices*. University of Toronto Press, Toronto.
- Daganzo, C.F., 1995. A Pareto optimum congestion reduction scheme. *Transportation Research Part B* 29 (2), 139–154.
- Daganzo, C.F., 2000. A Pareto improving strategy for the time-dependent morning commute problem. *Transportation Science* 34 (3), 303–311.
- Davis, L.W., 2008. The effect of driving restrictions on air quality in Mexico City. *Journal of Political Economy* 116 (1), 38–81.
- Dial, R., 1999. Minimal-revenue congestion pricing part I: a fast algorithm for the single origin case. *Transportation Research Part B* 33 (3), 189–202.
- Dial, R., 2000. Minimal-revenue congestion pricing part II: an efficient algorithm for the general case. *Transportation Research Part B* 34 (8), 645–665.
- Frey, B.S., Oberholzer-Gee, F., Eichenberger, R., 1996. The old lady visits your backyard: a tale of morals and markets. *Journal of Political Economy* 104 (6), 1297–1313.
- Friesz, T., Mookherjee, R., Yao, T., 2008. Securizing congestion: the congestion call option. *Transportation Research Part B* 42 (5), 407–437.
- Goddard, H.C., 1997. Using tradable permits to achieve sustainability in the world's large cities: policy design issues and efficiency conditions for controlling vehicle emissions, congestion and urban decentralization with an application to Mexico City. *Environmental and Resource Economics* 10 (1), 63–99.
- Goodwin, P., 1996. Empirical evidence on induced traffic: a review and synthesis. *Transportation* 23 (1), 35–54.
- Guo, X.L., Yang, H., 2010. Pareto-improving congestion pricing and revenue refunding with multiple user classes. *Transportation Research Part B* 44 (8–9), 972–982.
- Guo, X.L., Yang, H., 2009. User heterogeneity and bi-criteria system optimum. *Transportation Research Part B* 43 (4), 379–390.
- Hagstrom, J.N., Abrams, R.A., 2001. Characterizing Braess's paradox for traffic networks. In: *Proceedings of the Fourth IEEE Conference on Intelligent Transportation Systems*, August 25–29, 2001 Oakland, California, pp. 837–842.
- Han, D., Yang, H., Wang, X.L., 2010. Efficiency of the plate-number-based traffic rationing in general networks. *Transportation Research Part E* 46 (6), 1095–1110.
- Hansen, M., Huang, Y., 1997. Road supply and traffic in California urban areas. *Transportation Research Part A* 31 (3), 205–218.
- Hearn, D.W., Ramana, M.V., 1998. Solving congestion toll pricing models. In: Marcotte, P., Nguyen, S. (Eds.), *Equilibrium and Advanced Transportation Modeling*. Kluwer Academic Publishers, pp. 109–124.
- Hearn, D.W., Yildirim, M.B., 2002. A toll pricing framework for traffic assignment problem with elastic demand. In: Gendreau, M., Marcotte, P. (Eds.), *Current Trends in Transportation and Network Analysis – Papers in Honor of Michael Florian*. Kluwer Academic Publishers, pp. 135–145.
- Jara-Diaz, S.R., 2007. *Transport Economic Theory*. Elsevier, Amsterdam.
- Kockelman, K.M., Kalmanje, S., 2005. Credit-based congestion pricing: a policy proposal and the public's response. *Transportation Research Part A* 39 (7–9), 671–690.
- Larsson, T., Patriksson, M., 1999. Side constrained traffic equilibrium models: analysis, computation and applications. *Transportation Research Part B* 33 (4), 233–264.
- Lawphongpanich, S., Hearn, D.W., Smith, M.J. (Eds.), 2006. *Mathematical and Computational Models for Congestion Charging*. Springer, Berlin.
- Lawphongpanich, S., Yin, Y., 2010. Solving the Pareto-improving toll problem via manifold suboptimization. *Transportation Research Part C* 18 (2), 234–246.

²² For the schemes optimized with either fixed or elastic demand in this study, the benchmark price level shown in Eqs. (27) and (55) can be set to be unity. Then, the total amount of credits for next period should be in general increased (decreased) if the observed market price of credits in the current period is higher (lower) than unity.

- Lindsey, R., 2006. Do economists reach a conclusion on road pricing? The intellectual history of an idea. *Economic Journal Watch* 3 (2), 292–379.
- Lindsey, R., 2010. Reforming road user charges: a research challenge for regional science. *Journal of Regional Science* 50 (1), 471–492.
- Liu, Y., Guo, X., Yang, H., 2009. Pareto-improving and revenue-neutral congestion pricing schemes in bi-modal traffic networks. *Netnomics* 10 (1), 123–140.
- Mahendra, A., 2008. Vehicle restrictions in four Latin American cities: is congestion pricing possible? *Transport Review* 28 (1), 105–133.
- Meng, Q., Xu, W., Yang, H., 2005. Trial-and-error procedure for implementing a road-pricing scheme. *Journal of the Transportation Research Board, Transportation Research Record* 1923, 103–109.
- Nagurney, A., Dhanda, K., 1996. A variational inequality approach for marketable pollution permits. *Computational Economics* 9 (4), 363–384.
- Nagurney, A., 2000. Marketable pollution permits in oligopolistic markets with transaction costs. *Operations Research* 48 (3), 424–435.
- Nakamura, K., Kockelman, K.M., 2002. Congestion pricing and roadscape rationing: an application to the San Francisco Bay Bridge corridor. *Transportation Research Part A* 36 (5), 403–417.
- Nie, Y., Liu, Y., 2010. Existence of self-financing and Pareto-improving congestion pricing: impact of value of time distribution. *Transportation Research Part A* 44 (1), 39–51.
- Nie, Y., Zhang, M., Lee, D.H., 2004. Models and algorithm for the traffic assignment problem with link capacity constraints. *Transportation Research Part B* 38 (4), 285–312.
- Perrels, A., 2010. User response and equity considerations regarding emission cap-and-trade schemes for travel. *Energy Efficiency* 3 (2), 149–165.
- Pigou, A.C., 1920. *The Economics of Welfare*. MacMillan, London.
- Raux, C., 2004. The use of transferable permits in transport policy. *Transportation Research Part D* 9 (3), 185–197.
- Raux, C., Mariot, G., 2005. A system of tradable CO₂ permits applied to fuel consumption by motorists. *Transport Policy* 12 (3), 255–265.
- Raux, C., 2007. Tradable Driving Rights in Urban Areas: Their Potential for Tackling Congestion and Traffic-Related Pollution, Working Paper, Laboratoire d'économie des transports – CNRS, Université de Lyon, ENTPE.
- Robert, N.S., 1995. Transaction costs and tradable permits. *Journal of Environmental Economics and Management* 29 (2), 133–148.
- Seik, F.T., 1998. A unique demand management instrument in urban transport: the vehicle quota system in Singapore. *Cities* 15 (1), 27–39.
- Song, Z., Yin, Y., Lawphongpanich, S., 2009. Nonnegative Pareto-improving tolls with multiclass network equilibria. *Transportation Research Record* 2091, 70–78.
- Transek, A.B., 2002. Road Pricing in Urban Areas. Federation of European Transport and Environment and the Swedish National Road Administration. <<http://www.transportpricing.net/download/swedishreport.pdf>>.
- Tsekeris, T., Voss, S., 2009. Design and evaluation of road pricing: state-of-the-art and methodological advances. *Netnomics* 10 (1), 5–52.
- Teodorović, D., Triantis, K., Edara, P., Zhao, Y., Mladenović, S., 2008. Auction-based congestion pricing. *Transportation Planning and Technology* 31 (4), 399–416.
- US Environmental Protection Agency Office of Transportation and Air Quality, 2006. Greenhouse Gas Emissions from the US Transportation Sector, 1990–2003. Transportation GHG Emissions Report. <<http://www.epa.gov/otaq/climate/basicinfo.htm>>.
- Verhoef, E., Nijkamp, P., Rietveld, P., 1997. Tradable permits: their potential in the regulation of road transport externalities. *Environment and Planning B* 24 (4), 527–548.
- Viegas, J.M., 2001. Making urban road pricing acceptable and effective: searching for quality and equity in urban mobility. *Transport Policy* 8 (4), 289–294.
- Wang, X.L., Yang, H., Han, D., in press. Traffic rationing and short-term and long-term equilibrium. *Journal of the Transportation Research Board, Transportation Research Record*.
- Xiao, F., Zhang, H.M., 2010. On a Credit-Based Congestion Pricing Scheme for Transportation Networks. Working Paper, The University of California, Davis.
- Yang, H., Bell, M.G.H., 1997. Traffic restraint, road pricing and network equilibrium. *Transportation Research Part B* 31 (4), 303–314.
- Yang, H., Huang, H.J., 2005. Mathematical and Economic Theory of Road Pricing. Elsevier.
- Yang, H., Huang, H.J., 2004. The multi-class, multi-criteria traffic network equilibrium and systems optimum problem. *Transportation Research Part B* 38 (1), 1–15.
- Yang, H., Meng, Q., Lee, D.H., 2004. Trial-and-error implementation of marginal-cost pricing on networks in the absence of demand functions. *Transportation Research Part B* 38 (6), 477–493.
- Yang, H., Xu, W., He, B.S., Meng, Q., 2010. Road pricing for congestion control with unknown demand and cost functions. *Transportation Research Part C* 18 (2), 157–175.
- Yang, H., Zhang, X., 2002. Multi-class network toll design problem with social and spatial equity constraints. *Journal of Transportation Engineering, ASCE* 128 (5), 420–428.
- Yildirim, M.B., Hearn, D.W., 2005. A first best toll pricing framework for variable demand traffic assignment problems. *Transportation Research Part B* 39 (8), 659–678.
- Yin, Y., Lawphongpanich, S., 2006. Internalizing emission externality on road networks. *Transportation Research Part D* 11 (4), 292–301.