Managing Price Risk in a Multimarket Environment

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***Abstract*—In a competitive electricity market, a generation company (Genco) can manage its trading risk through trading electricity among multiple markets such as spot markets and contract markets. The question is how to decide the trading proportion of each market in order to maximize the Genco's profit and minimize the associated risk. Based on the mean-variance portfolio theory, this paper proposes a sequential optimization approach to electric energy allocation between spot and contract markets, taking into consideration the risks of electricity price, congestion charge, and fuel price. Especially, the impact of the fuel market on electric energy allocation is analyzed and simulated with historical data in respect of the electricity market and other fuel markets in the U.S. Simulation results confirm that the proposed analytic approach is consistent with intuition and therefore reasonable and feasible for a Genco to make a trading plan involving risks in an electricity market.**

***Index Terms*—Electricity market, mean-variance portfolio theory, risk management, utility theory.**

I. Introduction

A GENERATION company's (Genco's) objective, in a competitive environment, is to maximize its profit and minimize the associated risk. In order to achieve this goal, it is necessary and important for the Genco to make a trading plan with risk management before bidding into spot markets to get an expected high profit, since spot prices are substantially volatile. Various aspects of risk management have been applied to electricity markets [1]. For example, hedging the risk of spot price with forward contract [2] and futures contract [3] has been investigated; Monte Carlo simulation and decision analysis have been applied to find the optimal contract combination [4], [5]; Value at Risk (VaR) and Conditional Value at Risk (CVaR) have been adopted to value a trading portfolio [6]. Instead of controlling price risk by hedging with financial instruments such as forward contract and futures contract, this paper tries to reduce the price risk through diversification, i.e., trading electricity among multiple markets. Therefore, making a trading plan with risk management involves the determination of the trading quantity in different electricity trading markets. The mean-variance portfolio theory is applied to this allocation problem since it explicitly considers decision-makers' risk aversion and the statistical correlation among alternative outcomes compared to other allocation approaches such as Monte Carlo simulation and decision analysis. Based on the mean-variance portfolio theory, reference [7] has proposed a sequential optimization approach to electric energy allocation between spot and contract markets with considerations of the risks of electricity price and congestion charge. However, a Genco's profit depends on not only the revenue on the electricity market but also the production cost, which is mainly decided by the fuel price in a short run. In other words, the condition of fuel markets may affect a Genco's trading plan in the electricity market. In this paper, based on the model proposed in [7], some modifications are made to incorporate the risk of the fluctuation of fuel price. That is, this paper develops an approach to electric energy allocation with simultaneous considerations of risks of fuel price as well as electricity price and congestion charge. One of the key components of this approach is how to make a tradeoff between the expected profit and associated risk, as we all know that higher profit is often accompanied by higher risk.

In the financial literature, one of the approaches to making tradeoff between benefit (quantified in rate of return, thereafter "return" for short) and risk is the mean-variance approach, which was proposed by Markowitz in 1952 [8]. In this approach, an investment's profitability is indicated with the expected return; its risk is quantified with the variance of return. According to the mean-variance criterion, an option F dominates an option G if its expected return is greater than (or equal to) that of G, while its variance is smaller than (or equal to) the variance of G. Usually, option F with higher expected return may have higher variance of return compared to option G. The decision-maker should make a tradeoff between the expected return and variance. Practically, a risk-penalty factor named risk-aversion index is adopted to describe a decision-maker's risk preference, which can fulfill the function of making tradeoff between the expected return and variance. In the financial literature, the associated risk-aversion parameter has been derived based on historically statistical data. For example, $A = 3$ is derived and stands for moderate risk aversion [9] when an investment's benefit is quantified in "return." When "profit" is adopted to denote a Genco's benefit of trading electricity, the first question that should be solved is how to determine the corresponding risk-penalty factor. Reference [10] tried to make tradeoff between expected profit and risk with a risk factor when making purchase allocation but did not explain how to decide the value of this factor. In this paper, based on the mean-variance approach and the utility theory, a method to determine the risk-penalty factor of the profit approach is proposed.

The following sections are organized in this way: Section II describes how to make tradeoff between the expected profit and risk and determine the corresponding risk-penalty factor. Section III introduces the proposed approach to electric energy allocation between spot and contract markets. With historical

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data in respect of the electricity market and other fuel markets in the U.S., Section IV gives some numerical examples to demonstrate the proposed allocation model and especially, discuses the impact of fuel market on electric energy allocation. Finally, Section V draws the conclusion.

II. Determination of the Risk-Penalty Factor

In a competitive electricity market, obviously, a Genco has the intention of maximizing its benefit (profit or return) but has to consider the corresponding risk since a higher benefit is usually accompanied by a higher risk. How to balance benefit and risk depends on the Genco's risk preference. A general method of describing a decision-maker's preference is provided by the use of a utility function, a concept of the utility theory that was first proposed by mathematician D. Bernoulli and his contemporary G. Cramer and then developed by Von Neumann and Morgenstern [11], [12]. The value of a utility function is called utility value. Portfolios with higher utility values are preferred. In the mean-variance approach, a portfolio's utility is evaluated from a function that combines the reward (expected return) and risk preferences of a decision-maker into a simple relation [13]. Suppose that decision-makers are risk averse, i.e., higher reward and lower risk is preferred. A commonly used utility function in the financial literature is [9]

$$U(r) = E(r) - 0.5A \cdot Var(r) \quad (1)$$

where r is the return on an asset portfolio; $U(r)$ is the expected utility value of the portfolio; $E(r)$ and $Var(r)$ are expected return and variance of return of the portfolio, respectively; and A is an index of investors' risk-aversions. The factor 0.5 that appears in front of the variance is due to the commonly used example format in optimization involving a square term, $f(x) = ax + (1/2)bx^2$; rather, $f(x) = ax + bx^2$ is used. The factor (1/2) will cancel out the 2 in the square term when derivative in optimization is taken. Actually, the item $0.5A$ denotes the degree of an investor's risk-aversion, i.e., the extent that the investor "penalizes" the expected return with the corresponding risk into consideration. Based on historical data, a moderate index of risk-aversion is derived as $A = 3$. Naturally, $A > 3$ implies more risk averse and $A < 3$ indicates less risk averse [9]. Normally, in the field of finance, return is used to account investors' rewards, and this method is defined as "the return approach" in this paper.

For the convenience of establishing the energy allocation model, "profit" is applied to stand for a Genco's reward or benefit on electricity trading, and this is defined as "the profit approach." Similar to the return approach, a utility function is proposed to make tradeoff between the expected profit and risk in this paper

$$U(\pi) = E(\pi) - B \cdot Var(\pi) \quad (2)$$

where π is the profit on a Genco's trading portfolio in electricity markets, and B is a risk-penalty factor that denotes the extent that a Genco "penalizes" the expected profit considering the uncertainty (risk) of obtaining the corresponding profit. Clearly, a higher value of B indicates a higher degree of risk aversion and vice versa. The question is how to decide on the value of B for Gencos with different risk aversion. At least, what is the value of B for a Genco who is considered average risk averse?

According to the utility theory [11], a Genco's risk preference can be described with a utility function that is a way to ranking available portfolios, and a portfolio with the highest utility value will be selected. However, the utility function for a specific Genco is not unique. The absolute utility value has no meaning, and the relative utility value of any two portfolios makes sense. This is one of the attributes of the utility function, i.e., a utility function is defined up to a positive linear transformation, which states that the ranking of a group of alternative options will remain constant using the utility functions $U(x)$ or $U^*(x) = a + bU(x)$, independent of the value of x, so long as b is positive. Based on this principle, the risk-penalty factor B can be derived as follows.

Suppose that the production cost of a Genco during the planning period, denoted by $c(\cdot)$, is certain. Multiplying $c(\cdot)$ on both sides of (1), we get[1]

$$U(r) \cdot c(\cdot) = E(\pi) - \frac{A}{2c(\cdot)} Var(\pi). \quad (3)$$

According to the above attribute of the utility function, (3) is equivalent to (1) in terms of describing a Genco's risk preference. Furthermore, the Genco's risk preference is independent of the specific analysis approach (the return or profit approach), i.e., whatever approach is adopted, same Genco will make the same decision. In other words, utility function (2) used in the profit approach should be equivalent to utility function (1) used in the return approach in terms of describing the Genco's risk preference or ranking portfolios. Therefore, (3) is equivalent to (2). Comparing (3) and (2), B can be derived as

$$B = \frac{A}{2c(\cdot)}. \quad (4)$$

Naturally, B is equal to $1.5/c(\cdot)$ for a Genco who is considered average risk averse.

III. Methodology to Electric Energy Allocation

A. Scenario Consideration

Most electricity markets provide two types of markets in which energy is traded: spot market and contract market. In the contract market, Gencos trade energy by way of signing contract with their counterpartners (e.g., energy consumers). Specific details such as quantity, duration, price, and delivery point are bilaterally negotiated between Gencos and consumers or their agents. In the spot market, the market clearing price (MCP) depends on the specific pricing system. Three types of pricing systems have been adopted in the spot market: uniform marginal pricing, zonal pricing, and locational marginal pricing (LMP) [14]. From risk management point of view, a market with LMP system is general. In this paper, assume Gencos can trade energy both in contract market and spot market; LMP

[1] According to the definition of the return, $r = \pi/c$.

system is adopted to manage network congestion. That is, when there is no congestion anywhere on the system, there will be only one price in the market. When the transmission network is congested, prices will vary among locations or nodes on the network. The difference between locational prices represents congestion charges. Under this pricing system, from risk point of view, there are three types of trading approaches for a Genco to trade its electricity: 1) local contracts that are signed with local consumers[2] and involve the risks of electricity contract price and fuel price; 2) non-local contracts that are signed with non-local customers and involve the risks of not only electricity price and fuel price but also congestion charge; and 3) spot markets that involve the risks of electricity spot price and fuel price. In this section, a simple case, electric energy allocation between local contracts and spot markets, is first introduced to discuss the impacts of electricity spot price, electricity contract price, and fuel spot price on electric energy allocation. Then a general approach is developed to allocate electric energy among local contracts, non-local contracts, and spot markets taking into consideration the risks of fuel price, electricity price, and congestion charge with the aim of maximizing a Genco's profit and minimizing the associated risk. For the convenience of analytical deduction, generation cost function is simplified as a linear function, i.e., $c(\cdot) = be\lambda^F = bpt\lambda^F$, where b is the average fossil generation unit heat rate, i.e., average fuel consumption coefficient (MBtu/MWh); e is the amount of electric energy produced (MWh); p is the output power of units (MW); t is the trading time (hour); and λ^F is the fuel price ($/MBtu).

B. Simple Case—Electric Energy Allocation Between Local Contracts and Spot Markets

Suppose a Genco trades its electric energy with two trading approaches: local contracts and spot markets. Let y be the proportion of total electric energy allocated to spot markets and $1-y$ be the proportion allocated to local contracts. Then profit on the complete portfolio, denoted by C, is

$$\pi_C = ye\lambda^S + (1-y)e\lambda^B - be\lambda^F \tag{5}$$

where λ^B and λ^S are electricity contract price ($/MWh) and spot price ($/MWh), respectively. Three situations will be considered in the following: 1) considering the risk of electricity spot price; 2) considering the risks of electricity spot price and fuel spot price; and 3) considering the risks of electricity spot price, fuel spot price, and electricity contract price.

1) Considering the Risk of Electricity Spot Price: Suppose that λ^B and λ^F are deterministic; only λ^S is a random variable. Taking expectation and variance of the profit on the complete portfolio C, we get

$$E(\pi_C) = yeE(\lambda^S) + e\lambda^B - ye\lambda^B - be\lambda^F \tag{6}$$
$$Var(\pi_C) = (ye)^2 Var(\lambda^S). \tag{7}$$

[2]Here, local consumers refer to consumers located in the same pricing node as the Genco, while non-local consumers are consumers located in different pricing nodes from the Genco.

Mathematically, the Genco's objective can be achieved by maximizing its utility function with respect to the proportion allocated to the spot market, i.e.,

$$\underset{\mathrm{y}}{Max} \quad U = E(\pi_C) - B \cdot Var(\pi_C).$$

Let $\partial U/\partial y = 0$ and solve for this equation; the optimal energy allocation ratio to the spot market is derived as follows:

$$y_1^* = \frac{E(\lambda^S) - \lambda^B}{2B \cdot e \cdot Var(\lambda^S)}. \tag{8}$$

2) Considering the Risks of Electricity Spot Price and Fuel Spot Price: Suppose that only λ^B is deterministic; λ^S and λ^F are all random variables. Similarly, the corresponding optimal allocation ratio of the electricity spot market can be derived as

$$y_2^* = \frac{E(\lambda^S) - \lambda^B + 2B \cdot b \cdot e \cdot \rho_{S,F} \cdot \sigma(\lambda^S)\sigma(\lambda^F)}{2B \cdot e \cdot Var(\lambda^S)} \tag{9}$$

where $\rho_{S,F}$ is the correlation coefficient between λ^S and λ^F, and $\sigma(\lambda^S)$ and $\sigma(\lambda^F)$ are standard deviation of λ^S and λ^F, respectively. Comparing (9) and (8), the difference between y_2^* and y_1^* is

$$\Delta y = y_2^* - y_1^* = b \cdot \rho_{S,F} \cdot \frac{\sigma(\lambda^F)}{\sigma(\lambda^S)}. \tag{10}$$

A simple relation can be derived as

$$\begin{cases} \mathrm{y}_2^* = \mathrm{y}_1^*, & \rho_{S,F} = 0 \\ \mathrm{y}_2^* > \mathrm{y}_1^*, & 0 < \rho_{S,F} \le 1 \\ \mathrm{y}_2^* < \mathrm{y}_1^*, & -1 \le \rho_{S,F} < 0. \end{cases} \tag{11}$$

Equations (10) and (11) indicate that the impact of fuel price on a Genco's trading plan depends on the value of $\rho_{S,F}$ and the ratio of $\sigma(\lambda^F)$ and $\sigma(\lambda^S)$. The value of $\rho_{S,F}$ decides the direction of changing optimal allocation ratio y_2^* relative to y_1^*, i.e., increasing, decreasing, or staying put. If $\rho_{S,F} > 0$ (i.e., electricity price and fuel price are positively correlated), a Genco would trade more electricity in spot market because the increase of fuel price (i.e., the increase of production cost) will be compensated by the increase of electricity price in the spot market (i.e., the increase of revenue on electricity spot market). If $\rho_{S,F} = 0$, i.e., electricity price is independent on fuel price, fuel markets impose no influence on the Genco's trading decisions. If $\rho_{S,F} < 0$ (i.e., electricity price and fuel price are negatively correlated), the Genco would sell more electricity in contract market instead of spot market, since the increase of the fuel price is accompanied with the decrease of electricity prices, which makes transactions in spot market less attractive. As for the ratio of $\sigma(\lambda^F)$ and $\sigma(\lambda^S)$, practical experiences indicate that electricity spot price fluctuates much more so than any fuel spot prices, i.e., normally $\sigma(\lambda^F) < \sigma(\lambda^S)$. If

$\sigma(\lambda^F)$ is relatively small compared to $\sigma(\lambda^S)$, $\Delta y \approx 0$ i.e., fuel markets would have very little influence on the Genco's trading decisions.

3) Considering the Risks of Electricity Contract Price, Electricity Spot Price, and Fuel Spot Price: If a Genco cannot confirm electricity contract prices when making scheduling, λ^B should be considered as a random variable with a estimated value $E(\lambda^B)$ and a estimated error expressed with standard deviation or variance of the electricity contract price ($\sigma(\lambda^B)$ or $Var(\lambda^B)$). When λ^B, λ^F, and λ^S are all considered as random variables, the corresponding optimal allocation ratio of the electricity spot market is

$$y_3^* = \frac{E(\lambda^S) - E(\lambda^B) + 2BeVar(\lambda^B) - 2BeCov(\lambda^S, \lambda^B)}{2BeVar(\lambda^S) + 2BeVar(\lambda^B) - 4BeCov(\lambda^S, \lambda^B)} + \frac{2BbeCov(\lambda^S, \lambda^F) - 2BbeCov(\lambda^B, \lambda^F)}{2BeVar(\lambda^S) + 2BeVar(\lambda^B) - 4BeCov(\lambda^S, \lambda^B)} \quad (12)$$

where $Var(\lambda^B) = \sigma^2(\lambda^B)$, $Cov(\lambda^S, \lambda^B) = \rho_{S,B}\sigma(\lambda^S)\sigma(\lambda^B)$ $Cov(\lambda^F, \lambda^B) = \rho_{B,F}\sigma(\lambda^F)\sigma(\lambda^B)$. $\sigma(\lambda^B)$ is a value estimated by a decision-maker that indicates a fluctuating scope accepted when he/she negotiates contracts with distribution companies or other electricity purchasers. This value is therefore very small compared with the fluctuate extent of spot prices ($\sigma(\lambda^S)$ and $\sigma(\lambda^F)$). Based on this point, those items that are related with $Var(\lambda^B)$, $Cov(\lambda^S, \lambda^B)$, or $Cov(\lambda^F, \lambda^B)$ would be small. Then (12) can be approximated as follows:

$$y_3^* \approx \frac{E(\lambda^S) - \lambda^B + 2BbeCov(\lambda^S, \lambda^F)}{2BeVar(\lambda^S)} = y_2^*.$$

That is to say, the uncertainty of electricity contract price has little effect on the Genco's decisions and therefore may be ignored. In what follows, electricity contract price is considered as a deterministic variable.

C. General Case—Electric Energy Allocation in an LMP Market

Assume that there are $n+1$ areas, i.e., $n+1$ price-counting points, in an electricity market. A Genco is located in Area 0; other areas are labeled from 1 to n. To simply the problem, suppose that only one transaction is considered in each trading approach. That is, the Genco has $n+2$ potential transactions: one local contract, n non-local contracts, and one spot transaction.

1) Energy Portfolio: A combination of different types of transactions traded in different trading approaches is called an energy portfolio. Let w_i be the proportion of total electric energy allocated to the ith transaction. The profit on the complete portfolio, denoted by C, is π_C, i.e.,

$$\pi_C = \sum_{i=0}^{n+1} w_i \pi_i \quad (13)$$

where π_i is the profit on the ith transaction when total energy is allocated to this transaction ($i = 0$ denotes local contract, $i = 1 \sim n$ denotes non-local contract, and $i = n+1$ denotes spot transaction). Taking expectation and variance of the total profit π_C, we get

$$E(\pi_C) = \sum_{i=0}^{n+1} w_i E(\pi_i) \quad (14)$$

$$Var(\pi_C) = \sum_{i=0}^{n+1}\sum_{j=0}^{n+1} w_i w_j \sigma_{ij} \quad (15)$$

where $E(\pi_i)$ is the expected value of π_i, and σ_{ij} is the covariance between π_i and π_j, which can be calculated as follows.

2) Return Characteristics: Suppose that there are M trading intervals during planning period (one trading interval can be one hour, one day, one week, one month, or even one year, depending on the planning horizontal). Trading time for each trading interval is t (hour). The following notation will be used:

i, j	index of the trading area or pricing node;
k	index of the trading interval;
$\lambda_{i,k}^B$	kth trading interval's electricity contract price signed with customers of Area i;
$\lambda_{i,k}^S$	kth trading interval's electricity spot price of Area i;
λ_k^F	kth trading interval's fuel spot price;
π_i	profit on the ith trade, $i = 0$ denotes local contract; $i = 1 \sim n$ denotes non-local contract; $i = n+1$ denotes spot transaction;
e_k	kth trading interval's trading energy.

According to the definition of profit, profit = revenue − cost, if all the electric energy is traded through the local contract, the corresponding profit during the contract period is

$$\pi_0 = \sum_{k=1}^{M} \left(\lambda_{0,k}^B - b\lambda_k^F\right) \cdot e_k.$$

If all the energy is traded in the ith non-local contract, the corresponding cost includes not only the production cost but also the congestion charge. Generally, congestion charges should be paid by the associated bilateral transaction. However, who (Gencos or energy purchasers) should pay how many percentage of the involved congestion charges depends on the specific market rules. That is, from a Genco's point of view, its congestion charge is between zero and the complete congestion charge of the associated bilateral transaction. In this paper, a factor $\beta (0 \le \beta \le 1)$, which is decided by a specific electricity market, is used to denote the payment proportion of the Genco. Therefore, the profit on the ith non-local contract is

$$\pi_i = \sum_{k=1}^{M} \left(\lambda_{i,k}^B - \beta\left(\lambda_{i,k}^S - \lambda_{0,k}^S\right) - b\lambda_k^F\right) \cdot e_k \quad (i = 1 \sim n).$$

If all the energy is sold in the spot market, the corresponding profit is

$$\pi_{n+1} = \sum_{k=1}^{M} \left(\lambda_{0,k}^S - b\lambda_k^F\right) \cdot e_k.$$

The corresponding expectation, variance, and covariance of the above profits are derived as follows:

$$E(\pi_0) = \sum_{k=1}^{M} \left[\lambda_{0,k}^B - bE\left(\lambda_k^F\right)\right] \cdot e_k \tag{16}$$

$$Var(\pi_0) = \sum_{k=1}^{M} (be_k)^2 \cdot Var\left(\lambda_k^F\right) \tag{17}$$

$$E(\pi_i) = \sum_{k=1}^{M} \left[\lambda_{i,k}^B + \beta E\left(\lambda_{0,k}^S\right) - \beta E\left(\lambda_{i,k}^S\right) - bE\left(\lambda_k^F\right)\right] \cdot e_k \quad (i = 1 \sim n) \tag{18}$$

$$Var(\pi_i) = \sum_{k=1}^{M} e_k^2 \left[\beta^2 Var\left(\lambda_{0,k}^S\right) + \beta^2 Var\left(\lambda_{i,k}^S\right) + b^2 Var\left(\lambda_k^F\right) - 2\beta^2 Cov\left(\lambda_{0,k}^S, \lambda_{i,k}^S\right) - 2b\beta Cov\left(\lambda_{0,k}^S, \lambda_k^F\right) + 2b\beta Cov\left(\lambda_{i,k}^S, \lambda_k^F\right)\right] \quad (i = 1 \sim n) \tag{19}$$

$$E(\pi_{n+1}) = \sum_{k=1}^{M} \left[E\left(\lambda_{0,k}^S\right) - bE\left(\lambda_k^F\right)\right] \cdot e_k \tag{20}$$

$$Var(\pi_{n+1}) = \sum_{k=1}^{M} e_k^2 \left[Var\left(\lambda_{0,k}^S\right) + b^2 Var\left(\lambda_k^F\right) - 2bCov\left(\lambda_{0,k}^S, \lambda_k^F\right)\right] \tag{21}$$

$$\sigma_{0i} = -\sum b\beta e_k^2 Cov\left(\lambda_k^F, \lambda_{0,k}^S\right) + \sum b\beta e_k^2 Cov\left(\lambda_k^F, \lambda_{i,k}^S\right) + \sum b^2 e_k^2 Var\left(\lambda_k^F\right) \quad (i = 1 \sim n) \tag{22}$$

$$\sigma_{0,n+1} = -\sum be_k^2 Cov\left(\lambda_k^F, \lambda_{0,k}^S\right) + \sum b^2 e_k^2 Var\left(\lambda_k^F\right) \tag{23}$$

$$\sigma_{i,n+1} = \sum e_k^2 \left[\beta Var\left(\lambda_{0,k}^S\right) - \beta Cov\left(\lambda_{i,k}^S, \lambda_{0,k}^S\right) - bCov\left(\lambda_k^F, \lambda_{0,k}^S\right) - b\beta Cov\left(\lambda_k^F, \lambda_{0,k}^S\right) + b\beta Cov\left(\lambda_k^F, \lambda_{i,k}^S\right) + b^2 Var\left(\lambda_k^F\right)\right] \quad (i = 1 \sim n) \tag{24}$$

$$\sigma_{i,j} = \sum e_k^2 \left[\beta^2 Var\left(\lambda_{0,k}^S\right) - \beta^2 Cov\left(\lambda_{i,k}^S, \lambda_{0,k}^S\right) - 2b\beta Cov\left(\lambda_k^F, \lambda_{0,k}^S\right) - \beta^2 Cov\left(\lambda_{j,k}^S, \lambda_{0,k}^S\right) + \beta^2 Cov\left(\lambda_{i,k}^S, \lambda_{j,k}^S\right) + b\beta Cov\left(\lambda_k^F, \lambda_{i,k}^S\right) - b\beta Cov\left(\lambda_k^F, \lambda_{j,k}^S\right) + b^2 Var\left(\lambda_k^F\right)\right] \quad (i, j = 1 \sim n). \tag{25}$$

The statistics of prices involved in above equations, i.e., $E(\lambda_{i,k}^S)$, $Var(\lambda_{i,k}^S)$, $E(\lambda_k^F)$, $Var(\lambda_k^F)$, $Cov(\lambda_{i,k}^S, \lambda_{j,k}^S)$, and $Cov(\lambda_{i,k}^S, \lambda_k^F)$, can be estimated based on historical data according to the statistical method.

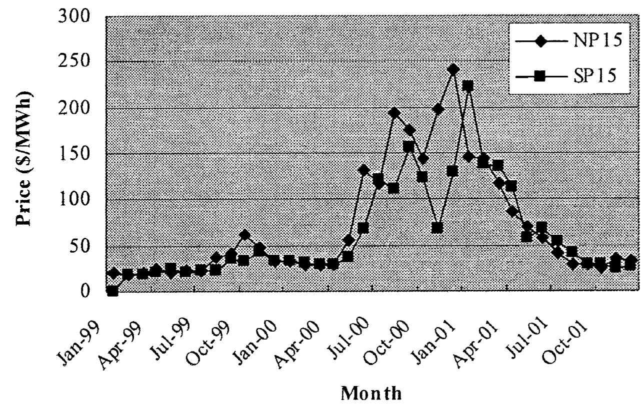

Fig. 1. Monthly spot price of electricity in California.

3) Optimal Allocation: The Genco's objective, maximizing profit and minimizing risk, can be achieved by maximizing its utility level, i.e.,

$$Max \quad U = E(\pi_C) - B \cdot Var(\pi_C). \tag{26}$$

Substituting $E(\pi_C)$ and $Var(\pi_C)$ with (14) and (15), respectively, the optimization problem becomes

$$\begin{aligned} \underset{w_i}{Max} \quad & U(w_0, w_1, \cdots, w_i, \cdots, w_{n+1}) \\ & = \sum_{i=0}^{n+1} w_i E(\pi_i) - B \sum_{i=0}^{n+1} \sum_{j=0}^{n+1} w_i w_j \sigma_{ij} \\ \text{s.t.} \quad & \sum_{i=0}^{n+1} w_i = 1 \end{aligned} \tag{27}$$

where B, $E(\pi_i)$, and σ_{ij} are calculated with (4) and (16)–(25). This optimization problem is a quadratic programming problem. There are standard algorithms [15] to solve it.

IV. Example

A Genco is making a long-term (say, a year) transaction plan. There are three approaches for this Genco to sell electricity: a one-year bilateral contract signed with local consumers; a one-year bilateral contract signed with non-local consumers; and the spot market. The Genco should decide the electric energy allocation ratio to each trading approach given electricity prices of bilateral contracts. Two cases are simulated in this section: 1) gas-fired plants' trading strategy and 2) coal-fired plants' trading strategy.

A. Trading Strategy of Gas-Fired Plants

Assume that the Genco owns a gas-fired generation unit with 600 MW capacity and 9.4 MBtu/MWh heat rates [16]. The following numerical simulation is performed based on historical data of electricity spot price of the California electricity market [17] and gas price sold to electricity utility in California [18]. Assume that this Genco's unit is located in Area 1 (SP15) and non-local consumers are located in Area 2 (NP15). Monthly spot price of electricity of Area 1 and 2 are shown in Fig. 1 [$E(\lambda_1^S) =$

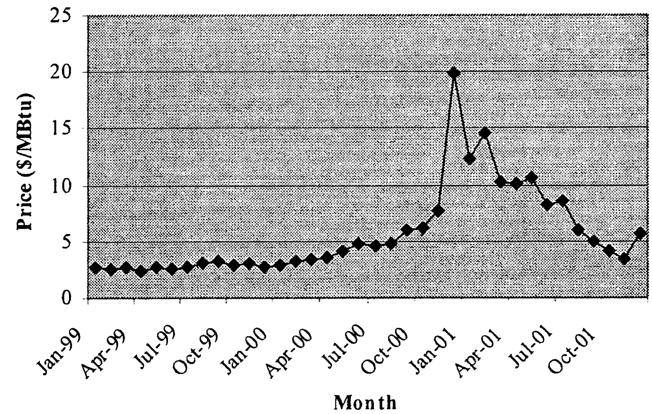

Fig. 2. Monthly spot price of natural gas.

TABLE I
DETERMINISTIC VERSUS RANDOM (GAS PRICES)

Contract prices	Gas prices	Allocation Ratios		
		Non-local contract	Local contract	Spot market
$\lambda_1^B = 58$	Deterministic	0.4175	0.3194	0.2632
$\lambda_2^B = 68$	Random	0.1237	0.1288	0.7476
$\lambda_1^B = 59$	Deterministic	0.2431	0.5912	0.1657
$\lambda_2^B = 69$	Random	0.1465	0.1579	0.6957

60.87, $\sigma(\lambda_1^S) = 58.2$ (95.62%), $E(\lambda_2^S) = 70.96$, $\sigma(\lambda_2^S) = 71.51$ (100.76%)]. Monthly spot price of gas from 1999 to 2001 is shown in Fig. 2 with $E(\lambda^F) = 5.65$ and $\sigma(\lambda^F) = 4.51$ (79.71%). Correlation coefficient between electricity spot price of Area 1 and gas price is calculated as $\rho_{S,F} = 0.77$ based on historical data.

1) Deterministic versus Random: Assume that $A = 3$, $\beta = 1$, for different contract prices, two situations: 1) gas prices are deterministic and 2) gas prices are random, are simulated with the developed energy allocation approach. Simulation results are shown in Table I, which indicates that more electric energy will be allocated to the spot market when the risk of fuel price is considered, as compared to the situation of fixed fuel price. The reason is that gas prices fluctuate quite significantly, and this risk can be better controlled by allocating more electric energy in the spot market, since the electricity spot price is positively correlated with the gas price ($\rho_{S,F} = 0.77 > 0$). Besides, higher contract prices result in less energy allocated to the spot market.

2) Impact of Risk Aversion on Trading Plan: Figs. 3 and 4 are simulation results of the relationship between allocation ratios and degrees of risk aversion when gas prices are deterministic and random, respectively. It is demonstrated that a Genco's trading decision depends on its risk preference to a great extent, and a relatively higher fluctuation of the gas price leads to different trading plan compared to the situation that gas prices are fixed.

Fig. 3 indicates that when gas price is deterministic, the allocation ratio to local contract increases toward 1, while the allocation ratios to non-local contract and spot market reduce toward 0 as the risk aversion increases. In other words, if the Genco is extremely risk averse, all the electric energy would be traded in the risk-free local contract, i.e., the Genco would not participate in the spot market. When the risk of gas price

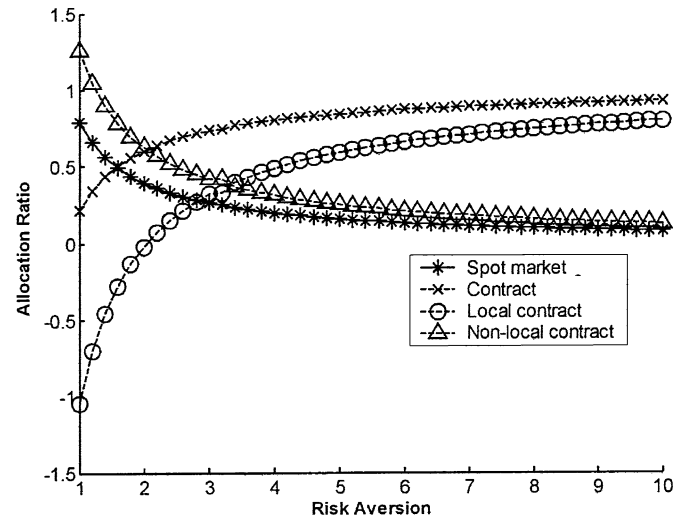

Fig. 3. Impact of risk aversion on allocation ratios (gas prices are deterministic).

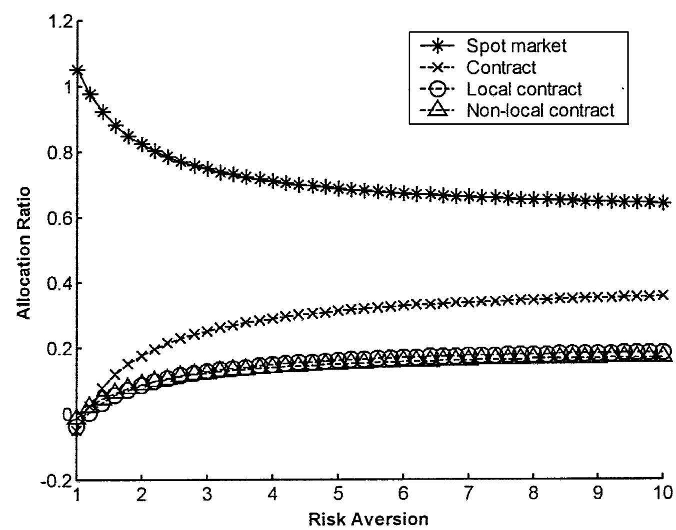

Fig. 4. Impact of risk aversion on allocation ratios (gas prices are random).

is considered, Fig. 4 shows that allocation ratio to spot market decreases and allocation ratios to local contract and non-local contract increase with the increase of risk aversion. However, still about 60% and 20% electric energy are allocated to the spot market and non-local contract, respectively, since these three transactions, including local contract, are all risky, and a combination of them is needed to reduce the total risk level.

B. Trading Strategy of Coal-Fired Plants

Assume that this Genco owns a coal-fired generation unit with 600 MW capacity and 8.9MBtu/MWh heat rates [16]. The following numerical simulation is performed based on historical data of electricity spot prices of the PJM electricity market[3] and coal prices in the U.S. [18]. The Genco's unit is located in Area 1 (PENELEC), and non-local consumers are located in Area 2 (PECO). Monthly spot price of electricity of Area 1, Area 2, and coal are showed in Figs. 5 [$E(\lambda_1^S) = 28.64$, $\sigma(\lambda_1^S) = 13.09$ (45.71%), $E(\lambda_2^S) = 29.47$, $\sigma(\lambda_2^S) = 14.28$ (48.46%)] and 6

[3]Monthly spot prices are integrated based on the hourly integrated LMP data, which are available on http://www.pjm.com.

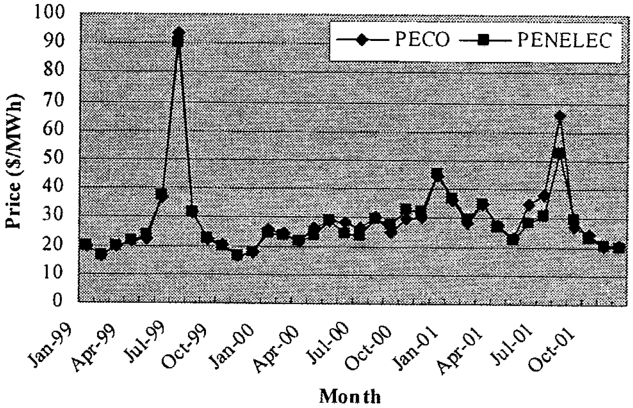

Fig. 5. Monthly spot price of electricity in PJM.

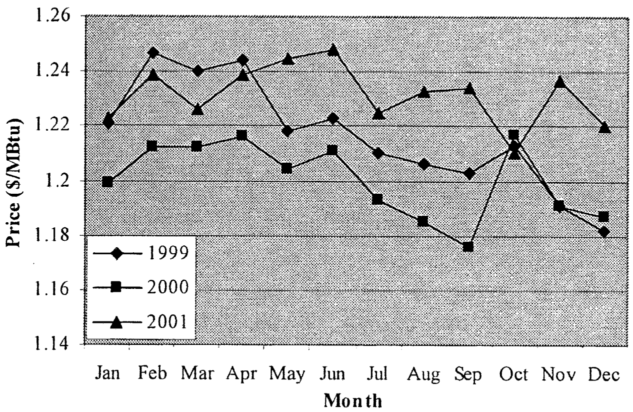

Fig. 6. Monthly spot price of coal.

TABLE II
DETERMINISTIC VERSUS RANDOM (COAL PRICES)

Fuel prices	Allocation Ratio		
	Non-local contract	Local contract	Spot market
Deterministic	0.3735	0.2032	0.4233
Random	0.3576	0.2183	0.4241

$[E(\lambda^F) = 1.2161, \sigma(\lambda^F) = 0.019462$ (1.6%)], respectively. The correlation coefficient of electricity spot price of Area 1 and coal price is calculated as $\rho_{S,F} = 0.06$.

Assume that $A = 3$, $\beta = 1$, $\lambda_1^B = 27$, $\lambda_2^B = 27.9$, two situations: 1) coal prices are deterministic and 2) coal prices are random, are simulated. Simulation results (shown in Table II) indicate that the fluctuation of the coal price imposes very little impact on the Genco's trading plan (the correlation degree between electricity price and coal price is very low, i.e., $\rho_{S,F} = 0.06$), although the allocation ratio to the spot market is actually increased a little (due to the positive correlation between electricity price and coal price). For different risk aversion degree, the energy allocation with the coal price risk is almost the same as that without the coal price risk (shown in Fig. 7).

C. Comparison of Trading Strategies

The following comparisons are made based on the situation that fuel prices are uncertain. As the Genco's risk aversion degree increases, the gas-fired plant allocates less energy to the spot market but more energy to the contract market, including local contract and non-local contract (see Fig. 4), while the coal-fired plant allocates more energy to local contract and less energy to non-local contract and the spot market (see Fig. 7). A common point of the trading strategies of these two plants is that the allocation ratio to the spot market decreases and the ratio to local contract increases as the Genco's risk aversion degree increases. The differential point is that the allocation ratio to non-local contract increases in gas-fired plant's trading strategy but decreases in coal-fired plant's trading strategy as risk aversion degree increases. This can be explained as follows. For the gas-fired plant, spot market, local contract, and non-local contract have comparable risk degrees (see Table III). As the risk aversion degree increases, which means that the Genco desires to reduce the risk of the complete portfolio further, more electric energy is needed to be traded with non-local contract since it is negatively correlated to the spot market and useful to reduce the risk of the complete portfolio. For the coal-fired plant, the risk of local contract is relatively small compared to the risks of non-local contract and spot market (see Table IV) and can be

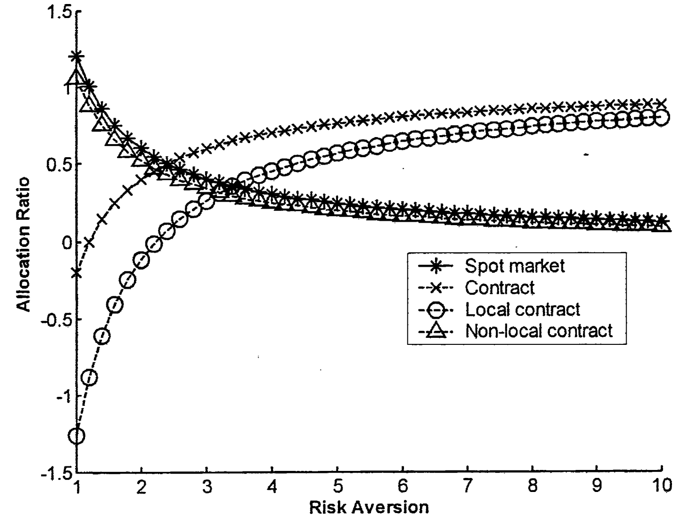

Fig. 7. Impact of risk aversion on allocation ratios (coal prices are deterministic or random).

TABLE III
COVARIANCE AMONG TRANSACTIONS (GAS-FIRED PLANT)

Transaction	Non-local contract	Spot market	Local contract
Non-local contract	6.2959×10^{15}	-1.0267×10^{15}	4.735×10^{15}
Spot market	-1.0267×10^{15}	3.1013×10^{15}	-2.3262×10^{14}
Local contract	4.735×10^{15}	-2.3262×10^{14}	4.0191×10^{15}

TABLE IV
COVARIANCE AMONG TRANSACTIONS (COAL-FIRED PLANT)

Transaction	Non-local contract	Spot market	Local contract
Non-local contract	1.8132×10^{13}	2.1375×10^{12}	4.4801×10^{11}
Spot market	2.1375×10^{12}	3.9494×10^{14}	-2.4076×10^{11}
Local contract	4.4801×10^{11}	-2.4076×10^{11}	7.4949×10^{10}

ignored. In other words, local contract can be considered as a risk-free transaction. Therefore, as the degree of risk aversion increases, more energy is allocated to the risk-free local contract, while less energy is allocated to risky trading approaches, including, of course, the non-local contract.

V. Conclusion

When making trading plan in markets involving risks, a Genco has to make tradeoff between the expected benefit and risk. This paper described this tradeoff with a utility function in which a risk-penalty factor is used to indicate the degree of a Genco's risk-aversion. According to the utility theory, this paper proposed a method to determine the value of this factor when "profit" is used to denote Gencos' rewards or benefits on electricity trading. Based on that, this paper developed an analytical and quantitative approach to electric energy allocation between spot and contract markets taking into considerations the risks of fluctuation in fuel price, electricity price and congestion charge.

Analyses and simulation results demonstrated that the impact of fuel market on electric energy allocation depends on the correlation degree between the fuel spot price and electricity spot price as well as the fluctuation degree of fuel prices compared to that of electricity prices. A higher positive correlation leads to more electricity traded in the spot market. The risk of fuel prices can be ignored if it is relatively small compared to that of electricity prices. Obviously, simulation results confirm that the proposed analytic approach is consistent with intuition and helpful for a Genco to achieve an optimal trading plan in electricity markets involving risks.

In this paper, the production cost of a generation unit was simplified, and improvement can be made by adopting a more practical model of the production cost in the future work. Furthermore, risk management (i.e., trading plan in this paper) as a part of the complete operation decision-making of a Genco is considered individually at present. The future work could be an integration of spot price forecasting, unit commitment, risk management, and participants' bidding strategy.

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