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# Manufacturing Cell Formation Under Random Product Demand

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# MANUFACTURING CELL FORMATION UNDER RANDOM PRODUCT DEMAND

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## ABSTRACT

The performance of cellular manufacturing systems is intrinsically sensitive to demand variations and machine breakdowns. A cell formation methodology that addresses, during the shop design stage, system robustness with respect to product demand variation is proposed. The system resources are aggregated into cells in a manner that minimizes the expected inter-cell material handling cost. The statistical characteristics of the independent demand and the capacity of the system resources are explicitly considered. In the first step of the proposed approach the expected value of the feasible production volumes, which respect resource capacities, are determined. Subsequently, the shop partition that results in near optimal inter cell part traffic is found. The applicability of the proposed approach is illustrated through a comprehensive example.

# 1 INTRODUCTION

Cellular manufacturing shops are arranged into production cells, each dedicated to the production of part families with similar processing requirements. A major benefit of this approach is the simplification of the material flow within the shop. Specifically, the inter-cell flow is decreased substantially while most of material handling is confined within the manufacturing cells. This fact coupled with reduced set-up times, which result from part similarities, yield shorter lead times and lower work-in-process (Kusiak, 1987). Furthermore, the decomposition of a large and complex manufacturing system into smaller subsystems simplifies scheduling and shop floor control (Flynn and Jacobs, 1986) and promotes automation (Martin, 1989). On the other hand, cellular manufacturing systems are highly sensitive to machine breakdowns (Flynn and Jacobs, 1986) and their performance rapidly deteriorates under changes in product mix (Seifoddini, 1990).

The problem of partitioning a manufacturing shop into cells is *NP*-hard (King and Nakornchai, 1982) and has been addressed extensively in the literature. The heuristic methods that have been proposed for its solution can be grouped in two main categories; those that are based on cluster analysis of the part-machine incidence matrix (King, 1980; McCormick *et al.*, 1972; Chan and Milner, 1982; Garcia and Proth, 1986), and those that employ mathematical programming formulations (Gunasingh and Lashkari, 1989; Harhalakis *et al.*, 1990; Kusiak and Cheng, 1990). Interesting comparisons and performance evaluations of these methods can be found in (Kusiak and Cheng, 1991). It is noted that the mathematical programming approaches consider explicitly many important system attributes, such as material handling traffic, set-up and run times, and machine capacities (Harhalakis *et al.*, 1990; Nagi *et al.*, 1990; Minis *et al.*, 1990).

Although the performance of a cellular manufacturing system is directly dependent upon the manufactured product mix, the issue of robustness of a cellular design to product demand changes has been addressed by only a few research studies. The cell formation approach proposed by Seifoddini (1990) considers the random nature of the product mix by assigning probabilities to discrete product mixes and to the associated machine-component incidence matrix. For each product mix under consideration a cell formation is determined, and the inter-cell material handling costs that correspond to this configuration, under all possible product mixes, are calculated. Subsequently, the expected inter-cell material handling cost for each configuration is evaluated and used to select a near optimal solution. This approach does not take into account shop characteristics that affect the probabilities assigned to each product mix, such as resource capacity constraints. Furthermore, the fact that the optimum shop partition is calculated for every product mix limits the number of alternative states to be considered. Minimization of the expected material handling cost has also been the target of related research conducted by Rosenblatt and Kropp (1992), who consider the stochastic nature of the product demand in their study of plant layout.

Vakharia and Kaku (1993) have studied the impact of demand changes on the performance of cellular manufacturing systems. They recognized that resource capacities limit the ability of a cellular system to adequately respond to such changes, and proposed a system redesign methodology to address the robustness issue. Their method is based on a zero-one mathematical programming formulation and attempts to allocate new parts, or reallocate those for which large demand changes have occured, among existing cells. Note that the manufacturing cells remain unchanged in composition and, therefore, system robustness is not addressed at the shop design stage. This technique may be viewed as complementary to a robust shop design methodology.

This paper focuses on the cell formation problem under random product demand and presents an approach to obtain robust shop decompositions, i.e. cellular designs with satisfactory performance over a certain range of demand variation. The statistical characteristics of the independent demand, as well as the capacity of the system resources are explicitly considered. The design objective is to minimize the expected inter-cell material handling traffic, a measure originally introduced by Seifoddini (1990). In the first stage of the proposed method the statistics of the feasible production volumes are determined given the statistics of the independent demand and are used to compute the design criterion for the candidate shop configurations. In the second stage, a near optimal cell formation is determined using an effective grouping method presented in (Harhalakis *et al.*, 1990).

The paper is organized as follows. In section 2 the cell formation framework is set up, the design criterion is introduced, and the case of infinite resource capacities is considered. Section 3 examines the cell formation problem under finite capacities. Section 4 describes the proposed cell formation algorithm. A sample application is presented in section 5 to illustrate the entire design procedure. Finally, the conclusions of this work and recommendations for future research are summarized in section 6.

## 2 PROBLEM FORMULATION

The cell formation problem consists of partitioning the manufacturing shop into a set of manufacturing cells $C=\{c_1,...,c_w\}$, such that the total inter-cell traffic of parts within the design time horizon $H$ is minimized. The following information concerning the shop operational characteristics is assumed available. It is noted that bold and underlined characters indicate random variables.

- The set of machines, $M=\{M_1,...,M_g\}$ and their capacities, $CM_j$, j=1,...,g.
- The set of all manufactured (make) components, $I=\{p_{11},...,p_{1u_1},p_{21},...,p_{2u_2},...,p_{n1},...,p_{nu_n}\}$. Note that $\{p_{11},p_{21},...,p_{n1}\}$ represent the finished products (final assemblies) while $\{p_{i2},...,p_{iu_i}\}$ represent the manufactured components or subassemblies of part $p_{i1}$, i=1,...,n.
- The random demand of the finished products over the time horizon $H$, $\mathbf{D}=\{\mathbf{D}_1,...,\mathbf{D}_n\}$.
- The processing sequence (routing) $r_{ia}$ of each make part $p_{ia}$, i=1,...,n, a=1,...,$u_i$. The production routing specifies a unique sequence of machines employed during the part manufacture as well as the corresponding processing times. Alternative routings that may employ functionally similar machines for the production of a certain part are not considered.

Throughout this study a lot-for-lot batch sizing strategy is assumed and, therefore, the demand of finished products defines the demand of all dependent make items. Due to machine capacity limitations and depending on the backlogging/holding policy of the particular manufacturing environment, the feasible production volume of finished products may or may not equal the demand. Let $\underline{\Delta}_i$ be the production volume of end product $p_{i1}$ within the entire design horizon $H$. Then the cell formation problem is formulated as follows :

$$\text{minimize} \qquad E\{\mathbf{T}(\underline{\Delta}_1, \underline{\Delta}_2,\ldots,\underline{\Delta}_n)\} = E\ \{\sum_{r=1}^{w}\ \sum_{s=1}^{w} [\ \sum_{i=1}^{n} \underline{\Delta}_i\ x_i(r,s)]\ \} \qquad (1)$$

subject to

$$q_r < Q\ , \qquad r=1,\ldots,w \qquad (2)$$

$$\sum_{i=1}^{n} \underline{\Delta}_{ij}\Theta_{ij} \leq CM_j\ , \quad j=1,\ldots,g \qquad (3)$$

where :

- $\mathbf{T}(\underline{\Delta}_1, \underline{\Delta}_2,\ldots,\underline{\Delta}_n)$ is the inter-cell traffic.
- $E\{\ldots\}$ is the expected value of the expression in brackets.
- $x_i(r,s)$ is the number of times the final product $p_{i1}$ *or any of its make components* $p_{i2},\ldots,p_{iu_i}$ have to be transported from cell $c_r$ to cell $c_s$ and is given by :

$$x_i(r,s) = \sum_{a=1}^{u_i} z_{ia}\ y_{ia}(r,s) \qquad (4)$$

  where $z_{ia}$ is the quantity of components $p_{ia}$ that are used to produce one unit of final product $p_{i1}$, $u_i$ is the total number of make components of end product $p_{i1}$, and $y_{ia}(r,s)$ is the number of times make component $p_{ia}$ is transported from cell $c_r$ to cell $c_s$.
- $q_r$ is the number of machines in cell r.
- Q is the maximum allowable number of machines per cell.
- $CM_j$ is the capacity of machine $M_j$.
- $\Theta_{ij}$ is the cummulative processing time of part type $p_{i1}$ and all its make items on machine $M_j$ and is given by :

$$\Theta_{ij} = [\sum_{a=1}^{u_i} \Theta_{iaj}^{su}] / bi + \sum_{a=1}^{u_i} z_{ia}\Theta_{iaj}^{run} \qquad (5)$$

  where $b_i$ is the average batch size for part $p_{i1}$ and $\Theta_{iaj}^{su}$ and $\Theta_{iaj}^{run}$ are the set up and run times of make item $p_{ia}$ on machine $M_j$, respectively.

All variables are expressed in terms of the same time horizon *H* .

Minimization of the objective function given by Eq. (1) ensures that the resulting machine to cell partition will yield minimal inter-cell traffic, on the average. It is emphasized that the traffic values with higher probability are weighted more by this criterion, while the entire spectrum of feasible production volumes is considered. The first constraint, given by Eq. (2), maintains the cell size below a predefined upper bound Q. The value of the latter is based on several factors, such as physical machine sizes, product envelope volumes and level of automation (Ang and Willey, 1984). The second set of constraints, given by Eq. (3), reflects the limited machine capacities. The workload depends on the set-up and run times of the make products and their production volumes.

This formulation does not address the case of alternative production routings and the distribution of the production volumes among them. The difficulties arising if alternative routings are to be considered are discussed in section 6.

The cellular design problem was first formulated in this manner in (Minis *et al.*, 1990; Nagi *et al.*, 1990), for the case of deterministic production volumes. If the machine capacity constraints are not active, it is shown below that the deterministic and random demand cases are equivalent.

Unlimited Backlogging within the Design Horizon

In this case it is assumed that the portion of the production which cannot be manufactured within a certain time segment of the design horizon $H$, due to capacity limitations, can be delayed and completed at a later time segment within $H$. Therefore, the demand $\mathbf{D}_i$ of part $p_{i1}$ over $H$, is equal to the production volume $\underline{\Delta}_i$ and the capacity constraints affect neither the production nor the inter-cell traffic. Remark 1 shows that in this case minimization of the mean inter-cell traffic is equivalent to minimizing the traffic that corresponds to the mean demands of the finished products.

Remark 1 : The following problems are equivalent

I) minimize $T(\mu_1,\dots,\mu_n)$
subject to $q_k < Q$ $\quad k=1,\dots,w$

II) minimize $E\{\mathbf{T}(\mathbf{D}_1,\dots,\mathbf{D}_n)\}$
subject to $q_k < Q$ $\quad k=1,\dots,w$

where $\mu_1=E\{\mathbf{D}_1\},\dots,\mu_n=E\{\mathbf{D}_n\}$ and the capacity constraints are not active.

Proof : Given a certain shop configuration, the inter-cell traffic is given by :

$$\mathbf{T}(\underline{\Delta}_1,\dots,\underline{\Delta}_n) = \mathbf{T}(\mathbf{D}_1,\dots,\mathbf{D}_n) = \sum_{r=1}^{w}\sum_{s=1}^{w}\sum_{i=1}^{n} \mathbf{D}_i\, x_i(r,s) =$$

$$= \sum_{i=1}^{n}\Big[\sum_{r=1}^{w}\sum_{s=1}^{w} x_i(r,s)\Big]\mathbf{D}_i = \sum_{i=1}^{n} A_i^c\, \mathbf{D}_i \qquad (6)$$

where the constants $A_i^c=\sum_{r=1}^{w}\sum_{s=1}^{w} x_i(r,s)$, $i=1,\dots,n$, depend on the shop configuration only. Taking the expected values of both sides of Eq. (6), we obtain :

$$E\{\mathbf{T}(\mathbf{D}_1,...,\mathbf{D}_n)\} = E\{\sum_{i=1}^{n} A_i^c \mathbf{D}_i\} = \sum_{i=1}^{n} A_i^c E\{\mathbf{D}_i\} = \sum_{i=1}^{n} A_i^c \mu_i \quad \Leftrightarrow$$

$$\Leftrightarrow \quad E\{\mathbf{T}(\mathbf{D}_1,...,\mathbf{D}_n)\} = T(\mu_1,...,\mu_n) \qquad \text{Q.E.D.}$$

The above relationship shows that problems (I) and (II) are equivalent. Problem (I) corresponds to the deterministic cell formation case, which has been extensively studied in the literature and will not be considered any further in this paper.

## 3 CELL FORMATION UNDER LIMITED CAPACITY

In the case of limited resource capacities, the objective function $E\{\mathbf{T}\}$ may no longer be determined using the mean values of the independent product demands $\mu_1,...,\mu_n$. This section outlines a systematic way to calculate the expected traffic value $E\{\mathbf{T}\}$ for a certain shop configuration, based on the evaluation of the actual production volumes that respect the resource capacity constraints. Having determined $E\{\mathbf{T}\}$ as a function of the shop configuration, any of the traffic-minimization grouping methods found in the literature can be employed to determine a near-optimal shop partition.

Consider the time horizon $H = fh$, where $h$ is the unit (elementary) production planning period beyond which no backlogging and no holding is allowed. For example, $H$ may be a ten year period for which the facility is designed, and $h$ an elementary six-month period. Let $\mathbf{d}_{it}$ be the demand for part $p_{i1}$ in period t, i=1,...,n, t=1,...,f. The statistics of $\mathbf{d}_{it}$ may be time invariant if the market is expected to exhibit relative stability for the demand of the manufactured products. Otherwise, the statistics of $\mathbf{d}_{it}$ may vary with time following anticipated market trends over the design period $H$.

Since backlogging or holding is not allowed in this case, the independent product demand in each unit production planning period may not be necessarily satisfied, due to capacity limitations. Let $\underline{\delta}_{1t},...,\underline{\delta}_{nt}$, represent the feasible production volumes of the final products $p_{11},...,p_{n1}$, respectively, in the time period t. If $\mathbf{d}_{1t}=d_{1t}...\mathbf{d}_{nt}=d_{nt}$ is a demand mix in elementary period t, for which all capacity constraints are satisfied, then $\underline{\delta}_{1t}=d_{1t},\ldots,\underline{\delta}_{nt}=d_{nt}$. If satisfaction of the independent demand $(d_{1t},\ldots,d_{nt})$ is not feasible within the period t, then the feasible production mix $(\delta_{1t},\ldots,\delta_{nt})$ is no longer a function of the independent demand alone, but also depends on a managerial decision that targets a certain objective, such as maximum profit.

Remark 2 : Let **T** be the total inter-cell traffic within the design horizon $H$ and $\underline{\Delta}_i$ the total production volume of part $p_{i1}$ in $H$ . Furthermore, let $\underline{\delta}_{it}$ be the production volume of part $p_{i1}$ within the elementary time period t of duration $h$. Then

$$E\{\mathbf{T}\} = \sum_{t=1}^{f} \sum_{i=1}^{n} A_i^c\, E\{\underline{\delta}_{it}\} \tag{7}$$

where $A_i^c$, i=1,...n, are the constants defined in Remark 1.

Proof : The total inter-cell traffic, within the design time horizon $H$, is given by [see also Eq.(6) of Remark 1] :

$$\mathbf{T} = \sum_{r=1}^{w} \sum_{s=1}^{w} \sum_{i=1}^{n} \underline{\Delta}_i\, x_i(r,s) = \sum_{i=1}^{n} A_i^c \underline{\Delta}_i \tag{8}$$

Considering the expected values of both sides, Eq.(8) yields

$$E\{\mathbf{T}\} = E\{\sum_{i=1}^{n} A_i^c \underline{\Delta}_i\} = \sum_{i=1}^{n} A_i^c\, E\{\underline{\Delta}_i\} \tag{9}$$

However, the total production volume of part $p_{i1}$, can be expressed as

$$\underline{\Delta}_i = \sum_{t=1}^{f} \underline{\delta}_{it} \tag{10}$$

and considering the expected values of both sides in Eq.(10) it follows that :

$$E\{\underline{\Delta}_i\} = E\{\sum_{t=1}^{f} \underline{\delta}_{it}\} = \sum_{t=1}^{f} E\{\underline{\delta}_{it}\} \tag{11}$$

Substituting Eq.(11) in Eq.(9) yields

$$E\{\mathbf{T}\} = \sum_{i=1}^{n} (A_i^c \sum_{t=1}^{f} E\{\underline{\delta}_{it}\}) = \sum_{t=1}^{f} \sum_{i=1}^{n} A_i^c\, E\{\underline{\delta}_{it}\} \tag{12}$$

Q.E.D.

The above remark shows that the mean inter-cell traffic $E\{\mathbf{T}\}$ over the entire time horizon $H$ can be determined, for a given shop configuration, from the expected values of the random variables $\underline{\delta}_{1t},\ldots,\underline{\delta}_{nt}$, t=1,...,f. The latter are given by :

$$E\{\underline{\delta}_{it}\} = \sum_{\delta_{1t}=0}^{\max \delta_{1t}} \cdots \sum_{\delta_{nt}=0}^{\max \delta_{nt}} \delta_{it}\ P\{\underline{\delta}_{1t}=\delta_{1t},\dots,\underline{\delta}_{nt}=\delta_{nt}\} \tag{13}$$

where $\max\delta_{1t}$ is the maximum possible value of the demand of end item $p_{i1}$.

The remaining portion of this section presents a method to determine the probability distribution $P\{\underline{\delta}_{1t},\dots,\underline{\delta}_{nt}\}$.

Evaluation of $P\{\underline{\delta}_{1t},\dots,\underline{\delta}_{nt}\}$

Consider the following regions in the independent demand space :

$$\mathcal{A}_1 = \{\ d_t=(d_{1t},\dots,d_{nt}) :\ \forall\ j\in\{1,\dots,g\},\ \sum_{i=1}^{n} d_{it}\ \Theta_{ij} < CW_j\ \} \tag{14}$$

$$\mathcal{A}_2 = \{\ d_t=(d_{1t},\dots,d_{nt}) :\ \exists\ j\in\{1,\dots,g\},\ \sum_{i=1}^{n} d_{it}\ \Theta_{ij} > CW_j\ \} \tag{15}$$

$$\mathcal{A}_3 = \Re^n{}_+ - (\mathcal{A}_1 \cup \mathcal{A}_2) \tag{16}$$

where $CW_j = CM_j / f$, is the capacity of each unique machine within period t [see Eq. (3)].

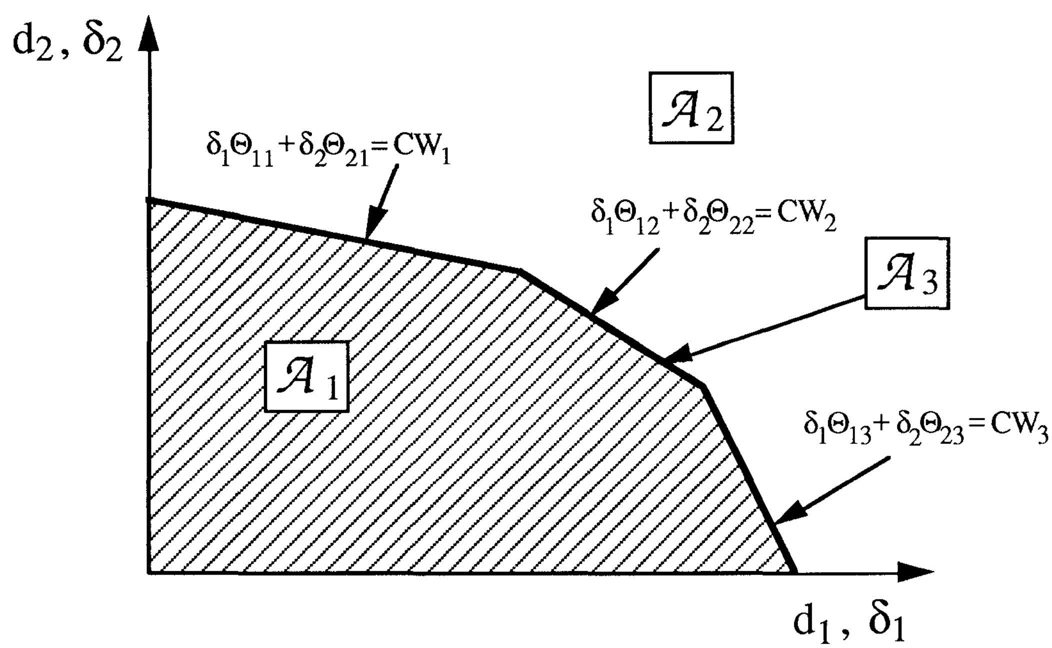


Figure 1 : Demand Space Partition for Two End Products and Three Machines

Figure 1 illustrates the three regions defined above, for the simple case of two final assemblies (end items) and three machines with limited capacities.

Case 1 : If $(\delta_{1t},...,\delta_{nt}) \in \mathcal{A}_1$,

$$P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}=P\{\mathbf{d}_{1t}=\delta_{1t},...,\mathbf{d}_{nt}=\delta_{nt}\}=\prod_{i=1}^{n} P\{\mathbf{d}_{it}=\delta_{it}\} \tag{17}$$

The last part of Eq. (17) is true, since the demands of the end items are assumed to be independent.

Case 2 : If $(\delta_{1t},...,\delta_{nt}) \in \mathcal{A}_2$,

$$P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}=0 \tag{18}$$

Case 3 : If $(\delta_{1t},...,\delta_{nt}) \in \mathcal{A}_3$, then both cases $(d_{1t},...,d_{nt}) \in \mathcal{A}_3$ and $(d_{1t},...,d_{nt}) \in \mathcal{A}_2$ must be considered to determine $P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}$. In the latter case, the capacity constraints are violated and the feasible production volume $(\delta_{1t},...,\delta_{nt})$ is determined by management on the basis of certain criteria. If an optimization procedure is employed to make this decision, the production volume $(\delta_{1t},...,\delta_{nt})$ will belong to the boundary $\mathcal{A}_3$.

The managerial problem, such as the one presented in Appendix A, will be referred to as $(\mathcal{P})$. Every solution of $(\mathcal{P})$ will map an infeasible demand point $d_t=(d_{1t},...,d_{nt}) \in \mathcal{A}_2$ to a feasible production volume $\delta_t=(\delta_{1t},...,\delta_{nt}) \in \mathcal{A}_3$, such that the appropriate criterion is optimized. Thus, if $\delta_t=(\delta_{1t},...,\delta_{nt}) \in \mathcal{A}_3$ then

$$P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}= P\{\mathbf{d}_{1t}=\delta_{1t},...,\mathbf{d}_{nt}=\delta_{nt}\}+P\{d_t \in \mathcal{A}_2 : d_t \rightarrow \delta_t \text{ through } (\mathcal{P})\} \tag{19}$$

where "$\rightarrow$" should be read as "is mapped to". The first term in Eq.(19) is given by

$$P\{\mathbf{d}_{1t}=\delta_{1t},...,\mathbf{d}_{nt}=\delta_{nt}\}=\prod_{i=1}^{n} P\{\mathbf{d}_{it}=\delta_{it}\} \tag{20}$$

To determine the second term, problem $(\mathcal{P})$ should be solved for every $d_t \in \mathcal{A}_2$. Let $\mathcal{D}_{\delta_t}$ be the set of infeasible points which are mapped to point $\delta_t$ through $(\mathcal{P})$, i.e.

$$\mathcal{D}_{\delta_t} = \{d_t \in \mathcal{A}_2 : d_t \rightarrow \delta_t \text{ through } (\mathcal{P})\}$$

Then, the second term of Eq.(19) is given by :

$$P\{d_t=(d_{1t},...,d_{nt}) \in \mathcal{A}_2 : d_t \rightarrow \delta_t \text{ through } (\mathcal{P})\} = \sum_{\mathcal{D}_{\delta_t}} [\prod_{i=1}^{n} P\{\mathbf{d}_{it}=d_{it}\}] \tag{21}$$

Substituting Eqs.(20) and (21) in Eq.(19), the probability function becomes

$$P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}=\prod_{i=1}^{n} P\{\mathbf{d}_{it}=\delta_{it}\} + \sum_{\mathcal{D}_{\delta_t}} [\prod_{i=1}^{n} P\{\mathbf{d}_{it}=d_{it}\}] \tag{22}$$

It is noted that ($\mathcal{P}$) is independent of the cellular configuration. However, even if the demand space is discretized by considering only a few distinct demand values for each end product, the total number of possible demand vectors that exceed the capacity constraints and for which ($\mathcal{P}$) should be solved, may be extremely large. For example, considering 10 end items and 10 discrete demand values for each one, the total number of demand vectors $d_t \in \mathcal{A}_2$, and consequently the number of problems ($\mathcal{P}$) to be solved, is of the order $10^{10}$. Thus, only the most critical end products, for which large demand uncertainty is anticipated, should be considered in the shop design phase (see also Appendix A for ways to efficiently determine $\mathcal{D}_{\delta_t}$ for a particular class of $\mathcal{P}$).

Having determined $P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}$ for all three regions $\mathcal{A}_1$, $\mathcal{A}_2$ and $\mathcal{A}_3$ of the demand space, Eq.(13) is employed to calculate the mean values of the production volumes in each elementary period, $E\{\underline{\delta}_{it}\}$, i=1,...,n, t=1,...,f. Subsequently, the mean value $E\{\mathbf{T}\}$ of the total inter-cell traffic $\mathbf{T}$ within the design horizon $H$ is determined from Eq. (7) for each candidate shop configuration. The overall design algorithm presented below is based on the results derived in this section.

## 4 DESIGN ALGORITHM

The cell formation algorithm consists of a two-step procedure that results in a near optimal partition of the manufacturing shop into production cells. In the first step, the mean values of the feasible production volumes are determined, as described in section 3. The latter are independent of the shop partition and are employed in the second step to determine the mean traffic for each candidate shop configuration. In the second step the existing approach of (Harhalakis *et al.*, 1990), briefly outlined in Appendix B, is applied to determine a near optimal system partition.

As mentioned in the previous section the number of managerial problems ($\mathcal{P}$) to be solved increases exponentially with the number of end items. Thus, it is critical for the implementation of this algorithm to consider random demand only for those final assemblies which are both financially important and expected to exhibit significant demand fluctuations over the elementary period t. Furthermore, when applicable, only bottleneck resources may be considered in order to simplify the capacity hull $\mathcal{A}_3$.

The inputs, outputs and major steps of the proposed design algorithm are listed below [see also Fig.2].

Inputs :

- Bills-of-material of each final assembly.
- Resources included in the manufacturing shop and their capacities.
- Sequence of operations and processing times for each end item and each make item.
- Cell size limit.
- The design Horizon $H$ and the number of elementary periods f of duration $h$.
- The demand statistics for each final assembly in each elementary period.
- Formulation of the managerial problem ($\mathcal{P}$) discussed in section 3 [see also Appendix A].

Algorithm :

(a) Consider elementary period t=1.

(b) For every $d_t=(d_{1t},...,d_{nt})\in \mathcal{A}_2$, solve the corresponding managerial problem ($\mathcal{P}$), an example of which is presented in Appendix A. This will provide the feasible production volume $\delta_t=(\delta_{1t},...,\delta_{nt})\in \mathcal{A}_3$.

(c) For every $\delta_t=(\delta_{1t},...,\delta_{nt})\in \mathcal{A}_1$ calculate the joint probability, $P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}$ from Eq.(17). For every $\delta_t=(\delta_{1t},...,\delta_{nt})\in \mathcal{A}_3$ calculate the joint probability $P\{\underline{\delta}_{1t}=\delta_{1t},...,\underline{\delta}_{nt}=\delta_{nt}\}$ from Eq.(22).

(d) Calculate the mean value of the production volume for each end item $p_{i1}$, i.e. $E\{\underline{\delta}_{it}\}$, $\forall$ i, using Eq.(13).

(e) Repeat steps b-d, for each elementary period t=2,...,f of duration $h$ .

(f) Obtain a near optimal shop configuration that minimizes the mean value of the inter-cell traffic based on the grouping algorithm found in (Harhalakis *et al.*, 1990). At each stage of this algorithm, Eq.(7) is used to calculate $E\{\mathbf{T}\}$ [see Remark 2].

Output :

- Partition of the manufacturing shop to cells that minimizes the expected inter-cell material handling traffic over the entire design horizon $H$.

Note that the degree of optimality of the resulting configuration depends on the performance of the grouping step (f).

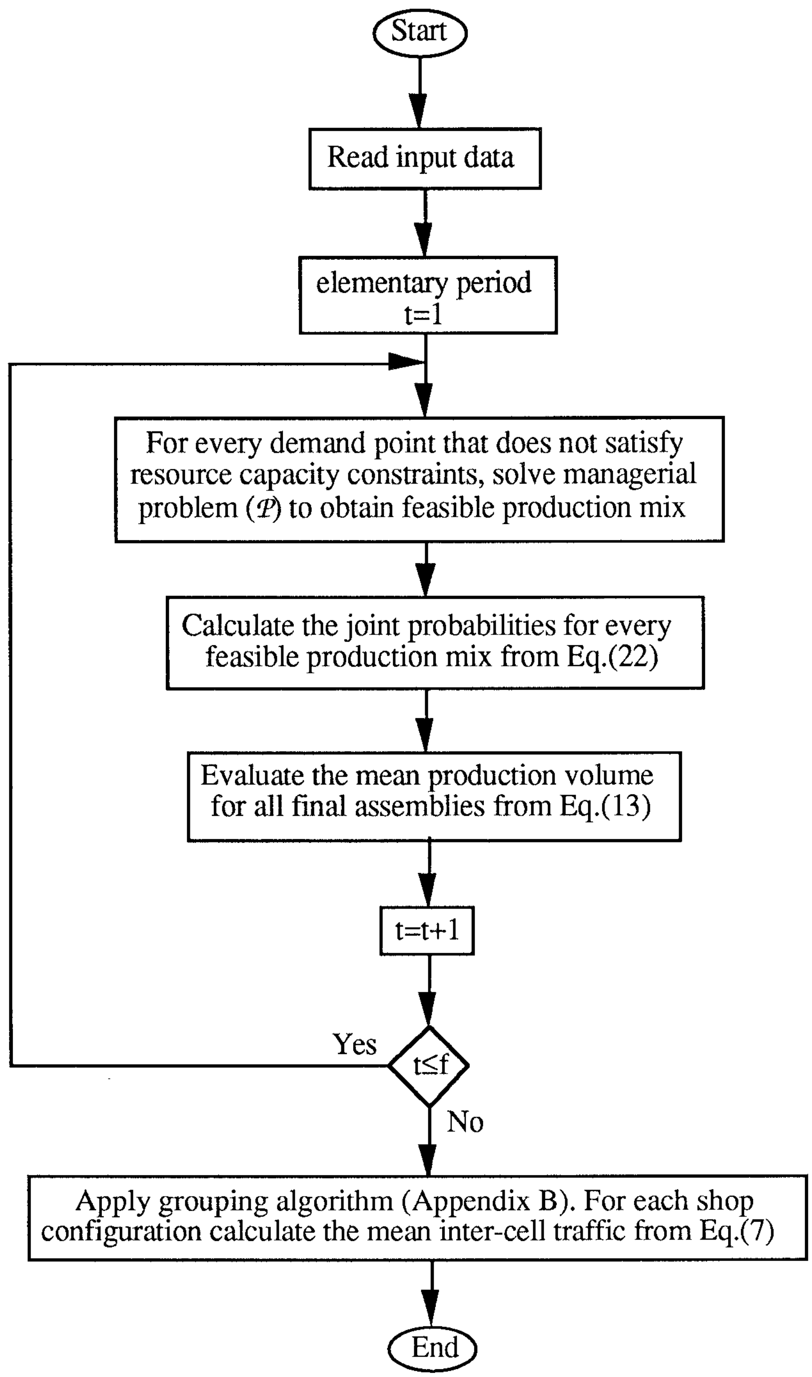


Figure 2 : Flowchart of the Design Algorithm

## 5 ILLUSTRATIVE EXAMPLE

In order to demonstrate the design approach described in the previous sections, a small-order example is presented. The sample manufacturing system produces three final assemblies and the total number of make items is twenty. Seventeen functionally unique machines are included in the shop. Only functionally unique machines are considered, in order to exclude alternative production routings which are not accounted for in the problem formulation of section 2.

The sequence of operations (routing) for each make item is presented in the part-machine incidence matrix of Table 1. The entries of this table identify the order of operations in the part production routing. The part numbers of the end items are 1,2, 3 and the part numbers of the remaining make items are 4 through 20. The machines are numbered from 1 to 17.

<u>Table 1</u> : Part-machine Incidence Matrix

| | Machine ID | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| Part | | | | | | | | | | | | | | | | | |
| 1 | . | 5 | . | . | 3 | . | . | 2 | . | . | . | . | 4 | . | . | 1 | . |
| 2 | . | . | . | . | . | 1 | . | . | . | . | 3 | . | . | 4 | . | . | 2 |
| 3 | 1 | . | 3 | . | . | . | . | . | . | . | 4 | 2 | . | . | . | . | . |
| 4 | . | . | . | 4 | . | . | . | . | 1 | 3 | . | . | . | . | 2 | . | . |
| 5 | . | . | . | 3 | . | . | . | . | . | 2 | . | . | . | . | 1 | 4 | . |
| 6 | 4 | . | 3 | . | . | . | 2 | . | . | . | . | 1 | . | . | . | . | . |
| 7 | . | . | . | . | 1 | . | . | 3 | . | . | . | . | 2 | . | . | . | . |
| 8 | . | . | . | . | . | 4 | . | . | . | . | 3 | . | . | 2 | . | . | 1 |
| 9 | . | . | 1 | . | . | . | 2 | . | . | . | . | 3 | . | . | . | . | . |
| 10 | . | 1 | . | . | 2 | . | . | 4 | . | . | . | . | 3 | . | . | . | . |
| 11 | . | . | . | 4 | . | . | . | . | 2 | . | . | . | . | . | 1 | 3 | . |
| 12 | 4 | . | 1 | . | . | . | 2 | . | . | . | . | 3 | . | . | . | . | . |
| 13 | . | . | . | 2 | . | . | . | . | 3 | 1 | . | . | . | . | 4 | . | . |
| 14 | 1 | . | 3 | . | . | . | . | . | . | . | 4 | 2 | . | . | . | . | . |
| 15 | . | 3 | . | . | 2 | . | . | 1 | . | . | . | . | 4 | . | . | . | . |
| 16 | . | . | . | 1 | . | . | . | . | 2 | 3 | . | . | . | . | . | 4 | . |
| 17 | . | . | . | . | . | 2 | . | . | . | . | . | . | . | 3 | . | . | 1 |
| 18 | 3 | . | 2 | . | . | . | 1 | . | . | . | 4 | . | . | . | . | . | . |
| 19 | . | 3 | . | . | 4 | . | . | 2 | . | . | . | . | 1 | . | . | 5 | . |
| 20 | . | . | 2 | . | . | . | 3 | . | . | . | . | 1 | . | . | . | . | . |

The processing times $\Theta_{ij}$ of each operation in the part production routings are given in Table 2. $\Theta_{ij}$ include both the run and set up times [see Eq.(5) of section 2]. The capacity limit of each machine for an elementary period of duration *h*, beyond which no backlogging or holding is allowed, is given in Table 3. Processing times and machine capacities are expressed in the same time units.

Table 2 : Part Processing Times

| | Machine ID | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 1 | . | 30 | . | . | 40 | . | . | 10 | . | . | . | . | 35 | . | . | 35 | . |
| 2 | . | . | . | . | . | 20 | . | . | . | . | 15 | . | . | 50 | . | . | 50 |
| 3 | 20 | . | 30 | . | . | . | . | . | . | . | 25 | 40 | . | . | . | . | . |
| 4 | . | . | . | 10 | . | . | . | . | 20 | 10 | . | . | . | . | 20 | . | . |
| 5 | . | . | . | 20 | . | . | . | . | . | 15 | . | . | . | . | 10 | 40 | . |
| 6 | 20 | . | 30 | . | . | . | 25 | . | . | . | . | 25 | . | . | . | . | . |
| 7 | . | . | . | . | 25 | . | . | 10 | . | . | . | . | 20 | . | . | . | . |
| 8 | . | . | . | . | . | 55 | . | . | . | . | 10 | . | . | 45 | . | . | 40 |
| 9 | . | . | 15 | . | . | . | 15 | . | . | . | . | 40 | . | . | . | . | . |
| 10 | . | 40 | . | . | 25 | . | . | 25 | . | . | . | . | 25 | . | . | . | . |
| 11 | . | . | . | 10 | . | . | . | . | 5 | . | . | . | . | . | 25 | 10 | . |
| 12 | 15 | . | 30 | . | . | . | 35 | . | . | . | . | 40 | . | . | . | . | . |
| 13 | . | . | . | 10 | . | . | . | . | 15 | 25 | . | . | . | . | 15 | . | . |
| 14 | 20 | . | 35 | . | . | . | . | . | . | . | 25 | 40 | . | . | . | . | . |
| 15 | . | 25 | . | . | 15 | . | . | 10 | . | . | . | . | 40 | . | . | . | . |
| 16 | . | . | . | 15 | . | . | . | . | 10 | 15 | . | . | . | . | . | 20 | . |
| 17 | . | . | . | . | . | 65 | . | . | . | . | . | . | . | 40 | . | . | 50 |
| 18 | 20 | . | 40 | . | . | . | 15 | . | . | . | 15 | . | . | . | . | . | . |
| 19 | . | 20 | . | . | 30 | . | . | 30 | . | . | . | . | 20 | . | . | 20 | . |
| 20 | . | . | 20 | . | . | . | 15 | . | . | . | . | 20 | . | . | . | . | . |

Table 3 : Machine Capacities over period *h*

| Machine | Capacity | Machine | Capacity | Machine | Capacity | Machine | Capacity |
|---|---|---|---|---|---|---|---|
| 1 | 16200 | 6 | 15500 | 11 | 18900 | 16 | 23100 |
| 2 | 30100 | 7 | 18700 | 12 | 22800 | 17 | 12500 |
| 3 | 34800 | 8 | 23400 | 13 | 24800 | | |
| 4 | 11200 | 9 | 10700 | 14 | 15500 | | |
| 5 | 35300 | 10 | 12600 | 15 | 14800 | | |

Table 4 presents the relationship between the three final assemblies and their make components in a single level fashion. For simplicity, without loss of generality, each make component is used with a quantity of one per unit parent product.

Table 4 : Parent-child Relationships and Demand Statistics of End Items

| End Product $p_{i1}$ | Mean Demand | Standard Deviation | Unit Profit | Make Items $p_{ij}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 175 | 40 | 500 | 4 | 7 | 10 | 13 | 15 | 19 |
| 2 | 80 | 10 | 700 | 5 | 8 | 11 | 16 | 17 | |
| 3 | 110 | 30 | 400 | 6 | 9 | 12 | 14 | 18 | 20 |

It is assumed that the demand within each elementary period of duration $h$ is distributed normally. Furthermore, the means and the standard deviations are assumed to be time invariant, i.e.

$$\mathbf{d}_{it} = \mu_{it} + \sigma_{it}\,\mathbf{e}_{it} \qquad i=1,...3,\ \ t=1,...,f$$

where,

- $\mu_{it} = \mu_i$ is the mean value of the demand of product $p_{i1}$ during period t.
- $\sigma_{it} = \sigma_i$ is the standard deviation of the demand of product $p_{i1}$ for the same period.
- $\mathbf{e}_{it}$ = random variable for which :
  - $E\{\mathbf{e}_{it}\}=0$
  - $E\{\mathbf{e}_{it}^2\}=1$
  - $E\{\mathbf{e}_{it}\mathbf{e}_{uv}\}=0$ if $i \neq u$ (different product) or $t \neq v$ (different time period)

Table 4 provides the mean values $\mu_i$ and the standard deviations $\sigma_i$, i=1,2,3. Note that due to the time invariant statistics, the design can be performed by taking into account the production variables of a single elementary period only.

In this example, it is assumed that the managerial problem ($\mathcal{P}$) maximizes the total profit. The resulting linear programming formulation is presented in Appendix A. The unit profit for each final assembly is given in Table 4. The mean feasible production volumes derived from the solution of ($\mathcal{P}$) were: $E\{\underline{\delta}_1\}=150$, $E\{\underline{\delta}_2\}=79$, $E\{\underline{\delta}_3\}=90$.

The design algorithm described in section 4 was applied to the above example, for a cell size limit Q=5 [see Eq.(2)] and the resulting machine to cell partition is shown in Table 5. Four cells are included in the optimal shop partition $L_A=\{C_{1A},C_{2A},C_{3A},C_{4A}\}$; $C_{1A}=\{2,5,8,13\}$, $C_{2A}=\{1,3,7,12\}$, $C_{3A}=\{4,9,10,15,16\}$ and $C_{4A}=\{6,11,14,17\}$. The expected value of the traffic that corresponds to this configuration is $E\{\mathbf{T}_A\}_{min}=570$.

<u>Table 5</u> : Manufacturing Cells for the Random Product Demand Case ($L_A$)

| | Machine ID | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part | 2 | 5 | 8 | 13 | 1 | 3 | 7 | 12 | 4 | 9 | 10 | 15 | 16 | 6 | 11 | 14 | 17 |
| 1 | 5 | 3 | 2 | 4 | . | . | . | . | . | . | . | . | 1 | . | . | . | . |
| 7 | . | 1 | 3 | 2 | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 10 | 1 | 2 | 4 | 3 | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 15 | 3 | 2 | 1 | 4 | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 19 | 3 | 4 | 2 | 1 | . | . | . | . | . | . | . | . | 5 | . | . | . | . |
| 3 | . | . | . | . | 1 | 3 | . | 2 | . | . | . | . | . | . | 4 | . | . |
| 6 | . | . | . | . | 4 | 3 | 2 | 1 | . | . | . | . | . | . | . | . | . |
| 9 | . | . | . | . | . | 1 | 2 | 3 | . | . | . | . | . | . | . | . | . |
| 12 | . | . | . | . | 4 | 1 | 2 | 3 | . | . | . | . | . | . | . | . | . |
| 14 | . | . | . | . | 1 | 3 | . | 2 | . | . | . | . | . | . | 4 | . | . |
| 18 | . | . | . | . | 3 | 2 | 1 | . | . | . | . | . | . | . | 4 | . | . |
| 20 | . | . | . | . | . | 2 | 3 | 1 | . | . | . | . | . | . | . | . | . |
| 4 | . | . | . | . | . | . | . | . | 4 | 1 | 3 | 2 | . | . | . | . | . |
| 5 | . | . | . | . | . | . | . | . | 3 | . | 2 | 1 | 4 | . | . | . | . |
| 11 | . | . | . | . | . | . | . | . | 4 | 2 | . | 1 | 3 | . | . | . | . |
| 13 | . | . | . | . | . | . | . | . | 2 | 3 | 1 | 4 | . | . | . | . | . |
| 16 | . | . | . | . | . | . | . | . | 1 | 2 | 3 | . | 4 | . | . | . | . |
| 2 | . | . | . | . | . | . | . | . | . | . | . | . | . | 1 | 3 | 4 | 2 |
| 8 | . | . | . | . | . | . | . | . | . | . | . | . | . | 4 | 3 | 2 | 1 |
| 17 | . | . | . | . | . | . | . | . | . | . | . | . | . | 2 | . | 3 | 1 |

To ensure optimality of this result, the traffic value, $E\{\mathbf{T}_A\}_{min}=570$, was used as an upper bound to the state-space search algorithm proposed by (Ghosh, *et al.*). This is essentially a branch-and-bound algorithm with a special state-space search that operates under memory and execution time constraints. Given very large values for these constraints, and for problems of small dimension, the algorithm is guaranteed to find the optimum. For the example of this section the optimum was verified to be 570.

The above results were compared to the partition derived when deterministic demand is assumed. The algorithm of Harhalakis *et al.* (1990), was applied on the same shop data using the means of the independent demands to define the constant production volumes [see Table 4]. The latter satisfy all capacity constraints. The traffic value that corresponds to the solution yielded by the grouping algorithm was found to be $T_B$=650. However, the state-space search algorithm of (Ghosh, *et al.*) showed that this is not the optimal configuration. The global optimum was found by the search algorithm and the corresponding traffic is $T_{Bmin}$=640. The resulting manufacturing cells are shown in Table 6. Four cells were also found in this case, $L_B=\{C_{1B},C_{2B},C_{3B},C_{4B}\}$, where $C_{1B}=\{2,5,8,13,16\}$, $C_{2B}=\{1,3,7,11,12\}$, $C_{3B}=\{4,9,10,15\}$ and $C_{4B}=\{6,14,17\}$. The difference between the two configurations, shown in Tables 5 and 6 is that machines 11 and 16 have been placed at different cells.

Table 6 : Manufacturing Cells for the Deterministic Product Demand Case ($L_B$)

| | Machine ID | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part | 2 | 5 | 8 | 13 | 16 | 1 | 3 | 7 | 11 | 12 | 4 | 9 | 10 | 15 | 6 | 14 | 17 |
| 1 | 5 | 3 | 2 | 4 | 1 | . | . | . | . | . | . | . | . | . | . | . | . |
| 7 | . | 1 | 3 | 2 | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 10 | 1 | 2 | 4 | 3 | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 15 | 3 | 2 | 1 | 4 | . | . | . | . | . | . | . | . | . | . | . | . | . |
| 19 | 3 | 4 | 2 | 1 | 5 | . | . | . | . | . | . | . | . | . | . | . | . |
| 3 | . | . | . | . | . | 1 | 3 | . | 4 | 2 | . | . | . | . | . | . | . |
| 6 | . | . | . | . | . | 4 | 3 | 2 | . | 1 | . | . | . | . | . | . | . |
| 9 | . | . | . | . | . | . | 1 | 2 | . | 3 | . | . | . | . | . | . | . |
| 12 | . | . | . | . | . | 4 | 1 | 2 | . | 3 | . | . | . | . | . | . | . |
| 14 | . | . | . | . | . | 1 | 3 | . | 4 | 2 | . | . | . | . | . | . | . |
| 18 | . | . | . | . | . | 3 | 2 | 1 | 4 | . | . | . | . | . | . | . | . |
| 20 | . | . | . | . | . | . | 2 | 3 | . | 1 | . | . | . | . | . | . | . |
| 4 | . | . | . | . | . | . | . | . | . | . | 4 | 1 | 3 | 2 | . | . | . |
| 5 | . | . | . | . | 4 | . | . | . | . | . | 3 | . | 2 | 1 | . | . | . |
| 11 | . | . | . | . | 3 | . | . | . | . | . | 4 | 2 | . | 1 | . | . | . |
| 13 | . | . | . | . | . | . | . | . | . | . | 2 | 3 | 1 | 4 | . | . | . |
| 16 | . | . | . | . | 4 | . | . | . | . | . | 1 | 2 | 3 | . | . | . | . |
| 2 | . | . | . | . | . | . | . | . | 3 | . | . | . | . | . | 1 | 4 | 2 |
| 8 | . | . | . | . | . | . | . | . | 3 | . | . | . | . | . | 4 | 2 | 1 |
| 17 | . | . | . | . | . | . | . | . | . | . | . | . | . | . | 2 | 3 | 1 |

In order to test configuration $L_B$ in a random demand environment, the means of the *feasible production volumes* were used to evaluate the actual mean traffic. Note that these values are, in general, lower than the expected values of the independent demand due to capacity limitations. However, as already shown in Remark 2, the expected value of the inter-cell traffic under random product demand is equal to the traffic value obtained when the mean feasible production volumes are used. For this example, an expected traffic value of $E\{\mathbf{T}_B\}=632$ was obtained for $L_B$, which is 9.81% higher than $E\{\mathbf{T}_A\}_{min}=570$, obtained for $L_A$ for the same production volumes. Thus, considerable savings in the material handling cost are achieved by grouping the machines into cells based on the mean production volumes, rather than the (deterministic) expected demand values. The savings are expected to increase if the standard deviations, $\sigma_i$, i=1,2,3, of the independent demand for the three final assemblies increase.

Table 7 : Total Traffic Evaluation for 20 Elementary Periods

| | Independent Demands | | | Production Volumes | | | Traffic | Traffic | Savings |
|---|---|---|---|---|---|---|---|---|---|
| Period | 1 | 2 | 3 | 1 | 2 | 3 | $L_A$ | $L_B$ | $(L_B-L_A)*100/L_B$ |
| 1 | 172 | 81 | 116 | 172 | 81 | 111 | 677 | 648 | -4.48% |
| 2 | 214 | 92 | 141 | 177 | 92 | 111 | 687 | 786 | 12.60% |
| 3 | 135 | 74 | 93 | 135 | 74 | 93 | 549 | 592 | 7.26% |
| 4 | 178 | 108 | 166 | 177 | 103 | 111 | 687 | 824 | 16.63% |
| 5 | 143 | 82 | 125 | 143 | 82 | 111 | 619 | 656 | 5.64% |
| 6 | 216 | 94 | 33 | 177 | 94 | 33 | 543 | 752 | 27.79% |
| 7 | 247 | 77 | 104 | 177 | 77 | 104 | 666 | 616 | -8.12% |
| 8 | 138 | 85 | 178 | 138 | 85 | 111 | 609 | 680 | 10.44% |
| 9 | 71 | 80 | 113 | 71 | 80 | 111 | 475 | 640 | 25.78% |
| 10 | 179 | 68 | 61 | 177 | 68 | 61 | 537 | 544 | 1.29% |
| 11 | 292 | 72 | 88 | 177 | 72 | 88 | 618 | 576 | -7.29% |
| 12 | 212 | 101 | 149 | 177 | 101 | 111 | 687 | 808 | 14.98% |
| 13 | 58 | 86 | 107 | 58 | 86 | 107 | 437 | 688 | 36.48% |
| 14 | 171 | 52 | 42 | 171 | 52 | 42 | 468 | 416 | -12.50% |
| 15 | 279 | 88 | 95 | 177 | 88 | 95 | 639 | 704 | 9.23% |
| 16 | 207 | 66 | 187 | 177 | 66 | 111 | 687 | 528 | -30.11% |
| 17 | 103 | 79 | 132 | 103 | 79 | 111 | 539 | 632 | 14.72% |
| 18 | 136 | 59 | 54 | 136 | 59 | 54 | 434 | 472 | 8.05% |
| 19 | 211 | 83 | 127 | 177 | 83 | 111 | 687 | 664 | -3.46% |
| 20 | 139 | 75 | 81 | 139 | 75 | 81 | 421 | 600 | 29.83% |
| | | | | Total Traffic | | | 11576 | 12826 | |

To demonstrate the appropriateness of the machine to cell partition obtained by the algorithm presented in this study, the traffic values corresponding to configurations $L_A$ and $L_B$ have been obtained for twenty elementary periods, which constitute the design horizon $H$. The results are shown in Table 7. The product demands for each of these periods have been generated from a random number generator using the mean and standard deviation values given in Table 4. Table 7 shows that the savings in material handling cost obtained using $L_A$ over $L_B$ are 9.75% over the entire design horizon $H$ . This is due to the activation of the resource capacity constraints that lower the mean production volumes, with respect to the mean demand values.

## 6 CONCLUSIONS

The manufacturing cell formation problem under random product demand has been set up in an optimization framework. The objective is to minimize the expected material handling cost, while constraints are imposed by resource capacities and cell size limits. It has been shown that the conventional deterministic formulation is applicable in the case of unlimited backlogging. The proposed design methodology addresses the case in which no backlogging or holding is allowed beyond a certain time period. Given the statistics of the independent demand the means of the production volumes, which respect the capacity constraints and maximize the overall profit, are determined by solving a set of linear programming problems. These mean values are utilized by the grouping algorithm to obtain a near optimal shop configuration.

Under accurate predictions of the independent demand statistics, the proposed algorithm will result in a shop design that offers substantial savings in the expected material handling cost as compared to the shop configuration obtained by using the mean values of the independent demand. This has been illustrated by a comprehensive example.

An important topic for further study is to extend the design methodology presented above to account for the presence of alternative production routings. This case is common in manufacturing shops that include functionally similar machines. The deterministic case of this problem has been addressed by (Nagi *et al.*, 1990) who distribute the part demand among the alternative production routings in a manner that inter-cell traffic is minimized. It is, however, a challenging problem to assign part production volumes to alternative production routings when random demand is considered. The main reason is that the assignment procedure should be performed for every product mix and system configuration. Furthermore, Remark 2 does not hold in this case, since the constants $A_i^c$ [see Eqs.(6) and (7) in sections 2 and 3, respectively] are not only dependent on the shop partition, but also depend on the assignment procedure (Nagi *et al.*, 1990).

## APPENDIX A

In this study it is assumed that the feasible production volume vector, $(\delta_{1t},\ldots,\delta_{nt})$, is selected such that the overall profit is maximized. The optimal production volume is determined by solving the following linear programming problem :

$$\text{maximize} \quad \beta_1\delta_{1t} + \ldots + \beta_n\delta_{nt} \tag{A1}$$

subject to

$$\sum_{i=1}^{n} \delta_{it}\,\Theta_{ij} \;\le\; CW_j,\; j=1,\ldots,g \tag{A2}$$

$$0\le \delta_{it} \le d_{it}, \qquad i=1,..,n \tag{A3}$$

where $\beta_i$, i=1,...,n, is the unit profit of end product $p_{i1}$ and $\beta_1\delta_{1t} + \ldots + \beta_n\delta_{nt}$ is the total profit in period t. The remaining symbols are explained in sections 2 and 3.

Equation (A1) represents the linear objective function. Constraint set (A2) prevents overloading of resources beyond their capacity limits. Constraint (A3) implies that holding is not allowed. The linear programing problem of Eqs. (A1), (A2) and (A3) forms the basis for the calculation of the mean feasible production volumes.

It is noted that in the case of only two final assemblies with independent random demand it is not required to solve the above problem for every infeasible demand mix. This is due to a special partition of the infeasible space that is possible with respect to the basic feasible solutions of the linear programming problem (LPP) of Eqs.(A1) and (A2).

For the example of Figure A-1, let point $A(\delta_{1A},\delta_{2A})$ represent the optimal solution of the LPP of Eqs.(A1) and (A2). Point A is the intersection of constraints $\delta_1\Theta_{11}+\delta_2\Theta_{21}=CW_1$ and $\delta_1\Theta_{12}+\delta_2\Theta_{22}=CW_2$. It can be easily shown that A is also the solution of the LPP of Eqs.(A1), (A2) and (A3) for every point $P_2$, the coordinates of which satisfy $\delta_1\geq\delta_{1A}$ and $\delta_2\geq\delta_{2A}$.

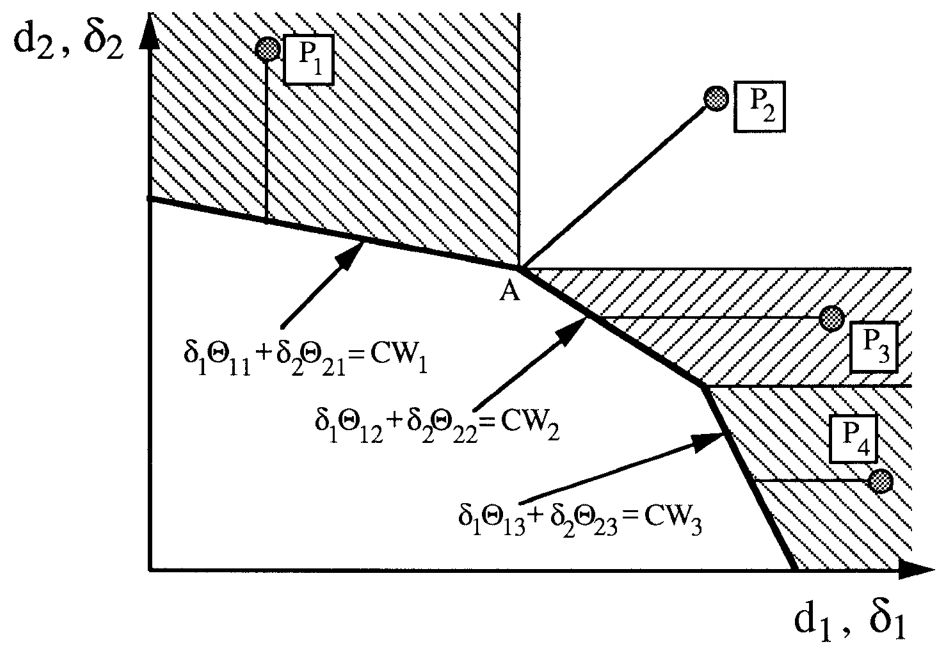


Figure A-1 : Solution of The Managerial LPP for the Case of Two End Items with Random Demand

Let $P_1$ be a point in the region defined by the inequalities $\delta_1\Theta_{11}+\delta_2\Theta_{21}>CW_1$ and $\delta_1\leq\delta_{1A}$ [see Fig.A-1]. For $P_1$ the optimal solution of LPP of Eqs.(A1), (A2) and (A3) is given by $\{(\delta_1,\delta_2)$: $\delta_1=d_{P_1}$, $\delta_2=(CW_1-d_{P_1}\Theta_{11})/\Theta_{21}\}$.

In a similar manner we can derive analytical expressions for the solution of the LPP of Eqs.(A1), (A2) and (A3) for every point $P_3$ and $P_4$ in the remaining shaded regions of Fig.A-1 (see Fig.A-1).

The fact that the LPP can be solved analytically results in a substantial reduction of the computational effort required to determine the expected values of the feasible production volumes $E\{\delta_{it}\}$, i=1,...,n, t=1,...,f [see section 3]. It is noted, however, that the infeasible space cannot be partitioned in a straight forward manner for those cases that i≥3.

## APPENDIX B

The bottom-up aggregation procedure proposed in (Harhalakis *et al.*, 1990) to minimize the inter-cell traffic in cellular manufacturing systems is briefly outlined in this appendix.

At the beginning of the procedure, each machine is placed in a separate cell. At each subsequent step of the minimization procedure, the normalized traffic (that is the value of the total inter-cell traffic divided by the total number of machines in both cells) is calculated for each feasible aggregation. Feasible aggregations are those that result in cells for which the number of machines does not exceed the cell-size limit Q. The two cells, between which the normalized traffic is maximum, are aggregated into a single cell. Every aggregation is accompanied by a reduction in the number of cells (w) by one, and a reduction of the total inter-cell traffic by the traffic corresponding to the two cells being merged. Subsequently, the traffic between the remaining cells is revised by the following rules : i) the part traffic between two unaffected cells remains the same; ii) the part traffic between an unaffected cell and the new aggregate is the summation of the traffic between the former and the components of the aggregate.

This procedure is repeated until it is either not possible to obtain a new feasible aggregation, or the traffic between each of the existing cells is zero (perfect decomposition).

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