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# Manifestly Covariant Canonical Formulation of Yang-Mills Field Theories. II[†]

## ——*SU*(2) *Higgs-Kibble Model with Spontaneous Symmetry Breaking*——

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The proporties of asymptotic fields are fully analysed for the $SU(2)$ Higgs-Kibble model with spontaneous symmetry breaking in covariant gauges. A general procedure to determine the symmetry transformations induced on asymptotic fields is presented for an arbitrary conserved charge. On the basis of the derived formula, the explicit asymptotic forms of $Q_B$ and $Q_c$ are obtained.

## § 1. Introduction

In the preceding paper[1] (hereafter we will refer to as I) we have presented a general formalism of the canonical theory of Yang-Mills (YM) fields in manifestly covariant manner. The detailed analysis of the asymptotic fields and the determination of the asymptotic forms of $Q_B$ and $Q_c$ have been left undone in I. It is the purpose of this second paper to present them in a typical YM model with the spontaneous breaking of local gauge symmetry. The model discussed here is the $SU(2)$ Higgs-Kibble model[2] in covariant gauges. The Lagrangian density is

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_s(A,\Psi) - i\partial^\mu \bar{c}^a D_\mu{}^{ab} c^b - \partial^\mu B^a \cdot A_\mu{}^a + \alpha_0 B^a B^a/2\,,\\ \mathcal{L}_s(A,\Psi) &= -\tfrac{1}{4}(\partial_\mu A_\nu{}^a - \partial_\nu A_\mu{}^a + g\varepsilon^{abc} A_\mu{}^b A_\nu{}^c)^2\\ &\quad + |\partial_\mu \Psi - \tfrac{1}{2} i g \tau^a A_\mu{}^a \Psi|^2 - V(\Psi^\dagger \Psi)\,, \end{aligned} \tag{1·1}$$

where $\Psi$ is a complex isospinor scalar field and, needless to say,

$$D_\mu{}^{ab} c^b \equiv (\partial_\mu \delta^{ab} + g\varepsilon^{acb} A_\mu{}^c) c^b \equiv (\partial_\mu c + g A_\mu \times c)^a\,.$$

The potential part $V(\Psi^\dagger\Psi)$ is adjusted so that the vacuum expectation value of $\Psi$ becomes $\langle\Psi\rangle = \binom{v}{0}/\sqrt{2}$. So the field $\Psi$ is parametrized as follows[2] in terms of $\phi$, called Higgs scalar, and $\chi^a\,(a=1,2,3)$, called Goldstone bosons:

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$$\Psi(x)=\frac{1}{\sqrt{2}}[(v+\psi(x))+i\chi^a(x)\tau^a]\begin{pmatrix}1\\0\end{pmatrix}. \qquad (1\cdot2)$$

As explained in I generally, this system possesses the symmetries under the scale transformation of the Faddeev-Popov (FP) ghost fields and the Becchi-Rouet-Stora (BRS) transformation.[3] Corresponding to the former transformation, one has the conserved charge $Q_c$:

$$Q_c=i\int d^3x(\bar{c}\overleftrightarrow{\partial}_0 c+g\bar{c}\cdot A_0\times c), \qquad (1\cdot3)$$

where $f\overleftrightarrow{\partial}_0 g\equiv f(\partial_0 g)-(\partial_0 f)g$. The BRS transformation in this case is

$$\begin{aligned}
&\delta A_\mu=\delta\lambda D_\mu c\,,\\
&\delta c=-\delta\lambda(g/2)c\times c\,,\\
&\delta\bar{c}=i\delta\lambda B\,,\\
&\delta B=0\,,\\
&\delta\psi=-\delta\lambda(g/2)\chi\cdot c\,,\\
&\delta\chi=\delta\lambda(g/2)[(v+\psi)c+\chi\times c], \qquad (1\cdot4)
\end{aligned}$$

where $\delta\lambda=i\delta\varepsilon\, e^{\pi Q_c}$ ($\delta\varepsilon$: real $c$-number). The BRS charge

$$Q_B{}^0=\int d^3x[B\overleftrightarrow{\partial}_0 c+gB\cdot A_0\times c+i(g/2)\partial_0\bar{c}\cdot(c\times c)],$$

generates the BRS transformation (1.4), namely,

$$[i\delta\lambda Q_B{}^0,\Phi]=\delta\Phi\,, \qquad (1\cdot5)$$

and satisfies the nilpotency property $(Q_B{}^0)^2=0$. By virtue of this BRS invariance of the system, one can derive the $\Gamma$-WT identity, i.e., the Ward-Takahashi identity for the generating functional $\Gamma$ of the one-particle-irreducible (1PI) vertices[3),4)] (see (I.2·34) and (I.2·35)):

$$\delta\Gamma/\delta B=\partial^\mu A_\mu+\alpha_0 B\,, \qquad (1\cdot6a)$$

$$\frac{\delta\tilde{\Gamma}}{\delta A_\mu}\frac{\delta\tilde{\Gamma}}{\delta K^\mu}+\frac{\delta\tilde{\Gamma}}{\delta\psi}\frac{\delta\tilde{\Gamma}}{\delta K_\psi}+\frac{\delta\tilde{\Gamma}}{\delta\chi}\frac{\delta\tilde{\Gamma}}{\delta K_\chi}+\frac{\delta\tilde{\Gamma}}{\delta c}\frac{\delta\tilde{\Gamma}}{\delta L}=0\,, \qquad (1\cdot6b)$$

$$\partial^\mu\frac{\delta\tilde{\Gamma}}{\delta K^\mu}+i\frac{\delta\tilde{\Gamma}}{\delta\bar{c}}=0\,. \qquad (1\cdot6c)$$

Here

$$\tilde{\Gamma}\equiv\Gamma-(-\partial^\mu B\cdot A_\mu+\alpha_0 B^2/2)$$

and the source terms $K$'s and $L$ are present in the (extended) action in the following form:

$$\int d^4x[K_\mu D^\mu c+(g/2)\{-K_\psi\chi\cdot c+K_\chi\cdot[(v+\psi)c+\chi\times c]-L\cdot c\times c\}]. \quad (1\cdot 7)$$

The content of this paper is organized as follows: Section 2 is devoted to the detailed analysis of the asymptotic fields on the basis of the $\Gamma$-WT identity (1·6). This analysis is made on the assumption of the asymptotic completeness with respect to the "elementary" fields. We determine all the commutation relations of asymptotic fields and develop the LSZ formalism including the case of dipole field which appears in the non-Landau gauges, $\alpha_0\neq 0$. In § 3, we deal, in quite a general manner, with the problem of how to determine the transformation induced upon the asymptotic fields by an arbitrary conserved charge $Q$. We will obtain a general formula to express the charge $Q$ in terms of the asymptotic fields. With the help of this general formula, the explicit asymptotic forms of the charges $Q_B$ and $Q_c$ are obtained in § 4. By these two results of this paper, the commutation relations of asymptotic fields and the asymptotic forms of $Q_B$ and $Q_c$, the proof of the physical $S$-matrix unitarity presented in I is really completed.

## § 2. Asymptotic fields and asymptotic states

The $\Gamma$-WT identity cited in § 1 furnishes us with information about 2-point functions. We begin with the definitions of 1PI-2-vertices (i.e., inverse propagators) in the momentum space:

$$\Gamma^{ab}_{\mu\nu}(k)\equiv\int d^4x\, e^{ik(x-y)}\frac{\delta^2\Gamma}{\delta A_a{}^\mu(x)\delta A_b{}^\nu(y)}\bigg|_0{}^{*)}$$

$$=\delta_{ab}\{(g_{\mu\nu}-k_\mu k_\nu/k^2)A(k^2)+B(k^2)k_\mu k_\nu/k^2\}, \quad (2\cdot 1)$$

$$\Gamma^{ab}_{A_\mu\chi}(k)\equiv i\delta_{ab}k_\mu C(k^2), \quad (2\cdot 2)$$

$$\Gamma^{ab}_{\chi\chi}(k)\equiv\delta_{ab}k^2F(k^2), \quad (2\cdot 3)$$

$$\Gamma^{ab}_{\bar{c}c}(k)\equiv -i\delta_{ab}k^2(1+\gamma(k^2)). \quad (2\cdot 4)$$

At the tree level, these functions $A$, …, $F$, $\gamma$ reduce to

$$A(k^2)=M^2-k^2,\quad B(k^2)=M^2,$$

$$C(k^2)=M,\quad F(k^2)=1,\quad \gamma(k^2)=0,$$

where $M\equiv gv/2$. Equation (1·6a), $\delta\Gamma/\delta B=\partial A+\alpha_0 B$, brings, at once, the following 2-vertices:

$$\Gamma_{A_\mu,B}(k)\equiv\int dx\, e^{ik(x-y)}\frac{\delta^2\Gamma}{\delta A^\mu(x)\delta B(y)}\bigg|_0=ik_\mu, \quad (2\cdot 5)$$

$$\Gamma_{B,B}(k)=\alpha_0,\quad \Gamma_{B,\chi}(k)=0. \quad (2\cdot 6)$$

*) $\cdots|_0$ represents to take the value setting all the arguments equal to zero.

Next, we need some 'partially amputated' Green's functions:

$$\frac{\delta^2 \widetilde{\Gamma}}{\delta K_\mu{}^a(x)\,\delta c^b(y)}\bigg|_0 = \int dz \langle 0|T(\partial^\mu c_a(x)\,\bar{c}_c(z) + g\varepsilon_{ade}A_d{}^\mu(x)\,c_e(x)\,\bar{c}_c(z))\,|0\rangle$$

$$\times\langle 0|T(c_c(z)\,\bar{c}_b(y))\,|0\rangle^{-1},^{*)}$$

$$\int dx\, e^{ik(x-y)}\frac{\delta^2 \widetilde{\Gamma}}{\delta K_\mu{}^a(x)\,\delta c^b(y)}\bigg|_0 \equiv -ik^\mu \delta_{ab} + g\varepsilon_{acd}\Gamma^{cdb}_{A^\mu c\bar{c}}(k)\,. \tag{2·7}$$

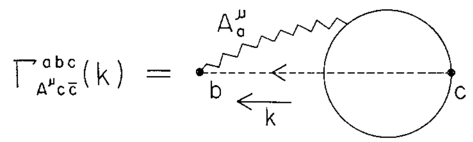


Fig. 1. The diagrammatical representation of the vertex function $\Gamma^{abc}_{A^\mu c\bar{c}}(k)$.

Diagrammatically, $\Gamma^{abc}_{A^\mu c\bar{c}}(k)$ is represented in Fig. 1, where the FP ghost propagator is amputated at the end point $c$. In a similar manner,

$$\frac{\delta^2 \widetilde{\Gamma}}{\delta K_\chi{}^a(x)\,\delta c^b(y)}\bigg|_0 = \int dx \langle 0|T([v+\phi(x)]\,c^a(x)\,\bar{c}^e(z) + \varepsilon_{acd}\chi^c(x)\,c^d(x)\,\bar{c}^e(z))|0\rangle$$

$$\times\langle 0|T(c^e(z)\,\bar{c}^d(y))\,|0\rangle^{-1},$$

and in the momentum space,

$$F.T.\frac{\delta^2 \widetilde{\Gamma}}{\delta K_\chi{}^a\delta c^b}\bigg|_0 = M\delta_{ab} + \frac{g}{2}\Gamma^{ab}_{\phi c\bar{c}}(k) + \frac{g}{2}\varepsilon_{acd}\Gamma^{cdb}_{\chi c\bar{c}}(k)$$

$$\equiv (M+\zeta(k^2))\,\delta_{ab}\,, \tag{2·8}$$

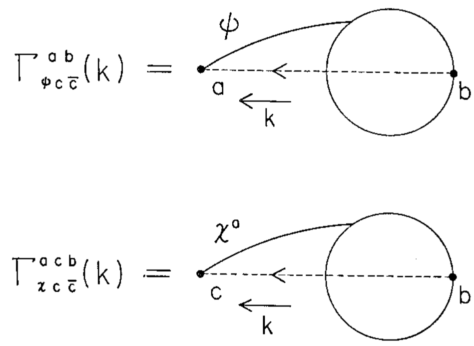


Fig. 2. The diagrams representing the vertices $\Gamma^{ab}_{\phi c\bar{c}}(k)$ and $\Gamma^{acb}_{\chi c\bar{c}}(k)$.

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*) $\langle 0|T(\cdots)\,|0\rangle^{-1}$ represents the inverse of $\langle 0|T(\cdots)\,|0\rangle$ in the sense of functionals, i.e., $\int dz \times\langle 0|T(c(x)\,\bar{c}(z))\,|0\rangle^{-1}\langle 0|T(c(z)\,\bar{c}(y))\,|0\rangle=\delta^4(x-y)$, and $\langle 0|T(c_a(x)\,\bar{c}_b(y)\,|0\rangle^{-1}=-i(\delta^2\Gamma/\delta\bar{c}_a(x)\,\delta c_b(y))$.

where the vertices $\Gamma^{ab}_{\psi c\bar{c}}$ and $\Gamma^{acb}_{\chi c\bar{c}}$ are depicted in Fig. 2.

Then, it follows from Eq. (1·6c) that

$$-\partial_x{}^\mu \frac{\delta^2\tilde{\Gamma}}{\delta K_\mu{}^a(x)\delta c^b(y)}\Big|_0 + i\frac{\delta^2\tilde{\Gamma}}{\delta\bar{c}^a(x)\delta c^b(y)}\Big|_0 = 0$$

or

$$ik^\mu(-ik_\mu\delta_{ab} + g\varepsilon_{acd}\Gamma^{cdb}_{A_\mu c\bar{c}}(k)) - k^2(1+\gamma(k^2))\delta_{ab} = 0\,,$$

and we obtain

$$g\varepsilon_{acd}\Gamma^{cdb}_{A_\mu c\bar{c}}(k) = -ik_\mu\gamma(k^2)\delta_{ab}\,. \tag{2·9}$$

Equation (1·6b) operated by $\delta/\delta c|_{c=\bar{c}=0}$ is

$$\left(\frac{\delta\tilde{\Gamma}}{\delta A_c{}^\nu}\frac{\delta^2\tilde{\Gamma}}{\delta K_\nu{}^c\delta c^b} + \frac{\delta\tilde{\Gamma}}{\delta\chi^c}\frac{\delta^2\tilde{\Gamma}}{\delta K_\chi{}^c\delta c^b} + \frac{\delta\tilde{\Gamma}}{\delta\psi}\frac{\delta^2\tilde{\Gamma}}{\delta K_\psi\delta c^b}\right)\Big|_{c=\bar{c}=0} = 0\,. \tag{2·10}$$

We obtain the following two equations, differentiating (2·10) with respect to $A_\mu{}^a$ and $\chi^a$ and taking account of (2·7), (2·8),

$$\Gamma^{ac}_{\mu\nu}[(-ik^\nu)\delta_{cb} + g\varepsilon_{cde}\Gamma^{deb}_{A^\nu c\bar{c}}(k)] + \Gamma^{ac}_{A_\mu\chi}(M+\zeta(k^2))\delta_{cb} = 0$$

and

$$\Gamma^{ac}_{\chi A_\nu}(-ik^\nu)(1+\gamma(k^2))\delta_{cb} + \Gamma^{ac}_{\chi\chi}(M+\zeta(k^2))\delta_{cb} = 0\,.$$

By (2·1) and (2·9), the former reduces to

$$B(k^2)(1+\gamma(k^2)) = C(k^2)(M+\zeta(k^2))\,, \tag{2·11a}$$

and the latter gives us, by (2·2) and (2·3),

$$C(k^2)(1+\gamma(k^2)) = F(k^2)(M+\zeta(k^2))\,. \tag{2·11b}$$

And thus,

$$B(k^2)F(k^2) = C^2(k^2)\,. \tag{2·12}$$

Now, the above equations and the symmetry properties (global $SU(2)$*) and ghost number conservation) tell us that inverse propagators are brought together into the following form:

$\Gamma^{(2)}_{ab} = \delta_{ab}\times$

*) It should be noted that this $SU(2)$ Higgs-Kibble model retains a global $SU(2)$ symmetry even after the breakdown of local symmetry, where $\psi$ and $\chi^a$ are iso-singlet and triplet, respectively.

$$
\begin{array}{c} \\ A_\mu \\ B \\ \chi \\ c \\ \bar{c} \end{array}
\begin{array}{ccccc}
A_\nu & B & \chi & c & \bar{c} \\
\end{array}
$$

$$
\begin{array}{c|ccccc|}
 & A_\nu & B & \chi & c & \bar{c} \\
A_\mu & (g_{\mu\nu}-k_\mu k_\nu/k^2)A(k^2)+B(k^2)k_\mu k_\nu/k^2 & ik_\mu & ik_\mu C(k^2) & 0 & 0 \\
B & -ik_\nu & \alpha_0 & 0 & 0 & 0 \\
\chi & -ik_\nu C(k^2) & 0 & k^2F(k^2) & 0 & 0 \\
c & 0 & 0 & 0 & 0 & ik^2(1+\gamma(k^2)) \\
\bar{c} & 0 & 0 & 0 & -ik^2(1+\gamma(k^2)) & 0
\end{array} . \tag{2·13}
$$

We have omitted here the parts containing $\psi$ which is decoupled from others.

Thus, inverting the matrix $\Gamma^{(2)}$, we obtain propagators:

$$
i^{-1}F.T.\langle 0|T(\Phi_i\Phi_j)|0\rangle = (\Gamma^{(2)})^{-1}_{ij}
$$

$$
= \begin{array}{c|ccccc|}
 & A_\nu & B & \chi & c & \bar{c} \\
A_\mu & \left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\Big/A(k^2) - \alpha_0\frac{k_\mu k_\nu}{(k^2)^2} & i\frac{k_\mu}{k^2} & i\alpha_0 k_\mu\frac{C(k^2)}{(k^2)^2F(k^2)} & 0 & 0 \\
B & -ik_\nu/k^2 & 0 & -C(k^2)/k^2F(k^2) & 0 & 0 \\
\chi & -i\alpha_0 k_\nu\frac{C(k^2)}{(k^2)^2F(k^2)} & -\frac{C(k^2)}{k^2F(k^2)} & \frac{1}{k^2F(k^2)}-\alpha_0\frac{B(k^2)}{(k^2)^2F(k^2)} & 0 & 0 \\
c & 0 & 0 & 0 & 0 & \frac{i}{k^2(1+\gamma(k^2))} \\
\bar{c} & 0 & 0 & 0 & -\frac{i}{k^2(1+\gamma(k^2))} & 0
\end{array} , \tag{2·14}
$$

where use has been made of the WT relation (2·12).

Next, one can deduce, from (2·14), the vacuum expectation values of commutation relations:

$$
\begin{aligned}
\langle 0|[A_\mu{}^a(x), A_\nu{}^b(y)]|0\rangle = \delta_{ab}[&-iZ_3(g_{\mu\nu}+m^{-2}\partial_\nu\partial_\nu)\varDelta(x-y;m^2) \\
&+iL\partial_\mu\partial_\nu D(x-y) - i\alpha_0\partial_\mu\partial_\nu E(x-y) \\
&-i\int_{+0}^\infty ds\,\sigma(s)\,(g_{\mu\nu}+s^{-1}\partial_\mu\partial_\nu)\,\varDelta(x-y;s)],
\end{aligned}
$$

$$
\langle 0|[A_\mu{}^a(x), B^b(y)]|0\rangle = -i\delta_{ab}\partial_\mu D(x-y),
$$

$$
\langle 0|[B^a(x), \chi^b(y)]|0\rangle = \delta_{ab}\left[-i\widetilde{M}D(x-y) - i\int_{+0}^\infty ds\,\sigma_{B\chi}(s)\,\varDelta(x-y;s)\right],
$$

$$
\langle 0|[A_\mu{}^a(x), \chi^b(y)]|0\rangle = \delta_{ab}\alpha_0\partial_\mu\Big[-i\widetilde{M}_1 D(x-y) + i\widetilde{M}E(x-y)
$$

$$-i\int_{+0}^{\infty}ds\,\sigma_{A\chi}(s)\,\Delta(x-y;s)\Big],$$

$$\langle 0|[\chi^a(x),\chi^b(y)]|0\rangle=\delta_{ab}\Big[(Z_\chi-\alpha_0\widetilde{M}_2)\,iD(x-y)+i\alpha_0\widetilde{M}^2E(x-y)$$

$$+i\int_{+0}^{\infty}ds\,\sigma_{\chi\chi}(s)\,\Delta(x-y;s)\Big],$$

$$\langle 0|[B^a(x),B^b(y)]|0\rangle=0,$$

$$\langle 0|\{c^a(x),\bar{c}^b(y)\}|0\rangle=\delta_{ab}\Big[-\widetilde{Z}_3D(x-y)-\int_{+0}^{\infty}ds\,\tilde{\sigma}(s)\,\Delta(x-y;s)\Big],\tag{2·15a}$$

where we have introduced the dipole invariant function $E(x)$,

$$E(x)\equiv-(\partial/\partial m^2)\,\Delta(x;m^2)\,|_{m^2=0}\,,\quad \Box E(x)=D(x).$$

In addition, because of the symmetry properties,

$$\langle 0|[A_\mu{}^a(x),\psi(y)]|0\rangle=\langle 0|[B^a(x),\psi(y)]|0\rangle=\langle 0|[\chi^a(x),\psi(y)]|0\rangle=0,$$

$$\langle 0|[c^a(x),\psi(y)]|0\rangle=\langle 0|[\bar{c}^a(x),\psi(y)]|0\rangle=0,\tag{2·15b}$$

and, finally, we can write

$$\langle 0|[\psi(x),\psi(y)]|0\rangle=iZ_\psi\Delta(x-y;m_\psi{}^2)+i\int ds\sigma_{\psi\psi}(s)\,\Delta(x-y;s).\tag{2·15c}$$

In the above, $m^2$ are defined as the zero of $A(s)$, i.e., $A(m^2)=0$, and various quantities are defined as follows:*)

$$\begin{cases}Z_3{}^{-1}\equiv-dA(s)/ds|_{s=m^2}\,,\\ \sigma(s)\equiv\pi^{-1}\,\mathrm{Im}\,(A^{-1}(s))-Z_3\delta(s-m^2),\\ L\equiv A^{-1}(0),\end{cases}\tag{2·16}$$

$$\begin{cases}\widetilde{M}\equiv C(0)/F(0),\\ \sigma_{B\chi}(s)\equiv-(\pi s)^{-1}\,\mathrm{Im}\,(C(s)/F(s)),\end{cases}\tag{2·17}$$

$$\begin{cases}\widetilde{M}_1\equiv\dfrac{d}{ds}\Big(\mathrm{Re}\,\dfrac{C(s)}{F(s)}\Big)\Big|_{s=0}\,,\\ \sigma_{A\chi}(s)\equiv-s^{-1}\sigma_{B\chi}(s),\end{cases}\tag{2·18}$$

---

*) Here we denote $(f(s))^n$ as $f^n(s)$. And we notice that, from the CCR,

$$Z_3=1-\int_{+0}^{\infty}ds\sigma(s)\quad\text{and}\quad L=Z_3/m^2+\int_{+0}^{\infty}ds\sigma(s)/s.$$

$$\begin{cases} Z_\chi \equiv F^{-1}(0)\,, \\ \widetilde{M}_2 \equiv \dfrac{d}{ds}\left(\mathrm{Re}\,\dfrac{B(s)}{F(s)}\right)\Big|_{s=0}\,, \\ \sigma_{\chi\chi}(s) \equiv -(\pi s)^{-1}\,\mathrm{Im}\,(F^{-1}(s)) + \alpha_0(\pi s^2)^{-1}\,\mathrm{Im}\,(B(s)/F(s))\,, \end{cases} \tag{2·19}$$

$$\begin{cases} \widetilde{Z}_3 \equiv (1+\gamma(0))^{-1}\,, \\ \widetilde{\sigma}(s) \equiv -(\pi s)^{-1}\,\mathrm{Im}\,((1+\gamma(s))^{-1})\,. \end{cases} \tag{2·20}$$

As stated in the Introduction, we assume that the LSZ asymptotic conditions hold, and, the following limiting relations are supposed, in the sense of weak convergence:

$$\int d^3x\, \Phi_i(x)\overleftrightarrow{\partial}_0 f_i(x) \to \int d^3x \sqrt{Z_i}\,\phi_i^{\mathrm{ex}}(x)\overleftrightarrow{\partial}_0 f_i(x) \quad \text{as} \quad x_0 \to \mp\infty\,, \tag{2·21}$$

where $f_i(x)$ is a positive frequency solution of the free equation of motion and 'ex' stands for 'in' or 'out'. We have introduced $\Phi_i$ and $\phi_i^{\mathrm{ex}}$ as the representatives of the Heisenberg fields and the corresponding asymptotic fields which we are concerned with. The renormalization constant $Z_B$ for the $B$ field is defined as $1/L$ for convenience. These asymptotic fields are, naturally, supposed to have their supports in time-like and/or light-like regions in the momentum space, so their commutation relations should be $c$-numbers, according to the Greenberg-Robinson theorem.[5),*)] On the basis of this remark, we obtain, from (2·15) and (2·21), the commutation relations:

$$[A_\mu^{\mathrm{ex}}(x), A_\nu^{\mathrm{ex}}(y)] = \langle 0|[A_\mu^{\mathrm{ex}}(x), A_\nu^{\mathrm{ex}}(y)]|0\rangle$$
$$= -i(g_{\mu\nu}+m^{-2}\partial_\mu\partial_\nu)\Delta(x-y;m^2) + iK\partial_\mu\partial_\nu D(x-y) - i\alpha\partial_\mu\partial_\nu E(x-y)\,, \tag{2·22}$$

$$[A_\mu^{\mathrm{ex}}(x), B^{\mathrm{ex}}(y)] = -i\sqrt{K}\partial_\mu D(x-y)\,, \tag{2·23}$$

$$[B^{\mathrm{ex}}(x), B^{\mathrm{ex}}(y)] = 0\,, \tag{2·24}$$

$$[B^{\mathrm{ex}}(x), \chi^{\mathrm{ex}}(y)] = -iD(x-y)\,, \tag{2·25}$$

$$[A_\mu^{\mathrm{ex}}(x), \chi^{\mathrm{ex}}(y)] = -[\chi^{\mathrm{ex}}(x), A_\mu^{\mathrm{ex}}(y)]$$
$$= -i\alpha N\partial_\mu D(x-y) + i\alpha K^{-1/2}\partial_\mu E(x-y)\,, \tag{2·26}$$

$$[\chi^{\mathrm{ex}}(x), \chi^{\mathrm{ex}}(y)] = (1-2\alpha NK^{-1/2})\,iD(x-y) + i\alpha K^{-1}E(x-y)\,, \tag{2·27}$$

$$\{c^{\mathrm{ex}}(x), \bar{c}^{\mathrm{ex}}(y)\} = -D(x-y)\,, \tag{2·28}$$

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*) Greenberg-Robinson theorem might well be applicable to the case of indefinite-metric spaces on the assumption that Wightman functions can be analytically continued also in these cases. Further, even in the indefinite-metric cases, the asymptotic completeness concludes the irreducibility of field algebras $\{\phi_i^{\mathrm{ex}}\}$ by virtue of the Fock space structure.[6)]

$$[\psi^{\rm ex}(x),\psi^{\rm ex}(y)]=i\Delta(x-y;m_\psi^2)\,, \tag{2·29}$$

and, $c^{\rm ex}$ and $\bar{c}^{\rm ex}$ commute with $A_\mu^{\rm ex}$, $B^{\rm ex}$, $\chi^{\rm ex}$ and $\psi^{\rm ex}$, while $\psi^{\rm ex}$ does with $A_\mu^{\rm ex}$, $B^{\rm ex}$, $\chi^{\rm ex}$, $c^{\rm ex}$ and $\bar{c}^{\rm ex}$. We have defined, in the above,

$$K\equiv L/Z_3=(Z_3Z_B)^{-1}\,, \tag{2·30a}$$

$$\alpha\equiv\alpha_0/Z_3\,, \tag{2·30b}$$

$$N\equiv(Z_3/Z_\chi)^{1/2}\widetilde{M}_1=(\sqrt{K}Z_3/Z_\chi)\widetilde{M}_2/2\,, \tag{2·30c}$$

and have used the WT relation (2·12) for $k^2=0$, $B(0)F(0)=C^2(0)$, and the equalities,

$$A(0)=B(0)\,, \tag{2·31a}$$

$$\frac{d}{ds}\left(\mathrm{Re}\,\frac{B(s)}{F(s)}\right)\Big|_{s=0}=2\frac{C(0)}{F(0)}\frac{d}{ds}\left(\mathrm{Re}\,\frac{C(s)}{F(s)}\right)\Big|_{s=0}\,. \tag{2·31b}$$

The equality (2·31a) is implied by regularity of $\Gamma_{\mu\nu}(k)$ at $k^2=0$, which makes sure of the one-particle irreducibility of $\Gamma_{\mu\nu}(k)$.*) Equation (2·31b) can be derived, by help of $B(k^2)/F(k^2)=(C(k^2)/F(k^2))^2$ from (2·12), on the assumption $\mathrm{Im}(C(s)/F(s))|_{s=0}=0$, which is reasonable from the perturbative viewpoint. The last equality in (2·30c) is a consequence of (2·31b).

The asymptotic completeness, which we assume here, means that the asymptotic fields $A^{\rm ex}$, $\psi^{\rm ex}$, $B^{\rm ex}$, $\chi^{\rm ex}$, $c^{\rm ex}$ and $\bar{c}^{\rm ex}$ are complete without bound states. Therefore, we can deduce from (2·23)∼(2·29) the following equations of motion for the asymptotic fields by help of their irreducibility:

$$\Box B^{\rm ex}=\Box c^{\rm ex}=\Box\bar{c}^{\rm ex}=(\Box+m_\psi^2)\psi^{\rm ex}=0\,, \tag{2·32a}$$

$$\Box\chi^{\rm ex}=-(\alpha/K)B^{\rm ex}\,. \tag{2·32b}$$

From (2·32b), $\chi^{\rm ex}$ turns out to be a dipole ghost field except for the Landau gauge case ($\alpha=0$). In order to separate physical modes completely from unphysical ones, we introduce a field $U_\mu^{\rm ex}$ in the following manner:[7)]

$$U_\mu^{\rm ex}\equiv A_\mu^{\rm ex}-(\sqrt{K}-\alpha N)\partial_\mu B^{\rm ex}-\sqrt{K}\partial_\mu\chi^{\rm ex}\,. \tag{2·33}$$

Then, we obtain, from (2·23)∼(2·27),

$$[U_\mu^{\rm ex}(x),B^{\rm ex}(y)]=[U_\mu^{\rm ex}(x),\chi^{\rm ex}(y)]=0\,,$$

$$[U_\mu^{\rm ex}(x),c^{\rm ex}(y)]=[U_\mu^{\rm ex}(x),\bar{c}^{\rm ex}(y)]=[U_\mu^{\rm ex}(x),\psi^{\rm ex}(y)]=0\,. \tag{2·34}$$

*) Of course, inverse propagators are one-particle-irreducible only with respect to the 'elementary' fields $A, B, \chi, c, \bar{c}$ and $\psi$. $\Gamma_{\mu\nu}$ would no longer be regular at $k^2\sim0$, in the presence of massless composite particles, the possibility of which we have excluded here by assumption.

It is an easy task to check*)

$$(\Box + m^2)\, U_\mu^{\text{ex}}(x) = 0 \quad \text{and} \quad \partial^\mu U_\mu^{\text{ex}}(x) = 0\,, \tag{2·35}$$

$$[U_\mu^{\text{ex}}(x),\, U_\nu^{\text{ex}}(y)] = -i\,(g_{\mu\nu} + m^{-2}\partial_\mu\partial_\nu)\, \Delta(x-y;\, m^2)\,, \tag{2·36}$$

using (2·22), (2·23), (2·26) and (2·32) $\sim$ (2·34). Thus, $U_\mu^{\text{ex}}$ is the Proca field with mass $m$ and, as a consequence, $A_\mu^{\text{ex}}$ satisfies the following equation of motion:

$$(\Box + m^2)\, A_\mu^{\text{ex}} = [(\sqrt{K} - \alpha N)\, m^2 - \alpha\sqrt{K^{-1}}]\,\partial_\mu B^{\text{ex}} + \sqrt{K} m^2 \partial_\mu \chi^{\text{ex}}\,,$$

$$\partial^\mu A_\mu^{\text{ex}} + \alpha\sqrt{K^{-1}} B^{\text{ex}} = 0\,.$$

Now, all this information enables us to construct the Fock space of asymptotic fields, which is identified with the total state vector space $\mathcal{V}$ on the assumption of asymptotic completeness. For the fields other than $\chi^{\text{ex}}$, since they are simple pole fields, the creation and annihilation operators are defined in the usual manner:

$$\phi_k^{(i)} \equiv i\int d^3x\, f_k^{(i)*}(x)\, \overleftrightarrow{\partial}_0 \phi_i^{\text{ex}}(x) \equiv (f_k^{(i)},\, \phi_i^{\text{ex}})\,,$$

$$\phi_k^{(i)\dagger} = i\int d^3x\, \phi_i^{\text{ex}\dagger}(x)\, \overleftrightarrow{\partial}_0 f_k^{(i)}(x) = (\phi_i^{\text{ex}},\, f_k^{(i)})\,. \tag{2·37}$$

Here $\phi_i^{\text{ex}}$ represents generically the asymptotic fields of simple pole and the index $i$ discriminates the field variety. As for the wave packet state $f_k^{(i)}(x)$, we introduce the following complete sets of wave packets:

$\{g_k(x)\}$ for $B^{\text{ex}}$, $c^{\text{ex}}$, $\bar{c}^{\text{ex}}$ (and $\chi^{\text{ex}}$):

$$\Box g_k(x) = 0\,, \quad (g_k,\, g_l) = \delta_{kl}\,,$$

$$\sum_k g_k(x)\, g_k^*(y) = D_+(x-y)\,; \tag{2·38}$$

$\{f_\alpha^{\ \mu}(x)\}$ for $U_\mu^{\text{ex}}$:

$$(\Box + m^2) f_\alpha^{\ \mu}(x) = 0\,, \quad \partial_\mu f_\alpha^{\ \mu}(x) = 0\,, \quad (f_\alpha^{\ \mu},\, f_{\beta,\mu}) = -\delta_{\alpha\beta}\,,$$

$$\sum_\alpha f_\alpha^{\ \mu}(x) f_\alpha^{\ \nu*}(y) = -g^{\mu\nu}\Delta_+(x-y;\, m^2)\,; \tag{2·39}$$

$\{q_\rho(x)\}$ for $\varphi^{\text{ex}}$:

$$(\Box + m_\varphi^{\ 2})\, q_\rho(x) = 0\,, \quad (q_\rho,\, q_\sigma) = \delta_{\rho\sigma}\,,$$

$$\sum_\rho q_\rho(x)\, q_\rho^*(y) = \Delta_+(x-y;\, m_\varphi^{\ 2})\,. \tag{2·40}$$

For the dipole ghost fields $\chi^{\text{ex}}$, we need another wave packet system $\{h_k(x)\}$, besides the above $\{g_k(x)\}$, which satisfies

*) The relation $L = \widetilde{M}^{-2} Z_\chi$ necessary for the derivation of (2·36) is guaranteed by the WT relation (2·12) and the equality (2·31a). Note also that the equality stated in (2·30c) plays an important role in the consistency of (2·36) with (2·33), (2·26) and (2·27).

$$\Box h_k(x) = g_k(x). \tag{2·41}$$

As the wave packet $h_k$ satisfying (2·41),

$$(\nabla^2)^{-1}(x_0\partial_0 - \omega)\, g_k(x)/2$$

can serve for an arbitrary constant $\omega$. Therefore taking account of the relation

$$E(x) = (\nabla^2)^{-1}(x_0\partial_0 - 1)\, D(x)/2\,, \tag{2·42}$$

we choose $\omega = 1/2$ and

$$h_k(x) \equiv (1/2)\,(\nabla^2)^{-1}(x_0\partial_0 - 1/2)\, g_k(x). \tag{2·43}$$

Omitting the infrared cutoff procedure,[8] we obtain

$$\sum_k (h_k(x) g_k^*(y) + g_k(x) h_k^*(y)) = E_+(x-y). \tag{2·44}$$

Further, the equation

$$(g_k, h_l) + (h_k, g_l) = 0 \tag{2·45}$$

follows from the identity

$$E_+(x-y) = i\int d^3z [D_+(x-z)\overleftrightarrow{\partial}_0^z E_+(z-y) + E_+(x-z)\overleftrightarrow{\partial}_0^z D_+(z-y)]. \tag{2·46}$$

Then, the annihilation operator for the dipole field $\chi^{\text{ex}}$ is defined as

$$\begin{aligned}\chi_k^{\text{ex}} &= i\int d^3x\,(g_k^*(x)\overleftrightarrow{\partial}_0\chi^{\text{ex}}(x) + h_k^*(x)\overleftrightarrow{\partial}_0\Box\chi^{\text{ex}}(x))\\ &= i\int d^3x\,(g_k^*(x)\overleftrightarrow{\partial}_0\chi^{\text{ex}}(x) - (\alpha/K)\, h_k^*(x)\overleftrightarrow{\partial}_0 B^{\text{ex}}(x)),\end{aligned} \tag{2·47}$$

where (2·32b) has been used. From the relation as

$$\chi^{\text{ex}}(x) = -\int d^3y\,(D(x-y)\overleftrightarrow{\partial}_0^y\chi^{\text{ex}}(y) + E(x-y)\overleftrightarrow{\partial}_0^y\Box\chi^{\text{ex}}(y)), \tag{2·48}$$

$$iD(x) = D_+(x) - D_-(x) = D_+(x) - D_+(-x)\,;\; iE(x) = E_+(x) - E_-(x), \tag{2·49}$$

and from (2·44) and (2·47), $\chi^{\text{ex}}(x)$ is expanded as follows:

$$\chi^{\text{ex}}(x) = \sum_k (\chi_k^{\text{ex}} g_k(x) - (\alpha/K)\, B_k^{\text{ex}} h_k(x) + \text{h.c.}). \tag{2·50}$$

Of course, (2·50) is consistent with (2·32b).

In virtue of (2·24)$\sim$(2·29), (2·34) and (2·36), the definitions of the creation and annihilation operators, (2·37) and (2·47), lead to the following commutation relations (or the metric matrix $\eta_1$ for 1-particle state):

$$\eta_1 \equiv ([\phi_k^{(i)}, \phi_l^{(j)\dagger}]_\mp) = (\langle \phi_k^{(i)} | \phi_l^{(j)} \rangle)$$

$$= \begin{array}{c} \\ U_\alpha \\ \phi_\rho \\ \chi_k \\ B_k \\ c_k \\ \bar{c}_k \end{array} \begin{array}{cc|cc|cc} U_\beta & \phi_\sigma & \chi_l & B_l & c_l & \bar{c}_l \\ \delta_{\alpha\beta} & 0 & & 0 & & \\ 0 & \delta_{\rho\sigma} & & & & \\ \hline & & (1-2\alpha N K^{-1/2})\delta_{kl} & -\delta_{kl} & & 0 \\ & 0 & -\delta_{kl} & 0 & & \\ \hline & & & 0 & 0 & i\delta_{kl} \\ & & & & -i\delta_{kl} & 0 \end{array} . \qquad (2\cdot51)$$

In the derivation of $(2\cdot51)$, use has been made of $(2\cdot45)$. By $(2\cdot51)$, we finish the proof of (I.4·1) which was already utilized in I: In this model, the *physical particles* are the massive Proca field $U_\alpha$ and the Higgs scalar $\psi_\rho$. The other modes, $\chi_k$ (Goldstone bosons), $B_k$, $c_k$ and $\bar{c}_k$, span the *unphysical particle* sector.

Now, $S$-matrix is defined as

$$S|\alpha \text{ out}\rangle = |\alpha \text{ in}\rangle, \qquad (2\cdot52)$$

and the relations between $\phi^{\text{in}}$ and $\phi^{\text{out}}$ directly follows from $(2\cdot52)$ and the asymptotic completeness:

$$S\phi_{k,\text{out}}^{(i)\dagger}S^{-1} = \phi_{k,\text{in}}^{(i)\dagger}, \qquad (2\cdot53\text{a})$$

$$S\phi_{k,\text{out}}^{(i)}S^{-1} = \phi_{k,\text{in}}^{(i)}. \qquad (2\cdot53\text{b})$$

Here the following important points should be noted. First, the asymptotic condition $(2\cdot21)$ or the Yang-Feldman equation as an equivalent expression of $(2\cdot21)$ requires that the hermiticity properties of $\Phi_i$, $\phi_i^{\text{in}}$ and $\phi_i^{\text{out}}$ must be identical. Then, $(2\cdot53)$ and the above remark together with the irreducibility of $\{\phi_i^{\text{ex}}\}$ conclude that the full $S$-matrix should be (pseudo-) unitary: $S^\dagger S = SS^\dagger = 1$, under the condition $S|0\rangle = |0\rangle$. Thus, it turns out that the conventional (wrong) hermiticity assignment to the FP ghosts,[3] (I.1·5),

$$C^\dagger = \overline{C} \quad \text{and} \quad \overline{C}^\dagger = C,$$

which makes the Lagrangian non-hermitian and breaks down the (pseudo-) unitarity of $S$, damages the asymptotic condition $(2\cdot21)$. Consequently, the vacuum and/or the asymptotic fields and states could never be defined consistently: $S|0\rangle \neq |0\rangle$ and/or, e.g., $A_\mu^{\text{in}\dagger} = (SA_\mu^{\text{out}}S^{-1})^\dagger = S^{-1\dagger}A_\mu^{\text{out}\dagger}S^\dagger \neq SA_\mu^{\text{out}\dagger}S^{-1}$.

At the end of this section, we comment upon the renormalized Heisenberg fields and the reduction formulae. The formers are defined as

$$\Phi_i^r(x) \equiv Z_i^{-1/2}\Phi_i(x). \qquad (2\cdot54)$$

Besides the usual reduction formulae for $B$, $c$ and $\bar{c}$, we note here the ones for $\chi$

and $U_\mu$:

$$\langle \chi_k \alpha\ \text{out}|T(\cdots)|\beta\ \text{in}\rangle - \langle \alpha\ \text{out}|T(\cdots)\chi_k{}^{\text{in}}|\beta\ \text{in}\rangle$$

$$= i\int d^4x g_k{}^*(x)\,\Box^x\langle \alpha\ \text{out}|T(\chi^r(x)\cdots)|\beta\ \text{in}\rangle \tag{2·55a}$$

$$-i(\alpha/K)\int d^4x h_k{}^*(x)\,\Box^x\langle \alpha\ \text{out}|T(B^r(x)\cdots)|\beta\ \text{in}\rangle$$

$$+i(\alpha/K)\int d^4x g_k{}^*(x)\langle \alpha\ \text{out}|T(B^r(x)\cdots)|\beta\ \text{in}\rangle,$$

$$\langle U_\alpha \beta\ \text{out}|T(\cdots)|\gamma\ \text{in}\rangle - \langle \beta\ \text{out}|T(\cdots)|\gamma - U_\alpha\ \text{in}\rangle$$

$$= i\int d^4x f_\alpha{}^{\mu *}(x)\,(\Box^x + m^2)\langle \beta\ \text{out}|T(A_\mu{}^r(x)\cdots)|\gamma\ \text{in}\rangle. \tag{2·55b}$$

## § 3. Symmetry transformation induced on asymptotic fields

In this section, we consider, in quite a general manner, the problem how to determine the asymptotic form of any conserved charge $Q$ (i.e., the form of $Q$ in terms of the asymptotic fields), when the symmetry corresponding to $Q$ is not broken spontaneously. On the basis of the general formula obtained in this section, we will give the desired asymptotic forms of $Q_B$ and $Q_c$ in the next section.

The arguments essentially depend on the assumption of completeness of the asymptotic fields. Let us denote these asymptotic in-fields as $\phi_I{}^{\text{in}} \equiv \phi_i{}^{(m_i)\text{in}}$ (where $m_i$ is the mass of the $i$-th particle), and the corresponding renormalized Heisenberg fields as $\Phi_I{}^r \equiv \Phi_i{}^{(m_i)r}$, similarly to those in § 2. First, for simplicity, consider the case without dipole (or more higher multi-pole) ghost fields. Then, the (anti-) commutation relations and the equations of motion of $\phi_I{}^{\text{in}}$'s are generally written as*)

$$[\phi_i{}^{(m_i)}(x),\, \phi_j{}^{(m_j)}(y)]_\mp = i\delta_{m_i m_j}\eta_{ij}^{(m_i)}\Delta(x-y\,;\, m_i^2), \tag{3·1a}$$

$$(\Box + m_i^2)\,\phi_i{}^{(m_i)}(x) = 0\,. \tag{3·1b}$$

By using (3·1) and the properties of $S$-matrix,

$$S^{-1}\phi_I{}^{\text{in}} S = \phi_I{}^{\text{out}} \quad \text{and} \quad S|0\rangle = |0\rangle, \tag{3·2}$$

we can easily show that the LSZ asymptotic condition (2·21),

$$\int d^3x\, \Phi_I{}^r(x)\overleftrightarrow{\partial}_0 f_I(x) \xrightarrow[x_0\to\mp\infty]{} \int d^3x\, \phi_I^{\substack{\text{in}\\ \text{out}}}(x)\overleftrightarrow{\partial}_0 f_I(x),$$

*) Here in (3·1a) we assume that $\eta_{ij}^{(m_i)}$ are constant matrices containing no derivatives. This is always possible for simple pole fields.

leads to the following expression of $S$-matrix, called GLZ formula:[9),*)]

$$S=\sum_{n=0}^{\infty}\frac{i^n}{n!}\sum_{I_1\cdots I_n}\sum_{J_1\cdots J_n}\int\prod_{a=1}^{n}d^4x_a:\phi_{I_1}^{\rm in}(x_1)\phi_{I_2}^{\rm in}(x_2)\cdots\phi_{I_n}^{\rm in}(x_n):$$
$$\times\prod_{b=1}^{n}(\eta_{I_bJ_b}^{-1}K_{J_b}(x_b))\langle 0|T\Phi_{J_n}^r(x_n)\Phi_{J_{n-1}}^r(x_{n-1})\cdots\Phi_{J_1}^r(x_1)|0\rangle\,,\qquad(3\cdot3)$$

where $\eta_{IJ}^{-1}\equiv\delta_{m_im_j}\eta_{ij}^{(m_i)-1}$ and $K_I\equiv K_{m_i}\equiv\Box+m_i^2$. We can rewrite this in a form:

$$S=:\exp\left[\int d^4x\phi_I^{\rm in}(x)\eta_{IJ}^{-1}K_J(x)\delta/\delta J_J(x)\right]:\langle 0|T\exp i\int d^4yJ_I(y)\Phi_I^r(y)|0\rangle|_{J=0}$$
$$\equiv:\exp(\phi^T\eta^{-1}K\delta/\delta J):\langle 0|T\exp iJ^T\Phi|0\rangle|_{J=0}$$
$$\equiv:\mathcal{K}:\langle 0|T\exp iJ^T\Phi|0\rangle,\qquad(3\cdot4)$$

where we have introduced the Klein-Gordon matrix $K$ as $(K)_{IJ}=\delta_{IJ}K_J$, and $\mathcal{K}$ operation for functional $F[J]$ as

$$:\mathcal{K}:F[J]\equiv:\exp(\phi^T\eta^{-1}K\delta/\delta J):F[J]|_{J=0}\,.\qquad(3\cdot5)$$

In quite a similar manner we can prove

$$S\mathcal{O}=:\mathcal{K}:\langle 0|T\mathcal{O}\exp iJ^T\Phi|0\rangle\qquad(3\cdot6)$$

for any polynomial $\mathcal{O}$ of local operators. For example, by taking $\mathcal{O}=\Phi_I^r(x)$, (3·6) gives us the Heisenberg field $\Phi_I^r$ in terms of the asymptotic fields $\phi^{\rm in}$.

Now consider an arbitrary *conserved* charge $Q$ which generates the following transformation on the Heisenberg fields $\Phi_I^r$:

$$[i\delta\theta\cdot Q,\Phi_I^r(x)]=\delta\Phi_I^r(x)\,.\qquad(3\cdot7)$$

Here the transformation parameter $\delta\theta$ is a $c$-number if $Q$ is an ordinary charge, but it is an 'anti-commuting (and anti-hermitian) number' if $Q$ is a super-type charge such as the BRS charge $Q_B$ (see § 2 of I).

We investigate the transformation induced by (3·7) on the asymptotic fields:

$$[i\delta\theta\cdot Q,\phi_I^{\rm in}(x)]=\delta\phi_I^{\rm in}(x)\,.\qquad(3\cdot8)$$

The form of $\delta\phi_I^{\rm in}(x)$ will be determined in such a way that the transformation (3·8) reproduces the original transformation (3·7). Before going into that problem, we cite some WT identities related to the transformation (3·7). Since the transformation (3·7) leaves the action of the system invariant, similar arguments as was given in § 2 of I lead to the WT identity:

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*) We comment here that this simple GLZ formula would have to be changed in some complicated fashion if we had anti-hermitian fields such as Faddeev-Popov anti-ghost field $\overline{C}$ in the usual convention. This is the reason why we have adopted the convention of unfamiliar *hermitian* field $\bar{c}$ defined as $\bar{c}\equiv-i\overline{C}$ in §2 of Ref. I.

$$\int d^4x \langle 0|T \sum_I J_I(x)\, \delta\Phi_I{}^r(x) \exp i \int J_J \Phi_J{}^r |0\rangle = 0\,. \tag{3·9}$$

We denote it simply as

$$\langle 0|T J^T \delta\Phi \exp iJ^T\Phi|0\rangle = 0\,. \tag{3·10}$$

Differentiating this with respect to $J_K(x)$, we obtain

$$\langle 0|T(\delta\Phi_K{}^r(x) + i\Phi_K{}^r(x) J^T\delta\Phi) \exp iJ^T\Phi|0\rangle = 0\,. \tag{3·11}$$

When the operator $:\phi^T\eta^{-1}K\delta/\delta J:$ is applied to (3·10) and (3·11), the external sources $J_I$ are replaced by the Klein-Gordon operator $K$ with coefficient of on-shell function $\phi$: $J_I \to (\phi^T\eta^{-1}K)_I$. Then, the WT identities (3·10) and (3·11) can be rather simplified. Because of the presence of the operator $(\phi^T\eta^{-1}K)_I$ instead of $J_I(x)$, the fields $\delta\Phi_I(x)$, which generally contains non-linear terms of fields also, can be replaced by the *linear* sums of fields with *same mass* $m_i$:

$$\delta\Phi_i{}^{(m_i)}(x) \underset{\text{on-shell}}{\longrightarrow} \bar{\delta}\Phi_i{}^{(m_i)}(x) = \delta\theta \cdot A_{ij}^{(m_i)} \Phi_j{}^{(m_j=m_i)}(x)\,. \tag{3·12}$$

In fact, considering the Feynman-like diagram shown in Fig. 3(a) (which is depicted according to the rule due to 't Hooft and Veltman[10]), we can easily convince ourselves that the coefficient $A_{ij}^{(m_i)}$ in (3·12) is explicitly given as

$$\delta\theta \cdot A_{ij}^{(m_i)} = \sum_k \int d^4z \langle 0|T\delta\Phi_i{}^{(m_i)}(x)\, \Phi_k{}^{(m_i)}(z)|0\rangle \langle 0|T\Phi_k{}^{(m_i)}(z)\, \Phi_j{}^{(m_i)}(y)|0\rangle^{-1}|_{\text{on-shell}}$$
$$= i\langle 0|T\delta\Phi_i{}^{(m_i)}(x)\, \Phi_k{}^{(m_i)}(y)|0\rangle \overleftarrow{K}_{m_i}(y)\, \eta_{kj}^{(m_i)-1}|_{\text{on-shell}}\,. \tag{3·13}$$

Diagrammatically this is shown in Fig. 3(b). This equation (3·13) indicates that $A_{ij}^{(m_i)}$ is some function of $p_\mu$ in momentum space in general and, therefore, may include some differential operators in $x$-space.

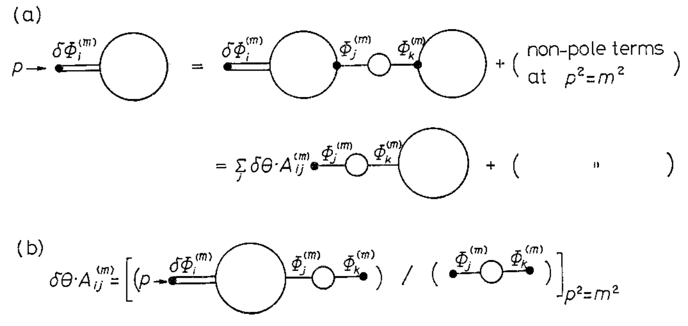


Fig. 3. (a) Diagram à la 't Hooft-Veltman;[10] pole contributions to the Green's functions containing $\delta\Phi_i{}^{(m_i)}$. (b) The diagrammatical representation of Eq. (3·13).

By the replacement (3·12), the identities (3·10) and (3·11) lead to the "on-shell WT identities", by using the notation (3·5),

$$:\mathcal{K}:\langle 0|TJ^T\delta\theta A\boldsymbol{\Phi}\exp iJ^T\boldsymbol{\Phi}|0\rangle=0\,, \tag{3·14}$$

$$:\mathcal{K}:\langle 0|T(\delta\boldsymbol{\Phi}_K{}^r(x)+iJ^T\delta\theta A\boldsymbol{\Phi}\boldsymbol{\Phi}_K{}^r(x))\exp iJ^T\boldsymbol{\Phi}|0\rangle=0\,. \tag{3·15}$$

Here we have introduced the matrix $A$ defined as $A_{IJ}\equiv\delta_{m_im_j}A_{ij}^{(m_i)}$, which commutes with the Klein-Gordon matrix $(K)_{IJ}=\delta_{ij}\delta_{m_im_j}(\Box+m_i^2)$:

$$KA=AK\,. \tag{3·16}$$

(3·14) and (3·15) are further rewritten as*)

$$:\mathcal{K}\cdot\phi^T:\eta^{-1}K\delta\theta A\langle 0|T\boldsymbol{\Phi}\exp iJ^T\boldsymbol{\Phi}|0\rangle=0\,, \tag{3·17}$$

$$\begin{aligned}&:\mathcal{K}:\langle 0|T\delta\boldsymbol{\Phi}_K{}^r(x)\exp iJ^T\boldsymbol{\Phi}|0\rangle\\ &\quad=-i:\mathcal{K}\cdot\phi^T:\eta^{-1}K\delta\theta A\langle 0|T\boldsymbol{\Phi}\boldsymbol{\Phi}_K{}^r(x)\exp iJ^T\boldsymbol{\Phi}|0\rangle.\end{aligned} \tag{3·18}$$

As we have finished preparations, let us determine the form of $\delta\phi_I{}^{\rm in}(x)$ in (3·8). We first note that $\delta\theta\cdot Q$ must commute with the $S$-matrix operator (3·3) because the charge $Q$ is conserved. That is, the commutator of $\delta\theta\cdot Q$ and $S$ must vanish: $[i\delta\theta\cdot Q,S]=0$. We find it sufficient to take $(\delta\phi)^T=-\phi^T\eta^{-1}A\eta\delta\theta$, i.e.,

$$\delta\phi_I{}^{\rm in}(x)=-\eta_{JI}A_{KJ}\eta_{LK}^{-1}\phi_L{}^{\rm in}(x)\,\delta\theta\,. \tag{3·19}$$

In fact the commutator,**)

$$\begin{aligned}[i\delta\theta\cdot Q,S]&=:[i\delta\theta\cdot Q,\phi^T]\eta^{-1}K\delta/\delta J\mathcal{K}:\langle 0|T\exp iJ^T\boldsymbol{\Phi}|0\rangle\\ &=:\mathcal{K}\cdot\delta\phi^T:\eta^{-1}K\langle 0|Ti\boldsymbol{\Phi}\exp iJ^T\boldsymbol{\Phi}|0\rangle,\end{aligned} \tag{3·20}$$

vanishes by the on-shell WT identity (3·17) and (3·19) together with (3·16). Further the choice (3·19) for $\delta\phi_I{}^{\rm in}(x)$ really *reproduces the original transformation* (3·7). The commutator of $\delta\theta\cdot Q$ with $S\boldsymbol{\Phi}_K{}^r(x)$ of (3·6),

$$[i\delta\theta\cdot Q,S\boldsymbol{\Phi}_K{}^r(x)]=:\mathcal{K}\cdot\delta\phi^T:\eta^{-1}K\langle 0|Ti\boldsymbol{\Phi}\boldsymbol{\Phi}_K{}^r(x)\exp iJ^T\boldsymbol{\Phi}|0\rangle,$$

becomes, by the choice (3·19) and by the on-shell WT identity (3·18),

$$[i\delta\theta\cdot Q,S\boldsymbol{\Phi}_K{}^r(x)]=:\mathcal{K}:\langle 0|T\delta\boldsymbol{\Phi}_K{}^r(x)\exp iJ^T\boldsymbol{\Phi}|0\rangle=S\delta\boldsymbol{\Phi}_K{}^r(x)\,. \tag{3·21}$$

*) Here, in the derivation of (3·17) and (3·18), the orders of the operation of $T$-product and the matrix $A$ have been exchanged. So, if $A$ contains time derivatives, there appear the additional terms like equal time commutators. These terms, however, give no contributions to (3·17) and (3·18) for lack of poles.

**) If the charge $Q$ mixed the positive and negative frequency parts of $\phi_I{}^{\rm in}$'s, the calculation (3·20) would not be valid because of the normal ordering. Here the validity of (3·20) is assured by the choice (3·19).

In the last step in (3·21), we have used a formula (3·6) for $\mathcal{O}=\delta\Phi_K{}^r(x)$. Since $S$ commutes with $\delta\theta\cdot Q$, (3·21) gives the desired transformation (3·7).

The induced transformation (3·19) of the in-fields can be simplified considerably. Define a matrix $B$:

$$\delta\phi=-(\eta^{-1}A\eta)^T\phi\delta\theta\equiv\delta\theta B\phi\,, \tag{3·22}$$

where the position of $\delta\theta$ should be noted because we are including the cases of anti-commuting $\delta\theta$. Note the Jacobi identity:

$$\begin{aligned}&-[[\phi_I(x),\phi_J(y)]_\mp, i\delta\theta\cdot Q]\\ &\quad=[\phi_I(x),[i\delta\theta\cdot Q,\phi_J(y)]]_\mp+[[i\delta\theta\cdot Q,\phi_I(x)],\phi_J(y)]_\mp\,.\end{aligned} \tag{3·23}$$

Here $[\ \ ,\ \ ]_\mp$ denotes commutator $(-)$ and anti-commutator $(+)$, the latter of which is taken only for the cases where both $\phi_I$ and $\phi_J$ obey Fermi-statistics. Since the (anti-) commutators (3·1a) of asymptotic fields are $c$-number, the left-hand side of (3·23) vanishes. So, (3·23) with (3·22) leads to

$$-[\phi_I(x),-(\eta^{-1}A\eta)^T_{JK}\phi_K(y)\delta\theta]_\mp=[\delta\theta B_{IK}\phi_K(x),\phi_J(y)]_\mp\,. \tag{3·24}$$

This is rewritten, by help of (3·1a), as $(\eta^{-1}A\eta)^T_{JK}\eta_{IK}=B_{IK}\eta_{KJ}$.*) Thus we obtain $A_{IJ}=B_{IJ}$ and have proved that

$$[i\delta\theta\cdot Q,\phi_i{}^{(m_i)\text{in}}]=\delta\theta\cdot A_{ij}^{(m_i)}\phi_j{}^{(m_j=m_i)\text{in}}=\bar{\delta}\phi_i{}^{(m_i)\text{in}}\,. \tag{3·25}$$

This formula (3·25) is proved for the cases without any multi-pole ghost fields. We now show that it is valid also even in the presence of multi-pole fields. First note that any multi-pole field can always be reduced to simple pole field, if we do not mind losing temporarily manifest Lorentz covariance. For example, for the dipole field $\chi^{\text{in}}$ in our model in § 2, which satisfies Eq. (2·32b)

$$\Box\chi^{\text{in}}=-(\alpha/K)B^{\text{in}}\,,$$

we can define a simple pole field $\tilde{\chi}^{\text{in}}$

$$\begin{aligned}&\tilde{\chi}^{\text{in}}(x)=\chi^{\text{in}}(x)+(\alpha/2K)(\nabla^2)^{-1}(x_0\partial_0-1/2)B^{\text{in}}(x)\,,\\ &\Box\tilde{\chi}^{\text{in}}(x)=0\,.\end{aligned} \tag{3·26}$$

It is an easy task to check

$$\tilde{\chi}^{\text{in}}(x)=\sum_k(\chi_k{}^{\text{in}}g_k(x)+\chi_k{}^{\text{in}\dagger}g_k{}^*(x))$$

by help of (2·43) and (2·50). Further, obviously, this $\tilde{\chi}^{\text{in}}$ field together with the other simple pole fields, $B^{\text{in}}$, $c^{\text{in}}$, $\bar{c}^{\text{in}}$, etc., spans the complete set of asymptotic fields.

*) One can easily check that this is valid even when $A$ contains differential operators.

So, generally, we have a complete set of simple-pole asymptotic fields $\{\tilde{\phi}_I{}^{\rm in}\}$ which is related to the original set of Lorentz covariant asymptotic fields $\{\phi_I{}^{\rm in}\}$ as

$$\tilde{\phi}_I{}^{\rm in}(\boldsymbol{x},t)=\int d^3\boldsymbol{y}\, M_{IJ}(\boldsymbol{x},\boldsymbol{y})\,\phi_I{}^{\rm in}(\boldsymbol{y},t)\equiv M_{IJ}\phi_J{}^{\rm in}(x)\,. \tag{3·27}$$

This relation represents similar one to (3·26). Note that this relation is invertible as $\phi_I{}^{\rm in}=M_{IJ}^{-1}\tilde{\phi}_J{}^{\rm in}$ because the set $\{\phi_I{}^{\rm in}\}$ also spans the complete set of asymptotic fields. The $M_{IJ}$'s may include time $t$ and finite order of time derivatives. The important point to be noted here is that the relation (3·27) is *essentially 'local' in time coordinate* $t$, although is non-local in space coordinate $\boldsymbol{x}$. Therefore, if we define the (non-covariant) Heisenberg fields $\tilde{\varPhi}_I{}^r$ by the same relation as (3·27),

$$\tilde{\varPhi}_I{}^r(\boldsymbol{x},t)\equiv\int d^3\boldsymbol{y}\, M_{IJ}(\boldsymbol{x},\boldsymbol{y})\,\varPhi_J{}^r(\boldsymbol{y},t)\equiv M_{IJ}\varPhi_J{}^r(x)\,, \tag{3·28}$$

the LSZ asymptotic conditions, $\tilde{\varPhi}_I{}^r(x)\xrightarrow[x_0\to\pm\infty]{}\tilde{\phi}_I{}^{\rm in}_{\rm out}(x)$, hold and the GLZ formula is also valid for the fields $\tilde{\phi}_I{}^{\rm in}(x)$ and $\tilde{\varPhi}_I{}^r(x)$. So, noticing that we have not made use of Lorentz covariance property anywhere in deriving the formula (3·25), we can obtain in this case

$$(\delta\tilde{\varPhi}_I{}^r(x))^{\rm in}\equiv[i\delta\theta\cdot Q,\tilde{\phi}_I{}^{\rm in}(x)]=\delta\theta\tilde{A}_{IJ}\tilde{\phi}_J{}^{\rm in}(x)\,, \tag{3·29}$$

corresponding to (3·25). $\tilde{A}_{IJ}$, of course, are given by similar equations to (3·13). This is rewritten, by help of (3·13), as

$$(\delta\tilde{\varPhi}_i{}^{(m_i)}(x))^{\rm in}=\langle 0|\delta\tilde{\varPhi}_i{}^{(m_i)}(x)\,|\phi_{j,\alpha}^{(m_j=m_i){\rm in}}\rangle\eta_{jk}^{(m_i)-1}\tilde{\phi}_k{}^{(m_k=m_i){\rm in}}+{\rm h.c.}\,,$$

or simply as

$$(\delta\tilde{\varPhi}_I{}^r(x))^{\rm in}=\sum{}'\langle 0|\delta\tilde{\varPhi}_I{}^r(x)\,|\phi_{J,\alpha}^{\rm in}\rangle\eta_{JK}^{-1}\tilde{\phi}_{K,\alpha}^{\rm in}+{\rm h.c.}\,, \tag{3·30}$$

where the suffix $\alpha$ runs over the complete set of wave packet states, and $\sum'$ means that the sum with respect to $J\equiv(j,m_j)$ is restricted only to that satisfying $m_j=m_i$. For the covariant fields $\varPhi_I(x)$, we obtain

$$(\delta\varPhi_I{}^r(x))^{\rm in}=\sum{}'\langle 0|\delta\varPhi_I{}^r(x)\,|\tilde{\phi}_{J,\alpha}^{\rm in}\rangle\eta_{JK}^{-1}\tilde{\phi}_{K,\alpha}^{\rm in}+{\rm h.c.} \tag{3·31}$$

by operating the inverse operator $M^{-1}$ of (3·28) on the both sides of (3·30). In order to obtain a formula which expresses the in-field transformation $(\delta\varPhi_I{}^r)^{\rm in}\equiv[i\delta\theta\cdot Q,\phi_I{}^{\rm in}]$ in terms of the *covariant fields* $\phi_I{}^{\rm in}$, we must substitute the inverse relation of*)

$$\phi_I{}^{\rm in}(x)=\sum{}'\langle 0|\varPhi_I{}^r(x)\,|\tilde{\phi}_{J,\alpha}^{\rm in}\rangle\eta_{JK}^{-1}\tilde{\phi}_{K,\alpha}+{\rm h.c.} \tag{3·32}$$

for $\tilde{\phi}_{K,\alpha}^{\rm in}$ in (3·31). This procedure requires an explicit construction of the whole set of wave packet states which is very complicated in general in the presence of

*) This relation (3·32) trivially follows from $\langle 0|\varPhi_I{}^r(x)|\tilde{\phi}_{J,\alpha}^{\rm in}\rangle=\langle 0|\phi_I^{\rm in}(x)|\tilde{\phi}_{J,\alpha}^{\rm in}\rangle$.

multi-pole ghost fields. Fortunately, we have a short cut for it as follows. Consider the two point functions $\langle 0|T\delta\Phi_I^r(x)\ \Phi_J^r(y)|0\rangle$. If we can find the coefficients $A_{IJ}$ which satisfy

$$\langle 0|T\delta\Phi_I^r(x)\Phi_J^r(y)|0\rangle|_{1P\text{-pole at } m_i}=\delta\theta\cdot A_{IK}\langle 0|T\Phi_K^r(x)\Phi_J^r(y)|0\rangle|_{1P\text{-pole at } m_i} \tag{3·33}$$

on the poles*) with mass $m_i$ due to single particle intermediate states, then, in view of (3·31) and (3·32), we can conclude that

$$(\delta\Phi_I^r(x))^{\text{in}}\equiv[i\delta\theta\cdot Q,\phi_I^{\text{in}}(x)]=\delta\theta\cdot A_{IJ}\phi_J^{\text{in}}(x)=\bar{\delta}\phi_J^{\text{in}}(x). \tag{3·34}$$

This is because the equality (3·33) holds if and only if (3·34) is valid, since the whole single particle space is connected to the vacuum by the complete set of covariant field operators $\{\Phi_J^r(y)\}$. Notice that (3·33) is nothing but the on-shell replacement and it gives a generalization of the first equation of (3·13). So we can generally find $A_{IJ}$ by the procedure illustrated in Fig. 3(b).

Thus we have proved quite a general formula (3·34) with the coefficient $A$ determined by (3·33). We should note that this formula itself is Lorentz covariant in spite of the use of noncovariant fields $\tilde{\phi}$ in the proof. By (3·34), the explicit form of $Q$ is given as**)

$$Q=\int d^3x:A_{IJ}\phi_J^{\text{in}}(x)\pi_I^{\text{in}}(x):. \tag{3·35}$$

The uniqueness of the forms (3·34) and (3·35) is assured by the irreducibility of the Heisenberg fields and the asymptotic fields, respectively. This result coincides with that obtained by Umezawa and his collaborators[11] in some specific cases of linear transformations and simple pole fields. Although the methods adopted here are similar to theirs, we have obtained the result without use of the path integral techniques but in the framework of canonical theory. Surprisingly enough, the formula (3·34) is valid even for the charges of non-linear transformations. In a sense, for any charges $Q$, when it is written in terms of the asymptotic fields, we can drop off the terms which produce non-linear terms of the transformation. The effect of those non-linear parts of the charge is just to "renormalize" the coefficients of the linear parts.

Although we have worked only about in-fields, the arguments given above are easily transformed into those for the case of out-fields by the use of the relation $S^{-1}\phi^{\text{in}}(x)S=\phi^{\text{out}}(x)$. So (3·34) and (3·35) hold also for the out-fields.

## §4. Asymptotic forms of $Q_B$ and $Q_C$

Now let us return to the problem to determine the asymptotic forms of our

*) These poles may be multi-poles in general.

**) When $A_{IJ}$ contains time derivatives, the terms such as $(\pi_I^{\text{in}})^2$ may appear in (3·35). Then, the factors 1/2 are needed in front of them.

charge $Q_B$ and $Q_C$. Due to the formula (3·34), all the problem which we have to do is to determine the coefficients $A_{IJ}$ by using (3·33) in these cases. The transformation (3·7) of the Heisenberg fields $\Phi_I^r(x)$ corresponding to the asymptotic fields $\phi_I^{\text{in}}(x)$ is given as follows in case of $Q_B$:

$$\begin{aligned}
\delta U_\mu{}^r &= [i\delta\lambda\cdot Q_B, U_\mu{}^r] = \delta\lambda\sqrt{K}Z_3\{D_\mu c^r - \sqrt{L/Z_\chi}\partial_\mu (g/2)[(v+\psi)+\chi\times]c^r\}, \\
\delta\psi^r &= [i\delta\lambda\cdot Q_B, \psi^r] = \delta\lambda Z_3\sqrt{L/Z_\psi}[-(g/2)\chi\cdot c^r], \\
\delta\chi^r &= [i\delta\lambda\cdot Q_B, \chi^r] = \delta\lambda Z_3\sqrt{L/Z_\chi}(g/2)[(v+\psi)+\chi\times]c^r, \\
\delta B^r &= [i\delta\lambda\cdot Q_B, B^r] = 0, \\
\delta c^r &= [i\delta\lambda\cdot Q_B, c^r] = -\delta\lambda Z_3\sqrt{L}(g/2)c^r\times c^r, \\
\delta \bar{c}^r &= [i\delta\lambda\cdot Q_B, \bar{c}^r] = i\delta\lambda B^r,
\end{aligned} \tag{4·1}$$

where use has been made of (1·4), (1·5) and the results on the renormalization constants obtained in § 2. We have defined the renormalized charge $Q_B$ which is related to the original Noether charge $Q_B{}^0$ as

$$Q_B \equiv \tilde{Z}_3{}^{1/2}Z_B{}^{-1/2}Q_B{}^0, \tag{4·2}$$

and have introduced the Heisenberg field $U_\mu{}^r$ which corresponds to the asymptotic Proca field $U_\mu{}^{\text{in}}$ defined in (2·33):

$$U_\mu{}^r \equiv A_\mu{}^r - (\sqrt{K}-\alpha N)\partial_\mu B^r - \sqrt{K}\partial_\mu\chi^r. \tag{4·3}$$

Consider the on-shell replacement formula (3·33) or (3·12). We can easily find

$$\bar{\delta} U_\mu{}^r = \bar{\delta}\psi^r = 0, \tag{4·4a}$$

$$\bar{\delta}\chi^r = \delta\lambda\tilde{Z}_3\sqrt{L/Z_\chi}(M+\zeta(0))c^r = \delta\lambda c^r, \tag{4·4b}$$

$$\bar{\delta} B^r = \bar{\delta} c^r = 0, \tag{4·4c}$$

$$\bar{\delta}\bar{c}^r = i\delta\lambda B^r, \tag{4·4d}$$

where we have used the relation $\tilde{Z}_3(M+\zeta(0))\sqrt{L/Z_\chi}=1$ derived from (2·11), (2·16), (2·19), (2·20) and (2·31a). (4·4a) follows because it is supposed that there are no single-particle poles at the masses $m$ and $m_\psi$ in the channels $D_\mu c^r$ or $[(v+\psi)+\chi\times]c^r$ and $\chi\cdot c^r$, respectively. (4·4b) is because the channel $(g/2)\times[(v+\psi)+\chi\times]c^r$ has a ghost pole with residue $(M+\zeta(0))$ at the mass of $\chi$, $m_\chi=0$. This is easily seen in the Feynman diagrams (see Fig. 4). $\bar{\delta}c^r=0$ follows from the fact that there are no massless poles in the channel $c^r\times c^r$.

The form $\bar{\delta}\Phi$ of (4·4) and the formulae (3·34) and (3·35) lead to the desired result

$$Q_B = \int d^3x : B^{\text{in}}(x)\overleftrightarrow{\partial}_0 c^{\text{in}}(x) : = i\sum_k (c_k{}^\dagger B_k - B_k{}^\dagger c_k). \tag{4·5}$$

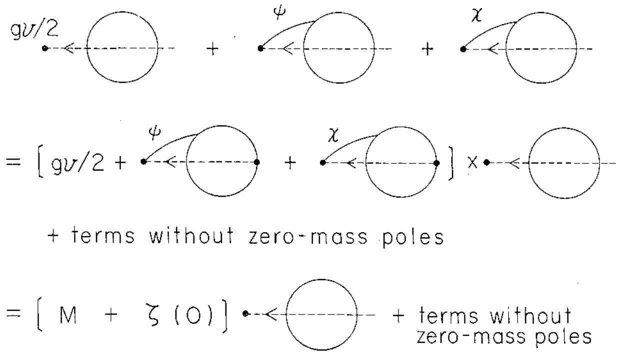


Fig. 4. The diagrams of $(g/2)[(v+\psi)+\chi\times]c$ contributing to the amplitudes on the mass-shell of $c$-fields.

For the case of charge $Q_c$, we trivially see that $\delta\Phi=\bar{\delta}\Phi$ because it is linear. So the asymptotic field transformation $\delta\phi^{\text{in}}$ has the same form as that of Heisenberg fields $\delta\Phi$. Thus, by (3·35), $Q_c$ is written as follows in terms of the asymptotic fields:

$$Q_c=i\int d^3x : \bar{c}^{\text{ex}}\overleftrightarrow{\partial}_0 c^{\text{ex}} : =\sum_k (\bar{c}_k{}^\dagger c_k + c_k{}^\dagger \bar{c}_k)\,. \qquad (4\cdot6)$$

Equations (4·5) and (4·6) together with (4·2) provide the proof of the asymptotic forms (I. 4·2) ~ (I. 4·4) used in I.

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