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Mapping Nonsquare and Unevenly Spaced 2-D SLDV Data of an Aircraft Fuselage by Using Spatial DFT-IDFT Techniques

The scanning laser Doppler vibrometry (SLDV) technique provides velocities of a structure at 2-dimensional (2-D) angularly evenly spaced (in the laser scanning sense) data points. This causes an unevenly spaced data point distribution on the surface of the test structure. In many cases evenly spaced data point distribution with square or rectangular grids is highly desirable. In this study the SLDV velocity data of a partial surface area of an aircraft fuselage were mapped to truly spatial evenly spaced coordinates by using the spatial DFT-IDFT technique with minimum distortion. This 2-D data mapping technique certainly is not limited to the fuselage, but can be very useful for many other 3-D structures. *© 1996 John Wiley & Sons, Inc.*

INTRODUCTION

Other than convenience, noncontact, and high accuracy, one of the main features of the Scanning Laser Doppler Vibrometry (SLDV) technique is that real and imaginary velocity data of tens of thousands of points on a vibrating surface can be gathered within a relatively short period of time (Oliver, 1991; Sriram, Craig, and Hanagud, 1990). The high spatial resolution provides great advantages that have contributed to the development of a number of new techniques in the areas of structural angular velocity extraction (Kochersberger, Mitchell, and Wicks, 1991; Sun and Mitchell, 1991), structural system identification (Li, Mitchell, and Lu, 1994), and spatial modal parameter estimation (Arruda, Sun, and Mitchell, 1992).

For typical SLDV data acquisition, the laser beam scans the surface of the structure with a constant scanning angle increment in the X and Y directions. Therefore, the grid of data points is equally spaced only in the sense of the scanning angle. For simple, small, and flat structures, such as beams and plates, the variations of the spaces between data points on the measured surface caused by the constant scanning angle increment are usually small due to the smaller scanning angle. If the operating shapes of the structure are of interest only in a qualitative sense, then these small variations are usually ignored. However, for large structures or structures with curved sur-

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faces, such as an aircraft fuselage, the spacing between the data points along the edges of the scanned area could be several times greater than the spacing between the data points near the center of the scanned area for the same scanning angle increment. This is due to the combined effects of the larger scanning angle and the curvature of the scanned surface. Figure 1 shows a scanned grid of data points from a section of a commercial aircraft fuselage with the units of scanning angle index in both the X (longitudinal) and Y (circumferential) directions. When this evenly spaced (in the sense of scanning angle) data point grid is converted to a grid along the surface of the fuselage (Fig. 2), the scanned area becomes pincushion shaped, the grid is no longer rectangular, and the space between data points is quite variable.

There are several applications for which the data format in Fig. 2 is unacceptable. First, because the fuselage has cylindrical shape plus large size, several scans are necessary to cover the entire fuselage surface. The curved edges of each scan make it difficult to patch these scans together. Second, in the aircraft industry, noise reduction is the primary task for structural system identification. The noise intensity is directly related to the structural wavenumber (spatial frequency) of the fuselage surface. To extract the structural wavenumber, the spaces between data points must be constant when a discrete Fourier transform (DFT) or a fast Fourier transform (FFT) is used. The third case is that, in general, the SLDV technique only measures the velocity along the line of sight of the laser to the specific data point. To obtain the true out-of-plane normal velocity or to obtain the full 3-dimensional (3-D) velocity information at a point, several measurements of that point from different viewing angles are needed. It is very difficult to scan the same points on the structure from a different viewing angle. Therefore, mapping the original data to a rectangular grid with constant space between the data points is an essential step for data processing. The spatial functionalization of each scan allows the determination of the velocity at any point in the space. In this way, three or more viewpoints of the same data position can be obtained.

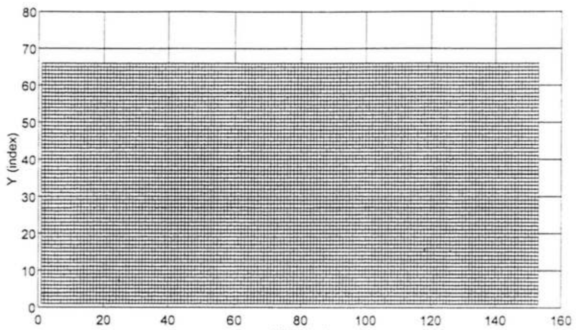

FIGURE 1 The original data point grid in the sense of scanning angle (66 × 153).

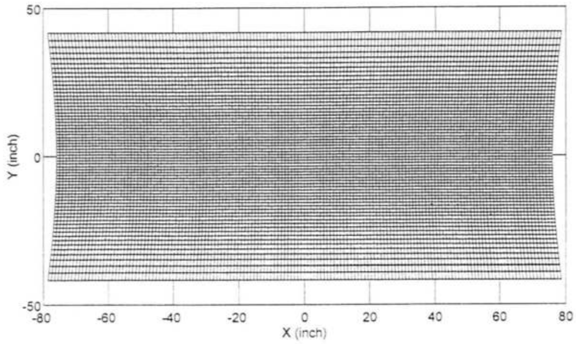

FIGURE 2 The actual data point grid appeared on the fuselage surface (66 × 153).

DFT-IDFT TECHNIQUE

The DFT-IDFT (DFT-inverse discrete Fourier transform) is a well-known technique. It has various applications in modal testing, such as data smoothing (Sun and Mitchell, 1991; Arruda, 1992), angular velocity extraction (Kochersberger et al., 1991; Sun and Mitchell, 1991), and discrete data functionalization (Neumann, 1993). There are two general requirements for the DFT. The first one is that the data points be equally spaced; the second is that the data must be periodic in the data space. Otherwise, leakage would occur. The first requirement can be easily met because the original SLDV data are evenly spaced in the sense of the scanning angle with today's commercial SLDV. The second one needs special treatment. A discrete data periodization technique, originally developed by L.D. Mitchell et al. (1991), is one solution. In this periodization scheme, the original nonperiodic data array (say N points) is doubled in its length with its own mirror and reversed image, and then sheared vertically so that it becomes periodic in a length of $(2N - 1)$ with a total $2(N - 1)$ number

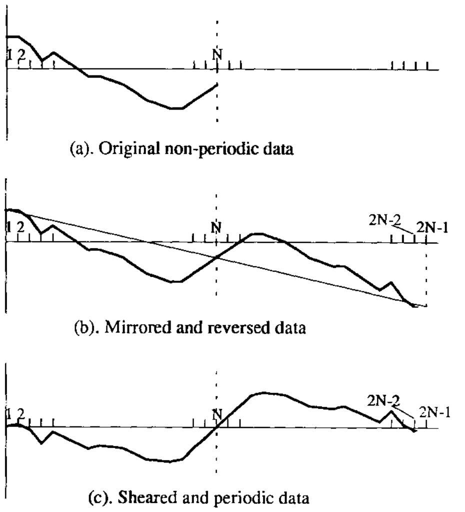

FIGURE 3 Periodization of nonperiodic data.

of data points. This procedure can be seen in Fig. 3.

For the fuselage problem, a two-step DFT-IDFT is needed. Suppose the original SLDV data matrix, such as the one in Fig. 1, has M rows and N columns; the first step is to perform rowwise (or columnwise) DFT-IDFT, and the second step is to perform columnwise (or rowwise) DFT-IDFT. Note that the acquired SLDV data are usually in the complex form of mobility that is defined as the velocity output over the force input. The rowwise DFT, from the extended data, will be

$$X_t(k) = \sum_{q=1}^{2(N-1)} x_{tq}\exp\left[\frac{-2\pi i}{2(N-1)}(q-1)(k-1)\right], \tag{1}$$

$$\text{for } t = 1 \cdots M,\ k = 1 \cdots N$$

where x_{tq} is the original SLDV data after periodization, X_t is the coefficient of Fourier series, t is the row index, and k is the index of the Fourier series terms. In this equation the implied scanning angle increment is unity.

The IDFT is used to generate new data points that are equally spaced on the measured surface. The coordinates of the new data points must be transferred into scanning angles. Figure 4 depicts this transformation. For the simplicity of computation, the home position (zero scanning angle) of the laser beam is set to be normal, to the fuselage surface with the distance, d, from the SLDV sensor to the point of incident, which should not be difficult in practice.

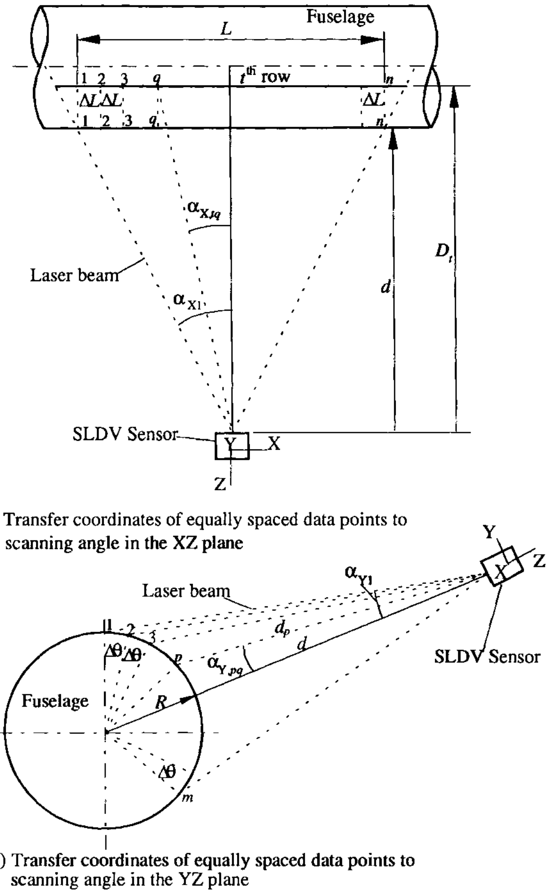

FIGURE 4 Transfer coordinates of equally spaced data points to scanning angles.

The X coordinate (in the sense of scanning angle), $\alpha_{X,tq}$, of the new data point (t, q) that has an equal interval of ΔL as shown in Fig. 4(a) is

$$\alpha_{X,tq} = \alpha_{X1} - \tan^{-1}\left\{\frac{d}{D_t}\tan(\alpha_{X1})\left[1 - \frac{2(q-1)}{n-1}\right]\right\}, \tag{2}$$

$$\text{for } t = 1 \cdots M,\ q = 1 \cdots n$$

where n is the new number of data points in the X direction (new number of columns), α_{X1} is the first X scanning angle (which can be easily measured or determined during data acquisition), and D_t is the distance from the SLDV sensor to the tth row of data points on the fuselage, which can be computed as

$$D_t = (R + d)\cos(\alpha_{Yt}) - \sqrt{[(R+d)\cos(\alpha_{Yt})]^2 - d(2R+d)}, \quad (3)$$

$$\text{for } t = 1 \cdots M$$

where R is the radius of the fuselage and α_{Yt} is the tth scanning angle in the Y direction. Once the coordinates in the sense of scanning angle are determined, the IDFT is just a reversed process of the DFT. To generate new data points that are evenly spaced, the rowwise IDFT will be

$$x_{tq} = \frac{1}{N-1}\sum_{k=1}^{N} X_t(k)\exp\left[\frac{\pi i}{2\alpha_{X1}}(k-1)\alpha_{X,tq}\right], \quad (4)$$

$$\text{for } t = 1 \cdots M,\ q = 1 \cdots n.$$

After the rowwise DFT-IDFT, the original SLDV data now are spatially evenly spaced in the X direction. Similarly, the columnwise DFT is:

$$X_q(j) = \sum_{t=1}^{2(M-1)} x_{tq}\exp\left[\frac{-2\pi i}{2(M-1)}(t-1)(j-1)\right], \quad (5)$$

$$\text{for } j = 1 \cdots M,\ q = 1 \cdots n.$$

The Y coordinate (in the sense of scanning angle), $\alpha_{Y,pq}$, of the new data point (p, q), which has an equal interval of $R\Delta\theta$, as shown in Fig. 4(b) is

$$\alpha_{Y,pq} = \alpha_{Y1} - \sin^{-1}\left(\frac{R}{d_p}\sin\left\{\left[1 - \frac{2(p-1)}{m-1}\right] \times \sin^{-1}\left[\frac{d_1}{R}\sin(\alpha_{Y1})\right]\right\}\right), \quad (6)$$

$$\text{for } p = 1 \cdots m,\ q = 1 \cdots n$$

where m is the new number of data points in the Y direction. Note that d_p is different from D_t and is determined as

$$d_p = \sqrt{d^2 + 2R(R+d)\left(1 - \cos\left\{\left[1 - \frac{2(p-1)}{m-1}\right]\sin^{-1}\left[\frac{d_1}{R}\sin(\alpha_{Y1})\right]\right\}\right)}, \quad \text{for } p = 1 \cdots m. \quad (7)$$

And the columnwise IDFT is

$$x_{pq} = \frac{1}{M-1}\sum_{j=1}^{M} X_q(j)\exp\left[\frac{\pi i}{2\alpha_{Y1}}(j-1)\alpha_{Y,pq}\right], \quad (8)$$

$$\text{for } p = 1 \cdots m,\ q = 1 \cdots n$$

where x_{pq} is the remapped mobility or velocity data of the points that are evenly spaced on the fuselage surface. A few points regarding Eqs. (4) and (8) need to be mentioned here. First, the total scanning angles in the X and Y directions are assumed to be $2\alpha_{X1}$ and $2\alpha_{Y1}$, respectively. This assumption is not necessary. However, the remapped points should lie inside the original scanned area because extrapolation of the Fourier series is not reliable. Second, to preserve maximum originality of the data (including the noise), all terms of the Fourier series (up to the Nyquist limit) are used to reconstruct the data. The Fourier series is guaranteed to exactly reproduce all the original data points under these conditions. The noise and the original signal will be reproduced. To filter out the noise, only the first few terms of X_t and X_q are needed to reconstruct the basic noise-free waveforms. Third, the analysis can be equally applied to a set of real data (such as velocities) or complex data (such as mobilities). Last, the two-step DFT-IDFT technique not only can be used to remap the original SLDV data to a rectangular and evenly spaced data point grid, but also can be used to arbitrarily change the original matrix size from $M \times N$ to $m \times n$, which could be more or less dense data points than the original. This certainly helps to compensate the limitation of the scanning resolution.

APPLICATION EXAMPLE

In this example, the mobility data of the cylindrical section of a Cessna Citation VI fuselage were acquired by using the Ometron VPI 9000 laser vibrometer system, which is based on the SLDV techniques (Oliver, 1991). Two sides of the fuselage were scanned from two opposite viewing points. The scanned area for each side is approximately 153 × 85 in. The two scans from two sides met along the top centerline of the fuselage. The distances from the SLDV to the scanned surface for the left side and right side of the fuselage were

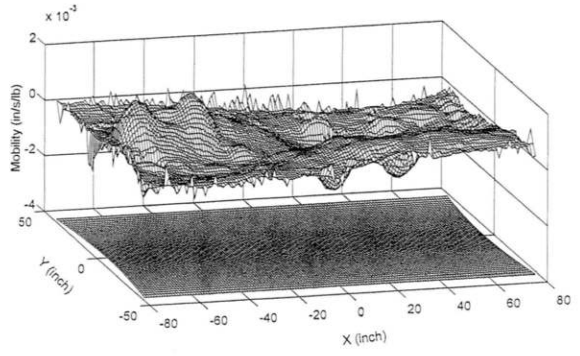

FIGURE 5 The original unevenly spaced SLDV data (66 × 153, real part) from the left side of the fuselage at 150 Hz.

610 and 590 in. respectively. This difference in distance was due to the limitation of the available space for the experiment setup and caused a different number of data points to be scanned for the two sides, which was 66 (rows) × 153 (columns) points for the left side and 68 (rows) × 158 (columns) points for the right side. To patch the scans from the left side and right side of the fuselage for structural wavenumber extraction, the number of columns of data points for both sides must be the same to utilize the DFT method.

Figure 5 shows the real part of the original mobility data (66 × 153 points) before mapping from their surfacewise unevenly spaced configuration. They were acquired from the left side of the fuselage with a 150-Hz excitation source at one of the engine mounts. These data were mapped to new evenly spaced coordinates (68 × 158 points) as shown in Fig. 6. The noise of the SLDV data were not filtered out. The point here is that the original mobility field was well preserved throughout this two-step DFT-IDFT data remapping process.

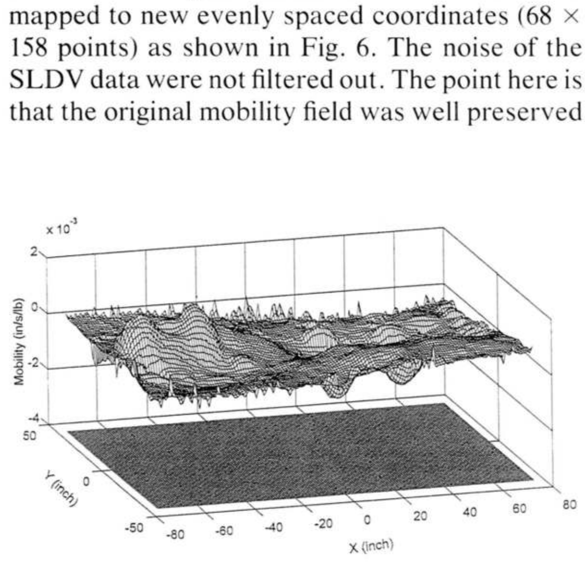

FIGURE 6 The remapped evenly spaced mobility data (68 × 158, real part) for the same data in Fig. 5.

This procedure was repeated on the right side of the fuselage. The data were remapped into a consistent 68 × 158 grid. This allows the left and right portions of the mobility field to be linked into one overall fuselage mobility data map around the circumference of the passenger compartment from floor line over the top of this fuselage to the floor line on the other side.

CONCLUSIONS

The spatial DFT-IDFT technique in conjunction with the discrete data periodization method is an ideal solution for high spatial density SLDV data remapping. The applications are not limited to flat or cylindrical surfaces. It can be used on any irregular surface as long as the surface coordinates functionalization with respect to the viewpoint can be determined. This done, these methods make it very easy to patch together remapped data to describe the entire velocity or mobility field of the structure, to extract a structural wavenumber, and/or to determine 3-D velocity components. Noise filtering could be a by-product of this data remapping procedure through the use of a limited number of Fourier terms in the IDFT process.

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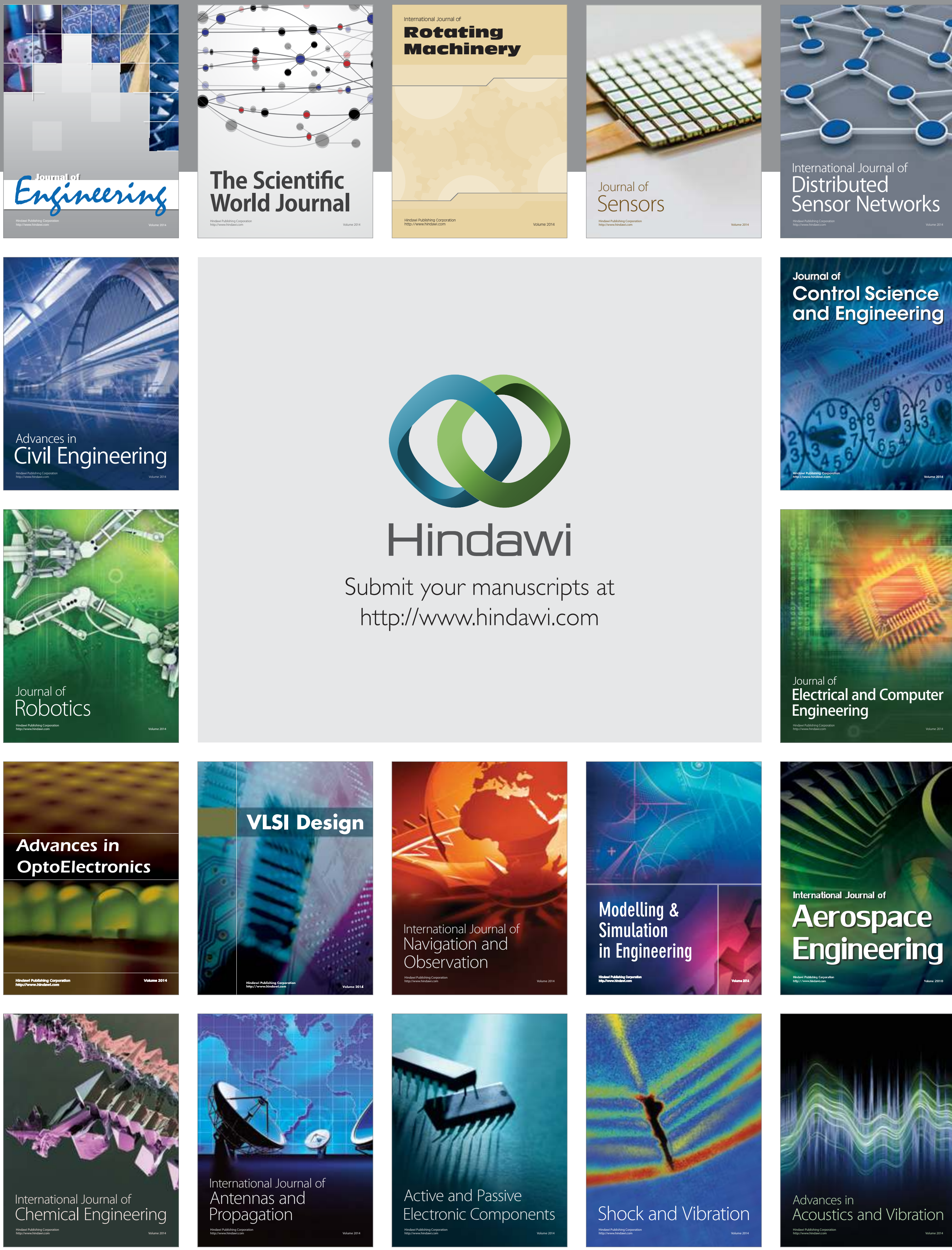
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