

Manifold Learning Theory and Applications

Yunqian Ma and Yun Fu



CRC Press

Taylor & Francis Group

Boca Raton London New York

CRC Press is an imprint of the
Taylor & Francis Group, an **Informa** business

Contents

List of Figures	xi
List of Tables	xvii
Preface	xix
Editors	xxi
Contributors	xxiii
1 Spectral Embedding Methods for Manifold Learning	1
<i>Alan Julian Izenman</i>	
1.1 Introduction	1
1.2 Spaces and Manifolds	3
1.2.1 Topological Spaces	3
1.2.2 Topological Manifolds	4
1.2.3 Riemannian Manifolds	5
1.2.4 Curves and Geodesics	6
1.3 Data on Manifolds	7
1.4 Linear Manifold Learning	7
1.4.1 Principal Component Analysis	8
1.4.2 Multidimensional Scaling	11
1.5 Nonlinear Manifold Learning	14
1.5.1 Isomap	15
1.5.2 Local Linear Embedding	20
1.5.3 Laplacian Eigenmaps	22
1.5.4 Diffusion Maps	23
1.5.5 Hessian Eigenmaps	26
1.5.6 Nonlinear PCA	27
1.6 Summary	32
1.7 Acknowledgment	32
Bibliography	32

2	Robust Laplacian Eigenmaps Using Global Information	37
	<i>Shounak Roychowdhury and Joydeep Ghosh</i>	
2.1	Introduction	37
2.2	Graph Laplacian	38
2.2.1	Definitions	38
2.2.2	Laplacian of Graph Sum	38
2.3	Global Information of Manifold	39
2.4	Laplacian Eigenmaps with Global Information	40
2.5	Experiments	40
2.5.1	LEM Results	43
2.5.2	GLEM Results	47
2.6	Summary	53
2.7	Bibliographical and Historical Remarks	53
	Bibliography	54
3	Density Preserving Maps	57
	<i>Arkadas Ozakin, Nikolaos Vasiloglou II, Alexander Gray</i>	
3.1	Introduction	57
3.2	The Existence of Density Preserving Maps	58
3.2.1	Moser's Theorem and Its Corollary on Density Preserving Maps	58
3.2.2	Dimensional Reduction to \mathbb{R}^d	60
3.2.3	Intuition on Non-Uniqueness	60
3.3	Density Estimation on Submanifolds	61
3.3.1	Introduction	61
3.3.2	Motivation for the Submanifold Estimator	61
3.3.3	Statement of the Theorem	62
3.3.4	Curse of Dimensionality in KDE	63
3.4	Preserving the Estimated Density:	
	The Optimization	64
3.4.1	Preliminaries	64
3.4.2	The Optimization	65
3.4.3	Examples	67
3.5	Summary	69
3.6	Bibliographical and Historical Remarks	69
	Bibliography	71
4	Sample Complexity in Manifold Learning	73
	<i>Hariharan Narayanan</i>	
4.1	Introduction	73
4.2	Sample Complexity of Classification on a Manifold	74
4.2.1	Preliminaries	74
4.2.2	Remarks	74
4.3	Learning Smooth Class Boundaries	74
4.3.1	Volumes of Balls in a Manifold	76
4.3.2	Partitioning the Manifold	77
4.3.3	Constructing Charts by Projecting onto Euclidean Balls	77
4.3.4	Proof of Theorem 2	78
4.4	Sample Complexity of Testing the Manifold Hypothesis	83

4.5	Connections and Related Work	84
4.6	Sample Complexity of Empirical Risk Minimization	85
4.6.1	Bounded Intrinsic Curvature	85
4.6.2	Bounded Extrinsic Curvature	85
4.7	Relating Bounded Curvature to Covering Number	86
4.8	Class of Manifolds with a Bounded Covering Number	86
4.9	Fat-Shattering Dimension and Random Projections	88
4.10	Minimax Lower Bounds on the Sample Complexity	89
4.11	Algorithmic Implications	91
4.11.1	k -Means	91
4.11.2	Fitting Piecewise Linear Curves	91
4.12	Summary	91
	Bibliography	92
5	Manifold Alignment	95
	<i>Chang Wang, Peter Krafft, and Sridhar Mahadevan</i>	
5.1	Introduction	95
5.1.1	Problem Statement	98
5.1.2	Overview of the Algorithm	98
5.2	Formalization and Analysis	99
5.2.1	Loss Functions	99
5.2.2	Optimal Solutions	103
5.2.3	The Joint Laplacian Manifold Alignment Algorithm	103
5.3	Variants of Manifold Alignment	103
5.3.1	Linear Restriction	104
5.3.2	Hard Constraints	106
5.3.3	Multiscale Alignment	106
5.3.4	Unsupervised Alignment	108
5.4	Application Examples	109
5.4.1	Protein Alignment	109
5.4.2	Parallel Corpora	111
5.4.3	Aligning Topic Models	114
5.5	Summary	117
5.6	Bibliographical and Historical Remarks	117
5.7	Acknowledgments	118
	Bibliography	119
6	Large-Scale Manifold Learning	121
	<i>Ameet Talwalkar, Sanjiv Kumar, Mehryar Mohri, Henry Rowley</i>	
6.1	Introduction	121
6.2	Background	122
6.2.1	Notation	123
6.2.2	Nyström Method	124
6.2.3	Column Sampling Method	124
6.3	Comparison of Sampling Methods	125
6.3.1	Singular Values and Singular Vectors	125
6.3.2	Low-Rank Approximation	125
6.3.3	Experiments	127
6.4	Large-Scale Manifold Learning	129

6.4.1	Manifold Learning	130
6.4.2	Approximation Experiments	132
6.4.3	Large-Scale Learning	132
6.4.4	Manifold Evaluation	136
6.5	Summary	140
6.6	Bibliography and Historical Remarks	140
	Bibliography	141
7	Metric and Heat Kernel	145
	<i>Wei Zeng, Jian Sun, Ren Guo, Feng Luo, and Xianfeng Gu</i>	
7.1	Introduction	145
7.2	Theoretic Background	147
7.2.1	Laplace–Beltrami Operator	147
7.2.2	Heat Kernel	148
7.3	Discrete Heat Kernel	149
7.3.1	Discrete Laplace–Beltrami Operator	149
7.3.2	Discrete Heat Kernel	149
7.3.3	Main Theorem	149
7.3.4	Proof Outline	150
7.3.5	Rigidity on One Face	151
7.3.6	Rigidity for the Whole Mesh	154
7.4	Heat Kernel Simplification	156
7.5	Numerical Experiments	158
7.6	Applications	159
7.7	Summary	162
7.8	Bibliographical and Historical Remarks	163
	Bibliography	163
8	Discrete Ricci Flow for Surface and 3-Manifold	167
	<i>Xianfeng Gu, Wei Zeng, Feng Luo, and Shing-Tung Yau</i>	
8.1	Introduction	167
8.2	Theoretic Background	170
8.2.1	Conformal Deformation	171
8.2.2	Uniformization Theorem	172
8.2.3	Yamabe Equation	174
8.2.4	Ricci Flow	175
8.2.5	Quasi-Conformal Maps	176
8.3	Surface Ricci Flow	177
8.3.1	Derivative Cosine Law	177
8.3.2	Circle Pattern Metric	178
8.3.3	Discrete Metric Surface	181
8.3.4	Discrete Ricci Flow	181
8.3.5	Discrete Ricci Energy	181
8.3.6	Quasi-Conformal Mapping by Solving Beltrami Equations	183
8.4	3-Manifold Ricci Flow	184
8.4.1	Surface and 3-Manifold Curvature Flow	184
8.4.2	Hyperbolic 3-Manifold with Complete Geodesic Boundaries	187
8.4.3	Discrete Hyperbolic 3-Manifold Ricci Flow	190
8.5	Applications	194

8.6	Summary	199
8.7	Bibliographical and Historical Remarks	202
	Bibliography	202
9	2D and 3D Objects Morphing Using Manifold Techniques	209
	<i>Chafik Samir, Pierre-Antoine Absil, and Paul Van Dooren</i>	
9.1	Introduction	209
	9.1.1 Fitting Curves on Manifolds	209
	9.1.2 Morphing Techniques	210
	9.1.3 Morphing Using Interpolation	210
9.2	Interpolation on Euclidean Spaces	211
	9.2.1 Aitken–Neville Algorithm on \mathbb{R}^m	211
	9.2.2 De Casteljau Algorithm on \mathbb{R}^m	212
	9.2.3 Example of Interpolations on \mathbb{R}^2	213
9.3	Generalization of Interpolation Algorithms on a Manifold M	213
	9.3.1 Aitken–Neville on M	214
	9.3.2 De Casteljau Algorithm on M	215
9.4	Interpolation on $SO(m)$	216
	9.4.1 Aitken–Neville Algorithm on $SO(m)$	216
	9.4.2 De Casteljau Algorithm on $SO(m)$	217
	9.4.3 Example of Fitting Curves on $SO(3)$	217
9.5	Application: The Motion of a Rigid Object in Space	218
9.6	Interpolation on Shape Manifold	224
	9.6.1 Geodesic between 2D Shapes	224
	9.6.2 Geodesic between 3D Shapes	225
9.7	Examples of Fitting Curves on Shape Manifolds	226
	9.7.1 2D Curves Morphing	226
	9.7.2 3D Face Morphing	227
9.8	Summary	229
	Bibliography	229
10	Learning Image Manifolds from Local Features	233
	<i>Ahmed Elgammal and Marwan Torki</i>	
10.1	Introduction	233
10.2	Joint Feature–Spatial Embedding	236
	10.2.1 Objective Function	237
	10.2.2 Intra-Image Spatial Structure	238
	10.2.3 Inter-Image Feature Affinity	238
10.3	Solving the Out-of-Sample Problem	239
	10.3.1 Populating the Embedding Space	240
10.4	From Feature Embedding to Image Embedding	240
10.5	Applications	240
	10.5.1 Visualizing Objects View Manifold	240
	10.5.2 What the Image Embedding Captures	241
	10.5.3 Object Categorization	243
	10.5.4 Object Localization	244
	10.5.5 Unsupervised Category Discovery	245
	10.5.6 Multiple Set Feature Matching	246
10.6	Summary	247

10.7 Bibliographical and Historical Remarks	247
Bibliography	248
11 Human Motion Analysis Applications of Manifold Learning	253
<i>Ahmed Elgammal and Chan Su Lee</i>	
11.1 Introduction	253
11.2 Learning a Simple Motion Manifold	256
11.2.1 Case Study: The Gait Manifold	256
11.2.2 Learning the Visual Manifold: Generative Model	258
11.2.3 Solving for the Embedding Coordinates	259
11.2.4 Synthesis, Recovery, and Reconstruction	260
11.3 Factorized Generative Models	262
11.3.1 Example 1: A Single Style Factor Model	264
11.3.2 Example 2: Multifactor Gait Model	265
11.3.3 Example 3: Multifactor Facial Expressions	265
11.4 Generalized Style Factorization	265
11.4.1 Style-Invariant Embedding	265
11.4.2 Style Factorization	266
11.5 Solving for Multiple Factors	267
11.6 Examples	269
11.6.1 Dynamic Shape Example: Decomposing View and Style on Gait Manifold	269
11.6.2 Dynamic Appearance Example: Facial Expression Analysis	271
11.7 Summary	272
11.8 Bibliographical and Historical Remarks	273
Acknowledgment	274
Bibliography	275
Index	281