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Abstract

Rather than organizing disposal of consumer-generated waste themselves, many states and countries have passed legislation that makes producers responsible for the proper disposal (i.e., recycling) of the products they bring to the market. We study the stability of producers' strategies emerging under such legislation. In our paper, the producers compete with multiple, differentiated products in consumer markets, but may consider cooperating when recycling those products in order to benefit from economies of scale. Products made by different producers or sold in different markets might still be considered for joint recycling. Our main question is when and whether firm-based recycling strategies (i.e., separately recycling products falling under same brand) or market-based recycling strategies (i.e., separately recycling products falling in the same product category) emerge as stable outcomes. To that end, we analyze a series of simple producer-market configurations. We first look at an asymmetric market model with two producers making three products in two markets, and then at a symmetric market model with two producers competing with four products in two markets. Our results show that with intense market competition and differentiated market sizes, producers may recycle their products on their own without cooperating with others. In some instances, they can add a product from their competitor to their recycling mix. As these outcomes are never socially optimal, they may reduce social welfare and may require government intervention. Otherwise, with less intense competition or more equitable market shares, all-inclusive (resp., market-based) recycling is most common stable outcome with high (resp., low) scale economies, and the firms' independent choices might lead to social optima.

Key words: Extended Producer Responsibility (EPR), firm-based recycling, economies of scale, dynamic stability.

1 Introduction

Historically, as governments bore the brunt of recycling-related costs, they have been active in proposing new solutions in order to reduce their financial burdens. In 1990, the Extended Producer Responsibility (EPR; see, e.g., Marques & Da Cruz [2016] and Lifset et al. [2013]) was introduced as a policy tool to reduce the waste stream generated by the increased volume and variety of consumer products. EPR financially encourages, motivates, or requires producers to take the environmental responsibility for products they bring to the market throughout their products' life cycles. While producers still determine the quantity of their products in the primary market, the implementation of EPR rebalances the market competition under the new cost structure. Currently, EPR-type strategies are widely deployed in different parts of the world. As early as 2002, the European Union led the way in collecting, recycling, and recovering electrical and electronic products through the Waste Electrical and Electronic Equipment Directive 2002/96/EC (WEEE), which imposes the responsibility for disposing of electronic wastes on their producers. This directive has become European law and has been implemented in all EU member countries by now. In the United States, legislation similar to the WEEE has not been approved by the federal government yet, but 25 states have passed legislation requiring the state-wide recycling of e-wastes (see, e.g., Souza [2013]).[1] For instance, Texas passed a computer take-back law in 2007 that requires producers selling new computer equipments in Texas to offer consumers a free and convenient recycling program. In 2011, a similar law was approved for televisions (see Shokouhyar & Aalirezaei [2017]).

Producers complying with the EPR-type legislation currently mostly contract with third-party recyclers. For example, Universal Recycling Technologies, Electronic Recyclers International, Inc., and MRM Recycling are Samsung's three primary recycling partners for consumer takeback. Producers pay recyclers for the collection, separation, disassembly, and recycling of their waste; such payment is considered to be the recycling cost for the producer and the income for the recycler. In addition, recyclers can generate income from recovered components or materials such as steel, precious metal, and plastic. As the disassembly of discarded products becomes more complicated, the recovered value from recycling decreases. For example, the recovery of high-value reusable components from a car, a television, or a cell phone may be too labor intensive, making recycling companies forgo disassembly and simply "grind up the product" to recover the less valuable raw materials instead of the more valuable components. Such reduced value is eventually transferred to the contracting producer as an increase in the producer's recycling cost. When recyclers contract with multiple producers or recycle multiple products, the producer's recycling cost can be influenced in two ways—through the unit recycling cost and through (dis)economies of scale.

Because of diversity in product designs, the heterogeneity of waste streams is a primary determinant of producers' unit recycling costs. The disassembly of valuable components and raw materials is more labor intensive when there are more variations in the way these components and raw materials are connected with each other. The increased task heterogeneity yields diseconomies of scope when products across different

[1] See Electronics Takeback Coalition: http://www.electronicstakeback.com/promote-good-laws

markets are recycled together (see, e.g., Gutowski & Dahmus [2005] and Dahmus & Gutowski [2007]). Indeed, upon examining prices charged by recyclers that focus only on certain types of products and comparing them with prices charged by more "universal" recyclers for recycling of miscellaneous products, we find that heterogeneous waste streams tend to exhibit higher unit recycling costs. For instance, in Earthworks Recycling, Inc., a recycling company in Washington State, we observe: i) Computer monitors, CPUs, televisions, laptops, and e-readers/e-books are recycled free of charge; ii) Refrigerators, freezers, air conditioners, and any other appliances that contain freon are recycled at \$0.10 per pound; and iii) Miscellaneous electronics are recycled at \$0.30 per pound. One approach to overcome these diseconomies of scope would be to recycle products at the level where they are more homogeneous, i.e., at the level of each market. For instance, the MRM is an electronic recycling company sponsored by companies including Panasonic, Toshiba, and Sharp, which recycles televisions, monitors, and laptops. In this paper, we refer to this recycling strategy as the *market-based* strategy.

Another way to influence producers' recycling costs is through (dis)economies of scale. The process of taking back products from consumers involves setting up a recycling network with shared resources that exhibit scale economies (see Gui et al. [2015]) or with various certifications that show scale diseconomies (see Atasu et al. [2013]). In order to leverage scale economies, some producers contract with a large comprehensive recycler with a lot of recycling resources and collectively recycle their products. Such policy mechanism is called the *Collective Producer Responsibility* (*CPR*; see, e.g., Atasu & Subramanian [2012]), which is an EPR category. For example, originally set up by Gillette, Braun, Electrolux, HP, and Sony, the European Recycling Platform (ERP) is a pan European producer recycling scheme for electronic wastes (see Butler [2008]). In this paper, we refer to this recycling strategy as the *all-inclusive* strategy. In order to reduce diseconomies of scale, some producers may contract with several small specialty recyclers, each of which focuses on recycling a certain type of products. In this paper, we refer to this recycling strategy as the *product-based* strategy.

We observe that despite scale economies some producers choose not to cooperate in recycling with other producers; instead, they recycle all of their products at the level of the individual producer, regardless of the product type (see, e.g., Dempsey et al. [2010]). Such policy mechanism is called the *Individual Producer Responsibility* (*IPR*; see, e.g., Tojo [2003] and Dempsey et al. [2010]), which is another category of EPR. For example, Samsung used to have an independent recycling system designed to take back only Samsung products in all states of the US that have EPR-type legislation. In this paper, we refer to this recycling strategy as the *firm-based* strategy.

Discussions of the current producer recycling programs focus mainly on the following questions:

1. Should producers join the CPR and recycle collectively, or should they adopt the IPR and set up individual recycling systems? In the EU, governments compel producers to set up Producer Compliance Schemes and recycle their products. Producers can either join an existing scheme (CPR) or establish an exclusive scheme (IPR) (see, e.g., Sachs [2006]). In the US, producers in Washington State have two

recycling options, Standard Plan (CPR) and Independent Plan (IPR) (see, e.g., Dempsey et al. [2010]); producers in New York State have to participate either in the Collective Electronic Waste-acceptance Program (CPR) or in the Individual Electronic Waste-acceptance Program (IPR).

2. Should recycling be organized based on product brands (firm-based) or categories (market-based)? Panasonic and Samsung are both producers of televisions, monitors, laptops, and toner cartridges. As one of the MRM founders, Panasonic recycles its televisions, monitors, and laptops through the MRM. However, it uses a separate take-back program to recycle its toner cartridges, organized by Office Depot. That is, Panasonic adopts the market-based recycling strategy. Samsung used to adopt the firm-based recycling strategy to recycle all Samsung-brand products through its own take-back program. Recently, it also switched to the market-based recycling strategy.

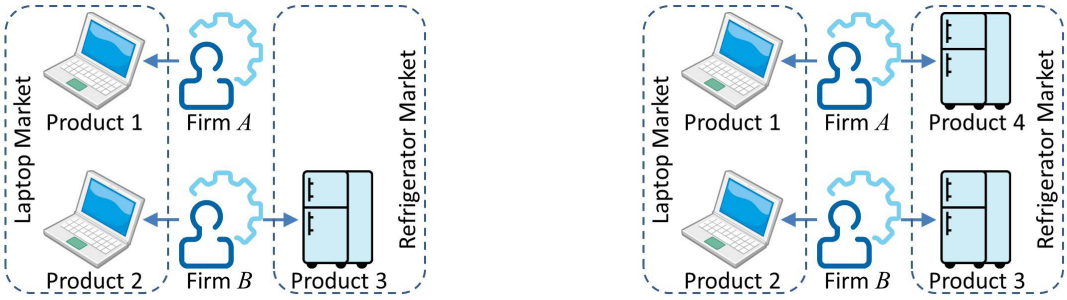
3. What recycling structure is preferred by the government? Should governments introduce specific regulations to guide how producers should recycle their products, or should producers be allowed to freely choose their recycling strategy? Currently, in most areas with EPR-type legislation, producers are only required to take on the responsibility for recycling, but they have the freedom to chose their own recycling strategy. However, producers' decisions may not maximize the social welfare. How big is the government' incentive to compel producers to adopt the socially optimal recycling strategy?

4. When producers are free to choose their recycling strategy, what is a stable outcome emerging as the result of their choices? Currently, producers are free to switch between the IPR and CPR. For instance, Vizio used to be one of the participating members of the MRM, but it has recently been removed from the MRM website. Another example of dynamic nature of recycling strategy selection is the abovementioned Samsung's move from firm-based to market-based recycling.

In addressing these questions and in comparing different recycling strategies, we are interested in an environment in which (a) multiple firms exist, (b) firms can make multiple products, (c) the products can belong to different markets, (d) firms can have different product portfolios, and (e) competing products may have different market shares. To capture this, we consider two models, *asymmetric manufacturing* and *symmetric manufacturing*. The asymmetric manufacturing model focuses on the recycling of three products made by two firms, as shown in figure 1a. One firm, for example A, is specialized and makes a single product, say product 1, in one market. The other firm, for example B, makes two products, say products 2 and 3, in two markets. On the one hand, firm B competes with A in the same market. On the other hand, firm B makes another product (3) in a separate market. The symmetric manufacturing model[2] focuses on the recycling of four products made by two firms, as shown in figure 1b. Both firms make products in two markets. Firm A makes products 1 and 4, and firm B makes products 2 and 3. The two firms are

[2]Although the two firms can have different market shares in the same market, we consider this model to be symmetric in the sense that both firms have products in both markets.

competitors in both markets: products 1 and 2 (respectively, 3 and 4) belong to the same market. These are the simplest models that capture features (a) to (e).



(a) Asymmetric Manufacturing

(b) Symmetric Manufacturing

Figure 1: Model Illustration

We study two settings, both with EPR-type legislation introduced: a benchmark scenario with government as the decision maker, and a scenario in which firms have more freedom. In the benchmark scenario, the government determines the recycling structure for all firms (before the latter determine their production quantities), and its objective is to maximize social welfare. Firms make products, pay for recycling of their products, and compete in the primary market. We refer to this scenario as the *Social Problem* (*SP*). In the second scenario, besides making products and paying for recycling, firms also determine their own recycling strategies. Although firms compete in the primary market, their recycling strategies in this setting may need to be made cooperatively (if they choose to have their products recycled together with the other firm's products). As all firms are individual decision makers, the cooperation process happens endogenously; we refer to this scenario as the *Endogenous Problem* (*EP*). In EP, any firm is free to change its decision, and the other firm may react by changing its decision in return. Therefore, we need to identify stable recycling structures in which no firm has the power or incentive to further defect. We capture this setting by using a dynamic stability concept, the largest consistent set (LCS; see Chew [1994]), described in Section 3.3.1 of this paper.

Our analysis shows that for the SP, results are rather intuitive. In the absence of economies of scale (including the case with diseconomies of scale), the product-based recycling structure generates the highest social welfare (because of more homogenous waste streams). In the presence of economies of scale, market-based recycling structure is optimal when scale economies are low, and all-inclusive recycling structure is optimal when they are high. With low scale economies, the effect of cost increase in a more heterogeneous waste stream is significant, hence marked-based recycling is optimal; when scale economies effect becomes more dominant than the cost increase due to heterogeneity, the best choice is to recycle all products together and increase the volume. Firm-based recycling is never preferred by the government because product characteristics are the main factors that determine the recycling costs—if any products should be recycled together, it is cheaper to do it for similar products belonging to the same market than for the potentially

diverse products made by the same producer. Hence, the government prefers the most cost-effective recycling organization based on markets or products, not based on firms. For the EP, we observe that in many cases results for the all-inclusive and market-based recycling structures carry over. However, it is interesting that in some cases the firm-based recycling structure does emerge as stable. These cases appear in both asymmetric manufacturing and symmetric manufacturing. In addition, in the symmetric manufacturing when both firms have multiple product choices, they may adopt different recycling strategies—for example, one firm adopts the firm-based recycling while the other adopts the product-based recycling, resulting in a hybrid recycling structure. We provide intuition about some less intuitive results in the main body of the paper.

This paper has three contributions to the literature. First, we analyze four applicable recycling structures: the product-based, the firm-based, the market-based, and the all-inclusive structure. In this paper, we consider not only different firms competing in the same market but also the same firm manufacturing across different markets. Our models are the simplest ones that enable us to study impacts of *competition between multiple firms and manufacturing across multiple markets*. To the best of our knowledge, this paper is the first work that analyzes these types of effects. Second, we consider impacts of (dis)economies of scale, product heterogeneity, market sizes, and market competition on the recycling structures. The economies of scale are considered as the incentive for all-inclusive (or joint) recycling, diseconomies of scale are considered as the incentive for product-based (or individual) recycling, product heterogeneity is considered as the incentive for market-based recycling, and market size and competition is considered as the incentive for firm-based recycling. Thus, we fill the gap of capturing interplay of incentives in the sustainable operations field. Finally, we use the game-theoretical methodology to study the implementation of EPR-type legislation (IPR and CPR). In EP, firms choose between recycling individually (product-based and firm-based) or collectively (market-based and all-inclusive), which involves the endogenous formation of coalitions. We study dynamic stability of recycling structures in order to better capture the possible actions and reactions of every firm.

Our results can provide conjectures for more general cases. We start our study from an asymmetric case with a limited number products, and then extend the discussion to a symmetric case with more products. All intuitive results in the asymmetric model carry over to the symmetric model. While there may seem that some stability results differ between the two models, our analysis of the endogenous process of coalition formation reveals consistence in firms' farsighted incentives. The two models show slightly different results only because they have different structures. For example, in the symmetric model, when one firm adopts firm-based recycling and the other firm adopts product-based recycling, we are faced with a hybrid structure, while in the asymmetric model, when the firm making multiple products adopts the firm-based recycling and the firm making single product adopts the product-based recycling, we are faced with a pure firm-based structure. As we have analyzed both the asymmetric case and the symmetric case and noticed consistency across models, we can make some conjectures about more general cases. For instance, when some firms make products across multiple markets, while remaining firms make products in a single market, it appears that the all-inclusive (resp., market-based) recycling should be adopted with high (resp., low) economies

of scale. If the market competition is intense, firms manufacturing across different markets are more likely to adopt the firm-based recycling, while the remaining firms may either recycle their products all together or recycle their products according to the market they belong to, depending on the scale economies. The implication is consistent with our findings in the two models and may explain some industrial phenomena. For example, Samsung used to recycle all of its products together because, compared with some other competitors, Samsung offers a greater selection of products across multiple markets.

2 Literature Review

Our work fits well in the closed-loop supply chain literature. Fleischmann [2001] and Esenduran et al. [2012] classify reverse logistics networks based on the form of reprocessing (remanufacturing vs. recycling vs. reuse), the driver for product recovery (economics vs. legislation), and the owner of recovery processes (producer vs. third-party). A variety of papers consider different combinations of these three characteristics. For instance, Savaskan et al. [2004] discuss an economics-driven model in which a producer either performs remanufacturing by itself or subcontracts remanufacturing to a third party. Toyasaki et al. [2011] introduce a model in which two producers outsource the WEEE-driven recycling to two recyclers, and recyclers either directly contract with producers or negotiate with a nonprofit organization. Alev et al. [2016] study the impact of EPR-type legislation on the strategy of a durable good producer in its secondary market, where used products are recovered. Esenduran & Atasu [2016] study a scenario in which producers compete with recyclers in e-waste recycling. Producers are motivated to engage in recycling by legislative factors, while recyclers are motivated by economic factors. Our work focuses on recycling driven by EPR-type legislation; producers take the financial responsibility of recycling and contract with third-party recyclers.

The stream of literature on legislation-driven producer-responsible recycling focuses on implementations of the IPR and CPR. Atasu et al. [2009] study how to implement the IPR in a model with a single product made by multiple producers. The authors suggest that when implementing IPR, besides the product quantities, one should also consider the recycling treatment costs, the market competition intensity, the product environmental impact, and customers' willingness to pay for the decrease in the environmental impact. Our paper considers all factors except the last two. Gui et al. [2015] study how to implement the CPR through a cost allocation mechanism in a large collection and recycling network, which consists of multiple producers with multiple products. The authors argue that, when implementing the CPR, a fair cost allocation mechanism is more likely to induce cooperation of all producers within a single network than a simple mechanism, which may bring higher costs to some producers and lower costs to others. The authors use the cooperative game-theoretical methodology to analyze the stability of networks or coalitions. Our paper also uses the cooperative game-theoretical framework, but we consider both the case in which all producers join a single network and the case in which producers form several networks. Atasu & Subramanian [2012] study how to select between the IPR and CPR in a recycling model with two products made by two

producers. Considering the case in which both producers follow the IPR and individually recycle vs. the case in which both follow the CPR and collectively recycle, the authors conclude that there is a trade-off between: (a) the reduction of recycling costs through improved design in the IPR and (b) the operational cost-efficiency under the CPR. Our paper considers both sides of the trade-off, but we also allow for simultaneous existence of IPR-based and CPR-based recycling. Esenduran & Kemahlıoğlu-Ziya [2015] discuss producers' choice between the IPR and CPR based on cost structures. These authors identify the size of the CPR coalition, the coalition composition (large or small producers), and the financial benefit from environmentally friendly product design as the key decision-making factors. While they discuss the effect of a producer's defection on the CPR coalition, they do not consider other stability issues, such as defections of multiple producers. Our paper also confirms the importance of the first two factors. Moreover, we provide a more comprehensive discussion of the stability of CPR coalitions.

The last important category of literature studies different notions of stability within the field of game theory. The earliest stability concepts in multilateral games are the von Neumann-Morgenstern stable set (see von Neumann & Morgenstern [1944]) and the Nash equilibrium (see Nash [1950]), which consider the instant payoff after an action as the incentive for that action. These stability concepts are myopic/static, as they only consider immediate consequences of players' actions. In addition, they consider players' competitive behavior because each player makes independent decisions and receives corresponding payoffs. Unlike these two concepts, the core (see Gillies [1959]), the coalition structure core (see Aumann & Dreze [1974], and the coalition-proof Nash equilibrium (see Bernheim et al. [1987]) allow players to form coalitions and cooperate in their decisions, but still under a static setting. More recently, researchers acknowledge that players consider how others react to their actions and develop stability concepts of a more dynamic nature. The bargaining set (see Aumann & Maschler [1964]) only considers two steps, objection and counter-objection. More recent work allows that players look further into the future and includes the largest consistent set (LCS; see Chwe [1994]) and the equilibrium process of coalition formation (EPCF; Konishi & Ray [2003]). Several papers in the operations-management area study coalition formation and stability in a dynamic sense; that is, when making their decisions, agents take into account how other agents react to their moves. For example, Granot and Sošić [2005], Sošić [2006], and Nagarajan & Sošić [2007] study horizontal cooperation among several retailers; Granot & Yin [2008], Nagarajan & Bassok [2008], and Nagarajan & Sošić [2009] study horizontal cooperation in assembly models; Kemahlıoğlu-Ziya & Bartholdi [2011] study cooperation among retailers who order from a common supplier; and Sošić [2010] studies vertical cooperation in a three-level supply chain. However, applications on producer take-back programs are very few in the literature. Gui et al. [2015] use the core to analyze the stability of the CPR coalition consisting of all producers. Our paper uses the LCS to analyze the stability of recycling coalitions for producers. We choose the LCS because (1) The dynamic concepts capture players' behavior more accurately, as they consider both actions and reactions, (2) The core can be empty, while the LCS always uniquely exists (non-empty), and (3) the LCS is an inclusive concept, which considers all possible deviations and following reactions. To the best of our knowledge, this

paper is the first work to apply the dynamic stability approach in the area of sustainable operations.

3 Asymmetric Manufacturing of Three Products

3.1 Model Setup

We start our analysis with a model in which *two* firms make a total of *three* products in *two* markets. In one of the markets, the two firms compete as duopolists, each with one product. In the other market, with the remaining product, one of the firms is the monopolist. This is the simplest model that allows us to both capture products that belong to the same market but are made by different firms, and capture products that are made by the same firm but belong to different markets. We refer to this model as *asymmetric manufacturing*. Bulow et al. [1985] adopt a similar model setting. They discuss the impact of the monopolist firm's actions in the monopoly market on its competitor's strategy in the duopoly market, and also on its own marginal costs in the duopoly market. We focus on how firms' cooperative strategies affect their payoffs and therefore determine the stable recycling structure.

Market Surplus. We assume that firm A makes product 1 and firm B makes products 2 and 3; in this section, we refer to firm A as a specialized, and to firm B as a multi-product firm. Products 1 and 2 compete in a duopoly market, and product 3 is standalone, in a monopoly market. $\gamma \in [0,1]$ is the *competition intensity* between products 1 and 2: $\gamma \to 1$ implies that products 1 and 2 are perfect substitutes and the market competition is intense; $\gamma \to 0$ implies that products 1 and 2 are not substitutable and there is no market competition. The model is illustrated in figure 1a. We use $q_i \geqslant 0$ to denote the output of product i, $i = 1, 2, 3$. Following Singh & Vives [1984], the market surplus brought by the three products is:

$$U(q_1, q_2, q_3) = \sum_{i=1}^{3} \alpha_i q_i - \frac{1}{2}\left(\sum_{i=1}^{3} q_i^2\right) - \gamma q_1 q_2, \tag{1}$$

where α_i is the market size of product i reduced by the unit production cost of product i, $i = 1, 2, 3$.

Unit Recycling Costs. To comply with EPR-type legislation, firms have to appropriately recycle all products they bring to the market. In general, firms contract with third-party recyclers to collect and process their products. We assume that recyclers can belong to one of the two types: specialty recyclers or universal recyclers. We define *specialty recyclers* as the ones who can only accept products from a certain market, while *universal recyclers* are able deal with products from different markets. Due to the increasing requirements on hardware (machines) and software (technology) when processing products from multiple markets, the unit recycling cost of universal recyclers is higher than that of specialty recyclers, even when they process the same product. Since products 1, 2, and 3 belong to two different markets, we consider three different types of third-party recyclers:

- x-type specialty recycler, accepting products from the duopoly market (i.e., products 1 and 2), at the unit recycling costs (for collection, separation, disassembly, and recycling) c_1 and c_2, respectively;

- y-type specialty recycler, only accepting products from the monopoly market (i.e., product 3), at the unit recycling cost of c_3;
- z-type universal recycler, accepting products from both markets (i.e., products 1, 2, and 3), at the unit recycling costs of λc_1, λc_2 and λc_3 ($\lambda > 1$), respectively.

Recycling Structures. Each firm can contract with one or more recyclers. The resulting recycling structure can belong to one of the following five cases:

- All products are recycled by one recycler. In other words, a recycler of type z recycles products 1, 2, and 3. We refer to this case as *all-inclusive* recycling, denoted by {123}, with unit recycling costs of products 1, 2, and 3 being λc_1, λc_2, and λc_3, respectively.
- Competing products 1 and 2 are recycled by one recycler; standalone product 3 is recycled by another recycler. That is, products from the same market are recycled together. We refer to this case as *market-based* recycling, denoted by {12}{3}. There are four possible scenarios for this case, depending on the type of recycler that is conducting recycling of specific products (that is, products 1 and 2 can be recycled by specialty or universal recycler; the same for product 3), and we only focus on the optimal one—a recycler of type x recycles products 1 and 2; a recycler of type y recycles product 3[3]. In this scenario, the unit costs for all products are the lowest: c_1, c_2, and c_3, respectively. For the complete analysis of unit costs in all four scenarios, see Section A of the Appendix.
- Firm A's product 1 is recycled by one recycler; Firm B's products 2 and 3 are recycled by another recycler. That is, products made by the same firm are recycled together. We refer to this case as *firm-based* recycling, denoted by {1}{23}. We only focus on the optimal scenario—a recycler of type x recycles product 1 and a recycler of type z recycles product 2 and 3, resulting in the unit costs of c_1, λc_2, and λc_3, respectively.
- Products 1 and 3 are recycled by one recycler; product 2 is recycled by another recycler. That is, the two products that are recycled together are from different markets and made by different firms. We refer to this case as *cross-market/firm* recycling, denoted by {13}{2}. The optimal scenario is that a recycler of type z recycles products 1 and 3 and a recycler of type x recycles product 2, resulting in the unit costs of λc_1, c_2, and λc_3, respectively.
- Each product is recycled by an individual recycler. We refer to this case as *product-based* recycling, denoted by {1}{2}{3}. When two recyclers of type x recycle products 1 and 2, respectively, and a recycler of type y recycles product 3, all products achieve the lowest unit costs—c_1, c_2, and c_3.

This is summarized in Table 1.

[3]In other words, we do not consider the case in which a universal recycler recycles products, as they unnecessarily lead to higher unit recycling cost, $\lambda > 1$.

All-inclusive: {123}	Firm-based: {1}{23}	Market-based: {12}{3}
Cross-market/firm: {13}{2}	Product-based: {1}{2}{3}	

Table 1: Nomenclature of Recycling Structures for Asymmetrically Manufactured Products.

(Dis)economies of Scale. A recycler's operations can generate (dis)economies of scale, based on the quantity of products it recycles, seen as benefits (or losses) to the recycler. If a recycler and a contracting firm engage in a long-term relationship, those benefits (or losses) are eventually transferred to the contracting firm as reductions (or increases) of the overall recycling cost. Following Amir [2003], we assume that such adjustments to the recycling-related costs are changed quadratically with the product quantity. We use κ to denote the factor of the quadratic form; positive (negative) κ means a decrease (increase) to the overall recycling cost, indicating (dis)economies of scale. For instance, if product 1 is recycled alone by a recycler, its overall unit cost is adjusted by $-\kappa q_1^2$; if products 1 and 2 are recycled together by the same recycler, their overall unit costs are adjusted by $-\kappa(q_1+q_2)^2$. Further, if different products are recycled together by the same recycler, the change stemming from scale economies should be apportioned to products by their quantities (see, e.g., Gui et al. [2015]). That is, if products 1 and 2 are recycled together, product i's cost is adjusted by $-\frac{q_i}{q_1+q_2}\kappa(q_1+q_2)^2$, $i=1,2$. The scheme that we propose, the quantity-based proportional rule, has some justification in both theory and practice. From practical point of view, proportional rules are easy to implement and hence used in practice (see, e.g., Electronic Product Collection, Recycling and Reuse Program For Washington State, `https://fortress.wa.gov/ecy/publications/documents/0607005.pdf`). From a theoretical point of view, as one example, Meca et al. (2004) consider economies of scale emerging when different firms cooperate and place joint orders in an EOQ system. In their model, they use a proportional rule to allocate ordering cost among firms.

Recycling Costs. We assume that the scale economies and the unit recycling costs are independent. Considering both effects, the overall recycling cost of product i depends on the recycling structure. Let us denote the set of all recycling structures by $\mathbf{X}$; then, $\mathbf{X}=\{\{123\},\{12\}\{3\},\{1\}\{23\},\{13\}\{2\},\{1\}\{2\}\{3\}\}$. Under a given recycling structure $X\in\mathbf{X}$, we let Z_i^X be the set of products recycled by product i's recycler. In other words, Z_i^X is a set of products that are recycled together by the same recycler and $i\in Z_i^X$. Note that if products i and j are recycled by the same recycler, $Z_i^X=Z_j^X$. Then, under the recycling structure X, the cost for recycling product i is

$$C_i^X(q_1,q_2,q_3)\doteq\overline{\lambda}_i^X c_i q_i-\frac{q_i}{\sum_{j\in Z_i^X}q_j}\kappa\left(\sum_{j\in Z_i^X}q_j\right)^2, \tag{2}$$

where $\overline{\lambda}_i^X=\begin{cases}1, \text{ if } Z_i^X=\{1\}\{2\}\{3\} \text{ or } \{12\}\{3\},\\ \lambda, \text{ otherwise.}\end{cases}$

We summarize our main and technical modeling assumptions:

Assumption 1 *(i) γ captures the degree of substitution or the competition intensity between product 1 (firm A) and product 2 (firm B). We assume that $\gamma \leq 1$.*
(ii) λ captures the increase in unit recycling costs when product 3 (made by firm B) is recycled together with products 1 and/or 2 (firm A and/or B). We assume that $\lambda \in [1, 2]$.
(iii) κ captures the potential recycling economies of scale. We assume that κ is low enough to ensure nonnegative quantities.
(iv) As products 1 and 2 are from the same market, we assume that their recycling costs are comparable, and for simplicity hereafter we use $c_1 = c_2$.

These restrictions are reasonable in practice and allow us obtaining analytic insight.

Next, we consider two problems corresponding to the two legislative choices of the government. In section 3.2, the government not only requires producers' to undertake recycling responsibilities but also determines the overall recycling structure for all firms, with the goal of achieving the maximum social welfare. In section 3.3, although firms are required to recycle their products, they have the freedom to determine their own recycling strategy; therefore, the formation of the recycling structure is an endogenous process.

3.2 Social Problem (SP)

By introducing the EPR-type legislation, the government requires firms to recycle all products they bring to the market. In many states, such as Maryland or Michigan, the state government organizes recycling and firms pay the state for the expenses. The government chooses the recycling structure that can generate the highest social welfare (taking firms' optimal production decisions for each option into account), while firms compete in the primary market and determine the production quantities, taking the recycling costs determined by recycling structure into consideration. We refer to this problem as the *Social Problem* (SP).

By taking the partial derivatives of the market surplus given in equation (1) (see Singh & Vives [1984]), the prices of the three products are

$$p_i = \frac{\partial U}{\partial q_i} = \alpha_i - q_i - \gamma q_j,\ i, j = 1, 2,\ i \neq j;\ \text{and } p_3 = \frac{\partial U}{\partial q_3} = \alpha_3 - q_3, i = 1, 2, 3.$$

The objective of each firm is to maximize its individual payoff, which is the revenue in the primary market reduced by the cost in the recycling market. Under the recycling structure $X \in \mathbf{X}$, the payoff that product i brings to its firm is $\pi_i^X = p_i q_i - C_i^X(q_1, q_2, q_3)$, where $C_i^X(q_1, q_2, q_3)$, given in equation (2), is the cost of recycling product i. With a certain $X \in \mathbf{X}$, the two firms choose their product quantities to optimize their respective profits: $\Pi_A^X = \max_{q_1} \pi_1^X$ and $\Pi_B^X = \max_{q_2, q_3} \left\{\pi_2^X + \pi_3^X\right\}$.

For instance, under the firm-based recycling structure, $X = \{1\}\{23\}$, firm A's objective is $\max_{q_1}\{q_1(\alpha_1 - q_1 - \gamma q_2) - (q_1 c_1 - \kappa q_1^2)\}$, where $q_1(\alpha_1 - q_1 - \gamma q_2)$ is the revenue from product 1, $q_1 c_1$ is the original cost of recycling product 1, and $-\kappa q_1^2$ is cost adjustment (discount or increase) stemming from the (dis)economies

of scale. Clearly, for a given recycling structure, specialized firm A can only determine the quantity of product 1, while the quantity of product 2 is the decision of firm B. Multi-product firm B's objective is $\max_{q_2,q_3}\{q_2(\alpha_2 - q_2 - \gamma q_1) + q_3(\alpha_3 - q_3) - [q_2\lambda c_2 + q_3\lambda c_3 - \kappa(q_2+q_3)^2]\}$, where $q_2(\alpha_2 - q_2 - \gamma q_1)$ and $q_3(\alpha_3 - q_3)$ are revenues from products 2 and 3, respectively, $q_2\lambda c_2$ and $q_3\lambda c_3$ are original costs of the universal z-type recycler for recycling products 2 and 3, respectively, and $-\kappa(q_2+q_3)^2$ is cost adjustment stemming from joint recycling of products 2 and 3. From the first order conditions, we can obtain the equilibrium quantities $q_1^{\{1\}\{23\}}$, $q_2^{\{1\}\{23\}}$ and $q_3^{\{1\}\{23\}}$. In Section A of the Appendix, we calculate q_i^X, $i = 1, 2, 3$ and $X \in \mathbf{X}$, the equilibrium quantities of all products under all recycling structures.

For any $X \in \mathbf{X}$, the social welfare is the market surplus reduced by the total recycling cost,

$$W^X(q_1, q_2, q_3) = U(q_1, q_2, q_3) - \sum_{i=1}^{3} C_i^X(q_1, q_2, q_3), \tag{3}$$

where $U(q_1, q_2, q_3)$ is given in equation (1) and $C_i^X(q_1, q_2, q_3)$ is given in equation (2). The government considers the abovementioned five recycling structures and determines a structure that maximizes the social welfare based on firms' equilibrium quantities: $\max_{X\in\mathbf{X}} W^X(q_1^X, q_2^X, q_3^X)$.

Proposition 1 *Consider the SP for asymmetric manufacturing. There exists $\kappa_0 > 0$ such that:*

- *When $\kappa \leqslant 0$, the product-based recycling, $\{1\}\{2\}\{3\}$, is optimal;*
- *When $0 \leqslant \kappa \leqslant \kappa_0$, market-based recycling, $\{12\}\{3\}$, is optimal;*
- *When $\kappa \geqslant \kappa_0$, all-inclusive recycling, $\{123\}$, is optimal.*

Intuitively, when $\kappa < 0$, recycling multiple products together increases the economic burden due to the diseconomies of scale. In addition, recycling products from different markets together also incurs higher unit recycling costs. Therefore, the product-based recycling is optimal. When $\kappa > 0$, joint recycling brings about economies of scale. As joint recycling of products from the same market does not increase the unit recycling cost, but joint recycling of products from different markets does, the optimal structure depends on whether the economies of scale can offset the (possibly) increased recycling costs. To make the all-inclusive recycling optimal, the economies of scale needs to be large enough (that is, we need $\kappa > \kappa_0$) to offset the increase in recycling costs that occurs because products from different markets are recycled by the same recycler. If the economies of scale have smaller impact (that is, $0 < \kappa < \kappa_0$), joint recycling of products from different markets increases the overall recycling costs. However, recycling products from the same market (i.e., products 1 and 2) can still reduce costs, and therefore the market-based recycling is optimal. The firm-based recycling and cross-market/firm recycling are not optimal in any cases with economies of scale because joint recycling of products from different markets (instead of products from the same market) is not efficient from the government's perspective.

In this subsection, we studied a benchmark scenario in which firms take the responsibility of recycling in a way determined by the government. This approach can lead to the highest social welfare. Next, we discuss a model in which firms can independently choose how to recycle their products. As the overall recycling structure depends on each individual firm's recycling choices, the process of determining a recycling structure can entail cooperation and defections. We refer to such an endogenous process as the Endogenous Problem.

3.3 Endogenous Problem (EP)

In this subsection, we assume that firms are required to recycle their products, but they have the freedom to choose which recycler to contract with and whether they want to cooperate with the other firm. On the one hand, firms compete in the primary market, while on the other hand, in order to take advantage of the economies of scale, products need to be recycled together and firms may need to cooperate in the recycling market. However, because of competition in the primary market, both cooperation and defections exist between firms, and firms endogenously form coalitions to recycle. We refer to this problem as the *Endogenous Problem* (EP).

Firms first determine their equilibrium quantities and payoffs under different recycling structures. The quantities, q_i^X, $i = 1, 2, 3$, correspond to those derived in the SP (shown in Section A of the Appendix). For a given structure, $X \in \mathbf{X}$, the equilibrium payoffs are

$$\Pi_A^X = (1-\kappa)(q_1^X)^2; \tag{4}$$

$$\Pi_B^X = \begin{cases} (1-\kappa)(q_2^X)^2 + (1-\kappa)(q_3^X)^2, & \text{if } X = (12,3),(13,2), \text{ or } (1,2,3) \\ (1-\kappa)(q_2^X)^2 + (1-\kappa)(q_3^X)^2 - 2\kappa q_2^X q_3^X, & \text{otherwise.} \end{cases}$$

In Section A of the Appendix, we calculate the expressions for the equilibrium payoffs under all recycling structures in terms of the parameters of the model.

Depending on their payoffs under different recycling structures, each firm has its most preferred structure. Clearly, there are instances in which we observe an inconsistency among structures that are most preferred by different firms. Thus, a firm may not end up in its most preferred recycling structure because it needs the participation of the other firm, which may have different preferences. Some of the common stability concepts, such as the core or the coalition structure core, turn out to be empty in this setting and are not useful in identifying stable outcomes. In these instances, various sequences of moves might occur. As a consequence, the process for determining equilibrium recycling structure is dynamic: every firm considers possible reactions (by others) to its actions. A solution concept that allows players to consider multiple possible further deviations is *the largest consistent set* (LCS), introduced by Chwe [1994]. It is introduced in more detail in the next subsection, and is used as a stability criterion in our analysis of stable alliance structures.

3.3.1 The Largest Consistent Set (LCS)

In this subsection, we introduce the LCS in our setting.

Let N denote the set of all firms and $\mathbf{X}$ denote the set of all partitions of N, also referred to as structures. For every firm $i \in N$, let Π_i^X denote i's payoff under structure $X \in \mathbf{X}$.

Let us denote by $\prec_i$ the strong preference relations among players, described as follows: for two structures, X_1 and $X_2 \in \mathbf{X}$, $X_1 \prec_i X_2 \Leftrightarrow \Pi_i^{X_1} < \Pi_i^{X_2}$. For $S \subseteq N$, if $X_1 \prec_i X_2$ for all $i \in S$, we write $X_1 \prec_S X_2$. Denote by $\rightharpoonup_S$ the defection of $S \subseteq N$: $X_1 \rightharpoonup_S X_2$ if structure X_2 is obtained when S deviates from structure X_1. We say that X_1 is *directly dominated* by X_2, denoted by $X_1 < X_2$, if there exists an $S \subseteq N$ such that $X_1 \rightharpoonup_S X_2$ and $X_1 \prec_S X_2$. We say that X_1 is *indirectly dominated* by X_m, denoted by $X_1 \ll X_m$, if there exist $X_1, X_2, X_3, \ldots, X_m$ and $S_1, S_2, S_3, \ldots, S_{m-1} \subseteq N$ such that $X_i \rightharpoonup_{S_i} X_{i+1}$ and $X_i \prec_{S_i} X_m$ for $i = 1, 2, 3, \ldots, m-1$.

The LCS assumes that the actual payoff is received only when firms reach a stable set. Thus, the defections might be seen as a mental exercise in which firms contemplate possible impacts of their moves. The underlying idea of the LCS is that a move by a set of firms to another structure, in which defecting firms can see an increase in their payoffs, is deterred if it triggers a sequence of defections that eventually ends in a stable structure in which some of the initially deviating firms are worse off than in the original structure. Similarly, a move by a set of firms to another structure, in which defecting firms can see a decrease in their payoffs, can happen if it triggers a sequence of defections that eventually ends in a stable structure in which all of the initially deviating firms are better off than in the original structure.

Following Chwe [1994], we define the LCS as follows. A set of coalition structures is called consistent if for each coalition in the consistent set all possible defections by any subset of players are deterred, as they may eventually lead to a member of the consistent set that is not preferred by some of the players who made the initial defection. More formally, $\mathcal{M} \subseteq \mathbf{X}$ is called consistent, if $X \in \mathcal{M}$ if and only if for all $Y \in \mathbf{X}$ and $S \subseteq N$ such that $X \rightharpoonup_S Y$, there is a $Z \in \mathcal{M}$, where $Y = Z$ or $Y \ll Z$, such that $X \not\prec_S Z$. The LCS is the largest consistent set. Chwe [1994] proves the existence, uniqueness, and non-emptiness of the largest consistent set. Since every coalition considers the possibility that, once it reacts, another coalition may react, and then yet another, and so on, the LCS incorporates *dynamic* coalition stability. The LCS describes all possible stable outcomes and has the merit of "ruling out with confidence". That is, if $X \notin$ the LCS, X cannot be stable. For a more detailed analysis of farsighted coalition stability, see Chwe [1994].

As mentioned earlier, dynamic stability may also be useful in identifying potentially stable outcomes in cases in which static stability concepts, such as the core and the coalition structure core, turn out to be empty. We illustrate this with some examples in the next subsection (see comments in Examples 1 and 2).

In the following subsection, we use the LCS to identify recycling structures that are stable from the dynamic perspective.

3.3.2 Stable Recycling Structures

We use expressions for profits from the Appendix (Section A) to evaluate firms' payoffs and identify stable structures. Note that the multi-product firm B controls two products, 2 and 3, while the specialized firm A controls only product 1, which gives more power to firm B. More precisely, if the current structure is product-based or firm-based recycling, $\{1\}\{2\}\{3\}$ or $\{1\}\{23\}$, specialized firm A cannot change it unilaterally, while multi-product firm B can (namely, $\{1\}\{2\}\{3\} \rightharpoonup_B \{1\}\{23\}$, $\{1\}\{23\} \rightharpoonup_B \{1\}\{2\}\{3\}$). As a result, whenever $\{1\}\{2\}\{3\}$ or $\{1\}\{23\}$ generates the highest payoff for firm B (compared to other structures), this structure is uniquely stable—B does not want to defect from it, A cannot change the structure on its own, and B can defect to either of these structures from any of the remaining possible structures.

Proposition 2 *Consider the EP for asymmetric manufacturing. When $\kappa = 0$, the market-based recycling, $\{12\}\{3\}$, and product-based recycling, $\{1\}\{2\}\{3\}$, generate identical payoff and are both stable.*

The statement of Proposition 2 is intuitive. When there is no economies of scale, firms have no incentives for joint recycling. Recall that the all-inclusive recycling, $\{123\}$, increases the unit recycling costs, but the market-based recycling, $\{12\}\{3\}$, does not. Consequently, both firms prefer the market-based recycling or product-based recycling. This result holds for arbitrary unit costs, $c_1 = c_2$ and c_3, and for any substitution level, γ. It is consistent with SP; that is, the endogenously formed recycling structure also achieves the highest social welfare.

Proposition 3 *Consider the EP for asymmetric manufacturing. When $\kappa < 0$, the product-based recycling, $\{1\}\{2\}\{3\}$, is the most common stable structure; firm-based recycling, $\{1\}\{23\}$, can be stable in some instances in which market size of the standalone product 3 is significantly smaller than the market sizes of the remaining products.*

Although product-based recycling is the only optimal structure that generates the highest social welfare when $\kappa < 0$ in SP, it is not always the most preferred structure for both firms in EP because of the market competition. However, due to the endogeneity of the structure formation, in most cases, product-based recycling emerges as uniquely stable; this result is not surprising for a model with diseconomies of scale. When the market size of the standalone product 3 is significantly smaller than those of competing products 1 and 2, the quantity of product 3 is much smaller than those of products 1 and 2. By jointly recycling the unrelated products 2 and 3 together, multi-product firm B also reduces the equilibrium quantity of product 2, leaving product 1 of the specialized firm with a large equilibrium quantity (generating diseconomies of scale), yielding lower profits for the specialized firm A, which makes the firm-based coalition attractive for the multiproduct firm B (and stable). This outcome is also a result of the market competition.

Proposition 4 *Consider the EP for asymmetric manufacturing. When $\kappa > 0$, all structures may emerge as stable (depending on parameter values):*

1. *when economies of scale are moderate to high or cost increase (λ) is low, the most common stable structure is all-inclusive recycling,* $\{123\}$*; when economies of scale are low and cost increase (λ) is moderate to high, the most common stable structure is market-based recycling,* $\{12\}\{3\}$*;*

2. *firm-based recycling,* $\{1\}\{23\}$*, can be uniquely stable when products are highly substitutable, market size of the standalone product 3 is significant compared with market sizes of products 1 and 2, market size of product 2 dominates that of product 1, economies of scale are low, and cost increase (λ) is low;*

3. *cross-market/firm recycling,* $\{13\}\{2\}$*, can be stable when products are highly substitutable, market size of product 1 dominates that of product 2, and either economies of scale are moderate to high, or economies of scale are low and market size of the standalone product 3 is low compared with products 1 and 2;*

4. *product-based recycling,* $\{1\}\{2\}\{3\}$*, can be uniquely stable when products are highly substitutable, market sizes of products are significantly different, and economies of scale are low.*

Similarly as with dis-economies, firm-based recycling can emerge as stable in EP, while it is never optimal in the SP, but, interestingly with economies of scale, cross-market/firm recycling can emerge as stable while it is never optimal in the SP. Below, we intuit the cases described above.

- We start with the first item. As universal recyclers who can recycle products from different markets charge higher prices ($\lambda > 1$), whether to use a universal recycler to recycle all products depends on the relationship between the scale economies and the cost increase. If the economies of scale dominate the cost increase (e.g., the scale economies, κ, are moderate to high, or the cost increase, λ, is low), the all-inclusive recycling emerges as stable. Otherwise, firms do not want to contract with universal recyclers but want specialty recyclers who charge lower prices; as the economies of scale still exist, the best solution is the market-based recycling.

- Now, we consider the second item. The firm-based recycling can emerge as uniquely stable only when multi-product firm B has the incentive to adopt it. When the market competition between products 1 and 2 is intense (i.e., γ is high), firm B has a stronger incentive to reduce the market share of product 1 by increasing the recycling cost (or, more specifically, restricting the economies of scale) of specialized firm A. On one side, when the market size of the standalone product 3 is significant, the quantity of product 3 can significantly impact economies of scale; in this case, not being able to take advantage of the cost reduction generated by recycling jointly with product 3 (which belongs to firm B) can lead to higher recycling cost for specialized firm A and make it less competitive. Therefore, firm B would not want its product 3 to be recycled together with the specialized firm A's standalone product 1. On the other side, although recycling competing products 1 and 2 together creates the economies of scale without incurring high unit cost, firm B would not want to jointly recycle product 2 with the specialized firm A's product 1 if the market size of product 2 dominates that of product 1, as firm A

would take advantage of the scale economies that are mostly created by firm B's product 2. Note that when the market size of product 1 dominates that of product 2, firm B wants to recycle competing products 1 and 2 together, as in this case, firm B would benefit from the scale economy that is mostly created by firm A's product 1.

Therefore, product 1 will be recycled individually. Then, multi-product firm B chooses between the firm-based recycling and the product-based recycling. Under the firm-based recycling, firm B benefits from the economies of scale but suffers from the high unit cost. It would adopt the firm-based recycling when both of its products (2 and 3) have large market sizes (and corresponding quantities) and the cost increase is low.

- Next, we consider the third item. When products are highly substitutable, both firms have strong incentives to reduce the market share of the other firm by increasing its recycling cost (or, more specifically, restricting the economies of scale). When the market size of product 1 dominates that of competing product 2, specialized firm A does not want its product 1 to be recycled together with product 2; otherwise, multi-product firm B would benefit from the scale economies that are mostly created by firm A's product 1. When the economies of scale are moderate to high, firm A would like to recycle product 1 and firm B's standalone product 3 together; that is, A prefers the cross-market/firm recycling. When the economies of scale are low and the market size of product 3 is low compared with products 1 and 2, specialized firm A still has the incentive to recycle together with product 3 because product 3 is from another independent market. In either case, firm B prefers the cross-market/firm recycling as it increases the unit recycling cost of product 1.

- Finally, we consider the last item. When market size of one competing product (1 or 2) dominates that of the other product, products are highly substitutable, and economies of scale are low, product-based recycling is either the most-preferred outcome of multi-product firm B, or it is its second favorite, after market-based recycling. At the same time, specialized firm A prefers product-based recycling to both all-inclusive and market-based. As a result, neither firm can unilaterally move from product-based recycling to the outcome it prefers to it, and product-based recycling is uniquely stable.

We illustrate the cases described above with a numerical example.

Example 1: In Table 2 we provide some illustrations of parameter values and corresponding stable outcomes; in all cases, $c_1 = c_2 = 2, c_3 = 5$.

Note that, for instance, when $\alpha_1 = 100, \alpha_2 = 50, \alpha_3 = 300, \gamma = 0.7, \kappa = 0.02, \lambda = 1.8$, cross-market/firm recycling is the most preferred structure for firm A, followed by the product-based recycling, while market-based recycling is the most preferred outcome for firm B, also followed by the product-based recycling, and static solution concepts do not help us in determining stable outcomes. However, the use of dynamic solution concepts help us determining that product-based recycling is stable in this setting.

Parameters						Stable	Parameters						Stable
α_1	α_2	α_3	γ	κ	λ	structure	α_1	α_2	α_3	γ	κ	λ	structure
100	200	300	0.5	0.1	≤ 2	{123}	100	200	300	0.75	0.02	1.2	{23}{1}
100	200	300	0.5	0.02	1.8	{12}{3}	100	200	300	0.75	0.02	1.8	{1}{2}{3}
100	100	300	0.9	0.1	≤ 2	{123}	100	100	300	0.9	0.02	1.8	{12}{3}
100	200	300	0.5	0.02	1.1	{123}	100	100	300	0.5	0.02	1.1	{123}
100	50	300	0.5	0.02	1.8	{12}{3}	100	50	300	0.7	0.02	1.8	{1}{2}{3}
100	50	300	0.6	0.17	≤ 2	{13}{2}	300	150	50	0.75	0.02	1.8	{1}{2}{3}
300	150	50	0.75	0.1	1.1	{13}{2}	300	150	50	0.75	0.02	1.1	{13}{2}

Table 2: Parameter Values and Corresponding (Unique) Stable Outcomes for Asymmetric Manufacturing

There are also cases in which multiple structures may emerge as stable, as shown in our next result.

Proposition 5 *Consider the EP for asymmetric manufacturing. When $\kappa > 0$,*

1. *all-inclusive,* {123}*, and market-based recycling,* {12}{3}*, can both emerge as stable when economies of scale are low, or when economies of scale are moderate and cost increase (λ) is high;*

2. *all-inclusive,* {123}*, and cross-market/firm recycling,* {13}{2}*, can both emerge as stable when market size of product 1 dominates that of product 2, the standalone product 3 has small market size compared to product 1, products are moderately substitutable, and economies of scale are moderate.*

In Proposition 4.1, the all-inclusive and market-based recycling are two major recycling structures. As the two firms conduct asymmetric manufacturing (firm A with one product, firm B with two), they have different preferences for the two recycling structures. Therefore, there exists a transitionary region between them in which both structures may emerge as stable; this is captured in Proposition 5.1 above.

When products are moderately substitutable, both firms have the incentive to reduce the market share of the other firm; however, the incentive is not strong enough to dominate the potential impact of scale economies if the all-inclusive recycling is adopted. When the economies of scale are moderate, firms balance the economies of scale on one side and the market competition on the other side. In addition, when the market size of product 1 dominates that of product 2, specialized firm A is reluctant to recycle product 1 together with its competing product 2; otherwise, multi-product firm B would benefit from the scale economies that are mostly created by product 1. As a result of the three factors, both all-inclusive and cross-market/firm recycling may emerge as stable. This is captured in Proposition 5.2 above.

We provide an illustration of some cases with multiple stable outcomes in our next numerical example.

Example 2: In Table 3 we provide some illustrations of parameter values and corresponding stable outcomes with multiple stable structures; in all cases, $c_1 = c_2 = 2, c_3 = 5$.

Parameters						Stable	Parameters						Stable
α_1	α_2	α_3	γ	κ	λ	structure	α_1	α_2	α_3	γ	κ	λ	structure
300	150	50	0.5	0.1	≤ 2	$\{123\},\{13\}\{2\}$	300	150	50	0.1	0.1	≤ 2	$\{123\}$
300	150	50	0.75	0.1	≤ 2	$\{13\}\{2\}$	300	300	100	0.9	0.02	1.8	$\{12\}\{3\}$
300	300	100	0.9	0.1	2	$\{123\},\{12\}\{3\}$	300	300	100	0.9	0.1	1.1	$\{123\}$
300	300	100	0.1	0.02	1.2	$\{123\},\{12\}\{3\}$	300	300	100	0.1	0.02	2	$\{12\}\{3\}$

Table 3: Parameter Values and Corresponding (Multiple) Stable Outcomes for Asymmetric Manufacturing

Note that, for instance, when $\alpha_1 = 300, \alpha_2 = 300, \alpha_3 = 100, \gamma = 0.9, \kappa = 0.1, \lambda = 2$, market-based recycling is the most preferred structure for firm A, followed by the all-inclusive recycling, while all-inclusive recycling is the most preferred outcome for firm B, followed by the market-based recycling, and—again—static solution concepts do not help us in determining stable outcomes. However, the use of dynamic solution concepts help us determining that both market-based and all-inclusive recycling emerge as stable in this setting. In addition, it is interesting to note that when all products have identical market sizes and face identical recycling cost (for example, $c_1 = c_2 = c_3 = 2$, $\alpha_1 = \alpha_2 = \alpha_3 = 100$), all-inclusive and market-based recycling can also both be stable (for instance, when $\lambda = 2$, $\gamma = 0.5$, and $\kappa = 0.05$).

Based on Propositions 1-5, we also compare the optimal outcome in SP and the stable outcome(s) in EP.

Proposition 6 *Consider the SP and EP for asymmetric manufacturing.*

1. *For $\kappa = 0$, the SP optimal outcome always coincides with the EP stable outcome;*
2. *For $\kappa < 0$, when the market size of the standalone product 3 is significantly smaller than those of products 1 and 2, the firm-based recycling is stable in EP but the product-based recycling is optimal in SP;*
3. *For $\kappa > 0$:*
   (a) *The switch from all-inclusive to market-based recycling occurs at lower value of scale economies in SP than in EP.*
   (b) *When market-based recycling is optimal in SP, it often emerges as the stable outcome in EP; the exception is the case with highly substitutable products with significantly different market sizes, in which case only product-based recycling is stable in EP.*
   (c) *When all-inclusive recycling is optimal in SP, any structure can emerge as stable in EP, depending on parameter values.*

The above result merits some discussion. We first look at the cases in which the optimal/stable outcomes coincide under both models. When the optimal structure in SP and the stable structure in EP are the same,

(for example, when there are no (dis)economies of scale, or in the presence of diseconomies of scale when the standalone product 3 has a considerable market size, or when products have low substitutability and the economies of scale are either low or high), both models achieve the same social welfare. In other words, there is no social welfare loss if the government let producers freely make recycling decisions.

Proposition 1 states that in the presence of economies of scale, the SP chooses one of the two options—market-based or all-inclusive recycling. Item 3.a in Proposition 6 indicates that in SP, the all-inclusive recycling can be optimal for a larger range of parameter values compared to its stability in the EP model. This happens because in SP, any firm benefiting from the all-inclusive recycling would contribute to the social welfare, hence all-inclusive recycling is optimal even when there is a modest benefit from scale economies. On the other hand, in EP, either both firms benefiting from the all-inclusive recycling or no firm losing too much are the main factors that lead to stability of that outcome, thus all-inclusive recycling requires higher impact of scale economies to emerge as stable.

Next, we consider what happens when the choices made in the SP and EP models may differ. As shown in Proposition 5, there are cases in which there exist multiple (two) stable recycling structures in EP; the SP optimal structure is one of them. In this case, if the SP optimal structure is chosen by firms, there is no social welfare loss. However, if firms adopt the other structure, the government faces the risk of losing some social welfare. For example, when products are less substitutable but the cost increase is significant, if firms adopt {12}{3}, there will be social welfare loss (about 8% in the example below). When products are more substitutable but the cost increase is small, if firms adopt {13}{2}, we can see a similar result (about 8% welfare loss in the example below). When this happens, government intervention can prevent social welfare loss, as we discuss below.

Finally, there exist cases in which the SP optimal structure is different from the (unique) EP stable structure. When this happens, having the producers independently choose their recycling mode always leads to the social welfare loss; depending on parameter values, this loss may be negligible, or it may have a larger impact. In particular, in the presence of high market competition, we are likely to see a decrease in social welfare regardless of the level of scale economies. In addition, with less intense market competition and moderate scale economies, the all-inclusive recycling dominates the market-based recycling in SP, but competition induces firms to prefer the market-based recycling in EP. Moreover, in the presence of diseconomies of scale, a scenario in which the standalone product 3 has significantly smaller market size than products in the duopoly market induces firms to adopt firm-based recycling, although product-based recycling generates the highest social welfare. As we illustrate in the example below, when there is a difference in optimal/stable outcomes between the two models, social welfare loss can be higher than 5%, especially when socially preferred outcome is all-inclusive recycling. This implies that government can benefit from adopting legislations that encourage formation of universal recyclers and all-inclusive recycling, which can capture the benefits that could otherwise be left on the table.

Example 3: In Table 4 we provide some illustrations of parameter values and corresponding (potential)

social welfare losses when comparing SP optimal outcome with the EP stable structure. In all examples, $c_1 = c_2 = 2, c_3 = 5$. The percentage loss depends on the parameter values.

α_1	α_2	α_3	γ	λ	κ	SP optimal structure	EP stable structure	Social welfare loss under EP stable structure
290	290	100	0.15	2	0.1	{123}	{123} and {12}{3}	8.17% ({12}{3})
300	150	280	0.5	1.2	0.1	{123}	{123} and {13}{2}	8.69% ({13}{2})
300	160	50	0.75	1.2	0.1	{123}	{13}{2}	4.41%
100	160	110	0.7	1.2	0.04	{123}	{1}{23}	2.14%
150	240	100	0.7	1.2	0.05	{123}	{1}{2}{3}	6.74%
165	300	50	0.6	1.8	0.03	{12}{3}	{1}{2}{3}	1.22%
150	300	50	0.1	2	0.1	{123}	{12}{3}	5.21%
300	300	100	0.1	2	-0.1	{1}{2}{3}	{1}{23}	3.94%

Table 4: Social Welfare Loss under the EP Stable Structure for Asymmetric Manufacturing

In this section, we studied firms' strategies when two firms (asymmetrically) make three products in two markets. Our results indicate that intense competition in the duopoly market may induce the firms to adopt a recycling structure that does not generate the highest social welfare. In most other scenarios, the two firms' farsighted decision is consistent with the SP optimal structure and maximizes the social welfare. In the next section, we study firms' strategies when the two firms both make two products, one in each of the two markets; we refer to this scenario as symmetric manufacturing.

4 Symmetric Manufacturing of Four Products

4.1 Model Setup

In Section 3, we discussed two firms that make three products—one firm makes products across two different markets, while the other firm makes a product in one of those markets. This setting captures situations in which a multi-product firm competes with a specialized firm. Now, we revert to markets wherein both firms are diversified. To that end, we assume that the specialized firm introduces a new product and thus competes with the multi-product firm in both markets. That is, two firms make a total of four products in two independent markets. Every firm makes a product in each of the markets, and every product made by one firm is (not necessarily perfectly) substitutable with a product made by the other firm. We refer to this model as *symmetric manufacturing*.

We assume that the new product, say 4, is independent from products 1 and 2, and is substitutable with product 3. Firm A makes products 1 and 4, and firm B makes products 2 and 3, as illustrated in figure 1b.

We assume the same substitution effect, γ, to hold between products 1 and 2 and between products 3 and 4. Assuming that q_i is the quantity of product i, $i = 1, 2, 3, 4$, following Singh & Vives [1984], the market surplus becomes

$$U(q_1, q_2, q_3, q_4) = \sum_{i=1}^{4} \alpha_i q_i - \frac{1}{2}\left(\sum_{i=1}^{4} q_i^2\right) - \gamma q_1 q_2 - \gamma q_3 q_4. \tag{5}$$

where α_i is the market size of product i reduced by the unit production cost of product i, $i = 1, 2, 3, 4$.

With four products, there are fifteen possible recycling structures. We refer to Table 5 for the nomenclature. Comparing Table 5 with recycling structures discussed in Section 3, the all-inclusive recycling, (full) market-based recycling, (full) firm-based recycling, (full) cross-market/firm recycling, and (full) product-based recycling are similar to the structures in Table 1. In those structures, all products are recycled according to the categorization schemes used in Section 3 (e.g., by markets, by firms, all-inclusive, by product, etc.). Some other structures partially use those categorization schemes. For instance, under {12}{3}{4}, a half market-based structure, only one market (of products 1 and 2) adopts the market-based recycling, while the other market does not. The remaining structures use new categorization schemes. For instance, under {1}{234}, an i-inclusive structure, product 1 is recycled individually, while the remaining three products are jointly recycled. However, this joint recycling does not fall into the market-based or firm-based category. In a word, the i-inclusive recycling refers to structures under which one product is recycled individually, while all other products are jointly recycled.

All-inclusive: {1234}	Full firm-based: {14}{23}
Full cross-market/firm: {13}{24}	Full product-based: {1}{2}{3}{4}
Full market-based: {12}{34}	Half market-based: {1}{2}{34} ,{12}{3}{4}
Half firm-based: {1}{23}{4}, {14}{2}{3}	Half cross-market/firm: {1}{24}{3}, {13}{2}{4}
i-inclusive: {1}{234}, {134}{2}, {124}{3}, {123}{4}	

Table 5: Nomenclature of Recycling Structures for Symmetrically Manufactured Products.

We use c_i to denote the cost of recycling each unit of product i, $i = 1, 2, 3, 4$, and we assume that products from the same market incur the same unit recycling costs: $c_1 = c_2$ and $c_3 = c_4$. Because of the product heterogeneity, when products 1 and/or 2 are recycled together with products 3 and/or 4, their unit costs increase by the factor of λ. Then, under the recycling structure X, the cost for recycling product i is

$$C_i^X(q_1, q_2, q_3, q_4) \doteq \overline{\lambda}_i^X c_i q_i - \frac{q_i}{\sum_{j \in Z_i^X} q_j} \kappa \left(\sum_{j \in Z_i^X} q_j\right)^2, \tag{6}$$

where $\overline{\lambda}_i^X = \begin{cases} 1, \text{ if } Z_i^X = \{1\}, \{2\}, \{3\}, \{4\}, \{12\}, \text{ or } \{34\}, \\ \lambda, \text{ otherwise.} \end{cases}$

We again start with the benchmark model, SP, in which firms determine production quantities while the government determines the recycling structure. Then, we study the Endogenous Problem, EP, in which firms competitively determine their production quantities and cooperatively choose the recycling structure.

4.2 Social Problem (SP)

For a given recycling structure X, the social welfare generated from the four products are

$$W^X(q_1, q_2, q_3, q_4) = U(q_1, q_2, q_3, q_4) - \sum_{i=1}^{4} C_i^X(q_1, q_2, q_3, q_4), \tag{7}$$

where $U(q_1, q_2, q_3, q_4)$ is given in equation (5) and $C_i^X(q_1, q_2, q_3, q_4)$ is given in equation (6). The objective of the government is to maximize the social welfare based on firms' equilibrium quantities: $\max_{X \in \mathbf{X}} W^X(q_1^X, q_2^X, q_3^X, q_4^X)$.

Proposition 7 *Consider the SP for symmetric manufacturing. There exists $\kappa_0 > 0$ such that*

- *When $\kappa < 0$, the full product-based recycling, $\{1\}\{2\}\{3\}\{4\}$, is optimal;*
- *When $\kappa = 0$, the full product-based, $\{1\}\{2\}\{3\}\{4\}$, half market-based, $\{12\}\{3\}\{4\}$ and $\{1\}\{2\}\{34\}$, and full market-based recycling $\{12\}\{34\}$ are optimal;*
- *When $0 < \kappa \leqslant \kappa_0$, full market-based recycling, $\{12\}\{34\}$, is optimal;*
- *When $\kappa \geqslant \kappa_0$, all-inclusive recycling, $\{1234\}$, is optimal.*

Proposition 7 shows that when the government organizes recycling, our results from the asymmetric manufacturing (Proposition 1) carry over. That is, the government in SP should recycle all products together (when the economies of scale are high), jointly recycle products according to their markets (when the economies of scale are low), or do not jointly recycle at all (in the presence of diseconomies of scale). As joint recycling and individual recycling do not make difference in the social welfare in the absence of (dis)economies of scale, having one firm recycle its products jointly while the other firm recycles them individually will also be optimal.

4.3 Endogenous Problem (EP)

The interesting question now is: When firms have a freedom to determine their recycling structures, do our results from asymmetric manufacturing carry over? For instance, when the market competition is intense, should products manufactured by one firm be recycled together in presence of high scale economies? Should firms recycle across markets/firms for moderate economies of scale?

We first obtain products' prices from equation (5): $p_i = \frac{\partial U}{\partial q_i} = 1 - \beta q_i - \gamma q_j$, $i, j = 1, 2$, $i \neq j$, or $i, j = 3, 4$, $i \neq j$. Under the recycling structure $X \in \mathbf{X}$, the payoff from product i is $\pi_i^X = p_i q_i - C_i^X$, where C_i^X is given in equation (6). The two firms choose their product quantities to optimize their profits:

$\Pi_A^X = \max_{q_1,q_4} \{\pi_1^X + \pi_4^X\}$ and $\Pi_B^X = \max_{q_2,q_3} \{\pi_2^X + \pi_3^X\}$. Based on the optimal payoffs under different recycling structures, the two firms agree on a stable outcome (recycling structure).

We start our analysis with the simplest case, in which quantity being recycled does not impact recycling cost.

Proposition 8 *Consider the EP for symmetric manufacturing, and assume $c_1 = c_2$, $c_3 = c_4$. When $\kappa = 0$, $\Pi_i^{\{1\}\{2\}\{3\}\{4\}} = \Pi^{\{12\}\{34\}} = \Pi^{\{1\}\{2\}\{34\}} = \Pi^{\{12\}\{3\}\{4\}}$, and these structures are the only stable outcomes.*

The statement of Proposition 8 is intuitive. When there is no economies of scale, firms have no incentives for joint recycling. Recall that the all-inclusive recycling, $\{1234\}$, increases the unit recycling costs, but when products from the same market are recycled together, or when products are recycled individually, the unit recycling cost do not increase. Consequently, firms may adopt the market-based recycling, or the product-based recycling, or a combination of the two. This result is carried over from the asymmetric manufacturing model (Proposition 2).

We now turn to the model with diseconomies of scale.

Proposition 9 *Consider the EP for symmetric manufacturing, and assume $c_1 = c_2$, $c_3 = c_4$. When $\kappa < 0$, the product-based recycling, $\{1\}\{2\}\{3\}\{4\}$, is the only stable outcome.*

According to Proposition 7, product-based recycling is the only optimal structure that maximizes social welfare in SP when $\kappa < 0$. At the same time, product-based recycling is not always the most preferred structure for both firms in EP because of market competition. However, due to the endogeneity of the stable structure formation, product-based recycling still emerges as uniquely stable; this result is not surprising for a model with diseconomies of scale.

Recall that in the asymmetric manufacturing model (Proposition 3) we showed that the firm making multiple products (firm B) has the incentive to adopt the firm-based recycling, if its market size in the monopoly market is small. As a result, specialized firm (firm A) has no other options except product-based recycling, since it only makes one product. In the symmetric manufacturing model wherein each of the two firms makes multiple products, we observe a similar phenomenon: each firm would want its competitor to adopt product-based recycling, and this outcome can be incentivized if the firm itself adopts firm-based recycling. However, as the same incentive exists on both sides, if both firms adopt firm-based recycling, neither of them creates the desired effect on its competitor's side. As a result, both firm converge and adopt product-based recycling. When comparing asymmetric and symmetric manufacturing, although the two results seem different, the coalition formation process follows the same logic justified by same incentives, and we consider the two results to be consistent.

Finally, we consider the model with scale economies and obtain the following result.

Proposition 10 *Consider the EP for symmetric manufacturing with $\kappa > 0$, and assume $c_1 = c_2$, $c_3 = c_4$. Without loss of generality, assume* $\max\{\alpha_1, \alpha_2\} \leq \max\{\alpha_3, \alpha_4\}$.

1. *When economies of scale are moderate to high, or when economies of scale are low and cost increase (λ) is low, the most common stable structure is all-inclusive recycling; when economies of scale are low and cost increase (λ) is moderate to high, the most common stable structure is market-based recycling;*

2. *firm-based and half firm-based recycling can be stable when products are highly substitutable, one firm dominates the other firm in both markets (through larger market sizes), and economies of scale are low to moderate;*

3. *i-inclusive recycling is the most common stable outcome when products are highly substitutable and in (at least) one market, market size of one product dominates that of the other product (through larger market size);*

4. *half market-based recycling can be stable when products are highly substitutable, in (at least) one market, market size of one product dominates that of the other product, economies of scale are low and cost increase (λ) is high.*

As we can see from above, when $\kappa > 0$, all results are either directly carried over from the asymmetric manufacturing model, or they are consistent with results in the asymmetric manufacturing model (Proposition 4). Let us discuss this in more detail.

- The first item is carried over from the finding in the asymmetric manufacturing (Proposition 4.1). As the scale economies intensify, or the unit cost increase declines, firms tend to move from the market-based recycling to the all-inclusive recycling.

- Consider the second item. When the market competition is intense, both firms have strong incentives to compete with each other. In the presence of large market size differentials, such incentive is further enhanced. With moderate economies of scale, both firms may adopt firm-based recycling (with high scale economies, all-inclusive recycling dominates firm-based recycling; with low scale economies, market-based recycling dominates firm-based recycling), hence firm-based recycling may be stable. When one firm dominates the other firm in both markets (through significantly larger market sizes), it has larger quantities than the dominated firm in both markets. As a result, when economies of scale are not too low, the dominating firm can still benefit from adopting firm-based recycling, while the same may not be true for the dominated firm because of the low economies of scale effect stemming from its smaller quantities. Consequently, this scenario leads to stability of the half firm-based recycling structure.

  In the asymmetric manufacturing (Proposition 4.2), when multi-product firm B's product has a larger market size, firm B chooses firm-based recycling, leaving specialized firm A to recycle on its own. From such perspective, the results in the asymmetric manufacturing and symmetric manufacturing are consistent.

- We now look at the third item. When products are highly substitutable and one firm (say, A) dominates the other firm (B) in (at least) one market, the larger firm (firm A) would not want to have both of its products recycled together with B's product that has larger market size. For example, if $\alpha_1 = 600$, $\alpha_2 = 300$, $\alpha_3 = 100$, and $\alpha_4 = 100$, firm A has a stronger market presence, and it would not want to recycle its products along with product 2, the one of two B's products with a larger market size. The reasoning behind this is that product 2, belonging to the less "powerful" firm, would enjoy higher scale economies due to its larger quantity. As a result, such product of the other firm is excluded from joint recycling of the other three products. In other words, product 2 is recycled individually, while the other three products are recycled together, leading to stability of i inclusive recycling ($\{134\}\{2\}$ in our example).

  In the asymmetric manufacturing (Proposition 4.3), when specialized firm A is more "powerful" than multi-product firm B (product 1 dominates product 2), A would not want to have its product recycled together with product 2 when product 2 has larger market size than product 3. As a result, product 2 is excluded from joint recycling of the other two products, leading to stability of cross market/firm recycling, $\{13\}\{2\}$. From such perspective, the results in the asymmetric manufacturing and symmetric manufacturing are consistent.

- Finally, consider the last item. When products are highly substitutable, if economies of scale are low and cost increase is high, joint recycling of products from different markets can hurt firms. Therefore, either market-based or product-based recycling should be adopted. If in (at least) one market a firm has market dominance over the other firm, the firm with the stronger market presence prefers to separately recycle its product in the more differentiated market and take advantage of scale economies at a higher extent than its competitor. At the same time, it wants to jointly recycle its product in the less differentiated market, where both firms experience more similar advantage from economies of scale. For example, if $\alpha_1 = 100$, $\alpha_2 = 200$, $\alpha_3 = 300$, and $\alpha_4 = 300$, market with products 3 and 4 is less differentiated than market with products 1 and 2, and firm B prefers to recycle product 2 independently from product 1, but to jointly recycle product 3 with product 4. As a result, the half market-based recycling, $\{1\}\{2\}\{34\}$, can emerge as stable.

  In the asymmetric manufacturing (Proposition 4.4), when product 2 dominates product 1 (through a larger market size), multi-product firm B has more incentive to individually recycle product 2. As specialized firm A cannot adopt the market-based recycling without collaboration of firm B, product-based recycling, $\{1\}\{2\}\{3\}$, emerges as stable. From such perspective, the results in the asymmetric manufacturing and symmetric manufacturing are consistent.

We now illustrate our results with some numerical examples.

EXAMPLE 4: In Table 6 we provide some illustrations of parameter values and corresponding unique stable outcomes; in all cases, $c_1 = c_2 = 2, c_3 = c_4 = 5$.

Parameters							Stable	Parameters							Stable
a_1	a_2	a_3	a_4	γ	κ	λ	structures	a_1	a_2	a_3	a_4	γ	κ	λ	structures
100	100	300	300	0.1	0.02	2	{12}{34}	100	100	300	300	0.5	0.1	≤ 2	{1234}
70	100	300	150	0.1	0.02	1.2	{1234}	100	100	300	150	0.5	0.1	2	{12}{34}
100	100	300	600	0.66	0.02	1.2	{124}{3}	100	100	300	600	0.66	0.02	2	{12}{3}{4}
100	100	300	150	0.75	0.1	2	{123}{4}	100	100	300	150	0.66	0.02	2	{12}{3}{4}
115	200	285	150	0.7	0.09	1.2	{14}{23}	78	100	300	155	0.8	0.1	2	{1}{23}{4}

Table 6: Parameter Values and Corresponding Unique Stable Outcomes for Symmetric Manufacturing

There are also cases in which multiple structures may emerge as stable, as shown in our next result.

Proposition 11 *Consider the EP for symmetric manufacturing. When $\kappa > 0$,*

1. *all-inclusive and market-based recycling can both emerge as stable when economies of scale are low, product substitutability is low, cost increase (λ) is high, and the market sizes differ;*

2. *all-inclusive and i-inclusive recycling can both emerge as stable when product substitutability is moderate to high;*

3. *i-inclusive and half cross-market/firm recycling or cross-market/firm recycling can both emerge as stable when product substitutability is high and market sizes are diverse in both markets.*

We now briefly discuss this result. In Proposition 10.1, the all-inclusive and market-based recycling are identified as two most common recycling structures. When the two firms have different market sizes, they may have different preference for these two recycling structures. As a result, there exists a transitionary region between them wherein both structures can emerge as stable, as shown in Proposition 11.1 above. This result is carried over from the asymmetric manufacturing (Proposition 5.1).

The results of Proposition 11.2-11.4 are consistent with results of Proposition 5.2. More precisely, when products are moderately substitutable, the incentive for exclusion of the product with a smaller market size from the three-product coalition in Proposition 11.2 (resulting in the i-inclusive recycling) is similar to the incentive for exclusion of a dominated product from the two-product coalition in Proposition 5.2 (resulting in the cross-market/firm recycling). When competition between firms is intense, there may be an incentive to exclude one more dominated product. The two dominated products may either be recycled together, resulting in cross-market/firm recycling, or be recycled separately, resulting in half cross-market/firm recycling, depending on the level of cost increase.

Example 5: In Table 7 we provide some illustrations of parameter values and corresponding multiple stable outcomes; in all cases, $c_1 = c_2 = 2, c_3 = c_4 = 5$.

The example below shows some transitions between stable outcomes with changes in parameter values.

Parameters							Stable	Parameters							Stable
a_1	a_2	a_3	a_4	γ	κ	λ	structures	a_1	a_2	a_3	a_4	γ	κ	λ	structures
100	200	300	300	0.1	0.02	1.85	$\{1234\},\{12\}\{34\}$	70	100	300	150	0.5	0.02	≤ 2	$\{1234\},\{123\}\{4\}$
70	100	200	400	0.5	0.02	1.2	$\{1234\},\{124\}\{3\}$	100	50	300	300	0.66	0.02	1.2	$\{1234\},\{134\}\{2\}$
100	100	300	300	0.9	0.02	1.2	$\{1234\},\{234\}\{1\},\{134\}\{2\}$	100	100	300	300	0.9	0.1	2	$\{1234\},\{234\}\{1\},\{134\}\{2\}$
100	70	300	150	0.75	0.07	1.2	$\{123\}\{4\},\{13\}\{2\}\{4\}$	100	70	300	150	0.75	0.08	2	$\{123\}\{4\},\{13\}\{24\}$

Table 7: Parameter Values and Corresponding Multiple Stable Outcomes for Symmetric Manufacturing

EXAMPLE 6: Suppose $\alpha_1 = 100, \alpha_2 = 200, \alpha_3 = \alpha_4 = 300, \gamma = 0.1$. In this case market size of product 2 dominates market size of product 1, and product substitutability is low. When economies of scale are low (say, $\kappa = 0.02$), $\{1234\}$ and $\{12\}\{34\}$ can both be stable for high λ, as there is small benefit from recycling all products together; when economies of scale are medium to high (say, $\kappa = 0.1$), $\{1234\}$ is uniquely stable as firms forgo market-based recycling to take advantage of higher economies of scale.

Based on propositions 7-11, we also compare the optimal outcome in SP and the stable outcome(s) in EP. This is summarized in our next result.

Proposition 12 *Consider the SP and EP for symmetric manufacturing.*

1. *For $\kappa \leq 0$, the SP optimal outcome always coincides with the EP stable outcome;*
2. *For $\kappa > 0$:*
   (a) *The switch from all-inclusive to market-based recycling occurs at lower value of scale economies in SP than in EP;*
   (b) *When market-based recycling is optimal in SP, it often emerges as the stable outcome in EP; the exception is the case when a firm is dominated by the other firm in both markets (through a significantly smaller market sizes), when the half firm-based recycling is stable;*
   (c) *When all-inclusive recycling is optimal in SP, any structure can emerge as stable in EP, depending on parameter values.*

The first item in the above result is completely carried over from Proposition 6.1 for $\kappa = 0$. For $\kappa < 0$, the result of the first item is slightly different from Proposition 6.1, because of different model setting (asymmetric vs. symmetric). The second item contains the case wherein the optimal structure in SP and the stable structure in EP are the same, when there exist multiple stable recycling structures in EP and the SP optimal structure is among them, and when the SP optimal structure is different from the EP stable structure(s).

Once again, we illustrate our result with numerical examples. The table below show potential welfare losses when the stable outcomes do not coincide with socially optimal results. As illustrated in the example

below, when there is a difference in optimal/stable outcomes between the two models, social welfare loss can be higher than 5%, especially when socially preferred outcome is all-inclusive recycling. Similarly to our conclusion in the asymmetric case, this implies that government can benefit from adopting legislations that encourage formation of universal recyclers and all-inclusive recycling.

Example 7: In Table 8 we provide some illustrations of parameter values and corresponding (potential) social welfare losses under the EP stable structure. The percentage loss depends on the parameter values. In all cases, $c_1 = c_2 = 2, c_3 = c_4 = 5$.

α_1	α_2	α_3	α_4	γ	λ	κ	SP optimal structure	EP stable structure	Social welfare loss under EP stable structure
100	100	325	600	0.6	2	0.02	{12}{34}	{12}{3}{4}	0.77%
100	200	300	150	0.7	1.2	0.04	{1234}	{1}{23}{4}	1.49%
115	200	285	150	0.7	1.2	0.09	{1234}	{14}{23}	5.43%
100	100	300	180	0.75	2	0.12	{1234}	{123}{4}	6.10%
100	200	300	300	0.1	1.85	0.02	{1234}	{1234} and {12}{34}	0.98% ({12}{34})
100	100	300	170	0.5	1.2	0.10	{1234}	{1234} and {123}{4}	8.59% ({123}{4})
100	76	300	160	0.75	2	0.08	{1234}	{123}{4} and {13}{24}	2.26% ({123}{4}) or 5.76% ({13}{24})
100	76	300	160	0.75	1.2	0.075	{1234}	{123}{4} and {13}{2}{4}	2.18% ({123}{4}) or 5.54% ({13}{2}{4})

Table 8: Social Welfare Loss under the EP Stable Structure for Symmetric Manufacturing

In this section, we studied firms' strategies when two firms (symmetrically) make four products in two markets. In particular, we compared our results with Section 3 to check if our results from the asymmetric case carry over when producers make more products. As shown in our discussion above, some intuitive results from Section 3 do carry over to the case with more products. For example, when the economies of scale are high/low, the all-inclusive/market-based recycling is most common stable outcomes; in the model with diseconomies of scale, the product-based recycling is usually stable.

What is of more interest are the cases with less intuitive results, for which some of the traditional methods of analysis (static concepts such as the core) would even fail to identify stable outcomes. Our analysis, based on the dynamic coalition formation process, enabled us not only to find stable outcomes, but also to confirm consistent incentives for firms in similar scenarios under both asymmetric and symmetric manufacturing.

As we mentioned in our discussion of Proposition 11, the results for the asymmetric and the symmetric model might seem different, but we can still confirm their consistency. For example, let us consider Propositions 5.2 and 11.2. When the economies of scale are moderate and the market competition is intense, large market size differentials incentivize firms to adopt firm-based recycling strategy. The stable recycling structure may be firm-based (when economies of scales are moderate) or half firm-based (when economies of scales are lower). In the asymmetric manufacturing, when multi-product firm B's product has a larger market size, it chooses firm-based recycling. No matter what decision specialized firm A makes, the firm-

based recycling is the unique stable outcome as A makes only one product. Although the two results seem different, the coalition formation process follows the same logic justified by same incentives, and we consider the two results to be consistent.

Similarly, let us consider Propositions 4.4 and 10.4. Suppose that market competition is intense, economies of scale are low, and cost increase is high. If in (at least) one market, market size of one product dominates that of the other product, the firm with the stronger market presence prefers to recycle individually (resp., jointly) in the more (resp., less) diversified market, resulting in the stability of the half market-based recycling. In asymmetric manufacturing, if the multi-product firm (firm B) is dominated (by firm A) in the duopoly market, it would individually recycle product 2; stand-alone product 3 is also individually recycled because it belongs to a monopoly market. As a result, the product-based recycling is stable. Once again, although the two results seem different, the coalition formation process follows the same logic justified by same incentives, and we consider the two results to be consistent.

5 Conclusion and Discussion

In this paper, we study the recycling of products belonging to different markets and made by different firms. Our analysis is based on two models, *asymmetric multi-product market,* in which two firms make three products in two markets, and *symmetric multi-product market*, in which two firms make four products in two markets. Recycling different products individually avoids the diseconomies of scale. Recycling different products together benefits from the economies of scale. However, because of product heterogeneity, when products from different markets are recycled together, the unit recycling cost increases as well. Important recycling structures discussed in the paper include: *all-inclusive* recycling structure (when all products are recycled together), *market-based* recycling structure (products from the same market are recycled together), *firm-based* recycling structure (products made by the same firm are recycled together), and *product-based* recycling structure (products made by different firms or from different markets are recycled independently).

For each model (asymmetric and symmetric multi-product market), we compare the results of two scenarios: the *Social Problem* (SP), in which the firms determine their product quantities purely based on the competition in the primary market while the government chooses a recycling structure in order to maximize the social welfare, and the *Endogenous Problem* (EP), in which firms not only competitively determine their own outputs but also cooperatively determine the recycling structure. In EP, the recycling structure is reached by taking into account each individual firm's payoff with the recycling costs included; therefore, we consider endogenously formed coalitions containing products made by individual firms with different payoffs. The objective in our paper is to study the interaction between multiple firms manufacturing across multiple markets, and the impact of (dis)economies of scale, product heterogeneity, market sizes, and multi-product market competition on the recycling structure.

As shown by our analysis, there is a significant consistency between stable results in asymmetric and

symmetric multi-product market. In both cases, most common stable outcomes in presence of scale economies are all-inclusive and market-based recycling, and they can be stable together. It is interesting to note that when competing products have same market sizes in the symmetric multi-product market, all-inclusive and market-based recycling structures are never stable together because firms have symmetric preferences and they always agree in their rankings of the two structures; thus, multiple stable outcomes in symmetric multi-product market occur purely because of differences in market presence.

One notable result from our analysis is that intense competition and market presence heterogeneity can induce firms to adopt some less intuitive recycling strategies, such as firm-based recycling. Although firm-based recycling increases firms' unit recycling cost, it can emerge as stable; this phenomenon can be observed in both asymmetric and symmetric multi-product markets, in presence of a dominating firm (in terms of higher market shares). This result is similar to the finding obtained by Esenduran & Kemahlıoğlu-Ziya [2015]. Although they look at a different model, they conclude that a large firm might prefer firm-based recycling to collaboration with multiple small firms. Similarly, in presence of intense competition and market dominance of one product, we can observe outcomes in which all but one products are jointly recycled, leading to structures wherein products from different markets made by different companies are recycled together. Both of these results are counterintuitive if we focus our attention on product heterogeneity and scale economies alone, and they are never the choice of a social planner; their stability comes as the result of market forces. These cases can lead to social welfare loss and deserve attention of social planners.

As one example, we mention in Section 3.2 that in states such as Maryland or Michigan, the state government organizes recycling and firms pay the state for the expenses. Such models may impose additional costs on government, which has to take on additional responsibilities. For instance, Washington State Department of Ecology (2006) mentions that "...it would be in the best interest of the citizens of Washington to require that manufacturers take responsibility for their brand products at end of life... Cost internalization, when used as the financing mechanism associated with the full program recommended herein:

- Minimizes government run programs and overhead costs; ...
- Shares responsibility for end of life management of consumer electronic products between those that create the problem rather than making it a problem of government."

Our results suggest that, indeed, the government does not need to intervene very often; its intervention is needed mostly in cases with high competition level among products and high differentiation between market sizes of different firms. If this is not the case, the government can let the firms choose their recycling options at will, and the outcome will not lead to inefficient recycling structures. An alternative choice of government intervention might be to impose taxes on recyclers implementing choices that lead to efficiency losses; this analysis is beyond the scope of this paper.

We mentioned above that all-inclusive and market-based recycling structures can emerge as stable together, due to different product market shares. We show that the same is true for i-inclusive recycling and

all-inclusive, half cross-market/firm, or cross-market/firm recycling (in the asymmetric case, this is reduced to all-inclusive and cross-market/firm recycling, which is a counterpart of the i-inclusive recycling in the symmetric case). As we discuss in this paper, the phenomenon of multiple potential stable outcomes is a result of the interplay of the level of market competition, differentiation in product market sizes, scale economies, and unit cost increase. When all-inclusive recycling is included in the set of stable outcomes, the firms may end up in a socially optimal structure; when this is not the case, we will always have welfare losses, regardless of the outcome that is eventually chosen by the two firms. Thus, in highly competitive markets governments should encourage formation of universal recyclers and all-inclusive recycling to avoid potential welfare loss.

Although our models are simple and capture the most essential elements of a market in which horizontally differentiated firms compete in primary markets, but, cooperate for recycling, their analysis is complex. Nevertheless, we believe that our study also provides insight in the more complex and realistic situations. Based on our results, we conjecture that the all-inclusive recycling should be adopted in markets with the intermediate level of competition when the potential of economies of scale is high, and that the market-based recycling should be preferred when that potential is low. When competition is intense and high recycling volume can significantly reduce recycling cost, firms with a rich product portfolio and strong market presence should adopt the firm-based recycling strategy, wherein they benefit from economies of scale and an increase in their market share. In some cases, when some of the competing firms have products that are close in terms of their market share, they can be added to the recycling mix, so that firm-based strategy becomes partially cross-market/firm. Other firms (with a smaller product selection) should cooperate and recycle their products together, and, to benefit from the economies of scale, should use either a marked-based or an all-inclusive strategy.

Apple, for instance, uses Brightsar, which specializes in mobile devices, to recycle its iPad, Apple watch and iPhone, and it uses Sims recycling solutions, a universal recycler of electronics and computers, to recycle its Apple TV, iPod, and older devices. This can be seen as consistent with some of our results. There is a significant level of market competition in both markets. In the smartphone market, Apple is a significant player (in 2017, it was ranked second overall, after Samsung[4]), while it is lagging in the streaming media devices (ranked after Roku, Amazon Fire, and Chromecast[5]). In addition, smartphone market is significantly bigger than the streaming media market (1,472 million smartphones sold in 2017 vs. 133 million active users of top four streaming providers in 2017[6]). It is, then, consistent with our results that Apple's product with smaller market share would be recycled jointly with other products made by other firms (hence, the use of universal recycler), while it would use market-based recycling for its product with a larger market share.

[4]https://www.idc.com/getdoc.jsp?containerId=prUS43548018

[5]https://techcrunch.com/2017/08/23/roku-is-the-top-streaming-device-in-the-u-s-and-still-growing-report-finds/

[6]https://techcrunch.com/2017/07/26/emarketers-2017-forecast-puts-roku-ahead-of-chromecast-fire-tv-and-apple-tv/

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Appendix

A Asymmetric Manufacturing

The complete list of all scenarios of recycling structures are shown as follows.

products 1, 2, and 3	product 1	product 2	product 3	Note
recycler z	λc_1	λc_2	λc_3	all-inclusive

products 1 and 2	product 3	product 1	product 2	product 3	Note
recycler x	recycler y	c_1	c_2	c_3	market-based
recycler z	recycler y	λc_1	λc_2	c_3	inferior to market-based
recycler x	recycler z	c_1	c_2	λc_3	inferior to market-based
recycler $z1$	recycler $z2$	λc_1	λc_2	λc_3	inferior to market-based

product 1	products 2 and 3	product 1	product 2	product 3	Note
recycler x	recycler z	c_1	λc_2	λc_3	firm-based
recycler $z1$	recycler $z2$	λc_1	λc_2	λc_3	inferior to firm-based

products 1 and 3	product 2	product 1	product 2	product 3	Note
recycler z	recycler x	λc_1	c_2	λc_3	cross-market/firm
recycler $z1$	recycler $z2$	λc_1	λc_2	λc_3	inferior to cross-market/firm

product 1	product 2	product 3	product 1	product 2	product 3	Note
recycler $x1$	recycler $x2$	recycler y	c_1	c_2	c_3	product-based
recycler z	recycler x	recycler y	λc_1	c_2	c_3	inferior to product-based
recycler x	recycler z	recycler y	c_1	λc_2	c_3	inferior to product-based
recycler $x1$	recycler $x2$	recycler z	c_1	c_2	λc_3	inferior to product-based
recycler $z1$	recycler $z2$	recycler y	λc_1	λc_2	c_3	inferior to product-based
recycler $z1$	recycler x	recycler $z2$	λc_1	c_2	λc_3	inferior to product-based
recycler x	recycler $z1$	recycler $z2$	c_1	λc_2	λc_3	inferior to product-based
recycler $z1$	recycler $z2$	recycler $z3$	λc_1	λc_2	λc_3	inferior to product-based

In the asymmetric model, let $\alpha_1 - c_1 = M_1$, $\alpha_2 - c_2 = M_2$, $\alpha_3 - c_3 = M_3$, $\alpha_1 - \lambda c_1 = P_1$, $\alpha_2 - \lambda c_2 = P_2$, $\alpha_3 - \lambda c_3 = P_3$, $1 - \kappa = X$, and $\kappa - \gamma = Y$. From the first order conditions, the equilibrium quantities are as follows:

- $q_1^{\{123\}} = [2(1-2\kappa)P_1 + (\kappa - \gamma + \kappa\gamma)P_2 + \kappa(1-\gamma)P_3]/[4 - 12\kappa + 6\kappa^2 + 2\kappa\gamma - (1-\kappa)\gamma^2]$, $q_2^{\{123\}} = [2(\kappa - \gamma + \kappa\gamma)P_1 + (4 - 8\kappa + 3\kappa^2)P_2 + \kappa(4 - 3\kappa - \gamma)P_3]/\{2[4 - 12\kappa + 6\kappa^2 + 2\kappa\gamma - (1-\kappa)\gamma^2]\}$, and $q_3^{\{123\}} = \{2\kappa(1-\gamma)P_1 + \kappa(4-3\kappa-\gamma)P_2 + [4(1-\kappa)^2 - (\kappa-\gamma)^2]P_3\}/\{2\left[4 - 12\kappa + 6\kappa^2 + 2\kappa\gamma - (1-\kappa)\gamma^2\right]\}$;
- $q_1^{\{12\}\{3\}} = [2(1-\kappa)M_1 + (\kappa-\gamma)M_2]/[4(1-\kappa)^2 - (\kappa-\gamma)^2]$, $q_2^{\{12\}\{3\}} = [(\kappa-\gamma)M_1 + 2(1-\kappa)M_2]/[4(1-\kappa)^2 - (\kappa-\gamma)^2]$, and $q_3^{\{12\}\{3\}} = M_3/[2(1-\kappa)]$;

- $q_1^{\{1\}\{23\}} = [2(1-2\kappa)M_1 - (1-\kappa)\gamma P_2 - \kappa\gamma P_3]/[(1-\kappa)(4-8\kappa-\gamma^2)]$, $q_2^{\{1\}\{23\}} = [-\gamma M_1 + 2(1-\kappa)P_2 + 2\kappa P_3]/[4-8\kappa-\gamma^2]$, and $q_3^{\{1\}\{23\}} = \{-2\kappa\gamma M_1 + 4\kappa(1-\kappa)P_2 + [4(1-\kappa)^2-\gamma^2]P_3\}/[2(1-\kappa)(4-8\kappa-\gamma^2)]$;
- $q_1^{\{13\}\{2\}} = [2(1-\kappa)P_1 - \gamma M_2 + \kappa P_3]/[4-8\kappa+3\kappa^2-\gamma^2]$, $q_2^{\{13\}\{2\}} = [-2(1-\kappa)\gamma P_1 + (4-8\kappa+3\kappa^2)M_2 - \kappa\gamma P_3]/[2(1-\kappa)(4-8\kappa+3\kappa^2-\gamma^2)]$, and $q_3^{\{13\}\{2\}} = \{2\kappa(1-\kappa)P_1 - \kappa\gamma M_2 + [4(1-\kappa)^2-\gamma^2]P_3\}/[2(1-\kappa)(4-8\kappa+3\kappa^2-\gamma^2)]$;
- $q_1^{\{1\}\{2\}\{3\}} = [2(1-\kappa)M_1 - \gamma M_2]/[4(1-\kappa)^2-\gamma^2]$, $q_2^{\{1\}\{2\}\{3\}} = [-\gamma M_1 + 2(1-\kappa)M_2]/[4(1-\kappa)^2-\gamma^2]$, and $q_3^{\{1\}\{2\}\{3\}} = M_3/[2(1-\kappa)]$.

The equilibrium profits are:

- $\Pi_A^{\{123\}} = (1-\kappa)[2(1-2\kappa)P_1 + (\kappa-\gamma+\kappa\gamma)P_2 + \kappa(1-\gamma)P_3]^2/[4-12\kappa+6\kappa^2+2\kappa\gamma-(1-\kappa)\gamma^2]^2$ and $\Pi_B^{\{123\}} = \{(1-\kappa)[2(\kappa-\gamma+\kappa\gamma)P_1 + (4-8\kappa+3\kappa^2)P_2 + \kappa(4-3\kappa-\gamma)P_3]^2 + (1-\kappa)\{2\kappa(1-\gamma)P_1 + \kappa(4-3\kappa-\gamma)P_2 + [4(1-\kappa)^2-(\kappa-\gamma)^2]P_3\}^2 - 2\kappa[2(\kappa-\gamma+\kappa\gamma)P_1 + (4-8\kappa+3\kappa^2)P_2 + \kappa(4-3\kappa-\gamma)P_3]\{2\kappa(1-\gamma)P_1 + \kappa(4-3\kappa-\gamma)P_2 + [4(1-\kappa)^2-(\kappa-\gamma)^2]P_3\}\}/4/[4-12\kappa+6\kappa^2+2\kappa\gamma-(1-\kappa)\gamma^2]^2$;
- $\Pi_A^{\{12\}\{3\}} = (1-\kappa)[2(1-\kappa)M_1 + (\kappa-\gamma)M_2]^2/[4(1-\kappa)^2-(\kappa-\gamma)^2]^2$ and $\Pi_B^{\{12\}\{3\}} = (1-\kappa)[(\kappa-\gamma)M_1 + 2(1-\kappa)M_2]^2/[4(1-\kappa)^2-(\kappa-\gamma)^2]^2 + M_3^2/4/(1-\kappa)$;
- $\Pi_A^{\{1\}\{23\}} = [2(1-2\kappa)M_1 - (1-\kappa)\gamma P_2 - \kappa\gamma P_3]^2/(1-\kappa)/(4-8\kappa-\gamma^2)^2$ and $\Pi_B^{\{1\}\{23\}} = \{4(1-\kappa)^2[-\gamma M_1 + 2(1-\kappa)P_2 + 2\kappa P_3]^2 + \{-2\kappa\gamma M_1 + 4\kappa(1-\kappa)P_2 + [4(1-\kappa)^2-\gamma^2]P_3\}^2 - 4\kappa[-\gamma M_1 + 2(1-\kappa)P_2 + 2\kappa P_3]\{-2\kappa\gamma M_1 + 4\kappa(1-\kappa)P_2 + [4(1-\kappa)^2-\gamma^2]P_3\}\}/4/(1-\kappa)/(4-8\kappa-\gamma^2)^2$;
- $\Pi_A^{\{13\}\{2\}} = (1-\kappa)[2(1-\kappa)P_1 - \gamma M_2 + \kappa P_3]^2/(4-8\kappa+3\kappa^2-\gamma^2)^2$ and $\Pi_B^{\{13\}\{2\}} = \{[-2(1-\kappa)\gamma P_1 + (4-8\kappa+3\kappa^2)M_2 - \kappa\gamma P_3]^2 + \{2\kappa(1-\kappa)P_1 - \kappa\gamma M_2 + [4(1-\kappa)^2-\gamma^2]P_3\}^2\}/4/(1-\kappa)/(4-8\kappa+3\kappa^2-\gamma^2)^2$;
- $\Pi_A^{\{1\}\{2\}\{3\}} = (1-\kappa)[2(1-\kappa)M_1 - \gamma M_2]^2/[4(1-\kappa)^2-\gamma^2]^2$ and $\Pi_B^{\{1\}\{2\}\{3\}} = (1-\kappa)[-\gamma M_1 + 2(1-\kappa)M_2]^2/[4(1-\kappa)^2-\gamma^2]^2 + M_3^2/4/(1-\kappa)$.

Proof of Proposition 1: With equilibrium quantities q_i^X, $i = 1,2,3$, shown above, we calculate and compare $W^X(q_1^X, q_2^X, q_3^X)$ (given in equation 3) for all $X \in \mathbf{X}$, and then obtain the optimal recycling structure with the highest $W^X(q_1^X, q_2^X, q_3^X)$. ■

Proof of Proposition 2: We first find expressions for payoffs when $\kappa = 0$:

$$\Pi_A^{\{1\}\{2\}\{3\}} = \left(\frac{2M_1 - \gamma M_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{1\}\{2\}\{3\}} = \left(\frac{-\gamma M_1 + 2M_2}{4-\gamma^2}\right)^2 + \left(\frac{M_3}{2}\right)^2;$$
$$\Pi_A^{\{13\}\{2\}} = \left(\frac{2P_1 - \gamma M_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{13\}\{2\}} = \left(\frac{-\gamma P_1 + 2M_2}{4-\gamma^2}\right)^2 + \left(\frac{P_3}{2}\right)^2;$$
$$\Pi_A^{\{1\}\{23\}} = \left(\frac{2M_1 - \gamma P_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{1\}\{23\}} = \left(\frac{-\gamma M_1 + 2P_2}{4-\gamma^2}\right)^2 + \left(\frac{P_3}{2}\right)^2;$$

$$\Pi_A^{\{12\}\{3\}} = \left(\frac{2M_1 - \gamma M_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{12\}\{3\}} = \left(\frac{-\gamma M_1 + 2M_2}{4-\gamma^2}\right)^2 + \left(\frac{M_3}{2}\right)^2;$$
$$\Pi_A^{\{123\}} = \left(\frac{2P_1 - \gamma P_2}{4-\gamma^2}\right)^2, \qquad \Pi_B^{\{123\}} = \left(\frac{-\gamma P_1 + 2P_2}{4-\gamma^2}\right)^2 + \left(\frac{P_3}{2}\right)^2.$$

Under our assumption that $c_1 = c_2$ and $\gamma < 1$, it is easy to verify that $\Pi_A^{\{13\}\{2\}} < \Pi_A^{\{123\}} < \Pi_A^{\{1\}\{2\}\{3\}} = \Pi_A^{\{12\}\{3\}} < \Pi_A^{\{1\}\{23\}}$, and that $\Pi_B^{\{1\}\{23\}} < \Pi_B^{\{123\}} < \Pi_B^{\{2\}\{13\}} < \Pi_B^{\{1\}\{2\}\{3\}} = \Pi_B^{\{12\}\{3\}}$. As mentioned in the body of the document, whenever product-based structure generates highest profit for B, it is uniquely stable. ■

Proof of Proposition 3: As mentioned above, when $\{1\}\{2\}\{3\}$ is strictly preferred by firm B to all other structures, it is uniquely stable. This is the most common outcome under different scenarios. When this does not hold, we can have examples like the ones below:

- $\Pi_A^{\{1\}\{23\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$ Under this scenario, firm A can defect from $\{2\}\{13\}$, $\{2\}\{13\} \rightharpoonup_A \{1\}\{2\}\{3\}$, to product-based structure, which is its second-most-preferred structure. As neither A nor B can unilaterally defect from this outcome to their most preferred outcome, product-based structure is stable. This can happen when market size of product 2 dominates that of product 1 and products are either moderately substitutable, or have low substitutability with high diseconomies of scale. In addition, this can occur when products are highly substitutable with similar market sizes, high diseconomies of scale and low λ.

- $\Pi_A^{\{1\}\{23\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{3\}\{12\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$. Under this scenario, firm A can defect from $\{2\}\{13\}$, $\{2\}\{13\} \rightharpoonup_A \{1\}\{2\}\{3\}$, to product-based structure, which is its second-most-preferred structure. As neither A nor B can unilaterally defect from this outcome to the outcomes they prefer more, product-based structure is stable. This happens when market size of product 2 dominates that of product 1 and products are highly substitutable with high diseconomies of scale.

- When market size of product 1 dominates that of product 2 and products are highly substitutable with high diseconomies of scale, both firms prefer firm-based recycling to all other outcomes and it is uniquely stable.

When the market size of product 3 is low compared with the other products, we can have cases in which $\Pi_A^{\{1\}\{23\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{1\}\{23\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$ or $\Pi_B^{\{2\}\{13\}} > \Pi_B^{\{123\}} > \Pi_B^{\{1\}\{23\}} > \Pi_B^{\{1\}\{2\}\{3\}} > \ldots$. The first case occurs, say, when $\alpha_1 = \alpha_2 = 300, \alpha_3 = 100, c_1 = c_2 = 2, c_2 = 5, \gamma = 0.5, \lambda = 1.5, \kappa = -0.1$, while the second case occurs, say, when $\alpha_1 = 150, \alpha_2 = 300, \alpha_3 = 50, c_1 = c_2 = 2, c_2 = 5, \gamma = 0.7, \lambda = 1.5, \kappa = -0.05$. Firm B would prefer cross market/firm recycling, which is preferred by A less than either product-based or firm-based recycling. Thus, A would defect from $\{2\}\{13\}$ to $\{1\}\{2\}\{3\}$, and the only joint defection that A would join is to $\{1\}\{23\}$. Thus, it is easy to see that firm-based recycling is the only stable outcome. ■

Proof of Proposition 4:

1. When both firms prefer all-inclusive recycling to all other outcomes, it is trivially the only stable outcome. This happens when product substitutability is low, or when product substitutability is moderate and market sizes of product 2 dominates market size of product 1.

   When products 1 and 2 have similar market sizes and product substitutability is moderate or high, we can observe $\Pi_A^{\{13\}\{2\}} > \Pi_A^{\{123\}} > \ldots$ and $\Pi_B^{\{123\}} > \Pi_A^{\{23\}\{2\}} > \ldots$. Then, $\{13\}\{2\}$ cannot be stable as B can always defect to $\{23\}\{1\}$, which it prefers more, and A cannot unilaterally change that outcome. Thus, all-inclusive recycling is uniquely stable again. The same is true when market size of product 1 dominates market size of product 2, product substitutability is moderate, and economies of scale are moderate to high.

   Under most scenarios with low economies of scale and moderate to high substitution level, the preferred outcome for firm B is market-based recycling, followed by the product-based recycling, while firm A favors all-inclusive recycling, but prefers market-based to product-based recycling. Under such scenario, the only possible defection by firm A from market based recycling leads to product-based recycling, which it prefers even less, and market-based recycling is stable.

2. When market size of product 2 dominates that of product 1, and market size of product 3 dominates that of product 2, economies of scale are low, cost increase is low, and products are highly substitutable, firm B prefers firm-based recycling to all other outcomes. As firm A cannot unilaterally change this outcome, firm-based recycling is uniquely stable.

3. When market size of product 1 dominates that of product 2, products are highly substitutable, and economies of scale are moderate to high, both firms prefer cross-market recycling to all other outcomes, and it is the unique stable outcome.

4. When market size of one product (1 or 2) dominates that of the other product, products are highly substitutable, and economies of scale are low, product-based recycling is either the most-preferred outcome of firm B, or it is its second favorite, after market-based recycling. At the same time, firm A prefers product-based recycling to both all-inclusive and market-based. As a result, neither firm can unilaterally move from product-based recycling to the outcome it prefers to it, and product-based recycling is uniquely stable. ■

Proof of Proposition 5:

1. Consider the case $\alpha_1 = \alpha_2 = 300, \alpha_3 = 100, c_1 = c_2 = 2, c_3 = 5, \gamma = 0.9, \lambda = 2, \kappa = 0.1$. We then have $\Pi_A^{\{12\}\{3\}} > \Pi_A^{\{123\}} > \ldots$ and $\Pi_B^{\{123\}} > \Pi_B^{\{12\}\{3\}} > \ldots$ It is then easy to evaluate that both all-inclusive and market-based recycling are stable. Similar preference ordering holds, for instance, when $\alpha_1 = \alpha_2 = 300, \alpha_3 = 100, c_1 = c_2 = 2, c_3 = 5, \gamma = 0.1, \lambda = 1.2, \kappa = 0.02$.

2. When market size of product 1 dominates that of product 2 and they both dominate market size of product 3, products are moderately substitutable, and economies of scale are moderate, one can evaluate that $\Pi_A^{\{13\}\{2\}} > \Pi_A^{\{123\}} > \Pi_A^{\{1\}\{2\}\{3\}} > \ldots$ and $\Pi_B^{\{123\}} > \Pi_B^{\{12\}\{3\}} > \Pi_B^{\{13\}\{2\}} > \ldots$ We can see that both all-inclusive and cross-market recycling are stable by considering following sequences: $\{13\}\{2\} \rightharpoonup_B \{23\}\{1\} \rightharpoonup_{A,B} \{13\}\{2\}$, $\{13\}\{2\} \rightharpoonup_B \{1\}\{2\}\{3\} \rightharpoonup_{A,B} \{13\}\{2\}$, $\{123\} \rightharpoonup_A \{23\}\{1\} \rightharpoonup_{A,B} \{123\}$. ■

B Symmetric Manufacturing

In the symmetric model, let $\alpha_1 - c_1 = M_1$, $\alpha_2 - c_2 = M_2$, $\alpha_3 - c_3 = M_3$, $\alpha_3 - c_3 = M_4$, $\alpha_1 - \lambda c_1 = P_1$, $\alpha_2 - \lambda c_2 = P_2$, $\alpha_3 - \lambda c_3 = P_3$, $\alpha_4 - \lambda c_4 = P_4$, $1 - \kappa = X$, and $\kappa - \gamma = Y$. From the first order conditions, equilibrium quantities and payoffs under different recycling structures are:

- $q_1^{\{1234\}} = \{8P_1 - 4P_4\kappa\gamma + 4P_3\kappa - 24P_1\kappa + 8P_4\kappa + 4\kappa P_2 - 4\gamma P_2 - 8\gamma P_3\kappa + 8\gamma\kappa P_2 + 6\kappa^2\gamma P_3 + 4\kappa\gamma P_1 - 6\kappa^2\gamma P_2 - 3\kappa\gamma^2 P_2 + 2\kappa\gamma^2 P_1 + \gamma^2 P_3\kappa + 2P_4\kappa\gamma^2 - 2P_1\gamma^2 + \gamma^3 P_2 - 12P_4\kappa^2 + 12P_1\kappa^2\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$, $q_2^{\{1234\}} = \{\gamma^3 P_1 - 2\gamma^2 P_2 + 2\kappa\gamma^2 P_2 - 3\kappa\gamma^2 P_1 + 2\gamma^2 P_3\kappa + P_4\kappa\gamma^2 + 6\gamma P_4\kappa^2 + 8\kappa\gamma P_1 - 4\gamma P_3\kappa - 6\gamma P_1\kappa^2 - 8P_4\kappa\gamma - 4P_1\gamma + 4\gamma\kappa P_2 + 8P_3\kappa + 4P_1\kappa + 4P_4\kappa - 12\kappa^2 P_3 - 24\kappa P_2 + 8P_2 + 12\kappa^2 P_2\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$, $q_3^{\{1234\}} = \{-4P_4\gamma + 8P_3 + 4\gamma P_3\kappa - 4\gamma\kappa P_2 - 2\gamma^2 P_3 + 4P_4\kappa + 4P_1\kappa - 24P_3\kappa + 8\kappa P_2 + 12\kappa^2 P_3 - 12\kappa^2 P_2 + 2\gamma^2 P_3\kappa + 2\kappa\gamma^2 P_2 + P_4\gamma^3 + 8P_4\kappa\gamma - 8\kappa\gamma P_1 - 3P_4\kappa\gamma^2 + \kappa\gamma^2 P_1 - 6\gamma P_4\kappa^2 + 6\gamma P_1\kappa^2\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$, and $q_4^{\{1234\}} = \{6\kappa^2\gamma P_2 + 8P_4 - 6\kappa^2\gamma P_3 - 4\gamma P_3 + 8\gamma P_3\kappa - 8\gamma\kappa P_2 - 2P_4\gamma^2 - 24P_4\kappa + 8P_1\kappa + 4P_3\kappa + 4\kappa P_2 + 12P_4\kappa^2 - 12P_1\kappa^2 - 3\gamma^2 P_3\kappa + \kappa\gamma^2 P_2 + \gamma^3 P_3 + 4P_4\kappa\gamma - 4\kappa\gamma P_1 + 2P_4\kappa\gamma^2 + 2\kappa\gamma^2 P_1\}/\{-8\gamma^2 + 16\kappa\gamma + 48\kappa^2 - 64\kappa + 16\kappa\gamma^2 + 16 + \gamma^4 - 12\kappa^2\gamma^2 - 4\kappa\gamma^3\}$.

$$\Pi_A^{\{1234\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \frac{q_1 + q_4}{q_1 + q_2 + q_3 + q_4}\kappa(q_1 + q_2 + q_3 + q_4)^2;$$
$$\Pi_B^{\{1234\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \frac{q_2 + q_3}{q_1 + q_2 + q_3 + q_4}\kappa(q_1 + q_2 + q_3 + q_4)^2.$$

- $q_1^{\{12\}\{34\}} = \{-M_2\kappa + 2M_1\kappa + \gamma M_2 - 2M_1\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_2^{\{12\}\{34\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma - M_1\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_3^{\{12\}\{34\}} = \{-M_4\kappa + 2M_3\kappa + \gamma M_4 - 2M_3\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, and $q_4^{\{12\}\{34\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3 - M_3\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$.

$$\Pi_A^{\{12\}\{34\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \frac{q_1}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \frac{q_4}{q_3 + q_4}\kappa(q_3 + q_4)^2;$$
$$\Pi_B^{\{12\}\{34\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \frac{q_3}{q_3 + q_4}\kappa(q_3 + q_4)^2.$$

- $q_1^{\{13\}\{24\}} = \{3\kappa^3 P_3 + 22P_1\kappa^2 - 6\kappa^3 P_1 - 5\kappa^2\gamma P_2 + 4P_3\kappa + 8P_1 - 24P_1\kappa - 4\gamma P_2 - 8\kappa^2 P_3 + 8\gamma\kappa P_2 + \gamma^2 P_3\kappa - 2P_1\gamma^2 + 2\kappa\gamma^2 P_1 + \gamma^3 P_2 - 4P_4\kappa\gamma + 4\gamma P_4\kappa^2\}/\{-8\gamma^2 + 16\kappa\gamma^2 - 10\kappa^2\gamma^2 + 9\kappa^4 + 88\kappa^2 - 48\kappa^3 + 16 - 64\kappa + \gamma^4\}$, $q_2^{\{13\}\{24\}} = \{3\kappa^3 P_4 - 6\kappa^3 P_2 + 4\kappa^2\gamma P_3 - 5\gamma P_1\kappa^2 - 8P_4\kappa^2 + 22\kappa^2 P_2 - 24\kappa P_2 + 2\kappa\gamma^2 P_2 + P_4\kappa\gamma^2 - 4\gamma P_3\kappa + 8\kappa\gamma P_1 + 4P_4\kappa + 8P_2 - 4P_1\gamma + \gamma^3 P_1 - 2\gamma^2 P_2\}/\{-8\gamma^2 + 16\kappa\gamma^2 - 10\kappa^2\gamma^2 + 9\kappa^4 + 88\kappa^2 - 48\kappa^3 + 16 - 64\kappa + \gamma^4\}$, $q_3^{\{13\}\{24\}} = \{4\kappa^2\gamma P_2 + \kappa\gamma^2 P_1 - 5\gamma P_4\kappa^2 + 2\gamma^2 P_3\kappa + P_4\gamma^3 - 4\gamma\kappa P_2 - 2\gamma^2 P_3 - 4P_4\gamma + 8P_4\kappa\gamma + 8P_3 + 22\kappa^2 P_3 +$

$4P_1\kappa-8P_1\kappa^2-24P_3\kappa+3\kappa^3P_1-6\kappa^3P_3\}/\{-8\gamma^2+16\kappa\gamma^2-10\kappa^2\gamma^2+9\kappa^4+88\kappa^2-48\kappa^3+16-64\kappa+\gamma^4\}$, and $q_4^{\{13\}\{24\}}=\{-5\kappa^2\gamma P_3-8\kappa^2P_2+3\kappa^3P_2-4\kappa\gamma P_1+4\gamma P_1\kappa^2+\kappa\gamma^2P_2+22P_4\kappa^2-6\kappa^3P_4+8\gamma P_3\kappa-2P_4\gamma^2+2P_4\kappa\gamma^2-4\gamma P_3+\gamma^3P_3+8P_4-24P_4\kappa+4\kappa P_2\}/\{-8\gamma^2+16\kappa\gamma^2-10\kappa^2\gamma^2+9\kappa^4+88\kappa^2-48\kappa^3+16-64\kappa+\gamma^4\}$.

$$\Pi_A^{\{13\}\{24\}}=q_1(P_1-q_1-\gamma q_2)+q_4(P_4-q_4-\gamma q_3)+\frac{q_1}{q_1+q_3}\kappa(q_1+q_3)^2+\frac{q_4}{q_2+q_4}\kappa(q_2+q_4)^2;$$
$$\Pi_B^{\{13\}\{24\}}=q_2(P_2-q_2-\gamma q_1)+q_3(P_3-q_3-\gamma q_4)+\frac{q_2}{q_2+q_4}\kappa(q_2+q_4)^2+\frac{q_3}{q_1+q_3}\kappa(q_1+q_3)^2.$$

- $q_1^{\{14\}\{23\}}=\{-4\gamma P_2-16P_4\kappa^2+16P_1\kappa^2+8P_1+2P_4\kappa\gamma^2+2\kappa\gamma^2P_1+8\kappa^2\gamma P_3-8\kappa^2\gamma P_2+\gamma^3P_2-2P_1\gamma^2-8\gamma P_3\kappa+8\gamma\kappa P_2+8P_4\kappa-24P_1\kappa\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$, $q_2^{\{14\}\{23\}}=\{-24\kappa P_2-4P_1\gamma+8P_2+2\gamma^2P_3\kappa+8\gamma P_4\kappa^2-8\gamma P_1\kappa^2+2\kappa\gamma^2P_2-8P_4\kappa\gamma+8\kappa\gamma P_1+8P_3\kappa-16\kappa^2P_3+16\kappa^2P_2+\gamma^3P_1-2\gamma^2P_2\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$, $q_3^{\{14\}\{23\}}=\{-8\gamma P_4\kappa^2+8\gamma P_1\kappa^2+16\kappa^2P_3-16\kappa^2P_2-8\kappa\gamma P_1+8P_4\kappa\gamma-24P_3\kappa+2\gamma^2P_3\kappa+2\kappa\gamma^2P_2+8\kappa P_2+P_4\gamma^3-2\gamma^2P_3+8P_3-4P_4\gamma\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$, and $q_4^{\{14\}\{23\}}=\{-8\gamma\kappa P_2+\gamma^3P_3+8P_4+8P_1\kappa-4\gamma P_3+2P_4\kappa\gamma^2+2\kappa\gamma^2P_1+8\gamma P_3\kappa-8\kappa^2\gamma P_3+8\kappa^2\gamma P_2-2P_4\gamma^2-24P_4\kappa+16P_4\kappa^2-16P_1\kappa^2\}/\{\gamma^4+16-64\kappa+64\kappa^2-16\kappa^2\gamma^2-8\gamma^2+16\kappa\gamma^2\}$.

$$\Pi_A^{\{14\}\{23\}}=q_1(P_1-q_1-\gamma q_2)+q_4(P_4-q_4-\gamma q_3)+\kappa(q_1+q_4)^2;$$
$$\Pi_B^{\{14\}\{23\}}=q_2(P_2-q_2-\gamma q_1)+q_3(P_3-q_3-\gamma q_4)+\kappa(q_2+q_3)^2.$$

- $q_1^{\{123\}\{4\}}=\{4\kappa P_2+8P_1-4\gamma P_2+4P_3\kappa+16P_1\kappa^2-4\kappa^2P_3-2\gamma\kappa M_4-4\kappa^2P_2+4\kappa^2\gamma P_3-4\kappa^2\gamma P_2-\kappa\gamma^2P_2-2P_1\gamma^2+\gamma^3P_2-24P_1\kappa-4\gamma P_3\kappa+8\gamma\kappa P_2+2\kappa\gamma^2P_1+2\gamma^2\kappa M_4\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, $q_2^{\{123\}\{4\}}=\{\gamma^3P_1+\gamma^2\kappa M_4-\kappa\gamma^2P_1+2\kappa\gamma^2P_2-2\gamma^2P_2+8\kappa\gamma P_1-4\gamma\kappa M_4+3\gamma\kappa^2M_4+2\kappa^2\gamma P_3-4\gamma P_1\kappa^2-4P_1\gamma-2\gamma P_3\kappa+8P_2+4P_1\kappa-24\kappa P_2+22\kappa^2P_2+6\kappa^3P_3+8P_3\kappa-4P_1\kappa^2-6\kappa^3P_2-14\kappa^2P_3\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, $q_3^{\{123\}\{4\}}=\{-14\kappa^2P_2+4P_1\kappa-24P_3\kappa+8P_3-2\gamma^2P_3-4\gamma M_4+8\kappa P_2-4\kappa\gamma P_1+4\gamma P_3\kappa-2\gamma\kappa P_2+8\gamma\kappa M_4+22\kappa^2P_3+\gamma^3M_4-6\kappa^3P_3-3\gamma\kappa^2M_4+6\kappa^3P_2+2\gamma^2P_3\kappa-2\gamma^2\kappa M_4-4P_1\kappa^2+4\gamma P_1\kappa^2-4\kappa^2\gamma P_3+2\kappa^2\gamma P_2\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, and $q_4^{\{123\}\{4\}}=\{3\kappa^2\gamma P_2+8M_4-24M_4\kappa-2\kappa\gamma P_1+8\gamma P_3\kappa-4\gamma P_3+12\kappa^2M_4+\gamma^3P_3-2\gamma^2M_4-4\gamma\kappa P_2+4\gamma\kappa M_4+2\kappa\gamma^2P_1-2\gamma^2P_3\kappa+\kappa\gamma^2P_2+2\gamma^2\kappa M_4-3\kappa^2\gamma P_3\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$.

$$\Pi_A^{\{123\}\{4\}}=q_1(P_1-q_1-\gamma q_2)+q_4(M_4-q_4-\gamma q_3)+\frac{q_1}{q_1+q_2+q_3}\kappa(q_1+q_2+q_3)^2+\kappa q_4^2;$$
$$\Pi_B^{\{123\}\{4\}}=q_2(P_2-q_2-\gamma q_1)+q_3(P_3-q_3-\gamma q_4)+\frac{q_2+q_3}{q_1+q_2+q_3}\kappa(q_1+q_2+q_3)^2.$$

- $q_1^{\{124\}\{3\}}=\{\gamma^3P_2+\gamma^2\kappa M_3-\kappa\gamma^2P_2+2\kappa\gamma^2P_1-2P_1\gamma^2-4\gamma P_2-2P_4\kappa\gamma+3\gamma\kappa^2M_3+8\gamma\kappa P_2-4\gamma\kappa M_3-4\kappa^2\gamma P_2+2\gamma P_4\kappa^2-24P_1\kappa+8P_1-6\kappa^3P_1-14P_4\kappa^2-4\kappa^2P_2+22P_1\kappa^2+8P_4\kappa+6\kappa^3P_4+4\kappa P_2\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$, $q_2^{\{124\}\{3\}}=\{4P_1\kappa-4P_1\gamma+4P_4\kappa+8P_2+4\gamma P_4\kappa^2-4\gamma P_1\kappa^2+16\kappa^2P_2-4P_4\kappa^2-4P_1\kappa^2-2\gamma\kappa M_3+2\kappa\gamma^2P_2-\kappa\gamma^2P_1+2\gamma^2\kappa M_3-4P_4\kappa\gamma+8\kappa\gamma P_1-24\kappa P_2+\gamma^3P_1-2\gamma^2P_2\}/\{16-64\kappa+\gamma^4+16\kappa\gamma^2-7\kappa^2\gamma^2-8\gamma^2+72\kappa^2-2\kappa\gamma^3-8\kappa^2\gamma+8\kappa\gamma-24\kappa^3\}$,

$q_3^{\{124\}\{3\}} = \{2\kappa\gamma^2 P_2 - 2P_4\kappa\gamma^2 + 3\gamma P_1\kappa^2 + \kappa\gamma^2 P_1 - 3\gamma P_4\kappa^2 + 2\gamma^2\kappa M_3 - 4P_4\gamma + 12\kappa^2 M_3 - 2\gamma\kappa P_2 + 8P_4\kappa\gamma - 4\kappa\gamma P_1 + 4\gamma\kappa M_3 + 8M_3 + P_4\gamma^3 - 2\gamma^2 M_3 - 24M_3\kappa\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, and $q_4^{\{124\}\{3\}} = \{-4\gamma\kappa P_2 + 4P_4\kappa\gamma - 14P_1\kappa^2 - 2\kappa\gamma P_1 + 22P_4\kappa^2 + 8\gamma\kappa M_3 + 8P_4 + 4\kappa P_2 - 24P_4\kappa + 8P_1\kappa + \gamma^3 M_3 - 2P_4\gamma^2 - 4\gamma M_3 - 6\kappa^3 P_4 + 6\kappa^3 P_1 - 3\gamma\kappa^2 M_3 + 2P_4\kappa\gamma^2 + 4\kappa^2\gamma P_2 - 4\gamma P_4\kappa^2 + 2\gamma P_1\kappa^2 - 4\kappa^2 P_2 - 2\gamma^2\kappa M_3\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$.

$$\Pi_A^{\{124\}\{3\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \frac{q_1 + q_4}{q_1 + q_2 + q_4}\kappa(q_1 + q_2 + q_4)^2;$$
$$\Pi_B^{\{124\}\{3\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_1 + q_2 + q_4}\kappa(q_1 + q_2 + q_4)^2 + \kappa q_3^2.$$

- $q_1^{\{134\}\{2\}} = \{8P_4\kappa - 4\gamma P_3\kappa + 4\kappa\gamma P_1 - 14P_4\kappa^2 - 2P_4\kappa\gamma + 8\gamma\kappa M_2 + 22P_1\kappa^2 + 6\kappa^3 P_4 - 6\kappa^3 P_1 - 3\gamma\kappa^2 M_2 + 4\kappa^2\gamma P_3 + 8P_1 + 4P_3\kappa - 24P_1\kappa - 2P_1\gamma^2 - 4\gamma M_2 + \gamma^3 M_2 - 4\gamma P_1\kappa^2 + 2\gamma P_4\kappa^2 - 4\kappa^2 P_3 + 2\kappa\gamma^2 P_1 - 2\gamma^2\kappa M_2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_2^{\{134\}\{2\}} = \{12\kappa^2 M_2 - 4P_4\kappa\gamma + 4\gamma\kappa M_2 + 2\gamma^2 P_3\kappa - 2\kappa\gamma^2 P_1 + 3\gamma P_4\kappa^2 + P_4\kappa\gamma^2 + 2\gamma^2\kappa M_2 - 3\gamma P_1\kappa^2 - 24M_2\kappa - 4P_1\gamma - 2\gamma P_3\kappa + 8\kappa\gamma P_1 + \gamma^3 P_1 - 2\gamma^2 M_2 + 8M_2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_3^{\{134\}\{2\}} = \{4P_4\kappa - 4P_4\gamma + 4P_1\kappa - 4P_4\kappa^2 + 4\gamma P_1\kappa^2 - 4\gamma P_4\kappa^2 + 16\kappa^2 P_3 - 4P_1\kappa^2 - 2\gamma\kappa M_2 + 8P_3 - 4\kappa\gamma P_1 + 8P_4\kappa\gamma - 24P_3\kappa - P_4\kappa\gamma^2 + 2\gamma^2\kappa M_2 + 2\gamma^2 P_3\kappa + P_4\gamma^3 - 2\gamma^2 P_3\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, and $q_4^{\{134\}\{2\}} = \{\gamma^3 P_3 - \gamma^2 P_3\kappa + 2P_4\kappa\gamma^2 - 2P_4\gamma^2 + \gamma^2\kappa M_2 + 8\gamma P_3\kappa - 2\kappa\gamma P_1 - 4\kappa^2\gamma P_3 - 4\gamma P_3 + 3\gamma\kappa^2 M_2 + 2\gamma P_1\kappa^2 - 4\gamma\kappa M_2 - 24P_4\kappa + 4P_3\kappa + 22P_4\kappa^2 + 6\kappa^3 P_1 + 8P_1\kappa - 6\kappa^3 P_4 + 8P_4 - 4\kappa^2 P_3 - 14P_1\kappa^2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$.

$$\Pi_A^{\{134\}\{2\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \frac{q_1 + q_4}{q_1 + q_3 + q_4}\kappa(q_1 + q_3 + q_4)^2;$$
$$\Pi_B^{\{134\}\{2\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \frac{q_3}{q_1 + q_3 + q_4}\kappa(q_1 + q_3 + q_4)^2.$$

- $q_1^{\{1\}\{234\}} = \{8M_1 - 4\gamma P_3\kappa + 4\gamma\kappa M_1 + 2P_4\kappa\gamma^2 - 2\kappa\gamma^2 P_2 + \gamma^2 P_3\kappa - 3\kappa^2\gamma P_2 + 2\gamma^2\kappa M_1 + \gamma^3 P_2 - 2\gamma^2 M_1 + 3\kappa^2\gamma P_3 - 24M_1\kappa - 2P_4\kappa\gamma + 8\gamma\kappa P_2 - 4\gamma P_2 + 12\kappa^2 M_1\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_2^{\{1\}\{234\}} = \{8P_3\kappa - 4P_4\kappa\gamma + 4\gamma\kappa P_2 - 2\gamma P_3\kappa + 22\kappa^2 P_2 + 8\gamma\kappa M_1 - 2\gamma^2 P_2 - 4M_1\gamma + \gamma^3 M_1 - 14\kappa^2 P_3 + 4P_4\kappa - 24\kappa P_2 + 6\kappa^3 P_3 - 3\gamma\kappa^2 M_1 - 6\kappa^3 P_2 + 4\gamma P_4\kappa^2 - 4\kappa^2\gamma P_2 + 2\kappa^2\gamma P_3 + 2\kappa\gamma^2 P_2 - 4P_4\kappa^2 - 2\gamma^2\kappa M_1 + 8P_2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, $q_3^{\{1\}\{234\}} = \{P_4\gamma^3 - P_4\kappa\gamma^2 + 2\gamma^2 P_3\kappa + \gamma^2\kappa M_1 - 2\gamma^2 P_3 + 8P_4\kappa\gamma - 4P_4\gamma + 3\gamma\kappa^2 M_1 - 2\gamma\kappa P_2 - 4\gamma P_4\kappa^2 + 2\kappa^2\gamma P_2 - 4\gamma\kappa M_1 - 14\kappa^2 P_2 - 24P_3\kappa - 6\kappa^3 P_3 + 4P_4\kappa + 22\kappa^2 P_3 + 8P_3 + 8\kappa P_2 + 6\kappa^3 P_2 - 4P_4\kappa^2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$, and $q_4^{\{1\}\{234\}} = \{-4\gamma P_3 + 4\kappa P_2 + 4P_3\kappa + 8P_4 - 4\kappa^2 P_3 + 4\kappa^2\gamma P_2 - 4\kappa^2\gamma P_3 - 2\gamma\kappa M_1 + 16P_4\kappa^2 - 4\kappa^2 P_2 - 4\gamma\kappa P_2 + 8\gamma P_3\kappa - 24P_4\kappa + \gamma^3 P_3 - \gamma^2 P_3\kappa - 2P_4\gamma^2 + 2\gamma^2\kappa M_1 + 2P_4\kappa\gamma^2\}/\{16 - 64\kappa + \gamma^4 + 16\kappa\gamma^2 - 7\kappa^2\gamma^2 - 8\gamma^2 + 72\kappa^2 - 2\kappa\gamma^3 - 8\kappa^2\gamma + 8\kappa\gamma - 24\kappa^3\}$.

$$\Pi_A^{\{1\}\{234\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \frac{q_4}{q_2 + q_3 + q_4}\kappa(q_2 + q_3 + q_4)^2;$$
$$\Pi_B^{\{1\}\{234\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \frac{q_2 + q_3}{q_2 + q_3 + q_4}\kappa(q_2 + q_3 + q_4)^2.$$

- $q_1^{\{12\}\{3\}\{4\}} = \{-M_2\kappa + 2M_1\kappa + \gamma M_2 - 2M_1\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_2^{\{12\}\{3\}\{4\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma - M_1\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, $q_3^{\{12\}\{3\}\{4\}} = \{\gamma M_4 - 2M_3 + 2M_3\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, and $q_4^{\{12\}\{3\}\{4\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$.

$$\Pi_A^{\{12\}\{3\}\{4\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \frac{q_1}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \kappa q_4^2;$$
$$\Pi_B^{\{12\}\{3\}\{4\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_1 + q_2}\kappa(q_1 + q_2)^2 + \kappa q_3^2.$$

- $q_1^{\{13\}\{2\}\{4\}} = \{8\gamma\kappa M_2 - 2\gamma\kappa M_4 - 8\kappa^2 P_3 + 4P_3\kappa + 24P_1\kappa^2 + 8P_1 - 24P_1\kappa + 2\gamma\kappa^2 M_4 + 2\kappa\gamma^2 P_1 - 4\gamma\kappa^2 M_2 + 4\kappa^3 P_3 - 8\kappa^3 P_1 - 4\gamma M_2 - 2P_1\gamma^2 + \gamma^3 M_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_2^{\{13\}\{2\}\{4\}} = \{-6\kappa^3 M_2 + 22\kappa^2 M_2 + 2\kappa^2\gamma P_3 - 4\gamma P_1\kappa^2 + \gamma^2\kappa M_4 - 2\gamma P_3\kappa + 8\kappa\gamma P_1 - 24M_2\kappa + 2\gamma^2\kappa M_2 - 4P_1\gamma + 8M_2 - 2\gamma^2 M_2 + \gamma^3 P_1\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_3^{\{13\}\{2\}\{4\}} = \{-2\gamma\kappa M_2 + 8\gamma\kappa M_4 + 24\kappa^2 P_3 - 24P_3\kappa - 8P_1\kappa^2 + 8P_3 - 2\gamma^2 P_3 - 4\gamma M_4 + \gamma^3 M_4 + 4P_1\kappa - 4\gamma\kappa^2 M_4 + 2\gamma^2 P_3\kappa + 2\gamma\kappa^2 M_2 - 8\kappa^3 P_3 + 4\kappa^3 P_1\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, and $q_4^{\{13\}\{2\}\{4\}} = \{\gamma^3 P_3 + \gamma^2\kappa M_2 - 2\gamma^2 M_4 + 2\gamma^2\kappa M_4 - 4\kappa^2\gamma P_3 + 22\kappa^2 M_4 - 6\kappa^3 M_4 - 4\gamma P_3 + 8M_4 - 24M_4\kappa + 8\gamma P_3\kappa - 2\kappa\gamma P_1 + 2\gamma P_1\kappa^2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$.

$$\Pi_A^{\{13\}\{2\}\{4\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \frac{q_1}{q_1 + q_3}\kappa(q_1 + q_3)^2 + \kappa q_4^2;$$
$$\Pi_B^{\{13\}\{2\}\{4\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \frac{q_3}{q_1 + q_3}\kappa(q_1 + q_3)^2.$$

- $q_1^{\{14\}\{2\}\{3\}} = \{\gamma^3 M_2 - 24P_1\kappa + 8P_1 - 16P_4\kappa^2 + 8P_4\kappa + 24P_1\kappa^2 + 8\gamma\kappa M_2 - 4\gamma\kappa M_3 + 8\kappa^3 P_4 - 8\kappa^3 P_1 - 4\gamma\kappa^2 M_2 + 4\gamma\kappa^2 M_3 + 2\kappa\gamma^2 P_1 - 4\gamma M_2 - 2P_1\gamma^2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_2^{\{14\}\{2\}\{3\}} = \{4\gamma P_4\kappa^2 + 16\kappa^2 M_2 - 4\gamma P_1\kappa^2 - 4P_4\kappa\gamma - 24M_2\kappa + 8\kappa\gamma P_1 + 2\gamma^2\kappa M_2 + 2\gamma^2\kappa M_3 + 8M_2 + \gamma^3 P_1 - 4P_1\gamma - 2\gamma^2 M_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_3^{\{14\}\{2\}\{3\}} = \{-4\gamma P_4\kappa^2 + 16\kappa^2 M_3 - 4P_4\gamma + 8M_3 - 24M_3\kappa + 8P_4\kappa\gamma - 4\kappa\gamma P_1 + 4\gamma P_1\kappa^2 + P_4\gamma^3 + 2\gamma^2\kappa M_2 - 2\gamma^2 M_3 + 2\gamma^2\kappa M_3\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, and $q_4^{\{14\}\{2\}\{3\}} = \{8P_1\kappa - 2P_4\gamma^2 - 4\gamma M_3 + \gamma^3 M_3 + 8P_4 + 24P_4\kappa^2 - 24P_4\kappa - 16P_1\kappa^2 - 4\gamma\kappa M_2 + 8\gamma\kappa M_3 - 8\kappa^3 P_4 + 8\kappa^3 P_1 + 2P_4\kappa\gamma^2 + 4\gamma\kappa^2 M_2 - 4\gamma\kappa^2 M_3\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$.

$$\Pi_A^{\{14\}\{2\}\{3\}} = q_1(P_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \kappa(q_1 + q_4)^2;$$
$$\Pi_B^{\{14\}\{2\}\{3\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \kappa q_3^2.$$

- $q_1^{\{1\}\{23\}\{4\}} = \{16\kappa^2 M_1 + 4\kappa^2\gamma P_3 - 4\kappa^2\gamma P_2 - 4\gamma P_3\kappa + 8\gamma\kappa P_2 - 24M_1\kappa + 2\gamma^2\kappa M_1 + 2\gamma^2\kappa M_4 - 4\gamma P_2 - 2\gamma^2 M_1 + 8M_1 + \gamma^3 P_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_2^{\{1\}\{23\}\{4\}} = \{8\gamma\kappa M_1 - 4\gamma\kappa M_4 - 16\kappa^2 P_3 + 8P_3\kappa + 24\kappa^2 P_2 + 4\gamma\kappa^2 M_4 + 2\kappa\gamma^2 P_2 - 4\gamma\kappa^2 M_1 + 8\kappa^3 P_3 - 8\kappa^3 P_2 - 2\gamma^2 P_2 + \gamma^3 M_1 - 4M_1\gamma - 24\kappa P_2 + 8P_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, $q_3^{\{1\}\{23\}\{4\}} = \{-2\gamma^2 P_3 - 4\gamma M_4 + \gamma^3 M_4 - 4\gamma\kappa M_1 + 8\gamma\kappa M_4 + 24\kappa^2 P_3 - 24P_3\kappa - 16\kappa^2 P_2 - 4\gamma\kappa^2 M_4 + 2\gamma^2 P_3\kappa + 4\gamma\kappa^2 M_1 - 8\kappa^3 P_3 + 8\kappa^3 P_2 + 8\kappa P_2 + 8P_3\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$, and $q_4^{\{1\}\{23\}\{4\}} =$

$\{\gamma^3 P_3 + 2\gamma^2\kappa M_1 - 2\gamma^2 M_4 + 2\gamma^2\kappa M_4 - 4\kappa^2\gamma P_3 + 16\kappa^2 M_4 - 4\gamma P_3 + 8M_4 - 24M_4\kappa + 8\gamma P_3\kappa - 4\gamma\kappa P_2 + 4\kappa^2\gamma P_2\}/\{80\kappa^2 - 32\kappa^3 - 64\kappa + 16\kappa\gamma^2 + 16 - 8\gamma^2 - 8\kappa^2\gamma^2 + \gamma^4\}$.

$$\Pi_A^{\{1\}\{23\}\{4\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \kappa q_4^2;$$
$$\Pi_B^{\{1\}\{23\}\{4\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(P_3 - q_3 - \gamma q_4) + \kappa(q_2 + q_3)^2.$$

- $q_1^{\{1\}\{24\}\{3\}} = \{-6\kappa^3 M_1 + 2\gamma P_4\kappa^2 - 4\kappa^2\gamma P_2 + 22\kappa^2 M_1 - 2P_4\kappa\gamma + 8\gamma\kappa P_2 + 2\gamma^2\kappa M_1 + \gamma^2\kappa M_3 - 24M_1\kappa + 8M_1 - 2\gamma^2 M_1 - 4\gamma P_2 + \gamma^3 P_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_2^{\{1\}\{24\}\{3\}} = \{8P_2 + 2\gamma\kappa^2 M_3 + 2\kappa\gamma^2 P_2 - 4\gamma\kappa^2 M_1 + 4\kappa^3 P_4 - 8\kappa^3 P_2 - 2\gamma^2 P_2 - 4M_1\gamma + 8\gamma\kappa M_1 - 2\gamma\kappa M_3 + \gamma^3 M_1 - 24\kappa P_2 - 8P_4\kappa^2 + 4P_4\kappa + 24\kappa^2 P_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, $q_3^{\{1\}\{24\}\{3\}} = \{P_4\gamma^3 + \gamma^2\kappa M_1 - 2\gamma^2 M_3 + 2\gamma^2\kappa M_3 - 4\gamma P_4\kappa^2 + 22\kappa^2 M_3 - 6\kappa^3 M_3 - 4P_4\gamma + 8M_3 - 24M_3\kappa + 8P_4\kappa\gamma - 2\gamma\kappa P_2 + 2\kappa^2\gamma P_2\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$, and $q_4^{\{1\}\{24\}\{3\}} = \{8P_4 - 4\gamma\kappa^2 M_3 + 2P_4\kappa\gamma^2 + 2\gamma\kappa^2 M_1 - 8\kappa^3 P_4 + 4\kappa^3 P_2 - 2\gamma\kappa M_1 + 8\gamma\kappa M_3 + 4\kappa P_2 + 24P_4\kappa^2 - 24P_4\kappa - 8\kappa^2 P_2 - 2P_4\gamma^2 - 4\gamma M_3 + \gamma^3 M_3\}/\{92\kappa^2 - 56\kappa^3 + 12\kappa^4 + \gamma^4 - 8\gamma^2 + 16\kappa\gamma^2 - 8\kappa^2\gamma^2 - 64\kappa + 16\}$.

$$\Pi_A^{\{1\}\{24\}\{3\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(P_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \frac{q_4}{q_2 + q_4}\kappa(q_2 + q_4)^2;$$
$$\Pi_B^{\{1\}\{24\}\{3\}} = q_2(P_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \frac{q_2}{q_2 + q_4}\kappa(q_2 + q_4)^2 + \kappa q_3^2.$$

- $q_1^{\{1\}\{2\}\{34\}} = \{\gamma M_2 - 2M_1 + 2M_1\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_2^{\{1\}\{2\}\{34\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_3^{\{1\}\{2\}\{34\}} = \{-M_4\kappa + 2M_3\kappa + \gamma M_4 - 2M_3\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$, and $q_4^{\{1\}\{2\}\{34\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3 - M_3\kappa\}/\{\gamma^2 - 2\kappa\gamma - 3\kappa^2 - 4 + 8\kappa\}$.

$$\Pi_A^{\{1\}\{2\}\{34\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \frac{q_4}{q_3 + q_4}\kappa(q_3 + q_4)^2;$$
$$\Pi_B^{\{1\}\{2\}\{34\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \frac{q_3}{q_3 + q_4}\kappa(q_3 + q_4)^2.$$

- $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{\gamma M_2 - 2M_1 + 2M_1\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{-2M_2 + 2M_2\kappa + M_1\gamma\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{\gamma M_4 - 2M_3 + 2M_3\kappa\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$, and $q_1^{\{1\}\{2\}\{3\}\{4\}} = \{-2M_4 + 2M_4\kappa + \gamma M_3\}/\{\gamma^2 - 4 + 8\kappa - 4\kappa^2\}$.

$$\Pi_A^{\{1\}\{2\}\{3\}\{4\}} = q_1(M_1 - q_1 - \gamma q_2) + q_4(M_4 - q_4 - \gamma q_3) + \kappa q_1^2 + \kappa q_4^2;$$
$$\Pi_B^{\{1\}\{2\}\{3\}\{4\}} = q_2(M_2 - q_2 - \gamma q_1) + q_3(M_3 - q_3 - \gamma q_4) + \kappa q_2^2 + \kappa q_3^2.$$

Proof of Proposition 7: With equilibrium quantities q_i^X, $i = 1, 2, 3, 4$, shown above, we calculate and compare $W^X(q_1^X, q_2^X, q_3^X, q_4^X)$ (given in equation 7) for all $X \in \mathbf{X}$, and then obtain the optimal recycling structure with the highest $W^X(q_1^X, q_2^X, q_3^X, q_4^X)$. ■

Proof of Proposition 8: Under our assumption that $c_1 = c_2, c_3 = c_4$, and $\gamma < 1$, it is easy to verify that, in most scenarios, outcomes that one player prefers more than product-based recycling, $\{1\}\{2\}\{3\}\{4\}$, are favored less then $\{1\}\{2\}\{3\}\{4\}$ by the other player; consequently, $\{1\}\{2\}\{3\}\{4\}$ emerges as stable. ■

Proof of Proposition 9: As mentioned above, under our assumption that $c_1 = c_2, c_3 = c_4$, and $\gamma < 1$, it is easy to verify that, in most scenarios, outcomes that one player prefers more than product-based recycling, $\{1\}\{2\}\{3\}\{4\}$, are favored less then $\{1\}\{2\}\{3\}\{4\}$ by the other player; consequently, $\{1\}\{2\}\{3\}\{4\}$ emerges as stable. ■

Proof of Proposition 10:

1. In most of the possible scenarios, for different parameter combinations, all-inclusive recycling dominates other structures for both firms, or firm A prefers $\{134\}\{2\}$ or $\{124\}\{3\}$ to all-inclusive recycling, and/or firm B prefers $\{123\}\{4\}$ or $\{234\}\{1\}$ to all-inclusive recycling. Under these scenarios, all-inclusive recycling is uniquely stable.

   Under the condition that economies of scale are low and product substitutability is high, a similar statement holds for market-based recycling.

2. Next, we consider firm-based recycling. Suppose that $\alpha_1 = 115, \alpha_2 = 200, \alpha_3 = 285, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.7, \lambda = 1.2,\ kappa = 0.09$. Then, we have $\Pi_A^{\{1234\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{23\}\{4\}}$ and $\Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{134\}\{2\}}$. If we consider B's most preferred outcome, $\{1\}\{23\}\{4\}$, A can defect unilaterally only to $\{14\}\{23\}$, which improves its payoff and is the second most preferred outcome for B. Any unilateral defection from firm-based recycling by B would only reduce its profit, and the same goes for A. In addition, any joint defection would reduce B's profit, hence firm-based recycling is stable. Now, if we consider $\{1234\}$, A's most preferred outcome, B can unilaterally defect to $\{14\}\{23\}$, which is stable. it is least preferred by B and any defection would improve B's profit. If we consider $\{134\}\{2\}$, A's second most preferred outcome, it is least preferred by B and any defection would improve B's profit. We can similarly see that for all other structures preferred to firm-based recycling by A, B can unilaterally defect to either $\{14\}\{23\}$, or to $\{1\}\{23\}\{4\}$, which will be followed by A's move to $\{14\}\{23\}$, hence firm-based recycling is the only stable outcome.

   When recycling becomes more costly, half-firm based recycling becomes stable. Suppose that $\alpha_1 = 78, \alpha_2 = 100, \alpha_3 = 300, \alpha_4 = 155, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.8, \lambda = 2, \kappa = 0.1$. Then, we have $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{134\}\{2\}}$. Similarly as above, we can show that in this case half firm-based recycling, $\{1\}\{23\}\{4\}$, is the only stable outcome, as A will always defect from its least preferred outcome, $\{14\}\{23\}$.

3. We now look at the i-inclusive recycling. Consider the case $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{134\}\{2\}}$, which occurs, for instance, when $\alpha_1 = \alpha_2 = 100, \alpha_3 = 300, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.75, \lambda = 1.2, \kappa = 0.02$. We can verify that in this case $\{123\}\{4\}$, the outcome most preferred by firm B, is the only stable outcome. The only possible defections from this outcome by firm A lead to $\{14\}\{23\}$ or $\{23\}\{1\}\{4\}$, the outcomes least preferred by A, so A does not want to defect. Consider, for instance, outcome most preferred by firm A, $\{134\}\{2\}$. This outcome is the least preferred by firm B and any defection can only improve its profit, so it cannot be stable. Next, consider firm-based recycling, $\{14\}\{23\}$. Both firms prefer $\{123\}\{4\}$ to $\{14\}\{23\}$ and it can be verified that firm-based recycling is not stable. Similarly, one can verify that all-inclusive recycling cannot be stable as firm B can unilaterally defect to $\{14\}\{23\}$. If the current outcome is $\{13\}\{24\}$, firm B can unilaterally defect to $\{1\}\{23\}\{4\}$, and firm A then benefits by defecting to $\{14\}\{23\}$. We can do the analysis for all the remaining possible defections.

4. Finally, we look at the half-market-based recycling. Consider, for example, case when $\alpha_1 = \alpha_2 = 100, \alpha_3 = 300, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.66, \lambda = 2, \kappa = 0.02$. Then, $\Pi_A^{\{12\}\{34\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}}$. We can verify that $\{12\}\{3\}\{4\}$, the outcome not most favored by either firm, is the only stable outcome in this case. It is easy to see that firm A can unilaterally defect from any of the three outcomes most preferred by firm B to product-based recycling, which it prefers to $\{14\}\{2\}\{3\}$, $\{1\}\{24\}\{3\}$ and $\{124\}\{3\}$. The only possible unilateral defection from product-based recycling by firm B leads to $\{1\}\{23\}\{4\}$, which is among least preferred outcome for B; hence, neither of these three structures is stable. A similar statement is true for four out of five outcomes most preferred by firm A and possible unilateral defection by firm B to product-based recycling. The one exception is $\{134\}\{2\}$, from which B can defect to $\{14\}\{2\}\{3\}$; as this is B's most preferred outcome and among A's least preferred outcome, A can defect from $\{14\}\{2\}\{3\}$ to product-based recycling. This leads us to $\{12\}\{3\}\{4\}$ as an outcome from which no firm can start a sequence of defection that would ultimately lead to a stable outcome that the defecting firm prefers to $\{12\}\{3\}\{4\}$; thus, $\{12\}\{3\}\{4\}$ is stable. Similar relationship hold in other cases with low economies of scale and high substitutability level in which products in one market have similar market sizes, while in the other market one product's market size dominates the other. ■

Proof of Proposition 11:

1. Consider the case $\Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{1234\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{13\}\{2\}\{4\}}$, which occurs, for instance, when $\alpha_1 = 100, \alpha_2 = 200, \alpha_3 = \alpha_4 = 300, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.1, \lambda = 2, \kappa = 0.02$. We can verify that in this case $\{1234\}$, the outcome most preferred by firm B, is stable. The only possible defections from this outcome by firm A lead to $\{14\}\{23\}$ or $\{23\}\{1\}\{4\}$, the outcomes with low preference by A, so A does not want to defect. Now, consider the outcome most preferred by firm A, $\{12\}\{34\}$. This outcome is the second-most preferred by firm B and any defection can only reduce its profit, so it emerges as stable as well. Thus, both all-inclusive and market-based recycling are stable in this case.

2. Next, consider the case $\Pi_A^{\{124\}\{3\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{134\}\{2\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{1\}\{234\}}$ and $\Pi_B^{\{1\}\{234\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{14\}\{2\}\{3\}}$, which occurs, for instance, when $\alpha_1 = 70, \alpha_2 = 100, \alpha_3 = 200, \alpha_4 = 400, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.5, \lambda = 1.2, \kappa = 0.02$. $\{124\}\{3\}$ is the most preferred outcome by firm A. Any possible sole defection by B results in one of the B's two least preferred outcomes, and $\{124\}\{3\}$ is stable. $\{1\}\{234\}$ is the most preferred outcome by firm B; A can defect to $\{14\}\{23\}$, which it prefers to $\{1\}\{234\}$, and if B unilaterally defects to $\{14\}\{2\}\{3\}$, it improves A's profit further, so $\{1\}\{234\}$ cannot be stable. $\{1234\}$ is the second most-preferred outcome by both firms. If A defects unilaterally, it leads to outcomes it prefers less, and B would not want to defect as it is cannot improve its profits. Hence, $\{1234\}$ is stable as well.

   Now, suppose that $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{1\}\{23\}\{4\}}$ and $\Pi_B^{\{1\}\{234\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{123\}\{4\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{134\}\{2\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{14\}\{2\}\{3\}}$, which occurs, for instance, when $\alpha_1 = \alpha_2 = 100, \alpha_3 = \alpha_4 = 300, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.9, \lambda = 2, \kappa = 0.1$. We can show stability of $\{1234\}$ similarly as above. $\{1\}\{234\}$ is most preferred outcome for B; any possible sole defection by A results in outcomes that both A and B prefer less than $\{123\}\{4\}$, and $\{123\}\{4\}$ is stable. Similar analysis holds for stability of $\{134\}\{2\}$, the favorite outcome of firm A. Thus, $\{1234\}, \{134\}\{2\}$ and $\{1\}\{234\}$ are all in the LCS.

3. We now look at the case $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{1\}\{234\}} >$

$\Pi_A^{\{14\}\{23\}} > \Pi_A^{\{1\}\{23\}\{4\}}$ and $\Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{134\}\{2\}}$, which occurs, for instance, when $\alpha_1 = 100, \alpha_2 = 70, \alpha_3 = 300, \alpha_4 = 150, c_1 = c_2 = 2, c_3 = c_4 = 5, \gamma = 0.75, \lambda = 1.2, \kappa = 0.07$. The outcome most preferred by A, $\{134\}\{2\}$, cannot be stable as a defection by B to $\{14\}\{23\}$ increases its payoff, and a further sole defection by A can only make B better off. If B defects from $\{13\}\{2\}\{4\}$ to $\{1\}\{23\}\{4\}$, which is its second most-preferred outcome, A would defect to $\{14\}\{23\}$, which it prefers most but, at the same time, makes B worse off. Thus, $\{13\}\{2\}\{4\}$ is stable. If A defects from $\{123\}\{4\}$, which is most preferred by B, it leads to one of its two least preferred outcomes, so $\{123\}\{4\}$ is stable as well.

If the cost increase in the above example changes from $\lambda = 1.2$ to $\lambda = 2$, we have $\Pi_A^{\{134\}\{2\}} > \Pi_A^{\{13\}\{2\}\{4\}} > \Pi_A^{\{13\}\{24\}} > \Pi_A^{\{1234\}} > \Pi_A^{\{12\}\{34\}} > \Pi_A^{\{1\}\{2\}\{34\}} > \Pi_A^{\{123\}\{4\}} > \Pi_A^{\{12\}\{3\}\{4\}} > \Pi_A^{\{124\}\{3\}} > \Pi_A^{\{1\}\{2\}\{3\}\{4\}} > \Pi_A^{\{1\}\{24\}\{3\}} > \Pi_A^{\{14\}\{2\}\{3\}} > \Pi_A^{\{1\}\{234\}} > \Pi_A^{\{1\}\{23\}\{4\}} > \Pi_A^{\{14\}\{23\}}$ and $\Pi_B^{\{123\}\{4\}} > \Pi_B^{\{1\}\{24\}\{3\}} > \Pi_B^{\{12\}\{3\}\{4\}} > \Pi_B^{\{1\}\{2\}\{3\}\{4\}} > \Pi_B^{\{13\}\{24\}} > \Pi_B^{\{1\}\{23\}\{4\}} > \Pi_B^{\{14\}\{2\}\{3\}} > \Pi_B^{\{124\}\{3\}} > \Pi_B^{\{14\}\{23\}} > \Pi_B^{\{13\}\{2\}\{4\}} > \Pi_B^{\{1234\}} > \Pi_B^{\{1\}\{234\}} > \Pi_B^{\{12\}\{34\}} > \Pi_B^{\{1\}\{2\}\{34\}} > \Pi_B^{\{134\}\{2\}}$. We can show stability of $\{123\}\{4\}$ and instability of $\{134\}\{2\}$ similarly as above. If we consider $\{13\}\{2\}\{4\}$, B can defect to $\{1\}\{4\}\{23\}$, which it prefers more than $\{13\}\{2\}\{4\}$. A may chose to move from $\{1\}\{4\}\{23\}$ to $\{14\}\{23\}$, which it prefers to $\{1\}\{4\}\{23\}$, and B would still be better off than in $\{13\}\{2\}\{4\}$. Consequently, $\{13\}\{2\}\{4\}$ is not stable in this case. If we now look at $\{13\}\{24\}$, we can consider the same sequence of defections: B moving to $\{1\}\{4\}\{23\}$, and A moving to $\{14\}\{23\}$. However, this sequence would make B worse off compared to $\{13\}\{24\}$, so $\{13\}\{24\}$ is stable, too. ■