Sustainable Lifestyles Research Group

Mapping rebound effects from sustainable behaviours

Key Concepts and Literature Review

by

Steve Sorrell

SLRG Working Paper 01-10

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The Sustainable Lifestyles Research Group (SLRG) is a multi-centre research network collaborating to further understanding of the opportunities and constraints on sustainable living in the UK. The SLRG is coordinated from the University of Surrey, and brings together project teams from Surrey's Centre for Environmental Strategy (CES) and Departments of Sociology and Psychology. It also includes teams at other internationally acclaimed research centres: the University of Sussex's Science Policy Research Unit; the Institute for Fiscal Studies; the University of Edinburgh; the University of Bath; and Brunel University.

SLRG is directed by Professor Tim Jackson, Professor of Sustainable Development at the University of Surrey, with Professor Andy Stirling of the University of Sussex as deputy director. The network is coordinated by Ian Christie, research fellow at CES.

Funded by the Department for Environment, Food and Rural Affairs (Defra), the Scottish Government and the Economic and Social Research Council (ESRC), SLRG builds on the pathbreaking research of the University of Surrey's previous Research, ESRC-funded programme RESOLVE – Research on values, lifestyles and environment, also directed by Professor Tim Jackson. SLRG works in partnership with its sister programme the Sustainable Practices Research Group (SPRG), funded by the ESRC and coordinated by Professor Dale Southerton at the University of Manchester.

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Contents

Figures

TABLES

1 Introduction

This document sets out some of the theoretical concepts underlying the project *Mapping rebound effects from sustainable behaviours* and reviews some of the previous studies in this area. The aim is to provide a starting point for the empirical research.

Both technical and behavioural changes to reduce environmental impacts can have unintended consequences. For example, more fuel-efficient vehicles make driving cheaper and hence can encourage greater car use. Any savings in fuel costs may be spent on other goods and services, the provision of which will also have environmental consequences. This generic problem of 'rebound effects' is widely neglected by policy makers, in part because such effects are difficult to quantify [1-4]. But these effects invariably offset and in some cases can entirely eliminate the environmental benefits from both technical and behavioural change.

The aim of this project is to estimate the rebound effects associated with various technical and behavioural changes by UK households. Rebound effects will be estimated and compared for energy consumption, carbon emissions and greenhouse gas (GHG) emissions. Rebound effects take a variety of forms [1] and this project will estimate both *direct* rebound effects (e.g. technical improvements make energy services cheaper, so the demand for those services may increase) and *indirect* rebound effects (e.g. consumers may use the cost savings from both technical improvements and behavioural changes to purchase other goods and services with associated environmental impacts). Key variables influencing the magnitude of direct rebound effects include the energy/carbon/GHG intensity and own-price elasticity of the relevant energy service, while key variables influencing the magnitude of indirect rebound effects include the energy/carbon/GHG intensity of other goods and services and the expenditure and cross-price elasticities associated with those goods and services. In this project, priority will be given to the estimation of *indirect* rebound effects, since the available evidence on these effects appears relatively weak.

The project will estimate rebound effects for UK households, distinguished by income levels, household type, socio-economic group and other variables. It will explore the impact of improvements in the energy efficiency of personal transport, heating, lighting and household appliances, together with the implications of more specific lifestyle choices, such as switching to public transport, changing diets and abstaining from flying. It will use a variety of techniques for estimating these effects, compare their results and highlight the implications for both voluntary actions to reduce environmental impacts and for energy and climate policy.

Data on the energy, carbon and greenhouse gas intensity of different categories of household goods and services, together with the consumption patterns of different socioeconomic groups will be derived from: a) the Surrey Environmental Mapping and Attribution (SELMA) framework, which attributes direct and indirect energy consumption to different categories of household goods and services and to different functional needs [5, 6]; and the Surrey Local Area Resource Analysis (LARA) model which estimates the direct and indirect energy consumption of households distinguished by type of dwelling, tenure, socio-demographic group and region [7]. Data on household consumption patterns will be derived from the UK Living Costs and Food Survey (LCFS) and other sources. The project will estimate and compare a range of econometric models of household consumption behaviour, using both cross-sectional and pooled cross-sectional data from the LCFS and with differing levels of commodity aggregation. It will use these to estimate expenditure and price elasticities for different categories of goods and services for different socioeconomic groups. These will be combined with the energy/carbon/GHG intensity estimates and used to estimate both direct and indirect rebound effects for different types of technical and behavioural change for different types of household.

The structure of the Note is as follows. Section 2 explains the nature and origin of rebound effects for households with the help of a simple graphical illustration of household consumption behaviour. Section 3 summarises some key concepts from the neoclassical theory of consumer demand systems which is the theory that underlies the majority of empirical work in this field. Section 4 summarises the research that has been conducted to date on indirect rebound effects for households and highlights both the strengths and weaknesses of these studies and the lessons that have been learned. Taken together, these reviews should provide a solid conceptual foundation for the empirical research.

2 Graphical illustration of rebound effects for households

For the purpose of this project, rebound effects may be understood as the unintended consequences of actions by households to reduce their energy consumption and/or greenhouse gas (GHG) emissions. The relevant actions may either be *technical*, such as purchasing a more fuel-efficient car, or *behavioural*, such as turning lights off in unoccupied rooms. Similarly, the relevant consequences may be measured in terms of energy consumption, carbon emissions or greenhouse gas emissions.

To date, most of the literature on rebound effects has focused upon on the impact of technical improvements in energy efficiency on energy consumption. Hence, one area where this project could make a contribution is through examining a broader range of behavioural changes by households and comparing the consequences for energy, carbon and GHGs. However, for the purpose of explaining rebound effects, the following three sections use the example of energy efficiency and energy consumption. Section 0 broadens the perspective to include rebound effects from behavioural change.

2.1 Rebound effects

The *rebound effect* is commonly used as umbrella term for a number of mechanisms which reduce the size of the 'energy savings' achieved from improvements in energy efficiency. On the micro level, the key question is whether improvements in the technical efficiency of energy use can be expected to reduce energy consumption by the amount predicted by simple engineering calculations. For example, will a 20% improvement in the fuel efficiency of passenger cars lead to a corresponding 20% reduction in motor-fuel consumption for personal automotive travel? Economic theory suggests that it will not. Since energy efficiency improvements reduce the marginal cost of energy services such as travel, the consumption of those services may be expected to increase. For example, since the cost per mile of driving is lower with a fuel-efficient car, consumers may choose to drive further and/or more often. This increased consumption of energy services may be expected to offset some of the predicted reduction in energy consumption.

This so-called *direct rebound effect* was first brought to the attention of energy economists by Daniel Khazzoom [8] and has since been the focus of much research [9]. But even if the direct rebound effect is zero for a particular energy service (e.g. even if consumers choose not to drive any further in their fuel efficient car), there are a number of other reasons why the economy-wide reduction in energy consumption may be less than simple calculations suggest. For example, the money saved on motor-fuel consumption may be spent on other goods and services that also require energy to provide. Depending upon the nature, size and location of the energy efficiency improvement, these so-called *indirect rebound effects* can take a number of forms that are briefly outlined in Box 1. Both direct and indirect rebound effects apply equally to energy efficiency improvements by consumers, such as the purchase of a more fuel efficient car, and energy efficiency improvements by producers, such as the use of energy efficient motors in machine tools. But in what follows, the focus is upon consumers.

Box 1 Indirect rebound effects

- *Embodied energy effects*: The equipment used to improve energy efficiency (e.g. thermal insulation) will itself require energy to manufacture and install and this 'embodied' energy consumption will offset some of the lifetime energy savings achieved.
- *Re-spending effects*: Consumers may use the cost savings from energy efficiency improvements to purchase other goods and services which themselves require energy to provide. For example, the cost savings from a more energy-efficient central heating system may be put towards an overseas holiday.
- *Output effects*: Producers may use the cost savings from energy efficiency improvements to increase output, thereby increasing consumption of energy inputs as well as capital, labour and material inputs which also require energy to provide. If the energy efficiency improvements are sector wide, they may lead to lower product prices, increased consumption of the relevant products and further increases in energy consumption. All such improvements will increase the overall productivity of the economy, thereby encouraging economic growth, increased consumption of goods and services and increased energy consumption.
- *Composition effects*: Both the energy efficiency improvements and the associated reductions in energy prices will reduce the cost of energy-intensive goods and services to a greater extent than non-energy-intensive goods and services, thereby encouraging consumer demand to shift towards the former.
- *Energy market effects:* Large-scale reductions in energy demand may translate into lower energy prices which will encourage energy consumption to increase. The reduction in energy prices will also increase real income, thereby encouraging investment and generating an extra stimulus to aggregate output and energy use.

As shown in Box 2, the *overall* or *economy-wide* rebound effect from an energy efficiency improvement represents the sum of these direct and indirect effects. It is normally expressed as a percentage of the *expected* energy savings from an energy efficiency improvement - where the latter is typically derived from simple engineering calculations. Hence, a rebound effect of 100% means that the expected energy savings are entirely offset, leading to zero net savings.

Rebound effects need to be defined in relation to particular *time frame* (e.g. short, medium or long term) and *system boundary* (e.g. household, firm, sector, national economy). The economy-wide rebound effect is normally defined in relation to a national economy, but there may also be effects in other countries through changes in trade patterns and international energy prices. Rebound effects may also be expected to increase in importance over time as markets, technology and behaviour adjusts. From a climate change perspective, it is the long-term effect on global energy consumption that is most relevant, but this is also the effect that is hardest to estimate.

The economy-wide rebound effect represents the net effect of a number of different mechanisms that are individually complex, mutually interdependent and likely to vary in importance both over time and from one type of energy efficiency improvement to another. Estimating the magnitude of the economy-wide rebound effect in any particular instance is therefore very difficult. Nevertheless, the general implication is that the energy 'saved' (and emissions avoided) from energy efficiency improvements will be less than is conventionally assumed.

Box 2 Classifying rebound effects

The economy-wide rebound effect represents the sum of the direct and indirect effects. For energy efficiency improvements by consumers, it is helpful to decompose the direct rebound effect into:

a) a *substitution effect*, whereby consumption of the (cheaper) energy service substitutes for the consumption of other goods and services while maintaining a constant level of 'utility', or consumer satisfaction; and

b) an *income effect*, whereby the increase in real income achieved by the energy efficiency improvement allows a higher level of utility to be achieved by increasing consumption of all goods and services, including the energy service.

Similarly, the direct rebound effect for producers may be decomposed into:

a) a *substitution effect*, whereby the cheaper energy service substitutes for the use of capital, labour and materials in producing a constant level of output; and

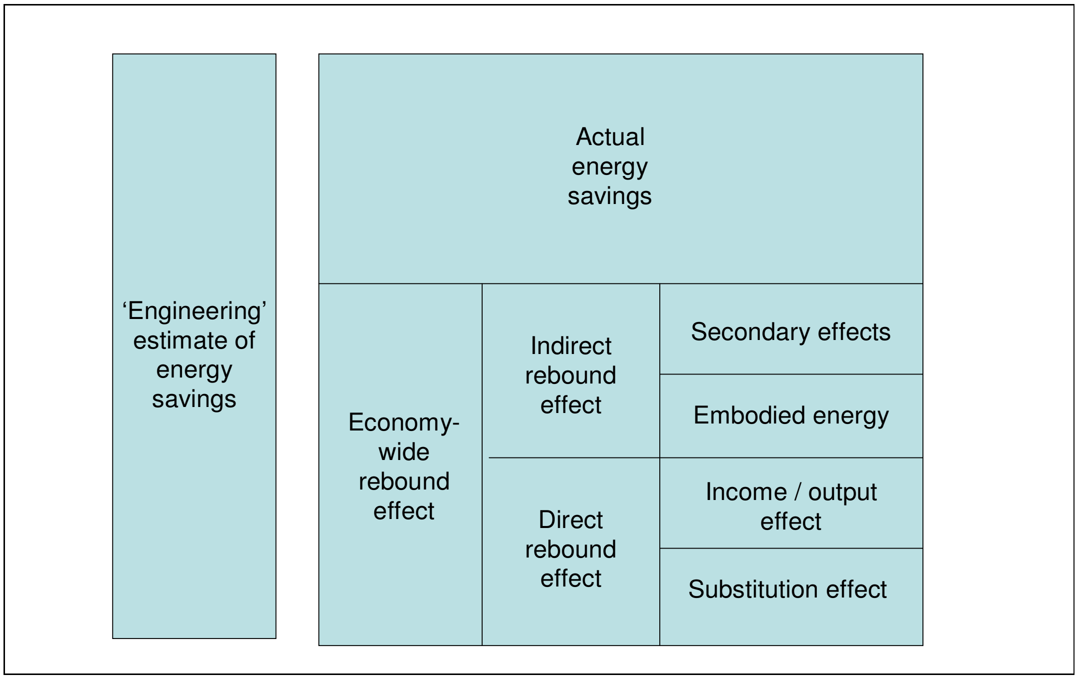
b) an *output effect*, whereby the cost savings from the energy efficiency improvement allows a higher level of output to be produced - thereby increasing consumption of all inputs, including the energy service.

It is also helpful to decompose the indirect rebound effect into:

a) the *embodied energy*, or indirect energy consumption required to <u>achieve</u> the energy efficiency improvement, such as the energy required to produce and install thermal insulation; and

b) the *secondary effects* that result as a <u>consequence</u> of the energy efficiency improvement, which include the mechanisms listed in Box 1.

A diagrammatic representation of this classification scheme is provided below. The relative size of each effect may vary widely from one circumstance to another and in some cases individual components of the rebound effect may be negative. For example, if an energy service is an 'inferior good', the income effect for consumers may lead to reduced consumption of that service, rather than increased consumption. It is theoretically possible for the economy-wide rebound effect to be negative ('super conservation'), although this appears unlikely in practice.



2.2 The direct rebound effect for consumers

2.2.1 Graphical illustration of the direct rebound effect

The direct rebound effect for consumers may be illustrated in a simple neoclassical framework, where consumers are assumed to be fully informed and perfectly rational and therefore act to maximise their utility. Utility is assumed to be derived from the consumption of a set of goods/services/commodities[1], a subset of which are may be defined as *energy services* (*ES*) such as thermal comfort, refrigeration and motive power. Energy services are delivered through a combination of energy commodities (*E*) and the associated energy systems, including energy conversion devices. In what follows, consumers are assumed to derive utility from consuming energy services (*ES*) such as heating, rather than from consuming energy commodities (*E*) such as gas. However, it should be noted that data limitations generally make it very difficult to capture and represent these energy services within an empirical model - for example, it is much easier to obtain data on the cost of gas than on the cost of heated space. Hence, many empirical applications focus solely upon the consumption of commodities, including energy commodities.

Following Sorrell and Dimitropoulos [10], we define an essential feature of an energy service as the *useful work* (*S*) obtained, which may be measured by a variety of thermodynamic or physical indicators [11, 12]. For example, the useful work from passenger vehicles may be measured in vehicle kilometres or passenger kilometres. But energy services also have broader *attributes* (*A*) that may be combined with useful work in a variety of ways. For example, all cars deliver passenger kilometres, but they may vary widely in terms of features such as speed, comfort, acceleration and prestige. The combination of useful work (*S*) with these associated attributes (*A*) may be considered to provide the full energy service: $ES = es(S, A)$.

[1] These terms will be used interchangeably in what follows.

The energy efficiency (π) of the relevant energy system is given by the ratio of useful work output to energy input: $\pi = S / E$. The *energy cost* of useful work (P_S) is then given by $P_S = P_E / \pi$, where P_E represents the unit price of energy. This is one component of the *generalised cost* of useful work (P_G), which also includes other costs, such as annualised capital costs, maintenance costs and time costs. Improvements in energy efficiency reduce the energy cost of useful work, but may also affect other costs. In what follows, these other costs are assumed to be unchanged following an energy efficiency improvement, together with the attributes (*A*) of the energy service - although in practice this is unlikely.

An improvement in the energy efficiency of the relevant system leads to a reduction in the energy cost of useful work (P_S) and hence the effective price of useful work. As a result, the consumption of useful work may be expected to increase. The consumer's response to this price reduction may be illustrated graphically, using *indifference curves*, which represent different combinations of goods/services to which a consumer is assumed to be *indifferent.* At each point on an indifference curve, a consumer has no preference for one combination of goods over another, so that each point provides the same level of *utility,* or satisfaction. The analysis rests upon a number of standard simplifying assumptions regarding indifference curves and consumer behaviour which are discussed in more detail in Section 3.3.

In Figure 1, the curves U_1 and U_2 represent indifference curves between the consumption of useful work for a particular energy service (*S*) and the consumption of another commodity (*Z*). As an illustration, the useful work may be passenger kilometres in a private automobile, while the other commodity may be restaurant meals. For illustrative purposes, the consumer is initially assumed to spend *all* of her income (*Y*) on just two commodities, *S* and *Z,* and the non-energy costs of the energy service are assumed to be zero ($P_G = P_S$).

The line S_0-Z_0 represents the consumer's budget constraint (*Y*). If P_S represents the energy cost of a unit of useful work and P_Z represents the unit price of the other commodity, the budget constraint may be written as $Y \geq P_S S_o + P_Z Z_o$. The slope of the budget curve is therefore equal to (P_Z/P_S).

At one extreme, the consumer could choose to consume S_0 useful work and none of the other commodity, while at the other extreme she could consume Z_0 of the other commodity and no useful work. The optimum consumption mix is given by (S_1, Z_1), where the budget constraint is tangential to the indifference curve U_1. At this point, utility is maximised and the *marginal rate of substitution (MRS)* of useful work for commodity *Z* is equal to the ratio of the price of useful work to the price of commodity *Z* (P_S/P_Z).

$$MRS = -\left.\frac{\partial Z}{\partial S}\right|_{U=constant} = \frac{P_S}{P_Z} \qquad (1)$$

The MRS measures the slope of the indifference curve, or the rate at which one service can be substituted for another while holding utility constant. Taking the total differential of the utility function we have:

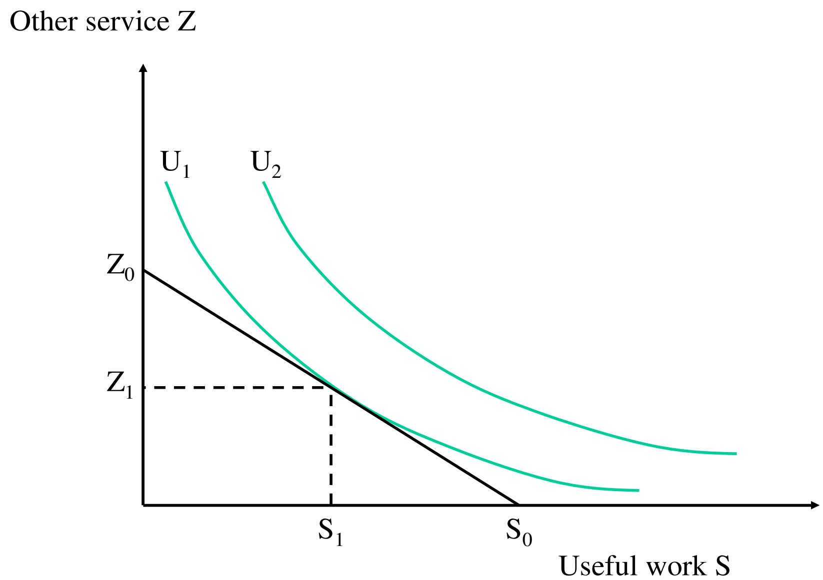
$$dU = \frac{\partial U}{\partial Z} dZ + \frac{\partial U}{\partial S} dS \qquad (2)$$

Along any particular indifference curve, *dU*=0. Hence, simple manipulation yields:

$$MRS = -\frac{\partial Z}{\partial S}\bigg|_{U=constant} = \frac{\partial U/\partial S}{\partial U/\partial Z} \tag{3}$$

Hence, marginal rate of substitution of *S* for Z is equal to the ratio of the *marginal utility* of *S* ($\partial U/\partial S$) to the marginal utility of *Z* ($\partial U/\partial Z$).

Figure 1 Trade-off between the consumption of useful work S and the consumption of another good or service Z

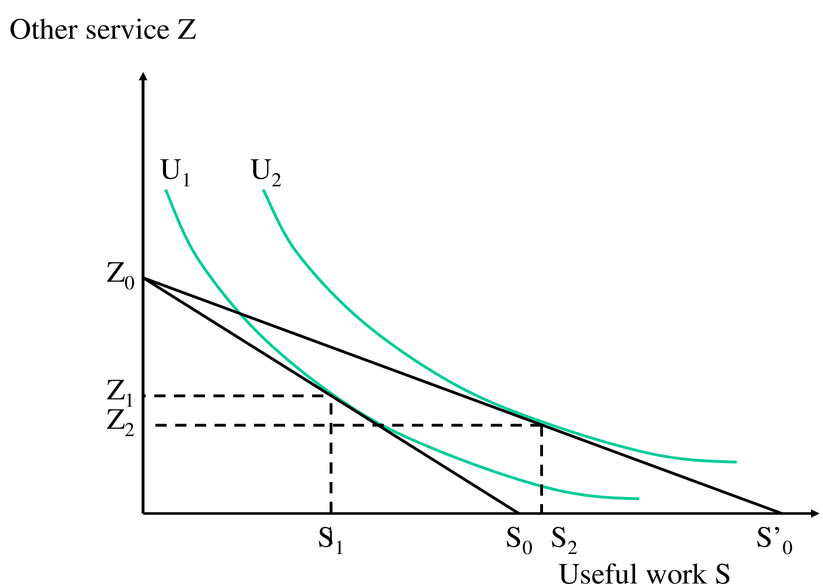


Let *E(s)* represent the energy consumption associated with consuming a quantity *s* of useful work (*E(y)>E(x) for y>x*). Then the initial level of energy consumption is given by *E*(S_1). Now suppose that there is an exogenous improvement in the energy efficiency of delivering this energy service. For example, suppose there is an improvement in the fuel efficiency of the vehicle. For simplicity, we ignore the costs associated with this technical improvement and assume that the other attributes of the energy service remain unchanged. Let the new energy consumption associated with consuming an amount of useful work *S* be given by *E*(S)* (where *E*(S) < E(S)*). An 'engineering' calculation of the percentage energy savings associated with this fuel efficiency improvement (*ENG*) would then be:

$$ENG = \frac{E(S_1) - E^*(S_1)}{E(S_1)} * 100\% \tag{4}$$

However, this overestimates the actual energy savings because it assumes that the consumption of useful work (*S*) is unchanged following the energy efficiency improvement. If the nominal prices of energy commodities are unchanged, the energy efficiency improvement will reduce the effective price of useful work (since $\pi' > \pi$ and $P_S = P_E/\pi$, then $P'_S < P_S$) and should therefore increase both the consumption of useful work and overall utility. As shown in Figure 2, if the consumer were to spend her entire budget on useful work, she would be able to consume a larger quantity S'_0. This may be represented by a shift of the budget line from Z_o-S_o to Z_o-S'_o. In conventional terminology, the consumers' *real income* has increased even though her *nominal income* is unchanged. The optimum consumption mix is now given by (S_2, Z_2) where the new budget constraint is tangential to the indifference curve U_2 which represents the maximum amount of utility that can be obtained from the new level of real income. Hence, consumption of useful work has increased ($S_2>S_1$), consumption of the other commodity has reduced ($Z_2<Z_1$) and the consumer obtains a higher level of utility ($U_2>U_1$).

Figure 2 Change in consumption following an energy efficiency improvement in a simple two-good model



The actual percentage saving in energy consumption (*ACT*) is then given by:

$$ACT = \frac{E(S_1) - E^*(S_2)}{E(S_1)} * 100\% \tag{5}$$

Since $E^*(S_2) > E^*(S_1)$, then $ACT \leq ENG$.

While energy consumption per unit of useful work has *reduced* $(\frac{E^*(S)}{E(S)} < 1)$, the consumption of useful work has *increased* $(S_2 > S_1)$. These two effects offset one another, with the result that the sign of ACT is ambiguous: the technical improvement in energy efficiency may either increase or decrease energy consumption for the energy service.

The direct rebound effect for the individual energy service (REB_d) may then be defined as:

$$REB_d = \frac{ENG - ACT}{ENG} * 100\% \quad (6)$$

If the actual savings in energy consumption equal the 'engineering' savings the direct rebound effect is zero, while if the actual savings are zero the direct rebound effect is 100%. If there is an increase in energy consumption $(ACT < 0)$, the direct rebound effect is >100% - a situation termed 'backfire'. Substituting, we have:

$$REB_d = \frac{\left(E(S_1) - E^*(S_1)\right) - \left(E(S_1) - E^*(S_2)\right)}{E(S_1) - E^*(S_1)} * 100\% \quad (7)$$

Or:

$$REB_d = \frac{E^*(S_2) - E^*(S_1)}{E(S_1) - E^*(S_1)} * 100\%$$

(8)

2.2.2 Decomposition of the direct rebound effect

A key measure relevant to the direct rebound effect is the percentage change in the consumption of useful work (*S*) that is 'caused' by a percentage change in the energy cost of useful work (P_S), holding nominal income, the price of other commodities and preferences constant. This 'own-price elasticity of demand' for useful work is defined as:

$$\varepsilon_{SS} = \frac{\partial S}{\partial P_S} \frac{P_S}{S} \quad (9)$$

A higher (lower) own-price elasticity leads to a greater (smaller) change in the quantity demanded in response to a change in price. Demand for useful work is said to be elastic when $|\varepsilon_{SS}| \geq 1$ and inelastic when $|\varepsilon_{SS}| \leq 1$. The own-price elasticity of useful work will be determined in part by the availability of substitutes. For example, the elasticity of demand for car travel may be expected to be higher if public transport alternatives are available. The elasticity will also depend upon the time frame under consideration and should be higher in the long-run since consumers have more time to adjust.

Following standard practice, the own-price elasticity of useful work may be decomposed into a *substitution* effect and an *income* effect [13]:

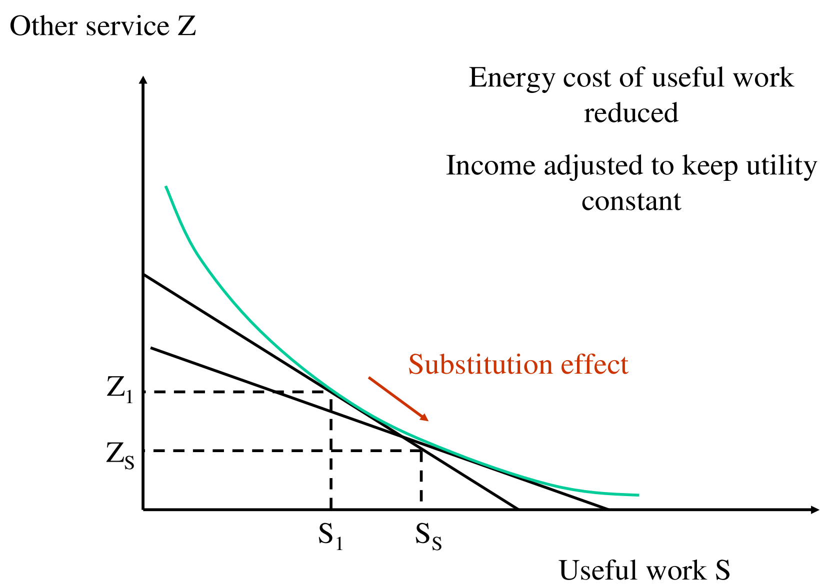
- *Substitution effect*: A decrease in the price of supplying useful work means that the rate at which the consumer can exchange consumption of useful work for consumption of other goods and services has increased. As a result, increased consumption of useful work will substitute for reduced consumption of other goods and services. The substitution effect is defined as the change in consumption that would result from the change in relative prices *if* nominal income were adjusted to keep utility constant. In effect, the change in consumption is artificially restricted to a movement along the original indifference curve.

- *Income effect:* Since useful work has become cheaper, the consumer's total purchasing power, or 'real income' has increased. This allows a shift from one indifference curve to another. The income effect may be defined as the change in consumption that would result exclusively from this change in real income, holding other prices and money income constant.

The relative size of these two effects may differ between the short and long-run. This decomposition is theoretical, in that only the sum of the two effects can be empirically observed, but is helpful in understanding the nature of the price response.

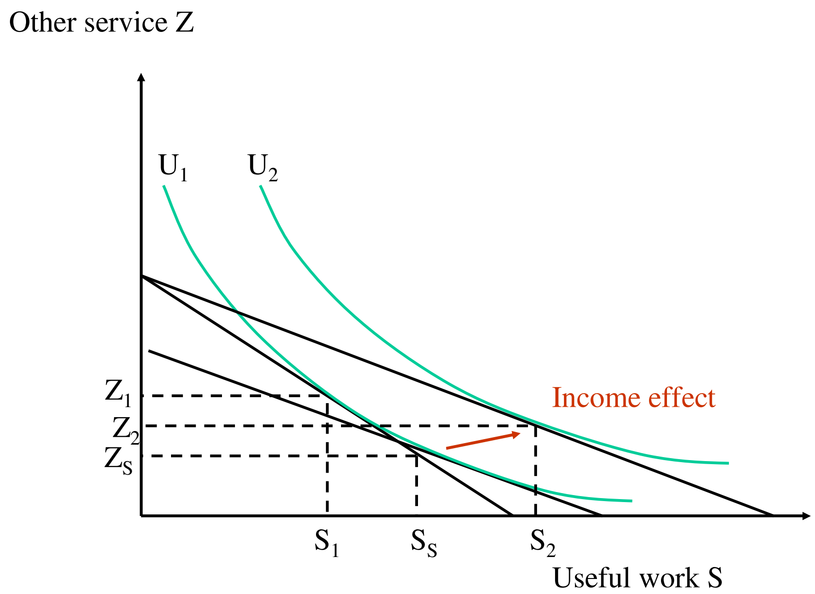
The substitution effect is illustrated in Figure 3. Here, the slope of the budget constraint has changed as result of the change in relative prices, but its location is artificially constrained to allow utility (U_1) to be unchanged. The consumption of useful work increases from S_1 to S_S, while the consumption of the other commodity decreases from Z_1 to Z_S.

Figure 3 Substitution effect following a decrease in the energy cost of useful work



The income effect is illustrated in Figure 4. Here, the budget constraint is shifted rightwards to reflect the increase in real income. The consumption of useful work increases from S_S (substitution effect alone) to S_2 (substitution effect + income effect = total effect). Also, the consumption of the other commodity increases from Z_S to Z_2 and utility increases from U_1 to U_2.

Figure 4 Income effect following a decrease in the energy cost of useful work

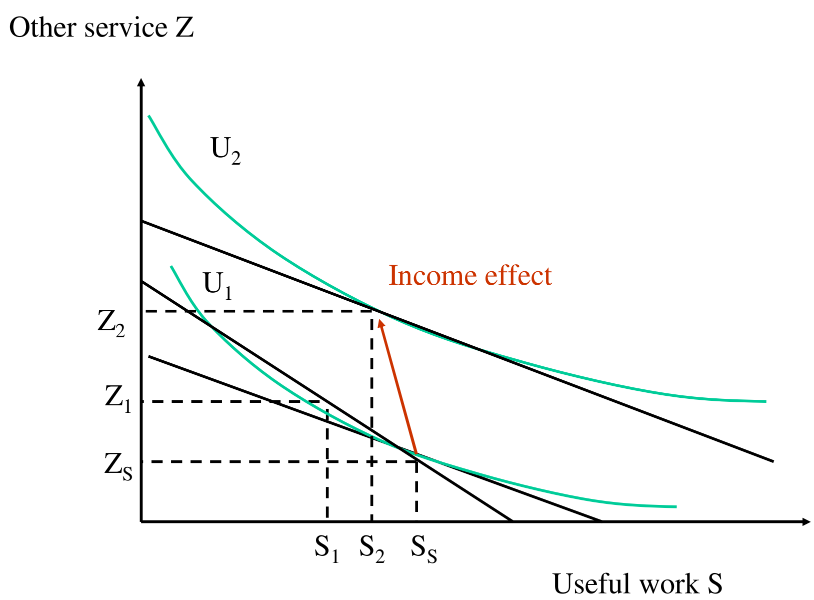


The substitution effect will always contribute to increased consumption of useful work following an improvement in energy efficiency. The size of this effect will depend upon the degree of substitutability between useful work and other goods and services and may be close to zero if there is limited substitutability. In contrast, the income effect may either increase or decrease consumption of useful work, depending upon whether useful work is a 'normal' good or an 'inferior' good [14]. Consumption of a normal good will increase following an increase in real income, while consumption of an inferior good will decrease. For example, it is possible that bus travel is an inferior good, since consumption may decline above a certain level of income.

The consequence of the income effect when useful work is an inferior good is illustrated in Figure 5. Here, the income effect leads to a reduction in the consumption of useful work from S_S to S_2. However, this is not sufficient to counteract the substitution effect, so there is still a net increase in the consumption of useful work from S_1 to S_2. In this case, the final demand for the other commodity *increases* from Z_1 to Z_2.

It is theoretically possible for the negative income effect for an inferior energy service to outweigh the substitution effect. In this case, a fall in the energy cost of useful work will lead to *reduced* consumption of useful work. The inferior energy service would be termed a 'Giffen good' in this instance [13]. In practice, however, this appears unlikely.

Figure 5 Income effect when useful work is an inferior good



2.2.3 Elasticity measures and the direct rebound effect

This decomposition into a substitution and an income effect has been formalised in the ***Slutsky equation*** [13]. This equation allows the magnitude of the substitution and income effects to be estimated, although only their sum is empirically observed. Let:

$\varepsilon_{SS} = \frac{\partial S}{\partial P_S}\frac{P_S}{S}$ measured or *uncompensated* own-price elasticity of demand for useful work;

$\eta_S = \frac{\partial S}{\partial Y}\frac{Y}{S}$ income (or expenditure) elasticity of demand for useful work; and

$w_S = \frac{P_S S}{Y}$ share of useful work in total expenditure.

Then define the *compensated* own-price elasticity of demand for useful work as:

$$\tilde{\varepsilon}_{SS} = \frac{\partial S}{\partial P_S}\frac{P_S}{S}\bigg|_{u=constant} \qquad (10)$$

The *compensated* own-price elasticity ($\tilde{\varepsilon}_{SS}$) of useful work is a measure of the change in the quantity demanded of useful work following a change in the price of useful work, holding utility constant. The Slutsky equation (derived in Section 3.5) states that the compensated own-price elasticity of useful work ($\tilde{\varepsilon}_{SS}$) is equal to the uncompensated own-price elasticity of useful work (ε_{SS}) plus the product of the income elasticity of demand for useful work (η_S) and the share of useful work in total expenditure (w_S):

$$\tilde{\varepsilon}_{SS} = \varepsilon_{SS} + \eta_S w_S \tag{11}$$

This is more commonly written as:

$$\varepsilon_{SS} = \tilde{\varepsilon}_{SS} - \eta_S w_S \tag{12}$$

Since ε_{SS}, η_S and w_S can be measured empirically (data permitting), the compensated own-price elasticity $\tilde{\varepsilon}_{SS}$ can be estimated.

The compensated own-price elasticity of useful work ($\tilde{\varepsilon}_{SS}$) is a measure of the *substitution* effect of the energy efficiency improvement, while the product of the income elasticity and the expenditure share ($\eta_S w_S$) is a measure of the *income* effect of the energy efficiency improvement. The first minus the second ($\varepsilon_{SS} = \tilde{\varepsilon}_{SS} - \eta_S w_S$) is a measure of the *total* effect.

It is important to appreciate the expected signs of each term in the Slutsky equation and the relationship of these to the expected sign of the direct rebound effect. The latter is normally expressed as *positive*, since it refers to an (unintended) *increase* in energy demand following an energy efficiency improvement. But it is theoretically possible for the direct rebound effect to be *negative*, implying that changes in consumption of useful work lead to an additional reduction in energy demand, over and above that achieved by the engineering improvement (i.e. $S_2<S_1$). Saunders [15] has termed this counterintuitive outcome 'super-conservation'.

Assuming the utility function confirms to certain standard and plausible conditions (Section 3.4), the compensated own-price elasticity of demand for useful work is always negative ($\tilde{\varepsilon}_{SS} < 0$). In other words, an increase (decrease) in the price of useful work should lead to a decrease (increase) in consumption of useful work, holding utility constant. Since an energy efficiency improvement *reduces* the price of useful work, the substitution effect will contribute to *increased* consumption of useful work ($S_S>S_1$) and hence to a positive direct rebound effect.

The income elasticity of a normal good is positive ($\eta_S > 0$) because an increase (decrease) in real income leads to an increase (decrease) in consumption of that good. Hence, if useful work is a normal good, the second term in the Slutsky equation will be negative ($-(\eta_S w_S) < 0$). Since an energy efficiency improvement *reduces* the price of useful work, the income effect will contribute to *increased* consumption of useful work ($S_2>S_S$) when the latter is a normal good and hence to a positive direct rebound effect. Therefore, when useful work is a normal good, the substitution and income effects *reinforce* one another and lead to increased consumption of useful work ($S_2>S_1$) and a positive direct rebound effect.

In contrast, the income elasticity of an inferior good is negative ($\eta_S > 0$), because an increase (decrease) in real income leads to a decrease (increase) in demand for that good. Hence, if useful work is an inferior good, the second term in the Slutsky equation will be *positive* $(-(\eta_S w_S) > 0)$. Since an energy efficiency improvement *reduces* the price of useful work, the income effect for an inferior good will contribute to *reduced* consumption of useful work (S_2<S_S) and hence to a negative direct rebound effect. Hence, when useful work is an inferior good, the substitution and income effects for useful work *counteract* one another. As a result, the consumption of useful work may either increase or decrease. The sign of the uncompensated own price elasticity of useful work (ε_{SS}) is therefore ambiguous, together with the sign of the direct rebound effect.

These different possibilities are summarised in Table 1. Since we would normally expect useful work to be a normal good, we would expect the uncompensated price elasticity to be negative ($\varepsilon_{SS} < 0$) and to be greater in magnitude than the compensated price elasticity ($|\varepsilon_{SS}| > \lfloor \tilde{\varepsilon}_{SS} \rfloor$ owing to the income effect ($-\eta_S w_S$). In these circumstances, the income and substitution effects will reinforce each other, the consumption of useful work will increase and the direct rebound effect will be positive. The only circumstance in which the direct rebound effect will be negative is when useful work is a 'Giffen good' – meaning that the negative income effect outweighs the substitution effect resulting in reduced consumption of useful work overall (S_2<S_1). In practice, this outcome appears extremely unlikely.

Generally, the substitution effect ($\tilde{\varepsilon}_{SS}$) will be larger when useful work (*S*) is a good substitute for other commodities, while the income effect will be larger when useful work accounts for a larger share of the overall budget. The compensated and uncompensated own price elasticities will be similar if the share of income devoted to useful work (w_S) is small, and/or the income elasticity of demand for useful work (η_S) is small. Either of these conditions will serve to reduce the importance of the income effect ($-\eta_S w_S$) relative to the substitution effect.

Table 1 Determinants of the sign of the direct rebound effect for useful work

Nature of S	**Sign of income elasticity**	**Sign of compensated own-price elasticity**	**Relative size of income and net substitution effects**	**Sign of uncompensated own-price elasticity**	**Sign of direct rebound effect**
Normal good	$\eta_S \geq 0$	$\tilde{\varepsilon}_{SS} \leq 0$	Not relevant	$\varepsilon_{SS} \leq 0$	Positive
Inferior good	$\eta_S \leq 0$	$\tilde{\varepsilon}_{SS} \leq 0$	$\lvert\tilde{\varepsilon}_{SS}\rvert > \lvert w_S \eta_S\rvert$	$\varepsilon_{SS} \leq 0$	Positive
Giffen good	$\eta_S \leq 0$	$\tilde{\varepsilon}_{SS} \leq 0$	$\lvert\tilde{\varepsilon}_{SS}\rvert < \lvert w_S \eta_S\rvert$	$\varepsilon_{SS} \geq 0$	Negative

2.3 The indirect rebound effect for consumers

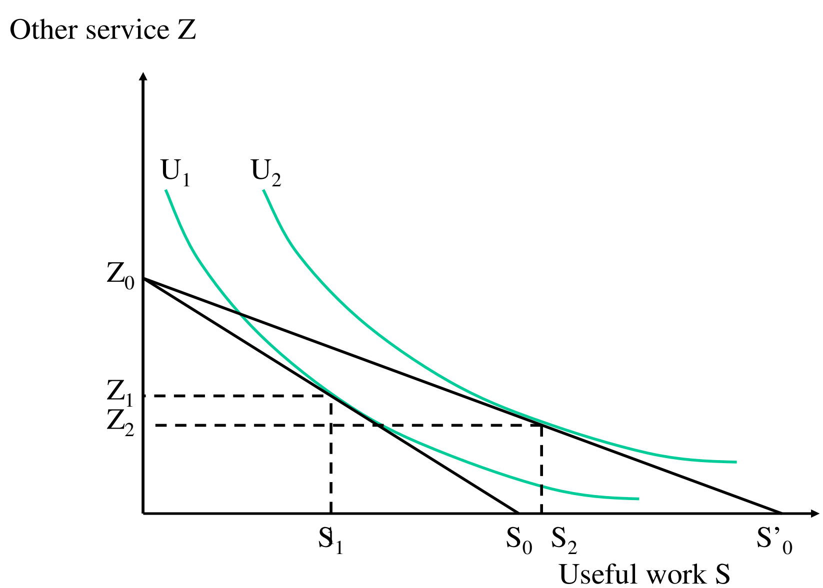
2.3.1 Graphical illustration of the indirect rebound effect

The direct rebound effect relates solely to the energy used to provide the energy service (*ES*) that benefited from the energy efficiency improvement. But such improvements will also change the quantity demanded of *other* goods and services (*Z*) that also require energy to provide. For example, the cost savings from a more energy-efficient central heating system may be put towards an overseas holiday. Changes in the consumption of these goods and services will lead to corresponding changes in the energy required to provide those goods and services. If the consumption of a commodity increases, the additional energy consumption that results will *offset* the energy savings from the energy efficiency improvement, while if consumption of a commodity falls, the energy saving that results will *add* to the energy savings from the energy efficiency improvement.

The commodities affected may include both *energy commodities* that are *directly* consumed by the household (e.g. petrol) and *non-energy commodities* whose provision requires the use of energy further back in the supply chain (e.g. food) and which therefore represent energy that is *indirectly* consumed by the household ('embodied energy'). For example, the cost savings from an energy-efficient heating system may be spent on greater car use (which leads to more direct energy consumption) and more restaurant meals (which leads to more indirect energy consumption). However, since in both cases this energy consumption is *separate* from that used for the provision of the relevant energy service, they may both be labelled as *indirect* energy consumption (despite the fact that some of this energy is directly consumed by the household). The corresponding changes in this energy consumption may similarly be labelled as *indirect rebound effects*.[2]

The origin of indirect rebound effects can be illustrated using the simple, two-commodity model introduced in the previous section. Here, the consumption of commodity *Z falls* when the energy service is a normal good ($Z_2<Z_1$) and *increases* when the energy service is an inferior good ($Z_2>Z_1$). The former case is illustrated again in Figure 6.

Figure 6 Change in consumption following an energy efficiency improvement in a simple two-good model



[2] The direct energy consumption may be relatively easy to estimate while the energy 'embodied' in the non-energy commodities may be estimated with the help of an Input-Output model (Section 4.1).

Since the energy intensity of commodity Z - $E(Z)$ – is unaffected by the energy efficiency improvement, the change in indirect energy consumption for commodity Z (IND) is determined solely by the change in quantity demanded (Z_2-Z_1):

$$IND = E(Z_2) - E(Z_1) \tag{13}$$

Since $Z_2<Z_1$ in this example, the indirect energy consumption associated with commodity Z has *fallen* and $IND<0$. These energy savings are *additional* to any savings in direct energy consumption from the energy efficiency improvement (ACT).

The *indirect* rebound effects (REB_i) associated with commodity Z may then be defined as the ratio of the change in indirect energy consumption (IND) to the expected energy savings from the energy efficiency improvement (ENG):

$$REB_i = \frac{IND}{ENG} * 100\% \tag{14}$$

Substituting, we have:

$$REB_i = \frac{E(Z_2) - E(Z_1)}{E(S_1) - E^*(S_1)} * 100\% \tag{15}$$

If IND is positive (i.e. $Z_2>Z_1$), indirect energy consumption has increased and the indirect rebound effect is positive, while if IND is negative (i.e. $Z_2<Z_1$) indirect energy consumption has fallen and the indirect rebound effect is negative. If $IND=0$ then the indirect rebound effect is zero, while if $IND>ENG$ the indirect rebound effect has led to 'backfire'.

The *total* rebound effect (REB_t) in this two-commodity example is equal to the sum of the direct and indirect effects:

$$REB_t = REB_d + REB_i \tag{16}$$

Or:

$$REB_t = \frac{ENG - ACT + IND}{ENG} * 100\% \tag{17}$$

Substituting, we have:

$$REB_t = \frac{\left(E(S_1) - E^*(S_1)\right) - (E(S_1) - E^*(S_2)) + \left(E(Z_2) - E(Z_1)\right)}{E(S_1) - E^*(S_1)} * 100\% \tag{18}$$

Or:

$$REB_t = \frac{\left(E^*(S_2) - E^*(S_1)\right) + \left(E(Z_2) - E(Z_1)\right)}{E(S_1) - E^*(S_1)} * 100\% \tag{19}$$

Compared to the equation for the direct rebound effect, there is an additional term in the numerator. If $E(Z_2) > E(Z_1)$, the energy savings from the energy efficiency improvement will be offset and the total rebound effect will be larger than the direct rebound effect, while if $E(Z_2) < E(Z_1)$, the energy savings from the energy efficiency improvement will be increased and the total rebound effect will be smaller than the direct rebound effect.

The above analysis is confined to a situation where the consumer chooses between only two goods/services – *S* and *Z*. In practice, a reduction in the energy cost of useful work will change the quantity demanded of *multiple* commodities (Z_k where k=1,2,....K). Some of these will be energy commodities that are directly consumed by the household while others will be non-energy commodities that represent 'embodied energy'. Consumption of some of these commodities may *decrease* following the energy efficiency improvement, while the consumption of others may *increase*. If the former is the case useful work (*S*) is said to be a *gross substitute* for commodity Z_k, while if the latter is the case useful work is said to be a *gross complement* to commodity Z_k. The former implies that increased consumption of useful work *substitutes* for reduced consumption of commodity Z_k, while the latter implies increased consumption of useful work leads to increased consumption of commodity Z_k. The prefix 'gross' indicates that the measured or uncompensated price responses are being referred to.

Let $E(Z_{kq})$ represent the indirect energy consumption associated with consuming a quantity *q* of commodity Z_k. Let Z_{k1} represent the quantity of commodity Z_k consumed *before* the energy efficiency improvement and Z_{k2} the quantity of commodity Z_k consumed *after* the improvement. Then the *total* change in indirect energy consumption associated with the change in demand for all other commodities is given by:

$$IND = \sum_k \left[E(Z_{k2}) - E(Z_{k1})\right] \tag{20}$$

As with the two-commodity example, the total change in indirect energy consumption (*IND*) following the energy efficiency improvement may be either positive or negative and will depend upon both the change in demand for each commodity (Z_{k2}-Z_{k1}) and the relative energy intensity of each commodity ($E(Z_k)$). The indirect rebound effect (REB_i) associated with the change in the quantity demanded of *all* other commodities is then given by:

$$REB_i = \frac{\sum_k \left[E(Z_{k2}) - E(Z_{k1})\right]}{E(S_1) - E^*(S_1)} * 100\% \tag{21}$$

Similarly, the total rebound effect (REB_t), taking into account both the direct change in energy consumption for the energy service *S* (REB_d) and the indirect change in energy consumption for all other commodities Z_k (REB_i) is given by:

$$REB_t = \frac{\left(E^*(S_2) - E^*(S_1)\right) + \sum_k \left[E(Z_{k2}) - E(Z_{k1})\right]}{E(S_1) - E^*(S_1)} * 100\% \tag{22}$$

2.3.2 Decomposition of the indirect rebound effect

A key measure relevant to the indirect rebound effect is the percentage change in the consumption of commodity Z that is 'caused' by a percentage change in the energy cost of useful work (P_S), holding nominal income, the price of other commodities and preferences constant. This 'cross-price elasticity of demand' for commodity Z is defined as:

$$\varepsilon_{ZS} = \frac{\partial Z}{\partial P_S}\frac{P_S}{Z} \qquad (23)$$

A higher (lower) cross-price elasticity leads to a greater (smaller) change in the consumption of Z in response to a change in the price of S. Conventionally, commodities are said to be *gross substitutes* if the cross-price elasticity is positive and *gross complements* if the cross-price elasticity is negative. For example, public transport is typically found to be a gross substitute for travel by car, since an increase in the cost of car travel ($P_S \uparrow$) typically leads to increased consumption of public transport ($Z \uparrow$).

As with the own-price elasticity, the cross-price elasticity may be decomposed into a *substitution* effect and an *income* effect [13]:

- *Substitution effect*: the change in consumption of commodity (Z) that would result from the change in the price of useful work (S) *if* nominal income was adjusted to keep utility constant.
- *Income effect:* the change in consumption of commodity Z that would result exclusively from the change in real income, holding other prices and money income constant.

The substitution effect in the two-good model is illustrated in Figure 7, where the consumption of commodity Z decreases from Z_1 to Z_S holding utility constant. The income effect is illustrated in Figure 8, where the consumption of commodity Z increases from Z_s to Z_2. Figure 8 also shows the total effect, where consumption of commodity Z has fallen from Z_1 to Z_2.

Figure 7 Substitution effect following a decrease in the energy cost of useful work

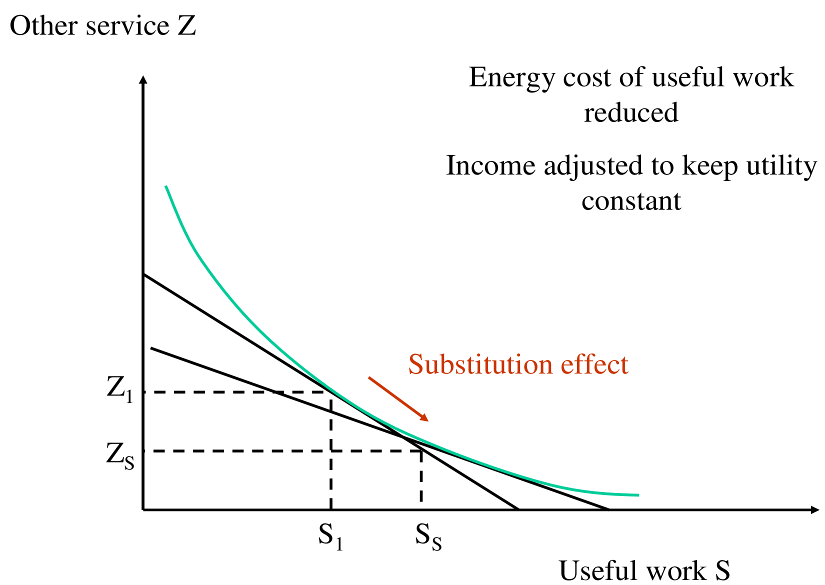
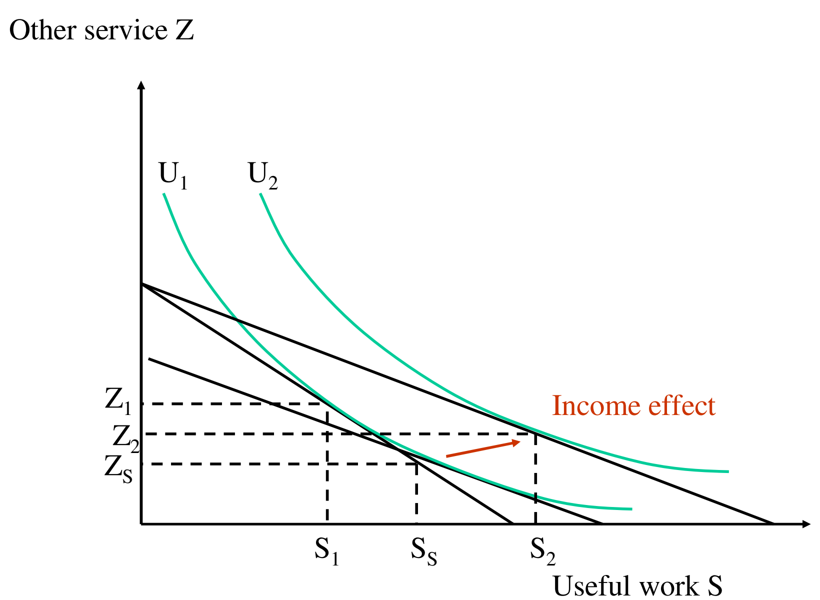


Figure 8 Income effect following a decrease in the energy cost of useful work



In a simple two-good model, the substitution effect for commodity *Z* is always negative – i.e. increased consumption of useful work must substitute for reduced consumption of *Z*. But in a multi-good world, the substitution effect for an individual commodity can be either positive or negative. Similarly, the income effect for that commodity can be either positive or negative, depending upon whether they are normal goods or inferior goods. Hence, the magnitude and sign of the total effect on a commodity will depend upon the magnitude and sign of the income and substitution effects.

2.3.3 Elasticity measures and the indirect rebound effect

As with the own-price elasticity, the cross-price elasticity may be decomposed into substitution and income effects with the help of the Slutsky equation:

$$\varepsilon_{ZS} = \tilde{\varepsilon}_{ZS} - w_S \eta_Z \quad (24)$$

This is more commonly written as:

$$\tilde{\varepsilon}_{ZS} = \varepsilon_{ZS} + w_S \eta_Z \quad (25)$$

The compensated elasticity of demand for commodity *Z* with respect to the price of useful work ($\tilde{\varepsilon}_{ZS}$) is a measure of the *substitution* effect of the energy efficiency improvement on the quantity demanded of commodity *Z*. The product of the income elasticity of demand for commodity *Z* and the expenditure share of useful work ($w_S \eta_Z$) is a measure of the *income* effect of the energy efficiency improvement on the quantity demanded of commodity *Z*. The first minus the second ($\varepsilon_{ZS} = \tilde{\varepsilon}_{ZS} - w_S \eta_Z$) is a measure of the *total* effect. The substitution effect will be larger when useful work is a good substitute for commodity *Z*, while the income effect will be larger when useful work accounts for a larger share of the overall budget and/or the income electricity for commodity Z is large.

Again, it is important to appreciate the expected signs of each term in the Slutsky equation and the relationship of these to the expected sign of the indirect rebound effect. The latter is normally expressed as *positive* if there is an increase in indirect energy consumption following an energy efficiency improvement and *negative* if there is a reduction in consumption. In contrast to the direct rebound effect, there is greater scope for the indirect rebound effect to be negative.

If the compensated cross-price elasticity between commodity *Z* and useful work is positive ($\tilde{\varepsilon}_{ZS} > 0$), commodity *Z* is said to be a *net substitute* for useful work*,* while if the compensated cross-price elasticity between commodity *Z* and useful work is negative ($\tilde{\varepsilon}_{ZS} < 0$), commodity *Z* is said to be a *net complement* to useful work. If commodity *Z* is a net *substitute* to useful work, the substitution effect following an energy efficiency improvement will lead to *reduced* consumption of commodity *Z* and hence contribute to a *negative* indirect rebound effect. In contrast, if commodity *Z* is a net *complement* to useful work, the substitution effect following an energy efficiency improvement will lead to *increased* consumption of commodity *Z* and hence contribute to a *positive* indirect rebound effect.

If commodity *Z* is a normal good it will have a positive income elasticity ($\eta_Z > 0$) while if it is an inferior good it will have a negative income elasticity ($\eta_Z < 0$). Hence, if commodity *Z* is a normal good, the second term in the Slutsky equation will be negative ($-(\eta_z w_S) < 0$). Since an improvement in energy efficiency corresponds to a *fall* in the price of useful work, the income effect will lead to *increased* consumption of commodity *Z* when the latter is a normal good and hence contribute to a *positive* direct rebound effect. In contrast, if commodity *Z* is an inferior good, the income effect will lead to *reduced* consumption of commodity *Z* and hence contribute to a *negative* direct rebound effect.

The sign of the uncompensated cross-price elasticity between useful work and commodity *Z* (ε_{ZS}) is therefore ambiguous, together with the sign of the indirect rebound effect for commodity *Z*. The substitution and income effects for commodity *Z* may either reinforce or counteract one another depending upon: a) whether commodity *Z* is a net substitute or net complement to useful work; b) whether commodity *Z* is a normal or inferior good; and c) the relative size of these substitution and income affects. The different possibilities are summarised in Table 2.

The total indirect rebound effect will be the sum of the cross-price effects on *all* the relevant commodities (Z_k). Given both the range of possible outcomes for each individual commodity (Table 2) and their wide variation in energy intensity, the magnitude and sign of the indirect rebound effect maybe expected to be both highly variable and difficult to predict.

Table 2 Determinants of the sign of the indirect rebound effect for commodity Z

Nature of *Z*	**Sign of income elasticity for *Z***	**Sign of compensated cross-price elasticity**	**Relative size of income and net substitution effects**	**Sign of uncompensated cross-price elasticity**	**Sign of indirect rebound effect for *Z***
Normal good	$\eta_Z \geq 0$	$\tilde{\varepsilon}_{ZS} < 0$ Net complements	Not relevant	$\varepsilon_{ZS} < 0$ Gross complements	Positive
Normal good	$\eta_Z \geq 0$	$\tilde{\varepsilon}_{ZS} > 0$ Net substitutes	$\lvert\tilde{\varepsilon}_{ZS}\rvert < \lvert w_S \eta_Z\rvert$	$\varepsilon_{ZS} < 0$ Gross complements	Positive
Normal good	$\eta_Z \geq 0$	$\tilde{\varepsilon}_{ZS} > 0$ Net substitutes	$\lvert\tilde{\varepsilon}_{ZS}\rvert > \lvert w_S \eta_Z\rvert$	$\varepsilon_{ZS} > 0$ Gross substitutes	Negative
Inferior good	$\eta_Z \leq 0$	$\tilde{\varepsilon}_{ZS} < 0$ Net complements	$\lvert\tilde{\varepsilon}_{ZS}\rvert > \lvert w_S \eta_Z\rvert$	$\varepsilon_{ZS} < 0$ Gross complements	Positive
Inferior good	$\eta_Z \leq 0$	$\tilde{\varepsilon}_{ZS} < 0$ Net complements	$\lvert\tilde{\varepsilon}_{ZS}\rvert < \lvert w_S \eta_Z\rvert$	$\varepsilon_{ZS} > 0$ Gross substitutes	Negative
Inferior good	$\eta_Z \leq 0$	$\tilde{\varepsilon}_{ZS} > 0$ Net substitutes	Not relevant	$\varepsilon_{ZS} > 0$ Gross substitutes	Negative

Rebound effects from changes in behaviour

The independent variable for most analyses of rebound effects is a *technical* improvement in the energy efficiency of delivering a particular energy service. For example, replacing traditional lightbulbs with compact fluorescents should improve the energy efficiency of a lighting system, while replacing an SUV with a fuel-efficient vehicle should improve the energy efficiency of personal travel. These improvements will reduce the energy cost of useful work (P_S). Such changes can be achieved through maintenance and operational measures, such as driving in a more fuel-efficient manner, but more commonly they require investment in durable goods. As a result, the potential saving in energy costs may be frequently be offset (and in some cases entirely outweighed) by the additional capital costs of energy-efficient equipment. As Henley *et al.* [16] have shown, this can have important implications for the size of any direct rebound effects.

An alternative approach to reducing direct energy consumption is through *behavioural* change, such as turning lights off in unoccupied rooms or replacing car trips with public transport or cycling. In contrast to improving energy efficiency, most changes of this type do not require investment in durable goods and hence are not associated with capital costs. However, these changes will also lead to rebound effects.

Behavioural changes such as these should not be associated with any *substitution* effects, since the effective price (P_S) of useful work for the relevant energy service (e.g. lighting) remains unchanged. However, they will be associated with *income* effects since the savings made on direct energy consumption for the relevant energy service (and possibly on associated expenditures such as maintenance) will be available to for re-spending. In most cases, it seems reasonable to assume that the re-spending will be directed towards *other* goods and services, *excluding* the energy service that was the focus of the behavioural change - because the consumer is assumed to be making a conscious decision to reduce consumption of that service. However, in some cases it is possible that a portion of the savings *will* be re-spent on the relevant energy service. For example, the cost savings from switching to CFLs may be used to increase the overall level of illumination in a household. Hence, behavioural change will always lead to *indirect* rebound effects and may in some cases lead to *direct* rebound effects as well. In both cases, these will be confined to *income* effects and will therefore differ in magnitude from those associated with energy efficiency improvements.

For commodities that are normal goods, the behavioural change will lead to a positive indirect rebound effect, while for commodities that are inferior goods it will lead to a negative indirect rebound effect. In both cases (unlike with an energy efficiency improvement) these income effects will neither be offset nor reinforced by any substitution effects. The aggregate effect will therefore depend on the relative proportion of normal and inferior goods, together with their relative energy intensity. In practice, there is likely to be significantly more normal goods than inferior goods, so we would expect the overall indirect rebound effect to be positive.

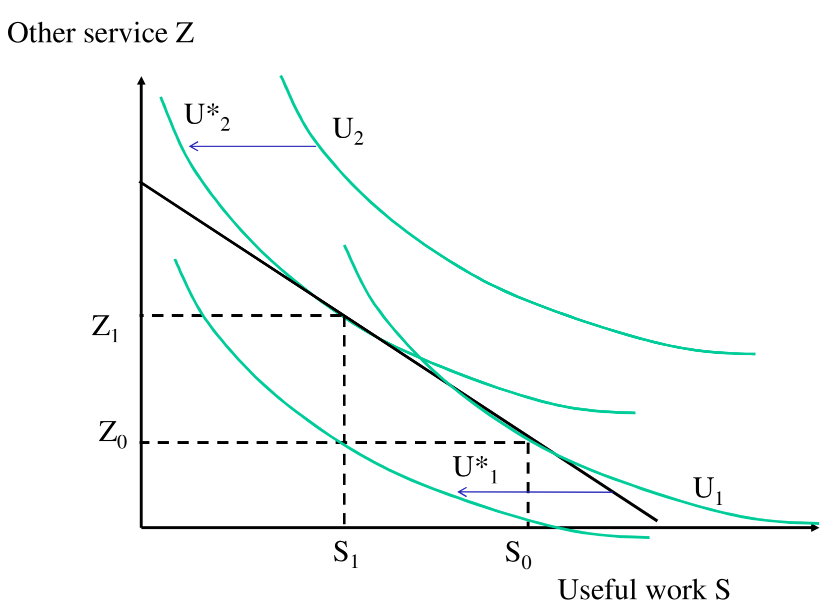
Table 3 summarises the difference between the rebound effects from behavioural change and those from improved energy efficiency.

Table 3 Components of total rebound effect from behavioural change and improved energy efficiency

	Direct rebound effect	Indirect rebound effect
Improved energy efficiency	Sum of substitution and income effects $\varepsilon_{SS} = \tilde{\varepsilon}_{SS} - \eta_S w_S$	Sum of substitution and income effects $\varepsilon_{ZS} = \tilde{\varepsilon}_{ZS} - w_S \eta_Z$
Behavioural change	Income effects alone $\eta_S w_S$ (or zero if no re-spending on *S*)	Income effects alone $\eta_Z w_S$

Behavioural changes of this type may best be understood and represented as a change in preferences – for example, towards cycling and away from car travel. In Figure 9, this is represented as a shift of the indifference curves U_1 and *U2* leftwards to U^*_1 and U^*_2. Prior to the behavioural change, they utility maximising position corresponded to consumption of S_0 of the energy service (e.g. lighting) and Z_0 of the other service. Following the behavioural change, the new utility maximising position corresponds to *reduced* consumption of the energy service ($S_1<S_0$) and *greater* consumption of the other service ($Z_1>Z_0$). Aggregate expenditure is unchanged, but has been redistributed between the other goods and services according to the change in preferences.

Figure 9 Rebound effect from behavioural change



3 The neoclassical theory of consumer demand systems: key concepts

Consumer demand analysis attempts to explain variations in household expenditure on different commodities using cross-sectional or pooled cross-sectional data on income, commodity prices, household composition and other relevant variables. While such analysis can focus upon a single commodity (*k*) , the interest here is household expenditure on a *group* of *K* commodities (*k=1,2,…..K*) and the associated interdependencies. This requires the formulation and estimation of a *system* of *K* equations that represents the demand for those commodities and the derivation of expenditure, own-price and cross-price elasticities from this system. For our purposes, these elasticities may be used for the estimation of direct and indirect rebound effects.

The neoclassical literature on consumer demand systems is well established, very extensive and highly sophisticated and only the 'bare bones' of the relevant theory will be summarised here.[3] The aim is to formulate a system of consumer demand equations that can be empirically estimated using household survey data. These systems typically comply with a number of theoretical restrictions on consumer behavior, but the primary purpose of these restrictions is to make statistical estimation more feasible by reducing the number of degrees of freedom. This advantage is particularly relevant when pooled cross-sectional data is being used to estimate price elasticities, since the degrees of freedom are constrained and only low levels of commodity disaggregation can be used. It is much less relevant when cross-sectional data is being used to estimate expenditure elasticities, since this provides more degrees of freedom and allows both higher levels of commodity disaggregation to be used, together with additional socioeconomic variables. For cross-sectional studies, more *ad hoc* approaches to specifying the functional form of the relevant equations are commonly employed.

Systems of equations for commodity demand can be formulated in a variety of ways and these may be more or less appropriate in different contexts and for different purposes. Most approaches assume that consumers choose a mix of commodities that maximise their 'utility' subject to a budget constraint. Preferences are taken as fixed and exogenous and consumption is assumed to be a function of income, prices and (sometimes) other demographic variables. These assumptions should not be viewed as a descriptively accurate model of consumer decision-making, but instead as a pragmatic approach to specifying a system of equations that can be statistically estimated. The validity of the resulting model is best judged by the extent to which fits the available data on consumer decision-making.

The section is structured as follows. Sections 3.1 and 3.2 introduce some of the key ideas behind the economic modelling of consumer demand, while Section 3.3 describes the estimation of Engel curves using cross-sectional data. Section 3.4 summarises the different ways in which a system of consumer demand equations can be derived while Section 3.6 reviews the standard properties of demand equations and the associated Slutsky conditions on the magnitude and sign of the various elasticities. Section 3.7 introduces the concepts of seperability and two-stage budgeting and provides empirical formula for the calculation of elasticities in a two-stage system. Section 3.8 introduces the Almost Ideal Demand System (AIDS) which is the most widely used system of consumer demand equations and has previously

[3] This section is based upon a number of sources, including in particular Deaton and Muelbauer [17]

been applied to the estimation of rebound effects [18, 19]. Alternative models and relevant extensions of the theory, such as the demand for durable goods, will be addressed at a later stage of the project, along with the limitations of and alternatives to the conventional neoclassical approach.

3.1 Key definitions

For a representative household, let k be an index for a commodity, good or service[4] (k=1,2......K). In empirical work, these typically refer to *aggregate* commodity groups such as total food, clothing or transport consumption over a specified period of time (typically one year) rather than discrete purchases. Indeed, most pooled cross-sectional studies are confined to less than ten major commodity groups ($K \leq 10$) and commonly to only three or four groups. The relevant household survey data is normally available for a number of years (t) and typically represents the *annual* expenditure on each commodity. But for ease of exposition, the subscript t is excluded in what follows.

Let:

p_k = price of commodity k ($k=1,2,....K$.)
q_k = quantity purchased of that commodity
x_k = expenditure on that commodity ($x_i = p_k q_k$)
x = total expenditure on the commodity set ($x = \sum_k x_k$)

Hence, $\mathbf{q} = (q_1, q_2,..q_K)$, $\mathbf{p} = (p_1, p_2,..p_K)$ and $\mathbf{x} = (x_1, x_2,..x_K)$ are vectors of quantities, prices and expenditures respectively for this commodity set. In addition, we define the *expenditure share* or *budget share* of commodity i as :

$$w_i = \frac{x_i}{x} = \frac{p_i q_i}{x} \tag{26}$$

So $\mathbf{w} = (w_1, w_2,..w_K)$ is a vector of expenditures shares for this commodity set. The expenditure share w_i is also known as the *average propensity to spend* on q_i.

In most empirical applications, x and $\mathbf{p}$ are assumed to be exogenous. Depending upon the empirical application, the commodity set $k=1,...K$ could represent:

- all expenditure, including saving and borrowing – so x is income
- non-financial commodities – so x is consumer expenditure
- non-durable commodities – so x is expenditure on non-durables
- a subset of commodities (e.g. food) – so x is expenditure on that subset

Subdividing expenditure in this way commonly relies upon the concept of *seperability*, discussed in Section 3.7. Many empirical applications focus solely upon non-durables.

Basic consumer demand theory assumes that households do not satisfy all their wants and the total amount to be spent (x) is decided separately from the amount

[4] These will be used interchangeably in what follows.

spent on each commodity. Under these assumptions, the household has a ***linear budget constraint*** as follows:

$$x = \sum_k x_k = \sum_k p_k q_k \qquad (27)$$

Or:

$$x = \mathbf{q'p} \qquad (28)$$

We assume that the consumer has rules for deciding how much of each commodity to purchase faced with vector of prices and a budget constraint:

$$q_i = g_i(x, \mathbf{p}) \qquad (29)$$

Or:

$$\mathbf{q} = \mathbf{g}(x, \mathbf{p}) \qquad (30)$$

This equation defines a set of ***Marshallian*** or ***uncompensated*** demand functions (g_i). These refer to the actual quantities purchased of each commodity. The terminology is used to differentiate them from the ***Hicksian*** or ***compensated*** demand functions (h_i) described in Section 3.4. It is standard to assume that Marshallian demand functions are continuously twice differentiable in both x and $\mathbf{p}$.

The Marshallian demand function for each commodity i gives the total quantity of that commodity (q_i) that is demanded at each possible price (p_i) for a given level of expenditure and other prices. A shift in the price of that commodity (Δp_i) will lead to a movement *along* the curve while a change in expenditure (x) or the prices of other commodities (p_k) will lead to a *shift* of the curve.

Marshallian demand functions may also be estimated for expenditures (x_i):

$$x_i = g_i^X(x, \mathbf{p}) \quad \text{or} \quad \mathbf{x} = \mathbf{g}^{\mathbf{X}}(x, \mathbf{p}) \qquad (31)$$

And expenditures shares (w_i):

$$w_i = g_i^W(x, \mathbf{p}) \quad \text{or} \quad \mathbf{w} = \mathbf{g}^W(x, \mathbf{p}) \qquad (32)$$

These formulations appear more appropriate for empirical applications, since the appropriate measure of quantity (q_i) will vary from one commodity to another. However, following Deaton and Muelbauer [17], the quantity equations ($q_i = g_i(x, \mathbf{p})$) will be used below for the purpose of developing the theory.

Empirical applications that estimate Marshallian demand functions for a sample of households often either: a) subdivide households into groups on the basis of variables such as such as household size, number of children and geographical region; b) include these as a vector of covariates (**z**); or c) reflect these variables through the use of 'equivalence scales'. These covariates will be ignored in what follows in order to simplify the exposition.

3.1.1 Expenditure elasticity and marginal budget share

Define the ***total expenditure elasticity*** of commodity *i* as:

$$\eta_i = \frac{\partial g_i}{\partial x}\frac{x}{q_i} \tag{33}$$

The appropriate interpretation of η_i will depend upon how total expenditure (*x*) is defined. In many empirical applications, expenditure is not coincident with total income, so η_i is not necessarily coincident with the *income elasticity* of demand, although these terms are often used interchangeably.

The expenditure elasticity is commonly used to classify commodities as *normal* goods or *inferior* goods:

Normal goods: $\eta_i > 0$ (purchases increase as total expenditure increases)
Inferior good: $\eta_i < 0$ (purchases decline absolutely as expenditure increases)

Normal goods may be further subdivided into *luxuries* or *necessities*:

Luxury goods: $\eta_i > 1$ (budget share increases with total expenditure)
Necessity good: $\eta_i < 1$ (budget share decreases with total expenditure)

The expenditure elasticity commonly varies with the level of expenditure - for example, commodities change from luxuries at low levels of income to necessities at higher levels. Hence, it may be misleading to estimate this elasticity at the mean level of expenditure ($\bar{x}$).

The marginal change in expenditure on a commodity following a marginal change in total expenditure is commonly termed the ***marginal propensity to consume*** (*MPC*) that commodity:

$$MPC = \frac{\partial x_i}{\partial x} = \frac{\partial (p_i q_i)}{\partial x} \tag{34}$$

This measure is sometimes referred to the *marginal propensity to spend* (*MPS*), but this acronym is more commonly used for the *marginal propensity to save*. With cross-sectional data from a single country or region, p_i may be assumed to be the same for each household so that:

$$MPC = p_i \frac{\partial q_i}{\partial x} \tag{35}$$

A related concept is the ***marginal expenditure share*** **or** ***marginal budget share*** (*MES* or *MBS*):

$$MBS = \frac{\partial w_i}{\partial x} \tag{36}$$

For necessities ($\eta_i < 1$), the marginal budget share is *less* than the expenditure share: $\partial w_i / \partial x < w_i$ while for luxuries ($\eta_i > 1$), the marginal budget share is *more* than the expenditure share $\partial w_i / \partial x > w_i$.

3.1.2 Own and cross-price elasticities

Define the Marshallian ***own-price elasticity*** of commodity *i* as:

$$\varepsilon_{ii} = \frac{\partial g_i}{\partial p_i} \frac{p_i}{q_i} \tag{37}$$

Define the Marshallian ***cross-price elasticity*** of commodity *i* with respect to commodity *j* as:

$$\varepsilon_{ij} = \frac{\partial g_i}{\partial p_j} \frac{p_j}{q_i} \tag{38}$$

The cross-price elasticity is commonly used to classify commodities as either *gross substitutes* (or Marshallian substitutes) or *gross complements*:

Gross substitutes: $\varepsilon_{ij} > 0$ (q_i and p_j move in the same direction)

Gross complements: $\varepsilon_{ij} < 0$ (q_i and p_i move in opposite directions)

We would normally expect an increase in the price of a commodity to lead to a reduction in consumption. So if the consumption of commodity *i* increases, it is *substituting* for the more expensive commodity *j*.

3.2 The adding-up and homogeneity restrictions

Systems of Marshallian demand functions are commonly assumed to meet two basic and highly plausible conditions known as the *adding up* and *homogeneity* restrictions. These in turn lead to associated conditions on the price and expenditure elasticities. The restrictions are commonly used to reduce the number of degrees of freedom in the empirical model and thereby improve efficiency, while the associated conditions are commonly used to estimate some elasticities given information about other elasticities. Both are introduced below.

3.2.1 The adding-up restriction

The ***adding-up restriction*** for Marshallian demand functions is given by:

$$\sum_k w_k = 1.0 \tag{39}$$

Or:

$$x = \sum_k x_k \tag{40}$$

Or:

$$x = \sum_k p_k g_k(x, \mathbf{p}) \qquad (41)$$

That is, the sum of expenditures on each commodity must equal the budget constraint. This restriction can only be satisfied for particular types of function g_k. In practice, the adding-up constraint leads to the redundancy of one of the Marshallian demand equations. For given *x* and **p**, the budget constraint and the set of Marshallian demand functions define a system of *K+1* relations among *K* unknowns. Hence, it is possible to delete one equation without loss of information on the demand behaviour for the commodity for which the equation has been dropped.

The adding-up restriction leads to two conditions on the price and expenditure elasticities that are commonly referred to as *Engel aggregation* and *Cournot aggregation*. These are commonly used to:

- test the assumptions embedded in econometric models;
- incorporate restrictions in econometric models to gain efficiency; and/or
- estimate some elasticities given information about other elasticities

3.2.1.1 Engel aggregation

This defines a relationship between the expenditure elasticities of several commodities. Take the first differentials of Equation
(41) wrt *x* :

$$\sum_k p_k \frac{\partial g_k}{\partial x} = 1 \qquad (42)$$

Hence, increases in income are completely allocated to all commodities. Re-write this equation as:

$$\sum_k \left[\frac{p_k q_k}{x}\right]\left[\frac{\partial g_k}{\partial x}\frac{x}{q_k}\right] = 1 \qquad (43)$$

Or:

$$\sum_k w_k \eta_k = 1 \qquad (44)$$

Hence, the sum of of expenditure elasticities for each commodity weighted by the budget share of each commodity is equal to unity.

This relation can be used to derive any single expenditure elasticity estimate (η_k) provided all the other expenditure elasticities are available.

3.2.1.2 Cournot aggregation

This defines a relationship between the own and cross-price elasticities of a commodity. Take the first differentials of Equation
(41) wrt p_i :

$$\sum_k p_k \frac{\partial g_k}{\partial p_i} + q_i = 0 \qquad (45)$$

So a change in prices does not violate the budget constraint. Rewrite the equation as:

$$\sum_k p_k \frac{\partial g_k}{\partial p_i} \frac{p_i}{x} \frac{q_k}{q_k} + q_i \frac{p_i}{x} \frac{q_i}{q_i} \qquad (46)$$

$$\sum_k \left[\frac{p_k q_k}{x}\right]\left[\frac{\partial g_k}{\partial p_i} \frac{p_i}{q_k}\right] + \left[\frac{p_i q_i}{x}\right] \qquad (47)$$

So:

$$\sum_k w_k \varepsilon_{ki} + w_i = 0 \qquad (48)$$

Or:

$$\sum_k w_k \varepsilon_{ki} = -w_i \qquad (49)$$

Hence, the sum of the cross-price elasticities between commodity i and commodity k weighted by the budget share of commodity k is equal to the negative of the average budget share of commodity i.

This equation can be used to derive any single cross-price elasticity estimate (ε_{ki}) provided all the other cross-price elasticities are available.

3.2.2 The homogeneity restriction

The ***homogeneity restriction*** for Marshallian demand functions is (for $\theta > 0$):

$$q_i = g_i(\theta x, \theta \mathbf{p}) = g_i(x, \mathbf{p}) \qquad \textit{(50)}$$

That is, the quantity demanded should remain unchanged if both prices and total expenditure change by an equal proportion. This means that the demand function is homogenous of degree zero. This property is sometimes called the 'absence of money illusion'.

The total differential of the Marshallian demand curve is:

$$dq_i = \sum_k \frac{\partial g_i}{\partial p_k} dp_k + \frac{\partial g_i}{\partial x} dx \qquad (51)$$

If prices and expenditure changed by an equivalent proportion, then $\frac{dx}{x} = \frac{dp_i}{p_i} = \alpha$ and $dq_i = 0$. So:

$$\sum_k \frac{\partial g_i}{\partial p_k} p_k + \frac{\partial g_i}{\partial x} x = 0 \quad (52)$$

Divide through by q_i:

$$\sum_k \frac{\partial g_i}{\partial p_k} \frac{p_k}{q_i} + \frac{\partial g_i}{\partial x} \frac{x}{q_i} = 0 \quad (53)$$

Which gives:

$$\sum_k \varepsilon_{ik} + \eta_i = 0 \quad (54)$$

Or:

$$\sum_k \varepsilon_{ik} = -\eta_i \quad (55)$$

Hence, the sum of all the own and cross-price elasticities for commodity i is equal to the negative of the expenditure elasticity of commodity i.

3.2.3 Summary

In summary, for a system of Marshallian demand equations that satisfy the adding-up and homogeneity restrictions, the following conditions apply to the magnitude of the price and expenditure elasticities:

Engel aggregation:
$$\sum_k w_k \eta_k = 1$$

Cournot aggregation:
$$\sum_k w_k \varepsilon_{ki} = -w_i$$

Homogeneity:
$$\sum_k \varepsilon_{ik} = -\eta_i$$

In empirical work, the adding-up and homogeneity restrictions are commonly *imposed* in order to increase the degrees of freedom when estimating a particular demand functions. It is also possible to *statistically test* these restrictions, although this will lose degrees of freedom. A key problem in consumer demand analysis is that the restrictions are commonly rejected in such tests [20].

3.3 Engel curves

An ***Engel curve*** describes how purchases vary with total expenditure (x) and other covariates (z_i), holding prices constant:

$$q_i = f_i\,(x, z_i) \quad (56)$$

Or:

$$\mathbf{q} = \mathbf{f}(x, \mathbf{z}) \tag{57}$$

This can also be expressed in terms of expenditures (x_i):

$$x_i = f_i^x(x, z_i) \tag{58}$$

Or expenditure shares:

$$w_i = f_i^w(x, z_i) \tag{59}$$

The covariates (***z***) may include household size, gender, number of children, geographical region, employment market status, seasonal effects and so on. In addition, the equations may be estimated for *household* expenditure, or *per capita* expenditure (*x/P* - where *P* is the number of people in the household) or household expenditure adjusted according to some *equivalence scale*.

Engel [21] was the first to study the relationship between household food expenditure and income using cross-sectional data on household expenditure. This kind of study was later extended to other commodities by authors such as Woking [22], Leser [23] and Prais and Houthakker [24]. Engel curves have been explored statistically using a wide range of functional forms which may be more or less consistent with the data in different circumstances [24, 25]. In contrast to the analysis of pooled cross-sectional data, the choice of functional form can be relatively *ad hoc* since (with the exception of the adding up constraint) there is less need to impose restrictions to increase the number of degrees of freedom.

It is common to use the logarithm of total expenditure in order to reduce the differences between households. In principle, the chosen functional form should: a) allow for saturation in commodity demand as expenditure increases; b) satisfy the adding-up criterion; and c) provide the best statistical fit to the data. However, it is unlikely that a single functional form will simultaneously satisfy all three requirements. Haque [25] demonstrates the importance of statistically comparing different functional forms, not least because the estimated expenditure electricity appear sensitive to the choice of functional form.[5] Options include the following:

$$q_i = \alpha_i + \beta_i x + z \quad \text{(linear)} \tag{60}$$

$$q_i = \alpha_i + \beta_i \ln x + z \quad \text{(semi-log)} \tag{61}$$

$$q_i = \alpha_i + \frac{\beta_i}{x} + z \quad \text{(hyperbolic)} \tag{62}$$

$$\ln q_i = \alpha_i + \beta_i \ln x + z \quad \text{(double log)} \tag{63}$$

$$\ln q_i = \alpha_i + \beta_i \ln x + \frac{\chi}{x} + z \quad \text{(log-log inverse)} \tag{64}$$

[5] Note that the goodness of fit for non-nested models such as these cannot be compared on the basis of adjusted R^2 – more sophisticated techniques are required [26].

$$q_i = \alpha_i + \beta_i x + \chi \ln x + z \qquad \text{(double semi-log)} \tag{65}$$

However, several of these functional forms do not satisfy the adding-up restriction $\sum_i w_i = 1$.. One widely used functional form that does comply with this restriction is the ***Working-Leser*** (WL) model. In expenditure share form, this relates expenditure shares to the log of total expenditure [23]:

$$w_i = \alpha_i + \beta_i \ln x + z \tag{66}$$

For the Woking-Leser model, the adding-up restriction implies that:

$$\sum_i \alpha_i = 1 \qquad \text{and} \qquad \sum_i \beta_i = 0 \tag{67}$$

For $\sum_i \beta_i = 0$, some β_i must be >0 (luxuries) and some <0 (necessities).

Using $w_i = p_i q_i / x$ and multiplying both sides of the Working-Leser equation by *x* gives:

$$x_i = \alpha_i x + \beta_i x \ln x + z \tag{68}$$

Then the marginal propensity to consume commodity *i* is given by:

$$\frac{\partial x_i}{\partial x} = \alpha_i + \beta_i (1 + \ln x) \tag{69}$$

Or:

$$\frac{\partial x_i}{\partial x} = w_i + \beta_i \tag{70}$$

Since prices are fixed, the expenditure elasticity for commodity *i* is given by:

$$\eta_i = \frac{\partial g_i}{\partial x} \frac{x}{q_i} = \frac{\partial x_i}{\partial x} \frac{x}{x_i} = \frac{\partial x_i}{\partial x} \frac{1}{w_i} \tag{71}$$

So the expenditure elasticity of commodity *i* in the Working-Leser model is:

$$\eta_i = 1 + \frac{\beta_i}{w_i} \tag{72}$$

This expression implies that the expenditure elasticity of a necessity falls when expenditure increases. This tends to fit the data better than models with constant elasticities and is consistent with Engel's original finding that the expenditure share of food declines with increasing expenditure ('Engel's law'). It implies that demand for some commodities can become saturated as a result of either absolute limits or declining marginal utility. However, a drawback with the Working-Leser model is that with sufficiently low or high values of expenditure, the budget share of a commodity can be less than zero more than one. Consequently, the model is only valid for a limited range of expenditure.

To estimate Engel curves from cross-sectional data it must be assumed that prices are constant across the sample of households. This assumption is more likely to hold if the survey data is collected over a relatively short period of time within a relatively small geographical region. Under these conditions, the expenditure-consumption relationship can be examined in isolation from changes in prices. Also, the large variation in income and expenditure between different households increases the precision of the estimates.

The shape of Engel curves will depend upon the level of aggregation of the relevant commodities. Estimates based upon narrowly defined commodities (e.g. apples) may exhibit considerable variation between households while estimates based upon aggregate commodities (e.g. food) may mix together luxuries and necessities. Other empirical challenges include unobserved variation in the quality of commodities purchased.

3.4 Specifying complete demand systems

For pooled cross-sectional data incorporating information on commodity prices it is necessary to take a more rigorous approach to specifying a system of equations. Neoclassical consumer theory provides a number of ways to specify a complete system of equations for consumer demand for *K* commodities, including:

1. Specifying a *utility function* that is consistent with certain conditions and maximising this subject to the budget constraint.
2. Specifying a *cost function* that is consistent with certain conditions and applying *Shephards's Lemma.*
3. Specifying an *indirect utility function* that is consistent with certain conditions and applying *Roy's identity*.

The last two approaches are effectively equivalent and rely upon the theory of *duality*. The following two sections summarise the 'primal' and 'dual' approaches in turn, introducing the relevant concepts along the way.

3.4.1 Maximising utility functions

The first way to generate demand equations is to maximise an assumed *utility function* subject to a budget constraint. The utility function reflects the preference ordering of different bundles of consumption by the consumer and hence is an *ordinal* function.

In neoclassical theory, individual preferences are assumed to be represented by utility function of the form $u(\mathbf{q})$ where $\mathbf{q} = (q_1, q_2, ...q_k)$ is a vector of the quantity purchased of each commodity. The following assumptions are commonly made about the utility function:

- *Completeness*: For any two bundles of commodities either $u(\mathbf{q}^1) \geq u(\mathbf{q}^2)$ or $u(\mathbf{q}^2) \geq u(\mathbf{q}^1)$

- *Consistency:* If $u(\mathbf{q}^1) \geq u(\mathbf{q}^2)$ and $u(\mathbf{q}^2) \geq u(\mathbf{q}^3)$ then $u(\mathbf{q}^1) \geq u(\mathbf{q}^3)$
- *Non-satiation*: $\frac{\partial u(\mathbf{q})}{\partial q_i} > 0$ for each *i*
- *Continuity*: the utility function forms a continuous surface that is continuously twice differentiable for each q_i.
- *Concavity*: the utility function is *quasi concave*, implying that the indifference curves are *strictly convex* to the origin. Formally, for two consumption bundles **q'** and **q''** and $1 \geq k \geq 0$; $u(k\mathbf{q}'+(1-k)\mathbf{q}'') \geq u(\mathbf{q}')$ and $u(k\mathbf{q}'+(1-k)\mathbf{q}'') \geq u(\mathbf{q}'')$

So the consumer's problem is to maximise $u(\mathbf{q})$ subject to $x = \sum_k p_k g_k(x, \mathbf{p})$. The solution to this is the Marshallian demand functions: $\mathbf{q} = \mathbf{g}(x, \mathbf{p})$

Given a utility function that satisfies the above conditions, the Marshallian demand functions can be derived through constrained maximisation, using the Lagrangian (L):

$$L = u(\mathbf{q}) + \lambda\left(x - \sum_k p_k g_k(x, \mathbf{p})\right) \quad (73)$$

The first order conditions for maximisation are:

$$\frac{\partial L}{\partial q_i} = \frac{\partial u}{\partial q_i} - \lambda p_i = 0 \quad (74)$$

And:

$$\frac{\partial L}{\partial \lambda} = x - \sum_k p_k g_k(x, \mathbf{p}) = 0 \quad (75)$$

The ratio of the marginal utilities of two commodities is equal to the ***marginal rate of substitution*** between them (*MRS*). Hence, it follows from Equation (74) that the condition for an optimal allocation of expenditure between commodities becomes:

$$MRS(q_i, q_k) = \frac{\partial u / \partial q_i}{\partial u / \partial q_k} = \frac{p_i}{p_k} \quad (76)$$

Hence, *for optimal allocation of expenditure, the marginal rate of substitution between commodity i and commodity j should be equal to the ratio of the price of commodity i to that of commodity j.*

Note also that, for all *i*:

$$\frac{\partial u}{\partial q_i} = \lambda p_i \quad (77)$$

If p_i is constant, this can be written as:

$$\lambda = \frac{\partial u}{\partial (p_i q_i)} \tag{78}$$

Hence, for optimal allocation of expenditure, *each additional £ spent on any commodity ($\partial(p_i q_i)$) should yield the same marginal utility (∂u)*. In other words, each commodity should have an identical marginal benefit to marginal cost ratio. Consequently, λ can be regarded as the *marginal utility of expenditure*.

Note that the first order conditions only hold when some positive amount of each commodity is purchased. But there may be situations where some commodities are not purchased (corner solutions). In this case we have:

$$\frac{\partial L}{\partial q_i} = \frac{\partial u}{\partial q_i} - \lambda p_i \leq 0 \tag{79}$$

If $\frac{\partial L}{\partial q_i} = \frac{\partial u}{\partial q_i} - \lambda p_i < 0$ then q_i=0

Hence, a complete set of Marshallian demand functions ($\mathbf{q} = g(x, \mathbf{p})$) may be obtained by:

- assuming a functional form for the utility function (u) that meets the conditions indicated above;
- performing a constrained maximisation.

The *Linear Expenditure System (LES)* introduced by Stone [27] can be derived using this route. Although widely used in the 1960s and 1970s, the LES has now been largely superseded by other models, so it will not be examined here.

3.4.2 Minimising cost functions

An alternative way of generating a system of demand equations is to specify a *cost function* (sometimes termed an *expenditure function*) and to apply *Shephards's Lemma*. This relies upon the theory of *duality*.

The consumers' *primary* problem is to *maximise utility* (u) subject to a given level of *expenditure* (x – the budget constraint):

Maximise $u(\mathbf{q})$ subject to $x = \sum_k p_k g_k(x, \mathbf{p})$ (80)

The consumers' *dual* problem is to *minimise expenditure* (x) subject to a given level of *utility* (u):

Minimise $x = \sum_k p_k g_k(x, \mathbf{p})$ subject to $u = v(\mathbf{q})$ (81)

In both cases, the optimal **q** is being sought and the values of u and x are the same.

The solution to the primary problem is the set of *Marshallian* or *uncompensated* demand equations which specify the quantity purchased as a function of total expenditure and a vector of prices:

$$\mathbf{q}=\mathbf{g}(x,\mathbf{p}) \tag{82}$$

The solution to the dual problem is the set of *Hicksian* or *compensated* demand equations which specify the quantity purchased as a function of utility and a vector of prices:

$$\mathbf{q}=\mathbf{h}(u,\mathbf{p}) \tag{83}$$

The *Hicksian demand function specifies the relationship between the quantity demanded and prices with utility (u) held constant*. In effect, the consumer is *compensated* for the effect of a price change in order to keep utility constant. This is a theoretical result since only uncompensated price responses are observed in practice. The Hicksian functions give the quantity of each commodity (q_i) that is demanded at each possible price (p_i) holding utility constant. The Hicksian function therefore only represents the *substitution* effects of a change in relative prices. The substitution effect is given by: $\partial h_i / \partial p_i$

If expenditure is minimised for a given *u*, we have:

$$q_i = h_i(u,\mathbf{p}) = g_i(x,\mathbf{p}) \tag{84}$$

Or:

$$\mathbf{q}=\mathbf{h}(u,\mathbf{p})=\mathbf{g}(x,\mathbf{p}) \tag{85}$$

The *cost function -* $c(u,\mathbf{p})$ *- is defined as the as the minimum cost of obtaining utility u given prices **p*****:**

$$c(u,\mathbf{p}) = Min\left[\sum_k p_k q_k\right] = x \text{ subject to } u = v(\mathbf{q}) \tag{86}$$

$$c(u,\mathbf{p}) = x = \sum_k p_k h_k(u,\mathbf{p}) \tag{87}$$

Cost functions are commonly assumed to be:

- Increasing in *u.*
- Increasing in at least one price
- Homogeneous of degree one in prices (i.e. for a scalar $\theta > 0$ $c(u,\theta\mathbf{p}) = \theta c(u,\mathbf{p})$). For example if prices double, expenditure doubles as well.
- Concave in prices (i.e. as prices rise, costs rise no more than linearly).
- Continuously twice differentiable in all prices (follows from the concavity assumption).

Commonly used cost functions that meet these conditions include the *Cobb-Douglas,* the *Translog* and those used to derive the *Almost Ideal Demand System* (AIDS – see Section 3.8). To obtain the cost-minimising demand for commodities from the cost function we use ***Shephard's Lemma.*** This states that the *derivative of the cost function with respect to the price of commodity j is equal to the quantity demanded of commodity j*:

$$\frac{\partial c(u,\mathbf{p})}{\partial p_j} = h_j(u,\mathbf{p}) = q_j \tag{88}$$

So the *Hicksian* or *compensated* demand functions ($q_i = h_i(u,\mathbf{p})$) may be obtained by first assuming a functional form for the cost function ($c(u,\mathbf{p})$) that meets the conditions indicated above and then applying Shephard's Lemma.

In practice, however, we more commonly wish to generate the *Marshallian* or *uncompensated* demand functions. This can be achieved with the additional use of the *indirect utility function* - $\psi(x,\mathbf{p})$. While the *cost function* specifies the minimum expenditure (*x*) as a function of utility (*u*) and prices (**p**) – $c(u,\mathbf{p})$ the *indirect utility function* specifies the maximum obtainable utility (*u*) as a function of expenditure (*x*) and prices (**p**) - $\psi(x,\mathbf{p})$. Hence, the indirect utility function is the *inverse* of the cost function - if you have one you can rearrange algebraically to obtain the other. Indirect utility functions therefore meet an analogous set of conditions to those for utility functions, namely:

- Non-decreasing in *x* and **p** (i.e. if $\mathbf{p'} \geq \mathbf{p}$, $\psi(x,\mathbf{p'}) \leq \psi(x,\mathbf{p'})$)
- Homogeneous of degree zero in expenditure and prices (i.e. for a scalar $\theta > 0$ $\psi(\theta x, \theta\mathbf{p}) = \psi(x,\mathbf{p})$).
- Quasi-convex in *x* and **p**.
- Continuously twice differentiable in all *x* and **p**.

To go from the indirect utility function to the Marshallian demand functions, we first write the indirect utility function as $\psi(c(u,\mathbf{p}),\mathbf{p})$ and differentiate with respect to p_i holding *u* constant:

$$\frac{\partial \psi}{\partial c}\frac{\partial c}{\partial p_i} + \frac{\partial \psi}{\partial p_i} = 0 \tag{89}$$

Using Shephard's Lemma and substituting:

$$\frac{\partial \psi}{\partial c} q_i + \frac{\partial \psi}{\partial p_i} = 0 \tag{90}$$

Rearranging gives ***Roy's Identity***:

$$q_i = g_i(x,\mathbf{p}) = -\frac{\partial \psi / \partial p_i}{\partial \psi / \partial x} \tag{91}$$

Roy's Identity states that *the quantity demanded of commodity i is given by ratio of the derivative of the indirect utility function with respect to the price of commodity i to the derivative of the indirect utility function with respect to total expenditure*.

Hence, an alternative route to obtaining the Marshallian demand functions is to:

- assume a functional form for the cost function that is consistent with the properties indicated above
- use Shephard's Lemma to obtain the Hicksian demand functions;
- invert the cost function to obtain the indirect utility function; and
- use Roy's identity to obtain the Marshallian demand functions.

Deaton and Muellbauer [28] use this approach to derive the Almost Ideal Demand System (AIDS), discussed below.

3.5 The Slutsky equation

The definitions and relationships introduced above may be used to derive the *Slutsky equation* that was introduced in Section 2. First, substitute the cost function into the Marshallian demand function:

$$q_i = h_i(u, \mathbf{p}) = g_i(c(u, \mathbf{p}), \mathbf{p}) \qquad (92)$$

Differentiate with respect to p_j:

$$\frac{\partial q_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} = \frac{\partial g_i}{\partial p_j} + \frac{\partial g_i}{\partial c}\frac{\partial c}{\partial p_j} \qquad (93)$$

But since $x = c(u, \mathbf{p})$

$$\frac{\partial g}{\partial c} = \frac{\partial g}{\partial x} \qquad (94)$$

Also, $\frac{\partial c}{\partial p_j} = q_j$ from Shepard's Lemma, so:

$$\frac{\partial h_i}{\partial p_j} = \frac{\partial g_i}{\partial p_j} + \frac{\partial g_i}{\partial x} q_j \qquad (95)$$

This is the ***Slutsky Equation,*** introduced earlier in elasticity form in Section 2.2. The first term on the *rhs* is the *uncompensated* price derivative of q_i with respect to p_j. To 'compensate' this and keep utility constant, an amount $\frac{\partial g_i}{\partial x} q_j$ must be added on (i.e. the product of the total expenditure derivative of g_i and the derivative of minimum cost with respect to p_j). Since everything on the right-hand side of the Slutsky equation can be estimated empirically, $\partial h_i / \partial p_j$ may also be derived. It is common to write this as:

$$s_{ij} = \frac{\partial h_i}{\partial p_j} \qquad (96)$$

Or:

$$\mathbf{S} = \frac{\partial \mathbf{h}}{\partial \mathbf{p}} \qquad (97)$$

This is termed the *substitution matrix* (**S**) or *Slutsky matrix* of compensated price responses.

The Slutsky Equation is more commonly written as:

$$\frac{\partial g_i}{\partial p_j} = \frac{\partial h_i}{\partial p_j} - \frac{\partial g_i}{\partial x} q_j \qquad (98)$$

The Slutsky Equation decomposes the uncompensated price response ($\partial g_i / \partial p_j$) into a *substitution* effect ($\partial h_i / \partial p_j$) and an *income* effect ($-\frac{\partial g_i}{\partial x} q_j$). The first effect is due to the fact that if the price of one commodity changes, its relative price also changes, with the result that less will be consumed of the commodity whose relative price increases (and more of the commodities which are substitutes for it), *if* one ignores the income effect. The second effect is due to the fact that a price change implies a change in the *real income* of the consumer (not the same as a change in nominal income). Both effects are the result of one and the same price change. This sum is equal to the observed change in quantity demanded.

The Slutsky equation can be converted into elasticity form. Denoting the *compensated* cross-price elasticity between commodity *i* and commodity *j* as $\tilde{\varepsilon}_{ij}$ we can rewrite the above equation as:

$$\tilde{\varepsilon}_{ij} = \frac{\partial h_i}{\partial p_j} \frac{p_j}{q_i} = \frac{\partial g_i}{\partial p_j} \frac{p_j}{q_i} + \frac{\partial g_i}{\partial x} q_j \frac{p_j}{q_i} \frac{x}{x} \qquad (99)$$

Which gives

$$\tilde{\varepsilon}_{ij} = \varepsilon_{ij} + w_j \eta_i \qquad (100)$$

Or:

$$\varepsilon_{ij} = \tilde{\varepsilon}_{ij} - w_j \eta_i \qquad (101)$$

Using this, the *compensated, or Hicksian price elasticity* ($\tilde{\varepsilon}_{ij}$) can be obtained from the measurable quantities ε_{ij}, w_j and η_i.

The compensated cross-price elasticity is commonly used to classify commodities as *Hicksian* or *net substitutes* or *net complements*:

Net substitutes: $\tilde{\varepsilon}_{ij} \geq 0$ ($s_{ij} \geq 0$)

Net complements: $\tilde{\varepsilon}_{ij} \leq 0$ ($s_{ij} \leq 0$)

'Independent'commodities: $\tilde{\varepsilon}_{ij} = 0$ ($s_{ij} = 0$)

Note that independence does *not* mean that the demand for commodity *i* is independent of the price of commodity *j* since $\tilde{\varepsilon}_{ij}$ does not include income effects.

For own-price elasticities:

$$\varepsilon_{ii} = \tilde{\varepsilon}_{ii} - w_i \eta_i \tag{102}$$

Although the compensated own-price elasticity is non-positive ($\tilde{\varepsilon}_{ii} \leq 0$), it is possible for this to be outweighed by a *positive* income effect (i.e. $-q_i \eta_i > 0$) leading to a positive uncompensated own-price elasticity ($\varepsilon_{ii} > 0$) . In other words, the demand for commodity *i increases* as the price of *i* increases. For this to occur, *i* must be an inferior commodity ($\eta_i < 0$) but not all inferior commodities exhibit this unusual behaviour. Those few that do are termed *Giffen goods*.

The Slutsky equation suggests that the compensated own-price elasticity ($\tilde{\varepsilon}_{ii}$) will be similar to the uncompensated own-price elasticity (ε_{ii}) if the share of expenditure devoted to commodity *i* is small (w_i) and the expenditure elasticity of that commodity is small (η_i).

3.6 Properties of systems of demand equations

We are now in a position to state four general properties of systems of Marshallian and Hicksian demand equations.

3.6.1 Adding-up

The total value of both Marshallian and Hicksian demands is total expenditure:

$$\sum_k p_k h_k(u, \mathbf{p}) = \sum_k p_k g_k(x, \mathbf{p}) = x \tag{103}$$

This leads to the Engel and Cournot aggregation conditions on the uncompensated price and expenditure elasticities that were derived earlier in Section 3.2:

$$\sum_k w_k \eta_k = 1 \qquad \text{(Engel aggregation condition)} \tag{104}$$

$$\sum_k w_k \varepsilon_{ki} = -w_i \qquad \text{(Cournot aggregation condition)} \tag{105}$$

Substituting the Slutsky equation into the Cournot aggregation condition:

$$\sum_k w_k \varepsilon_{ki} = \sum_k w_k \left[\tilde{\varepsilon}_{ki} - w_i \eta_k\right] = -w_i \tag{106}$$

Given the Engel aggregation condition, this implies that the following conditions on the compensated price elasticities:

$$\sum_k w_k \tilde{\varepsilon}_{ki} = 0 \qquad \text{(Slutsky aggregation condition)} \qquad (107)$$

This is known as the *Slutsky aggregation condition*. It states that *the sum of the compensated cross-price elasticities between commodity i and commodity k weighted by the budget share of commodity k is equal to zero.*

3.6.2 Homogeneity

The Hicksian demands are homogeneous of degree zero in prices while the Marshallian demands are homogeneous degree zero in both prices and expenditure.

$$h_i(u, \theta\mathbf{p}) = h_i(u, \mathbf{p}) = g_i(\theta x, \theta\mathbf{p}) = g_i(x, \mathbf{p}) \qquad (108)$$

As illustrated in Section 3.2.2, this leads to the following condition on the uncompensated cross-price elasticities:

$$\sum_k \varepsilon_{ik} = -\eta_i \qquad \text{(Heterogeneity condition)} \qquad (109)$$

It also leads to an analogous condition on the compensated price elasticities:

$$\sum_k \tilde{\varepsilon}_{ik} = 0 \qquad \text{(Heterogeneity condition)} \qquad (110)$$

Hence, *the sum of the compensated cross-price elasticities between commodity i and commodity k is equal to zero*

3.6.3 Symmetry

The cross-price derivatives of the Hicksian demands are symmetric. That is, for all $i \neq j$

$$\frac{\partial h_i(u, \mathbf{p})}{\partial p_j} = \frac{\partial h_j(u, \mathbf{p})}{\partial p_i} \qquad (111)$$

Or:

$$s_{ij} = s_{ji} \qquad \text{(Symmetry condition)} \qquad (112)$$

In other words, *the compensated impact on the quantity demanded of commodity i of a unit increase in the price of commodity j should equal the compensated impact on the quantity demanded of commodity j of a unit increase in the price of commodity i*. Note that the symmetry condition reduces the number of independent s_{ij} terms by one half.

The symmetry condition can be proved as follows. From Shepard's Lemma we have:

$$h_i(u,\mathbf{p}) = \frac{\partial c(u,\mathbf{p})}{\partial p_i} \tag{113}$$

So:

$$\frac{\partial h_i(u,\mathbf{p})}{\partial p_j} = \frac{\partial^2 c(u,\mathbf{p})}{\partial p_j \partial p_i} \quad \text{and} \quad \frac{\partial h_j(u,\mathbf{p})}{\partial p_i} = \frac{\partial^2 c(u,\mathbf{p})}{\partial p_i \partial p_j} \tag{114}$$

But from Young's theorem:

$$\frac{\partial^2 c(u,\mathbf{p})}{\partial p_j \partial p_i} = \frac{\partial^2 c(u,\mathbf{p})}{\partial p_i \partial pj} \tag{115}$$

So:

$$\frac{\partial h_i(u,\mathbf{p})}{\partial p_j} = \frac{\partial h_j(u,\mathbf{p})}{\partial p_i} \tag{116}$$

Although it is far from obvious, the symmetry property follows from the requirement that preferences are consistent (i.e. if $u(\mathbf{q}^1) \geq u(\mathbf{q}^2)$ and $u(\mathbf{q}^2) \geq u(\mathbf{q}^3)$ then $u(\mathbf{q}^1) \geq u(\mathbf{q}^3)$)

The symmetry condition is commonly expressed in elasticity form:

$$w_i \tilde{\varepsilon}_{ij} = w_j \tilde{\varepsilon}_{ji} \tag{117}$$

Hence, *the compensated cross-price electricity between commodity i and commodity j weighted by the expenditure share of commodity i is equal to the compensated cross-price electricity between commodity j and commodity i weighted by the expenditure share of commodity j.*

3.6.4 Negativity

The Slutsky matrix **S** is symmetric *negative semi-definite*. The technical meaning of this is involved, but the most important implication is that the main diagonal terms of the Slutsky matrix are non-positive:

$$s_{ii} \leq 0 \tag{118}$$

Or in elasticity form:

$$\tilde{\varepsilon}_{ii} \leq 0 \tag{119}$$

3.6.5 Summary

Hence, systems of demand equation that are consistent with neoclassical theory comply with four general conditions, namely:

- The expenditure on each commodity adds up to total expenditure;
- they are homogeneous of degree zero in prices and total expenditure;
- their compensated price responses are symmetric
- their compensated price responses form a negative semidefinite matrix - **S**.

Taken together, these are commonly termed the *Slutsky conditions*. The elasticity relationships that follow from these conditions are summarised in Box 3.

Box 3 Summary of the Slutsky elasticity conditions

$\varepsilon_{ij} = \tilde{\varepsilon}_{ij} - w_j \eta_i$	(Slutsky equation)
$\sum_k w_k \eta_k = 1$	(Engel aggregation)
$\sum_k w_k \varepsilon_{ki} = -w_i$	(Cournot aggregation)
$\sum_k w_k \tilde{\varepsilon}_{ki} = 0$	(Slutsky aggregation)
$\sum_k \varepsilon_{ik} = -\eta_i$	(Homogeneity)
$\sum_k \tilde{\varepsilon}_{ik} = 0$	(Homogeneity)
$w_i \tilde{\varepsilon}_{ij} = w_j \tilde{\varepsilon}_{ji}$	(Symmetry)
$\tilde{\varepsilon}_{ii} \leq 0$	(Negativity)

Note also that:

- The adding-up and homogeneity conditions are a consequence of the linear budget constraint
- The symmetry condition is a consequence of the assumption of consistent preferences.
- The negativity condition is a consequence of the assumption of utility maximisation, or cost minimisation

None of these conditions are dependent upon the choice of a particular functional form for the utility function, but all imply certain conditions on that functional form.

3.7 Seperability and multi-stage budgeting

A key difficulty in estimating a system of demand equations is gaining sufficient degrees of freedom. For example, suppose the Marshallian demand equations took the form:

$$\ln q_i = \alpha_i + \eta_i \ln x + \sum_{j=1,n} \varepsilon_{ij} \ln p_j \qquad (120)$$

Where is η_i is the expenditure elasticity for commodity *i* and ε_{ij} are the uncompensated price elasticities. In this system of *n* equations there are *n* intercepts

$(\alpha_1, \alpha_2 .... \alpha_n)$, *n* expenditure elasticities $(\eta_1, \eta_2 ... \eta_n)$ and n^2 price elasticities $(\varepsilon_{ij}\, i, j = 1, ... n)$. Hence, the total number of coefficients to estimate is $n+n+n^2=n(2+n)$. So for example, if there were ten commodity groups (*n*=10) there would be 120 coefficients to estimate. Even if the homogeneity, symmetry and adding up restrictions were imposed, there would still be of the order of 100 (n^2) coefficients to estimate. This number may be reduced by setting some of the cross price elasticities to zero on the basis of prior evidence or a-priori reasoning. But are more common approach is to *aggregate* commodities into a small number of groups that can be dealt with as a single unit (e.g. food or transport) and to *separate* consumer decisions on one set of commodities (e.g. food) from those of another set of commodities (e.g. transport). For example, we may assume that the decision of how much of the total budget to allocate to transport is separate from the decision of how much of this transport budget should be spent on individual modes of transport. Similarly, we may assume that the decision of how to allocate total current expenditure into different categories of commodities can be made separately from the decision of how to arrange expenditure over time. While theoretical justifications can be provided for this procedure, they nevertheless raise some difficulties [17].

One justification for aggregating individual commodities into broad groups is the *composite commodity theorem*. This states that if a group of prices move in parallel, then the corresponding group of commodities can be treated as a single commodity. In practice however, this condition may rarely hold. For example, the fact that the price of fish is relatively volatile would prevent its classification with other foods [17].

Hence, the process is more commonly justified through the notion of *weak seperability* of preferences. If this holds, commodities can be partitioned into groups so that preferences within groups can be described independently of the quantities purchased in other groups. For example, the preferences for different types of food may be assumed to be independent of preferences for different types of entertainment. This leads to the concept of *two-stage budgeting*, where the consumer is assumed to allocate total expenditure in two stages. At the first stage, expenditure is allocated to broad groups of commodities such as food, shelter and entertainment. At the second stage, expenditures for each group are allocated to individual commodities within those groups, such as different types of food. The procedure can be extended if necessary to *multi-stage budgeting*. For this to be justified, the results of the multi-stage budgeting should be identical to those for single-stage budgeting with perfect information.

Two-stage budgeting implies that the decisions at each stage are equivalent to an independent utility maximisation problem. Then *the quantities purchased within any group can be written as a function of the group expenditure and prices within the group alone*. Hence, prices or expenditures for commodities outside the group are only relevant to the extent that they influence the overall expenditure for that group. Furthermore, while both stages of the budget allocation process can be considered and estimated, there is no need to do so. If necessary, attention can be limited to just one group at the second stage: for example, the allocation of 'food expenditure' between individual food items. But a drawback of this approach is that the estimated expenditure elasticity refers to 'food expenditure' rather than expenditure as a whole.

Consider the allocation of total consumption between K commodities ($k=1,2....K$). The single stage Marshallian demand functions for these commodities are given by:

$$\mathbf{q} = \mathbf{g}(x, \mathbf{p}) \tag{121}$$

Or:

$$q_k = g_k(x, \mathbf{p}) \tag{122}$$

Two-stage budgeting implies that allocation takes place in two independent steps [29]. In the *first stage*, total expenditure (x) is allocated between R groups of commodities. The allocation of expenditure to the rth group ($r=1,2....R$) may be expressed as:

$$x^r = \boldsymbol{\gamma}^{\mathbf{r}}(\mathbf{P}, x) \tag{123}$$

Or:

$$\mathbf{x} = \boldsymbol{\gamma}(\mathbf{P}, x) \tag{124}$$

Where:
$\mathbf{x}$ is a Rx1 vector of group expenditures ($\mathbf{x} = (x^1, x^2......x^R)$)
P is a Rx1 vector of group price indices ($P = (P^1, P^2......P^R)$)

The *second stage* involves the allocation of expenditure between the individual commodities within each group. Let $\mathbf{q^r}$ represent the vector of commodities within the rth group, so q_i^r represents the quantity demanded of the ith commodity ($i=1,2....n_r$) within the rth group and p_i^r represents the price of that commodity. The *conditional*, or *within group* demand function for this ith commodity may be expressed as:

$$q_i^r = f_i^r(p_i^r, x^r) \tag{125}$$

Or for the vector of commodities within this group:

$$\mathbf{q^r} = \mathbf{f^r}(\mathbf{p^r}, x^r) \tag{126}$$

Where:
$\mathbf{q^r}$ is the n_r x1 subvector of **q** corresponding to the commodities in the rth group
$\mathbf{p^r}$ is the equivalent subvector of **p**
x^r is the rth element of x, or the expenditure on the rth group

So:

$$x = \sum_r x^r \tag{127}$$

$$K = \sum_r n_r \tag{128}$$

$$x^r = \mathbf{q'^r}\,\mathbf{p^r} = \sum_{i \in r} q_i^r p_i^r \qquad (129)$$

For two-stage budgeting to be valid, the conditional and unconditional demand functions must be equivalent:

$$q_i^r = g_i^r(\mathbf{p}, x) = f_i^r(\mathbf{p^r}, x^r) = f_i^r(\mathbf{p^r}, \gamma^r(\mathbf{P}, x)) \qquad (130)$$

Or:

$$\mathbf{q^r} = \mathbf{g^r}(\mathbf{p}, x) = \mathbf{f^r}(\mathbf{p^r}, x^r) = \mathbf{f^r}(\mathbf{p^r}, \boldsymbol{\gamma}^\mathbf{r}(\mathbf{P}, x)) \qquad (131)$$

So the *unconditional* system of equations ($q_i^r = g_i^r(\mathbf{p}, x)$) has q_i^r as a function of total expenditure (*x*) and the prices of *all* the individual commodities (**p**). In contrast, the *conditional* system of equations ($\mathbf{q^r} = \mathbf{f^r}(\mathbf{p^r}, \boldsymbol{\gamma}^\mathbf{r}(\mathbf{P}, x))$) has q_i^r as a function of the prices of commodities within that group ($\mathbf{p^r}$), together with the expenditure on that group (x^r). The latter, in turn is a function of total expenditure (*x*) and the aggregate price index for each group (**P**).

For this condition to hold for the second stage of allocation requires ***weak seperability*** of preferences between the relevant groups. Weak seperability means that the *K* commodities can be sorted into *R* groups in such a way that the *preference ordering over the commodities in one group is independent of the quantities of the commodities in the other groups*. In these circumstances, the utility function can be written as:

$$u(\mathbf{q}) = \varsigma\left[v_1(\mathbf{q^1}), v_2(\mathbf{q^2}), \ldots. v_r(\mathbf{q^r})\right] \qquad (132)$$

Equivalently, weak seperability implies that *the marginal rate of substitution between two commodities in one group is independent of the quantities of other commodities in other groups*. So for all $s \neq r$

$$MRS(q_i^r, q_k^r) = \frac{\partial}{\partial q_j^s}\left[\frac{\partial u / \partial q_i^r}{\partial u / \partial q_k^r}\right] = 0 \qquad (133)$$

Under these conditions, the demand for commodities within one group can be written solely as a function of the prices of other commodities within that group and the overall expenditure on that group: $\mathbf{q^r} = \mathbf{f^r}(\mathbf{p^r}, x^r)$.

Note that this condition does *not* say that the quantities demanded in group *r* are independent of either the prices of commodities in other groups or of total expenditure (*x*). Instead, it implies that total expenditure (*x*) and the prices of commodities in other groups (p_j^s) enter into the demand function for commodities in group *r* only through their effect on the expenditure share of group *r* ($x^r = \gamma(\mathbf{P}, x)$). Hence, once x^r is known, we can ignore the prices of commodities outside group *r*. To see this, differentiate Equation 130 with respect to the price of a commodity *j* in another group *s* (p_j^s)

$$\frac{\partial q_i^r}{\partial p_j^s} = \frac{\partial f_i^r}{\partial x^r} \frac{\partial x^r}{\partial p_j^s} \tag{134}$$

So the change in the consumption of a commodity in group *r* caused by a change in the price of a commodity in group *s* is proportional to the change in the expenditure share of group *r* (x^r) caused by that price change.

Similarly for a change in total expenditure

$$\frac{\partial q_i^r}{\partial x} = \frac{\partial f_i^r}{\partial x^r} \frac{\partial x^r}{\partial x} \tag{135}$$

So the change in the consumption of a commodity in group *r* caused by a change in total expenditure (*x*) is proportional to the change in the resulting expenditure share of group *r* (x^r).

Weak seperability is a *sufficient* condition for the second stage of two-stage budgeting – the estimation of the commodity shares within the group as a function of commodity prices and the expenditure share of that group. As Edgerton [29] notes, this "...is quite a rigorous condition, but not completely implausible" – *provided* the groups are chosen sensibly. However, weak seperability is *not* sufficient to justify the *first* stage of two-stage budgeting - the allocation of total expenditure into broad groups using group price indices. In general, the assumptions required to replace the prices of all commodities in a group with a single price index appear implausible [29, 30]. Nevertheless, two-stage and multi-stage budgeting is widely used in order to make econometric estimation feasible. Fortunately, Edgerton [29] shows that it can lead to an *approximately* correct allocation if:

- preferences are weakly separable between the relevant groups; and
- the group price indices (**P**) being used do not vary 'too greatly' with the utility (or, equivalently, expenditure) level.

Under these conditions, empirical formula for the relevant elasticities can be derived. These widely used formulae are summarised below.

3.7.1 Defining two-stage elasticities

The total expenditure elasiticity for the *i*th commodity in the *r*th group can be defined as:

$$\eta_i = \frac{\partial g_i^r}{\partial x} \frac{x}{q_i^r} \tag{136}$$

Similarly, the within group or *conditional* expenditure elasiticity for this commodity may be defined as:

$$\eta_i^r = \frac{\partial f_i^r}{\partial x^r} \frac{x^r}{q_i^r} \tag{137}$$

Defining an expenditure elasticity for the *r*th group of commodities is more difficult since the individual commodities in that group will not be measured in the same units. So instead, a quantity index for the group is defined using the group price index:

$$Q^r = \frac{x^r}{P^r} \tag{138}$$

Then we can write this index as a function of total expenditure and the group price index:

$$\mathbf{Q} = \varphi(\mathbf{P}, x) \tag{139}$$

Where **Q** and **P** are vectors of quantity and price indices for the *R* groups. The *group expenditure elasiticity* for the *r*th commodity group is then defined as:

$$\eta^r = \frac{\partial \varphi^r}{\partial x} \frac{x}{Q^r} \tag{140}$$

The expenditure shares at different levels may be defined as follows:

$$w_i = \frac{p_i^r q_i^r}{x} \qquad \text{(overall)} \tag{141}$$

$$w_i^r = \frac{p_i^r q_i^r}{x^r} \qquad \text{(within group)} \tag{142}$$

$$w^r = \frac{P^r Q^r}{x} \qquad \text{(between group)} \tag{143}$$

The uncompensated total price elasticity between the *i*th commodity in the *r*th group and the *j*th commodity within the *s*th group can be defined as:

$$\varepsilon_{ij} = \frac{\partial f_i^r}{\partial p_j^s} \frac{p_j^s}{q_i^r} \tag{144}$$

Similarly, the uncompensated total price elasticity between the *i*th and *j*th commodities within the *r*th group can be defined as:

$$\varepsilon_{ij}^r = \frac{\partial f_i^r}{\partial p_j^r} \frac{p_j^r}{q_i^r} \tag{145}$$

And the uncompensated group price elasticity for the *r*th and *s*th group can be defined as:

$$\varepsilon^{rs} = \frac{\partial \varphi^r}{\partial P^s} \frac{P^s}{Q^r} \tag{146}$$

Using the Slutsky equation, we can define the equivalent *compensated* price elasticities:

$$\tilde{\varepsilon}_{ij} = \varepsilon_{ij} + w_j \eta_i \tag{147}$$

$$\tilde{\varepsilon}_{ij}^{r} = \varepsilon_{ij}^{r} + w_j^{r} \eta_i^{r} \tag{148}$$

$$\tilde{\varepsilon}^{rs} = \varepsilon^{rs} + w^{s} \eta^{r} \tag{149}$$

3.7.2 Empirical formulae for estimating two-stage elasticities

Assuming that: a) preferences are weakly separable between the relevant groups; and b) the group price indices do not vary 'too greatly' with the expenditure level; Edgerton [29] derives some empirical formula for estimating expenditure and price elasticities within a two-stage budgeting system. These formulae are widely used.

The *total* expenditure elasiticity for the *i*th commodity in the *r*th group (η_i) is simply the product of the *conditional expenditure elasiticity* for the *i*th commodity in the *r*th group (η_i^r) and the group expenditure elasticity for the *r*th group (η^r):

$$\eta_i = \eta_i^r \eta^r \tag{150}$$

In a similar manner, Edgerton [29] derives the following formula for estimating the uncompensated cross-price elasticity (ε_{ij}) in a two-stage budgeting system:

$$\varepsilon_{ij} = \delta_{rs} \varepsilon_{ij}^{r} + \eta_i^r w_j^s \left[\delta_{rs} + \varepsilon_{rs}\right] \tag{151}$$

Where δ_{rs} (Kronecker's delta) is equal to unity when *r=s* and is zero elsewhere. The formula for the uncompensated cross-price elasticity (ε_{ij}) shows that, for two commodities in the *r*th group, the total price elasticity (ε_{ij}) is equal to the within group price elasticity (ε_{ij}^r) plus a factor given by $\eta_i^r w_j^s \left[1 + \varepsilon_{rs}\right]$. This factor comprises the product of the:

- the effect of a change in the price of the *j*th commodity on the *r*th group price index (*P^r*) – given by w_j^s;
- the effect of a change in the price of the *j*th commodity on the expenditure on group *r* –given by $\left[1 + \varepsilon_{rs}\right]$,
- the within-group expenditure elasticity of the *i* commodity - η_i^r

Edgerton derives a similar formula for the compensated cross-price elasticity ($\tilde{\varepsilon}_{ij}$)

$$\tilde{\varepsilon}_{ij} = \delta_{rs} \tilde{\varepsilon}_{ij}^{r} + \eta_i^r w_j^s \tilde{\varepsilon}_{rs} \tag{152}$$

Many studies have employed these empirical formulae, including Brannlund *et al.* [18] in their study of rebound effects.

3.8 The Almost Ideal Demand System (AIDS)

The 'Almost Ideal Demand System' (AIDS) has become the model of choice in most contemporary investigations of consumer demand. Introduced by Deaton and Muellbauer [28, 31], AIDS is claimed to have a number of advantages over competing models, including

- allowing for aggregation over consumers without requiring the Engel curves to be linear;
- being simple to estimate, with coefficients that are easy to interpret;
- avoiding – in a commonly used variation known as the Linear Almost Ideal Demand System (LAIDS) - the need for non-linear estimation techniques;
- automatically satisfying the adding-up restrictions; and
- allowing straightforward imposition of the homogeneity and symmetry restrictions, or the statistical tests of those conditions

Deaton and Muellbauer [28] derive the AIDS model by assuming the following functional form for the cost function

$$\ln c(u,\mathbf{p}) = (1-u)\ln a(\mathbf{p}) + \ln b(\mathbf{p}) \qquad (153)$$

Since *u* lies between zero ('subsistence') and unity ('bliss'), the functions *a*(**p**) and *b*(**p**) can be interpreted as the cost of subsistence and bliss respectively. For reasons explained in [28], Deaton and Muellbauer assume the following functional forms for *a*(**p**) and *b*(**p**):

$$\ln a(\mathbf{p}) = a_0 + \sum_k \alpha_k \ln p_k + \frac{1}{2}\sum_k \sum_j \gamma_{kj} \ln p_k \ln p_j \qquad (154)$$

$$\ln b(\mathbf{p}) = \ln a(\mathbf{p}) + \beta_0 \prod_k p_k^{\beta_k} \qquad (155)$$

With various restrictions on the parameter values, this cost function is linearly homogeneous in prices. Deaton and Muellbauer then: a) use Shepheard's Lemma to derive an expression for the budget share of each commodity (w_i) as a function of utility and prices; b) invert the cost function to give the indirect utility function (*u* as a function of *x* and *p*); and c) substitute the latter into the former to give the following equation for the budget share of each commodity as a function of prices and total expenditure:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(x/P) = 0 \qquad (156)$$

Where α_i is the constant coefficient in the *i*th share equation, γ_{ij} is the slope coefficient associated with the price of commodity *j*, β_i is the slope coefficient associated with total expenditure in the *i*th share equation and *P* is a price index defined as:

$$\ln P = \alpha_0 + \sum_k \alpha_j \ln p_j + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j = 0 \qquad (157)$$

The β_i parameters measure the change in budget share for commodity *i* following a proportional change in total expenditure *(x).* If $\beta_i > 0$, then *i* is a *luxury* commodity since the budget share w_i increases with expenditure (x). Similarly, if $\beta_i < 0$, then *i* is a *necessity.*

The γ_{ij} parameters measure the change in budget share for commodity *i* following a proportional change in p_j with (x/P) held constant. Each γ_{ij} represents the 100 times the effect on the budget share of commodity of a % increase in the price of commodity *j* with x/P held constant.

In many practical situations, where prices are relatively collinear, *P* will be approximately proportional to any appropriately defined price index (i.e. different price indices are highly correlated with each other). Hence, to simplify the econometric estimation, most empirical applications use *Stone's price index* - $\ln P^*$ - instead. This is weighted average of log-prices across all commodities, where the weights are the shares of expenditure on each commodity:

$$\ln P^* = \sum_j w_j \ln p_j \qquad (158)$$

When this approximation is used, the AIDS model is commonly termed the linear approximate AIDS model (LAIDS). This model is relatively easy to interpret and estimate, although the use of the Stone's index can lead to bias [32].

The *adding-up* restriction requires that:

$$\sum_i \alpha_i = 1, \ \sum_i \gamma_{ij} = 0, \ \sum_i \beta_i = 0$$

The *homogeneity* restriction requires that:

$$\sum_j \gamma_{ij} = 0,$$

The *symmetry* restriction requires that:

$$\gamma_{ij} = \gamma_{ji}$$

The adding-up restriction is normally imposed by deleting any one of the equations and calculating its coefficients from the restriction.

The homogeneity and symmetry restrictions can also be imposed, thereby increasing the number of degrees of freedom. Alternatively, the unconstrained model can be estimated and these restrictions statistically tested to see whether they are empirically justified.

There are some difficulties in calculating the expenditure and price-elasticities in the LAIDS model and most authors use simplified version of the correct equations [33]. The expenditure elasticity of commodity *i* is commonly estimated from:

$$\eta_i = 1 + \frac{\beta_i}{w_i} \tag{159}$$

The price elasticities are commonly estimated from:

$$\varepsilon_{ij} = \frac{\gamma_{ij} - \beta_i w_j}{w_i} - \delta_{ij} \tag{160}$$

$$\tilde{\varepsilon}_{ij} = \frac{\gamma_{ij}}{w_i} + w_j - \delta_{ij} \tag{161}$$

Where δ_{ij} (Kronecker's delta) is equal to unity when *i=j* and is zero elsewhere. These elasticities depend upon the expenditure shares and are sometimes evaluated at the sample means.

A two-stage AIDS can be specified as follows [18]. First, the budget share of commodity group *r* can be written as:

$$w^r = \alpha^r + \sum_j \gamma^{rs} \ln p^s + \beta^r \ln(x/P) = 0 \tag{162}$$

Where:

$$\ln P = \sum_r w^r \ln P^r \tag{163}$$

Second, the share of commodity *i* within the budget for group *r* can be written as:

$$w_i^r = \alpha_i^r + \sum_{j=1,n_r} \gamma_{ij}^r \ln p_i^r + \beta_i^r \ln(x^r / P^r) = 0 \tag{164}$$

Where:

$$\ln P^r = \sum_{i=1,n_r} w_i^r \ln p_i^r \tag{165}$$

The standard AIDS model is static and implies instantaneous adjustment following changes in prices and income. This may be inappropriate, given the habitual nature of much household consumption, the costs of adjustment and incorrect expectations regarding future prices. Many studies using the AIDS model exhibit serial correlation which could indicate misspecification. Hence, numerous studies have developed dynamic versions of the AIDS model, for example by making the intercept term of the share equations a function of the lagged budget share ($\alpha_{it} = \alpha_{it}^* + \alpha_{it}^{**} w_{it-1}$) [34, 35].

4 Review of studies estimating indirect rebound effects for households

This section provides a review of studies that have estimated the *indirect* rebound effects from either technical or lifestyle changes by households. Many of of these studies also include estimates of the *direct* rebound effects from these changes. However, studies that are confined *solely* to the estimation of direct rebound effects are excluded. A comprehensive review of the latter (which are much larger in number) is provided by Sorrell *et al*. [36].

The section first outlines how indirect rebound effects may be estimated using a combination of household survey, Life-Cycle Analysis (LCA) and Input-Output (IO) data. It then summarise the methods, assumptions and results of seven studies that use this broad approach. Each study combines estimates of the energy, carbon and/or greenhouse gas intensity of household goods and services with estimates of the expenditure and/or price elasticities associated with those goods and services. However, they differ substantially in terms of their data sources, methodology, level of commodity aggregation, technical and/or behavioural changes examined, rebound effects covered and the magnitude of effects found.

4.1 The basic approach

The indirect rebound effects associated with both technical improvements and lifestyle changes by different categories of household may be estimated by combining estimates of:

- the energy consumption, carbon emissions and/or greenhouse gas emissions embodied within different categories of household goods and services; and
- the expenditure and/or price elasticities associated with those goods and services.

The former may be derived from input-output analysis (I-O), life-cycle analysis (LCA), or a combination of the two [37-39], while the latter may be derived from the econometric analysis of household expenditure data using the concepts and frameworks developed in Section 3. The commodity classification (k) must be consistent between the two data sources, and since expenditure data is frequently available to a higher level of commodity disaggregation than I-O/LCA data, some reconciliation is generally required.

4.1.1 Energy intensity estimates

The total (direct + indirect) energy consumption (E) associated with household purchase of commodity k may be estimated from:

$$E_k = x_k e_k \qquad (166)$$

Where x_k is the *expenditure* on that commodity (in £) and e_k is the estimated *energy intensity* of that commodity (in kWh/£). The latter includes both the energy consumed within all stages of the supply chain of the relevant commodity (i.e. *indirect* or *embedded* energy consumption) and, in the case of energy commodities such as petrol and gas, the energy that is *directly* consumed by the household. Similar expressions may be used for carbon emissions ($C_k = x_k c_k$) and/or GHG emissions ($G_k = x_k g_k$), but the following discussion focuses upon energy consumption. The total energy consumption (E) associated with the household purchase of a group of K commodities can be estimated from:

$$E = \sum_k e_k x_k \tag{167}$$

Energy intensity estimates (e_k) for individual goods and services may be derived from life-cycle analysis (LCA), but a full LCA is data-intensive and time consuming to conduct. It must also address two key problems [40, 41]:

- *Truncation* - or uncertainty over the appropriate system boundary. For example, the indirect energy costs of a refrigerator should include the energy used to make the aluminium casing, but what about the energy used to mine the alumina or to make the machinery used within the mine? There is no standard procedure for determining when energy costs become small enough to neglect
- *Joint production* – or how to attribute energy consumption to two or more products from a single sector. For example, the energy used in the production of biodiesel needs to be split between the biodiesel itself and the co-product of oilseed meal which is used as feed for livestock.

In contrast to LCA, input-output (I-O) analysis can be used to simultaneously estimate the energy intensity of *all* the goods and services purchased by households. I-O analysis avoids the truncation problem, but at the expense of using rather aggregate categories of commodities that do not reflect the full range of variation within each commodity group (e.g. failing to distinguish between organic and conventionally produced food). Some applications use a *hybrid* approach that applies LCA techniques to the 'principal' (e.g. most energy intensive) commodities and I-O techniques to the remaining ('residual') commodities [42]. In all cases, these techniques are readily extended to include carbon and/or greenhouse gas emissions.

Estimates of the energy (or emissions) intensity of a commodity are contingent upon the definition of the production system and supply chain, the spatial and temporal boundaries applied and the methodology and assumptions used. Of particular importance are the methods for aggregating different types of energy carrier [43], for treating energy consumed at different times [44], for accounting for the energy costs of capital goods [45], for accounting for the energy cost of labour [46] and for distinguishing between domestically produced and imported goods - including the extent to which the energy and emission intensity of different producing regions is accounted for [47]. Methodological differences on these and other issues have contributed to wide variations in the results of different studies. Nevertheless, it is typically found that: a) there is wide variation in the direct and indirect energy consumption of different categories of household; b) for households above a certain threshold of income, indirect energy consumption typically exceeds the direct consumption;[6] c) indirect energy consumption increases with income, but at a declining rate; and d) direct energy consumption shows signs of saturation with income [50]. This suggests that indirect energy consumption should become an increasing component of total energy consumption over time, if incomes continue to increase.

[6] For example, Herendeen [48] found that indirect energy consumption in Norway accounted for one third of total energy consumption for a poor family and approximately two thirds for a rich family, while Vringer and Blok [49] found that 54% of total energy demand in Dutch households was indirect

4.1.2 Expenditure and price elasticity estimates

The broad approach to estimating the expenditure and price elasticities of different household commodities from household survey data was described in Section 3. In practice, different studies vary widely in their level of sophistication and method of approach.

The marginal changes used for the estimation of rebound effects may refer to either the *quantity* purchased of a commodity (q_k), the *expenditure* on that commodity ($x_k = p_k q_k$), or the *budget share* of that commodity ($w_k = x_k / x$) - where $x = \sum_k x_k$.

The different possibilities are listed in Table 4. Different studies label these quantities in different ways. For example, $\partial x_k / \partial x$ is commonly termed the *marginal propensity to consume (MPC)* or sometimes *the marginal propensity to spend (MPS),* but the latter is easily confused with the *marginal propensity to save*. Similarly, $\partial w_k / \partial x$, is commonly termed the *marginal budget share* (MBS).

Table 4 Measures of the marginal change in commodity demand used in the estimation of rebound effects

	Dependent variable		
Independent variable	Quantity (q_k)	Expenditure (x_k)	Expenditure share (w_k)
Total expenditure (x)	$\frac{\partial q_k}{\partial x}$	$\frac{\partial x_k}{\partial x}$	$\frac{\partial w_k}{\partial x}$
Commodity price (p_j)	$\frac{\partial q_k}{\partial p_j}$	$\frac{\partial x_k}{\partial p_j}$	$\frac{\partial w_k}{\partial p_j}$

The elasticities corresponding to these marginal changes are indicated in Table 5, using the notation introduced in Section 3. The expenditure and uncompensated price elasticities can be calculated from the coefficients of the estimated system of demand equations, while the compensated price elasticity can be calculated with the help of the Slutsky equation:

$$\tilde{\varepsilon}_{ij} = \varepsilon_{ij} + w_j \eta_i \quad (168)$$

Table 5 Elasticity measures used in the estimation of rebound effects

	Dependent variable		
Independent variable	Quantity (q_k)	Expenditure (x_k)	Expenditure share (w_k)
Total expenditure (x)	η_i	η_i^X	η_i^W
Commodity price (p_j)			
Uncompensated	ε_{ij}	ε_{ij}^X	ε_{ij}^W
Compensated	$\tilde{\varepsilon}_{ij}$	$\tilde{\varepsilon}_{ij}^X$	$\tilde{\varepsilon}_{ij}^W$

Rebound effects may be estimated by combining intensity, expenditure elasticity and price elasticity estimates, but manner in which these are estimated and used will depend upon the type of data that is available, the model that is being estimated and the nature of the changes that are being explored. Three factors are particularly relevant, namely:

- whether the model includes energy services such as heating, or is confined solely to commodities;
- whether the model uses cross-sectional or pooled cross-sectional data; and
- whether the model is being used to explore energy efficiency improvements or behavioural changes (or both).

These are discussed below.

4.1.3 Services versus commodities

As described in Section 2.2, households combine durable and no-durable goods to produce various *energy services*. For example, they combine motor vehicles (*K*) with gasoline (*E*) and other commodities to produce the energy service (*ES*) of personal automotive travel. An essential feature of an energy service is the *useful work* (*S*) obtained which may be measured by a variety of thermodynamic or physical indicators. For example, the useful work from motor vehicles may be measured in vehicle kilometres or passenger kilometres. The combination of useful work (*S*) with other attributes (*A*) such as acceleration and legroom provides the full energy service: $ES = es(S, A)$ - although for many purposes these other attributes may be ignored. The energy efficiency (π) of the relevant energy system is given by the ratio of useful work output to energy input: $\pi = S / E$. The *energy cost* of useful work (P_S) is then given by $P_S = P_E / \pi$, where P_E represents the unit price of energy. This is one component of the *generalised cost* of useful work (P_G), which also includes the annualised capital, maintenance and time costs.

To accurately estimate the direct and indirect rebound effects from energy efficiency improvements, it is therefore necessary to model the useful work (*S*) associated with the relevant energy services. Given data on the energy efficiency of the relevant capital stock (π) and the energy consumed for the relevant energy service (*E*), the household consumption of useful work may be estimated from: $S = \pi E$. Such a model should allow the energy intensity and own-price elasticity of useful work to be estimated, together with the cross-price elasticities between useful work and other goods and services. The estimated system of demand equations will therefore need to combine equations for *energy services* such as heating and lighting with equations for *non-durable commodities* such as food. This will therefore allow the cross-price elasticities between these two categories to be estimated. So for example, *i* could represent household heating while *j* could represent other commodities such as food. However, estimating a demand system such as this is far from straightforward and typically requires data on the ownership and energy efficiency of the durable goods required to provide those services (such as private vehicles) and possibly data on their capital costs and rate of replacement as well [51].

Rebound effects can nevertheless be estimated by a simpler model that is confined solely to expenditure on non-durable commodities. So for example, *i* could represent expenditure on gas, the bulk of which is used for household heating, while *j* represents other non-durable commodities. An improvement in the energy efficiency of heating (i.e. a lower P_S) may then be estimated by simulating a reduction in the price of gas (a lower P_E). But the substitution and income effects that are estimated with such a model will be those associated with lower priced gas, rather than lower-priced heating. Rebound effects can be calculated from these elasticities, but they will provide a biased estimate of the 'actual' rebound effect [10].

4.1.4 Cross-sectional versus pooled cross-sectional data

Expenditure elasticities may be estimated from cross-sectional data, but price elasticities can only be estimated from pooled cross-sectional data – since this includes variations in prices between periods.

As described in 2.2, the direct rebound effect from an energy efficiency improvement can be decomposed into a substitution effect and an income effect. This decomposition is theoretical, but if estimates of ε_{ii} and η_i can be obtained, the magnitude of each effect can be estimated with the help of the Slutsky equation. So $\tilde{\varepsilon}_{ii}$ gives an estimate of the substitution effect while $-\eta_i w_i$ gives an estimate of the income effect. Similar comments apply to the indirect rebound effects from an energy efficiency improvement. Here, $\tilde{\varepsilon}_{ij}$ gives an estimate of the substitution effect, $-\eta_j w_i$ gives an estimate of the income effect and $\tilde{\varepsilon}_{ij}$ gives an estimate of the total effect.

A model estimated from cross-sectional data can only be used to estimate the *income* effects of an energy efficiency improvement. Such improvements lead to cost savings (e.g. lower petrol bills) which can be treated as equivalent to an increase in income (*x*). These cost savings can be assumed to be re-spent on household goods and services (*k*) in accordance with the estimated marginal propensity to spend ($\partial x_k / \partial x$) (or marginal budget share - $\partial w_k / \partial x$) of each good and service. Re-spending on the good or service that benefited from the energy efficiency improvement (e.g. car travel) leads to a direct rebound effect, while re-spending on other goods and services (e.g. food) leads to an indirect rebound effect.

Such a model is likely to *underestimate* the direct rebound effect, however, since only the income effect of the energy efficiency improvement will be captured and the substitution effect will be ignored. If the energy service is a normal good, the substitution and income effects should reinforce each other and lead to increased consumption of the energy service. In these circumstances, neglect of the substitution effect will lead to an underestimate of the direct rebound effect. In contrast, if the energy service is an inferior good, the income effect should offset the substitution effect. If the former is larger than the latter, the magnitude of the direct rebound effect will be underestimated, while if the former is smaller than the latter the sign of the effect will be wrongly estimated (Table 1). In practice, most energy services are likely to be normal goods.

Whether such a model will model overestimate or underestimate the indirect rebound effect is more difficult to judge (Table 2). If the commodity is both a normal good and a net complement to the energy service, the substitution effect for commodity *j* should reinforce the income effect (i.e. demand for commodity *j* should increase because it is a complement to the energy service and because the consumer's real income has increased). In these circumstances, neglect of the substitution effect will lead to an *underestimate* of the indirect rebound effect for commodity *j*. In contrast, if commodity *j* is a normal good and a net substitute to the energy service, the substitution effect should offset the income effect (i.e. demand for commodity *j* should fall because it is a net substitute for the energy service, but should increase because the consumer's real income has increased). In these circumstances, neglect of the substitution effect could lead the indirect rebound effect for commodity *j* to be either over or underestimated, depending upon the size of this effect relative to the income effect. For the same reason, the sign of the indirect rebound effect for commodity *j* may be wrongly estimated. These conclusions will be reversed if the energy service is an inferior good. Since the overall indirect rebound effect is the sum of the substitution and income effects on *all* the other commodities (*j=1,2,....K*) and since these vary widely in their relative energy intensity, the magnitude and sign of the bias in estimating the overall indirect rebound effect will vary from one situation to another.

A model estimated from pooled cross-sectional data, in contrast, can be used to estimate both the substitution and income effects of an energy efficiency improvement. In principle, this should allow both the direct and indirect rebound effects to be accurately calculated and for each to be decomposed into substitution and income effects. However, owing to constraints on the number of degrees of freedom (Section 3.7), models estimated from pooled cross-sectional data typically use much higher levels of commodity aggregation and leave less scope for incorporating demographic variables. Hence, for the purpose of estimating rebound effects, there is a trade-off between cross-sectional and pooled cross-sectional studies. It is by no means obvious that the more accurate representation of income and substitution effects that is achievable with pooled cross-sectional studies will outweigh the drawbacks of a much higher level of commodity aggregation and/or the neglect of other demographic variables.

4.1.5 Energy efficiency improvements versus behavioural change

The potential advantages of pooled cross-sectional studies are primarily relevant to modelling improvements in energy efficiency. For the rebound effects associated with behavioural changes (such as turning lights off in unoccupied rooms), models estimated from cross-sectional data are likely to be sufficient. This is because such changes are unlikely to lead to any substitution effects, since the effective price of useful work (e.g. lighting) remains unchanged. However, they will be associated with income effects since the savings made on direct energy consumption will be available for re-spending. These income effects may be calculated from expenditure elasticities estimated from cross-sectional data. Moreover, the greater level of commodity disaggregation achievable with cross-sectional studies should allow the investigation of more nuanced behavioural changes – such as shifting towards a vegetarian diet.

A consumer that is making a conscious decision to reduce consumption of a particular energy service (e.g. lighting) seems unlikely to re-spend the saved income on that energy service. Hence, in most cases, it seems reasonable to assume that any re-spending will be directed towards *other* goods and services. However, in some cases it is possible that a portion of the savings *will* be re-spent on the relevant energy service. For example, the cost savings from switching to CFLs may be used to increase the overall level of illumination in a household. Hence, behavioural change will always lead to *indirect* rebound effects and may in some cases lead to *direct* rebound effects as well. In both cases, these will be confined to *income* effects and will therefore differ in magnitude from those associated with energy efficiency improvements.

For commodities that are normal goods, the behavioural change will lead to a positive indirect rebound effect, while for commodities that are inferior goods it will lead to a negative indirect rebound effect. In both cases (unlike with an energy efficiency improvement) these income effects will neither be offset nor reinforced by any substitution effects. The aggregate effect will therefore depend on the relative proportion of normal and inferior goods, together with their relative energy intensity. In practice, there is likely to be significantly more normal goods than inferior goods, so we would expect the overall indirect rebound effect to be positive.

4.1.6 Direct versus indirect rebound effects

Finally, the classification of rebound effects as direct or indirect deserves a brief comment. This classification should be based upon whether the energy is used for the provision of the relevant energy service and *not* whether the energy is consumed directly or indirectly by the household. For example, take the case of improving the energy efficiency of a gas boiler. This will reduce the marginal cost of heating and thereby encourage increased consumption of heating. The increased use of gas that results will offset the reduction in gas consumption per unit of heating. This is a *direct* rebound effect since the energy in question (gas) is used for the energy service that benefits from the energy efficiency improvement (heating).

At the same time, the savings from lower cost heating may be put toward increased car travel and thereby increased petrol consumption. This should be classified as an *indirect* rebound effects since the energy in question (petrol) is *not* used for the energy service that benefited from the energy efficiency improvement (heating) but instead for another energy service (car travel). This is despite the fact that petrol is *directly* consumed by the household. The cost savings may also be put towards increased consumption of other goods and services, such as food which represent energy that is *indirectly* consumed by the household (i.e. embodied energy). The indirect rebound effect derives from the sum of the two therefore includes energy that is both directly and indirectly consumed by the household.

Note, however, that the classification and estimation of rebound effects may depend upon the level of commodity disaggregation that is available in the empirical model. For example, the cost savings from using more a fuel-efficient vehicle could be spent upon both increased car travel and increased air travel. If separate elasticity estimates are available, both impacts could be estimated and the former could be classified as a direct rebound effect and the latter as an indirect rebound effect. However, if elasticity estimates are only available for an aggregate 'transport' category, it becomes much more difficult to separate the two effects.

4.1.7 Limitations

The approach to estimating rebound effects described above cannot capture the full range of price and quantity adjustments that may follow technical or behavioural changes by households. This is because it neglects wider and longer-term adjustments such as producer responses to changes in energy prices and patterns of demand. For example, widespread adoption of fuel-efficient or hybrid vehicles may reduce the demand for gasoline and hence the price of gasoline. But this in turn would encourage greater gasoline consumption, by both households that adopted fuel-efficient vehicles and those that did not [52].

Nevertheless, the approach outlined here appears promising and should allow the consequences of energy efficiency improvements and lifestyle changes to be usefully explored. At present, however, the estimation of rebound effects by this route appears to be in its infancy and only a handful of studies have been identified. One reason for this could be the scarcity of embodied energy estimates at a sufficient level of commodity disaggregation. For example, Kok, *et al.* [37] argue that micro-level investigations such as this require embodied energy estimates derived from a hybrid of I-O and LCA studies [45], but only three of the 19 studies they review take this approach. However, valuable results can still be obtained from studies that use relatively aggregate commodities categories (see below). Furthermore, the extensive development of I-O techniques in recent years [47, 53, 54] should have increased the potential for estimating rebound effects.

The following sections summarise the methods, assumptions and results of seven studies that use this broad approach to estimate rebound effects for technical and behavioural changes by households. Table 6 classifies these studies according to the type of data used, whether energy services are modelled and the type of change that is investigated.

Table 6 Methodological approach of seven studies of indirect rebound effects for households

	Expenditure survey data	**Energy/emission intensity data**	**Commodities or Services**	**Efficiency improvement or behavioural change**
Lenzen and Dey [55]	XS	I-O	C	E and B
Thiesen, *et al*. [56]	XS	Hybrid	C	B
Alfredsson [57, 58]	XS	Hybrid	C	B
Nassen and Holmberg [59]	XS	I-O	C and S	E
Brännlund *et al.* [18]	PXS	I-O	C	E
Mizobuchi [19]	PXS	I-O	C	E
Kratena [60, 61]	PXS	I-O	C and S	E

Note: XS= cross-sectional; PXS= pooled cross-sectional; I-O= input-output; Hybrid = combination of I-O and life-cycle analysis (LCA)

4.2 Lenzen and Dey

Lenzen and Dey [55] estimate the implications for energy consumption and greenhouse gas emissions of a number of different consumption choices for Australian consumers, including both energy efficiency improvements and behavioural change. In two of these cases they provide estimates of the rebound effects associated with re-spending the associated cost savings.

Lenzen and Dey use an I-O model to estimate the energy and greenhouse gas intensity of different categories of final consumption within the Australian economy.

The aggregate energy and emission intensities of Australian households are found to vary considerably with income which means that rebound effects will vary depending on whether the household is rich or poor. Lenzen and Dey's subsequent analysis is therefore based upon three different income quintiles, namely lowest 20%, middle 20% and highest 20%. They calculate the average energy and greenhouse gas intensities for the consumption patterns of each quintile, but note that these are inappropriate for estimating rebound effects since they reflect expenditure upon both necessities and luxuries. Hence, for the calculation of rebound effects they employ estimates of the marginal energy and greenhouse gas intensities of each quintile. To obtain these, Lenzen and Dey first regress total (direct plus indirect) energy consumption (*E*) on *per capita* household expenditure (*x*) - $E = Ax^{\theta_E}$ - then calculate the marginal intensity of energy consumption from:

$$\frac{\partial E}{\partial x} = \theta_E A x^{(\theta_E - 1)} \qquad (169)$$

Where $\theta_E = \frac{\partial E}{\partial x}\frac{x}{E}$ is the expenditure elasticity of per capita energy consumption. The marginal intensity of per capita greenhouse gas emissions ($\frac{\partial G}{\partial x}$) is calculated in the same manner. Lenzen and Dey then estimate the energy and emissions that are associated with re-spending the cost savings from specific consumption changes by multiplying the mean cost saving within each quintile by the relevant marginal intensity for that quintile. However, since the re-spending effect is measured at the marginal intensity of the *previous* consumption pattern rather than the *new* consumption pattern, this procedure will overestimate the rebound effect when the consumption changes have a higher than average energy/greenhouse gas intensity and an underestimate the effect when they have a lower than average intensity.

Lenzen and Dey use this approach to estimate the energy and greenhouse gas consequences of shifting from the current Australian diet to one based upon recommended dietary intake (RDI). The new diet involves less food consumption in weight terms, a 30% reduction in total food expenditure and significant reductions in food-related energy consumption and greenhouse gas emissions. However, once the re-spending effect is allowed for, the net effect is to *increase* overall energy consumption by 4 to 7% – although greenhouse gas emissions are still reduced by ~20% partly as a result of reduced livestock emissions. If *DIR* represents the energy savings associated solely with the consumption of food products and *ACT* represent the total saving in energy consumption after allowing for re-spending, the rebound effect can be estimated from: $(DIR - ACT)/DIR$). With this definition, the rebound effect from the dietary change varies from 112 to 123% for energy consumption and from 45 to 50% for greenhouse gas emissions.

Lenzen and Dey use a similar approach to explore the consequences of shifting from electric storage heaters (with low capital but high energy costs) to more efficient gas storage heaters (with higher capital but lower energy costs). Their estimate of the associated cost savings allows for both operating costs and discounted capital costs The rebound effect is much smaller in this case (between 6 and 8%), broadly the same for all income quintiles and higher for energy and the carbon. The difference is related to the smaller cost savings and the fact that most of these savings are spent on goods and services with a lower energy and carbon intensity than household heating.

Rebound effects are not estimated for the other consumption changes analysed in the paper since these do not result in cost savings. However, these examples do provide some interesting results. For example, Lenzen and Dey compare the energy and emission consequences of buying a new car to those of repairing and extending the life of an existing, ten-year old car for a further ten years. Given the energy required to produce a new car, they estimate that this car would need to be at least 13% more fuel-efficient than the existing car for this option to provide net energy savings over the ten-year period. A comparable calculation for clothes washers leads to the dramatic result that the new washer would need to be 91% more energy efficient to deliver any net energy savings over a ten year period.

4.3 Thiesen, et al.

Thiesen, *et al*. [56] make the case that most existing LCA studies are misleading since they do not allow for rebound effects. Such studies often compare the life-cycle environmental impacts of one commodity with a second, comparable commodity, but do not consider the price differences between these commodities and hence how any associated cost savings may be re-spent. To illustrate this, they choose the example of two Danish cheese products that are broadly comparable in terms of taste and quantity, but differ in packaging - with one having a 'traditional' packaging and the second a 'convenience' packaging. Since the convenience product is 8.6% more expensive, purchasers of the traditional product will save money that can be spent upon other goods and services.

Thiesen, *et al*. use life-cycle analysis to compare the environmental impact of the two cheeses and combine household survey (188 product groups) and I-O data (48 product groups) to estimate the environmental impact of re-spending the cost savings. Households are allocated to one of five income groups and their marginal propensity to spend ($\partial x_k / \partial x$) on each of the I-O product groups (k=1 to 48) is estimated by comparing the consumption patterns of adjacent income groups. The logic of this approach is that households, in theory, move towards the next income group by choosing the cheaper product and thereby having more money available for other consumption. The MPS for Danish households as a whole is estimated by weighting these results by the relative purchasing power of each income group (i.e. the product of the average income and the number of households in each group). This procedure leads to the result that the MPS on each product for Danish households as a whole is very similar to the average budget share ($w_k = x_k / x$) of those products for households as a whole, although there are large differences in the consumption patterns of different income groups.

The results demonstrate that allowing for the rebound effect of the price difference can have a dramatic influence on the relative environmental performance of the two spending options. For example, on the basis of the LCA analysis alone, the cheaper cheese has three times the global warming impact of the convenience cheese. But this increases to seven and half times when the the consequences of re-spending the cost savings are allowed for. In the case of nutrient enrichment and acidification, allowing for the re-spending effects leads to a *reversal* of the relative environmental performance of the two cheeses. These differences are all the more dramatic since the price difference between the two cheeses is relatively small (8.6%). However, the variations in the carbon and energy intensity of the different product groups may be smaller than the corresponding variations in GHG intensity, nutrient enrichment and other environmental impacts. Hence, although Thiesen, *et al*. do not report rebound effects for energy and carbon, it is possible that these will be smaller than those for GHGs.

The authors provide several caveats to their results, including the level of aggregation of income and product groups, the neglect of other demographic factors and the unsophisticated method of estimating marginal consumption - although they show that using a different weighting factor, based upon household expenditure on cheese, leads to broadly comparable results. They also note that more expensive products need not necessarily result in less consumption in the economy as a whole, since they could be associated with higher profits for producers who in turn will spend those profits on other goods and services. Nevertheless, their results provide a strong case for including re-spending effects within life-cycle appraisals.

4.4 Alfredsson

Alfredsson [57, 58] explores the potential environmental benefits of 'green' consumption patterns using a complex, three-stage model of Swedish household expenditure. Data from a cross-sectional survey of the expenditure patterns of 1104 households is combined with estimates of the energy and carbon intensity of ~300 commodity categories. These, in turn, are derived from a hybrid IO-LCA analysis of Dutch consumption patterns combined with separate analysis of the carbon intensity of Swedish industrial sectors.

In the first stage, Alfredsson estimates total household expenditure as a function of earned income,[7] number of children and employment status. In the second stage, she estimates vehicle ownership and the type and size of the house (or flat) as a function of the number of persons in the household, the age of the head of household and the house location (urban or rural). In the third stage, she estimates how household expenditure is distributed between eight aggregate groups of commodities as a linear function of the independent and dependent variables of stages one and two. The eight commodity groups (*j*) are travel, housing, food, recreation, clothes, furniture, services and health. Since each of these corresponds to a 'basic' or 'functional' need, the scope for substitution between them should be limited.

The third-stage regression amounts to estimating linear Engel curves using the following functional form:

$$x_j = a_{oj} + a_{1j}x + \mathbf{a_{2j}D} + \mathbf{a_{3j}L} \tag{170}$$

Where:

j	aggregate commodity category
x_j	expenditure on commodity category *j*
x	total expenditure
D	a vector of demographic variables (e.g. number of children)
L	a vector of 'lifestyle choice' variables (e.g. car ownership, size of house)

An adding-up constraint is imposed when estimating this equation to ensure that the sum of expenditure on each commodity category equals the total expenditure ($\sum_j x_j = x$). The coefficient on household expenditure (a_{1j}) represents the marginal propensity to consume (*MPC*) commodity *j* ($\partial x_j / \partial x$). Alfredsson uses these estimated MPC to estimate how any monetary savings from changes in consumption patterns are re-spent. In other words, these savings are treated as an increase in

[7] On average, expenditure is 95% of disposable income.

income, although total expenditure (*x*) is held fixed. The linear Engel curves do not allow for luxury and inferior goods, but this may not be a problem at this level of commodity aggregation.

To explore the implications of ‘greener’ consumption patterns, Alfredsson disaggregates the food, travel and housing categories into a larger number of subcategories. For example, food consumption is subdivided into 18 commodity groups. She then explores the energy and emission consequences of shifting consumption patterns *within* each group. For example, in the case of food consumption, the expenditure pattern is shifted towards locally grown fruits and vegetables and away from meat and dairy products. This changes the average energy and carbon intensity of food expenditure, as well as reducing the total expenditure on food products. The resulting cost savings are then redistributed across the aggregate consumption groups (*j*) according to their estimated MPC, leaving total expenditure (*x*) unchanged. The energy and carbon emissions associated with the ‘greener’ expenditure pattern are then compared with those for the original expenditure pattern.

With this approach, the ‘first stage’ estimates of energy and carbon savings (i.e. excluding re-spending) are based upon a breakdown of commodities within a single group, while the second stage estimates uses the aggregate groups to calculate the effect of the re-spending. This calculation is confined to income effects, although in principle there should be substitution effects as well since the changed consumption patterns change the unit price of the ‘greened’ aggregate group (e.g. food).

The calculation includes the income component of the direct rebound effect because a proportion of the re-spending is allocated to the commodity group for which the cost savings were obtained. For example, households who adopted the greener and cheaper diet are assumed to spend some of the resulting cost savings on more food. However, the energy and carbon consequences of this re-spending are estimated using the energy and carbon intensities of the ‘greened’ diet - implying that the re-spending is confined to the ‘greener’ food categories. Alfredsson argues that a more detailed reallocation of the cost savings between individual food products would make little difference to the overall results, since the additional energy consumption from re-spending in the ‘greened category’ is relatively small compared to the size of the first stage reduction.

The results for the ‘green diet’ echo those of Lenzen and Dey. In the absence of any re-spending, the greener diet would reduce energy consumption by 5% and carbon emissions by 13%. This is largely a result of reduced food expenditure since the greener diet has a *higher* aggregate energy and carbon intensity (i.e. food expenditure is reduced by more than food-related indirect energy consumption). But once the 15% saving on food expenditure is reallocated across the commodity groups, total energy consumption and carbon emissions are estimated to have *increased* by 2%. This suggests a rebound effect of 140% for energy consumption and more than 300% for carbon emissions– with increased spending on travel accounting for most of the increase (indeed, increased spending on travel alone takes back all of the energy saved from the greener diet).[8]

[8] Alfredsson find that the rebound effect for energy is greater than that for carbon emissions following a dietary change, while Lenzen and Dey find the rebound effect for greenhouse gas emissions is greater than that for energy. The difference is most likely due to the role of non-CO_2 GHGs and illustrates the importance of estimating and comparing all three measures.

Alfredsson's green travel scenario involves a mix of behavioural changes (e.g. increased use of public transport, walking and cycling) and technical changes (improved vehicle fuel efficiency) which are assumed to become gradually more ambitious over time. Energy and carbon savings are estimated for 2010, 2020 and 2050, assuming no change in either total expenditure or the energy/carbon intensity of commodities. The results reported in Alfredsson's tables suggest a rebound effect of ~80% for energy consumption and ~60% for carbon emissions, depending upon the year. However, the figures in the tables appear inconsistent with those in Alfredsson's graphs and text, where she reports a rebound effect of only 28% for energy and 12% for carbon. A very similar approach is followed for the 'green housing' scenario which includes behavioural and technical changes to reduce hot water and electricity consumption. This leads to estimated rebound effects of ~14% for energy consumption and ~20% for carbon. The reason why rebound effects are lower for green travel and green housing than for a green diet is that: first, the cost savings are smaller in absolute terms; and second, the cost savings are made in a relatively high energy/carbon intensity category and re-spent in relatively low intensity categories - whereas the reverse is the case for the green diet.

Alfredsson also combines all three sets of lifestyle changes in a single 'green lifestyle' scenario. When re-spending is allowed in all commodity categories (i.e. including those from which the savings were made), the estimated rebound effect is 33% for energy and 20% for carbon emissions. But when the re-spending is confined to the residual categories (i.e. excluding direct rebound effects), this reduces to 17% for energy and 7% for carbon emissions - since travel and housing are relatively energy/carbon intensive. Hence, the inclusion or exclusion of direct rebound effects can have a significant impact on the results.

The overall 'green lifestyle' scenario represents a comprehensive set of technical and behavioural changes that are estimated to reduce carbon emissions by 17% in 2010, 27% in 2020 and 30% 2050. Alfredsson examines how quickly these savings could be eroded by increases in income. She finds that income growth of 1%/year would more than offset all of the energy savings by 2020 and lead to a 5% increase in total energy consumption. Carbon emissions would still be 7% below the baseline in this scenario, but if incomes grew by 2%/year, carbon emissions would be 13% higher by 2020. In other words, even quite modest rates of income growth could rapidly erode the environmental benefits of fairly substantial technical and behavioural changes. These estimates are pessimistic, since the energy and carbon intensity of commodities is held fixed - implying no efficiency improvements or changes in fuel mix within the rest of the economy. Nevertheless, this result demonstrates the limitations of 'green consumption' within the context of a growing economy.

Alfredsson's results appear sensitive to the dataset and commodity classification that are used and the particular mix of technical and behavioural changes that are assumed. For example, a more recent study [62] using a similar model and approach, but employing Swedish rather than Dutch data on energy intensity, finds that a shift to 'green' food consumption could *reduce* overall energy consumption. Closer examination reveals that this result follows largely from the assumption that greener diets are more expensive (owing to the higher cost of locally produced and organic food), thereby leading to a negative income effect. These results highlight the importance of conducting sensitivity tests, using a high level of commodity disaggregation, exploring a range of consumption changes and comparing the rebound effects for energy, carbon and greenhouse gases.

4.5 Nassen and Holmberg

Nassen and Holmberg [59] develop generic equations for estimating direct and indirect rebound effects from both technical improvements and lifestyle changes. Using cross-sectional data for Swedish households, they use these equations to examine how rebound effects vary with variables such as the capital cost of the energy efficiency measure. Their approach combines the estimation of direct rebound effects for individual energy services with the estimation of indirect rebound effects for other goods and services. However, it relies in part upon exogenous information for the former.

The starting point is to assume that annual household expenditure (*x*) is divided between *N* 'goods' (*i*). One of these (*i=1*) represents an energy service such as heating that is affected by either a technical improvement or behavioural change. Nassen and Holmberg use the term *price effect* to refer to the direct rebound effect for this energy service because it derives from a behavioural response to the lower price of the energy service [10]. As described above, the price effect includes both substitution and income effects. The price effect is calculated using exogenous estimates for the own-price elasiticity of different energy services (ε_{11}) taken from Greening and Greene [9] and other sources.

Nassen and Holmberg use the term *income effect* to refer to the re-spending of the cost savings from the energy efficiency improvement since these are equivalent to an increase in real income. The income effect is calculated using estimates of the marginal propensity to consume (*MPC*) different goods and services, together with estimates of the energy intensity of those goods and services.

Nassen and Holmberg estimate the income effect for all goods and services, *including* the energy service itself. This would appear to lead to double counting, since the exogenous estimates of own-price elasticities for these energy services *already reflect* both substitution and income effects. Hence, Nassen and Holmberg's approach seems to double count the income effect for the energy service (*i=1*) and therefore to overestimate the direct rebound effect.

However, if the energy service was *excluded* when estimating income effects, the latter may be overestimated for *other* goods and services. This is because the actual increase in expenditure on these goods and services would be lower than this calculation assumes – since some of the cost savings would be spent on increased consumption of the energy service itself. Whether either method would lead to an under or overestimate of the total rebound effect will depend upon: first, the relative magnitude of substitution and income effects for the energy service and for other goods and services; and second, the ratio of the energy intensity of the energy service to that for other goods and services.

The source of these difficulties is that Nassen and Holmberg are combining an exogenous estimate of the direct rebound effect for individual energy services (i.e. substitution + income effect) with an endogenous calculation of income effects for all commodities. This contrasts with the studies by Alfredsson [57, 58] and Thiesen, *et al*. [56] which are confined to income effects alone and ignore substitution effects altogether. Arguably, the focus upon income effects is appropriate in these cases since only behavioural changes are being examined (although, as noted above, these do lead to changes in the unit price of aggregate commodity groups). But Nassen and Holmberg are primarily exploring the effect of energy efficiency improvements, so both substitution and income effects are clearly relevant. A better approach would be to use pooled cross-sectional data and estimate a system of equations that allowed both substitution and income effects to be simultaneously calculated. The studies by Brannlund *et al.* [18], Mizobuchi [19] and Kratena *et al.* [60, 61] discussed below attempt to do this, but they necessarily use a much higher level of commodity aggregation.

Nassen and Holmberg derive expressions for what they term the price effect (R_P), income effect (R_I) and total effect (R_T=R_P+R_I) as a function of four dimensionless parameters:

- ε_{11}: the own-price elasticity of the relevant energy service (i=1);
- β: the fractional reduction in energy consumption for the relevant energy service that would be expected in the absence of any direct rebound effect ($0 \le \beta \le 1$);
- $\bar{e}/e_1$:the ratio of the energy intensity of marginal consumption ($\bar{e}$) to the energy intensity of the relevant energy service (e_1). The energy intensity of marginal consumption is given by: $\bar{e} = \sum_i \frac{\partial w_i}{\partial x} e_i$ where $\frac{\partial w_i}{\partial x}$ is the marginal budget share of commodity i ($\sum_i \frac{\partial w_i}{\partial x} = 1$) and e_i is the energy intensity of that commodity.
- q/q_{BE}: the ratio of the annualised capital cost of the energy efficiency improvement (q) to the annualised capital cost that required for the investment to break even (q_{BE}). If $q/q_{BE} = 1$, this means the annualised capital cost is equal to the annual reduction in energy costs. If $q/q_{BE} = 0$ the capital cost is zero and if $q/q_{BE} < 0$ the efficiency improvement is cheaper than the inefficient alternative.

The relevant equation for the price effect, R_P is:

$$R_P = 1 - \frac{1}{\beta}\left[1 - (1-\beta)^{\varepsilon_{11}+1}\right] \quad (171)$$

And the relevant equation for the total effect, R_T is:

$$R_T = R_P + \frac{\bar{e}}{e_1}\left[1 - R_P - \frac{q}{q_{BE}}\right] \quad (172)$$

They also modify this equation to represent the total rebound effects from behavioural change (R_B) – such as lowering indoor temperatures or driving fewer miles. In this case, they assume that both the price effect (R_P) and the capital cost (q) are zero and there is no income effect for the energy service itself. These modifications lead to:

$$R_B = \frac{\sum_{i=2,N} w_i e_i}{(1-w_1)e_1} \quad (173)$$

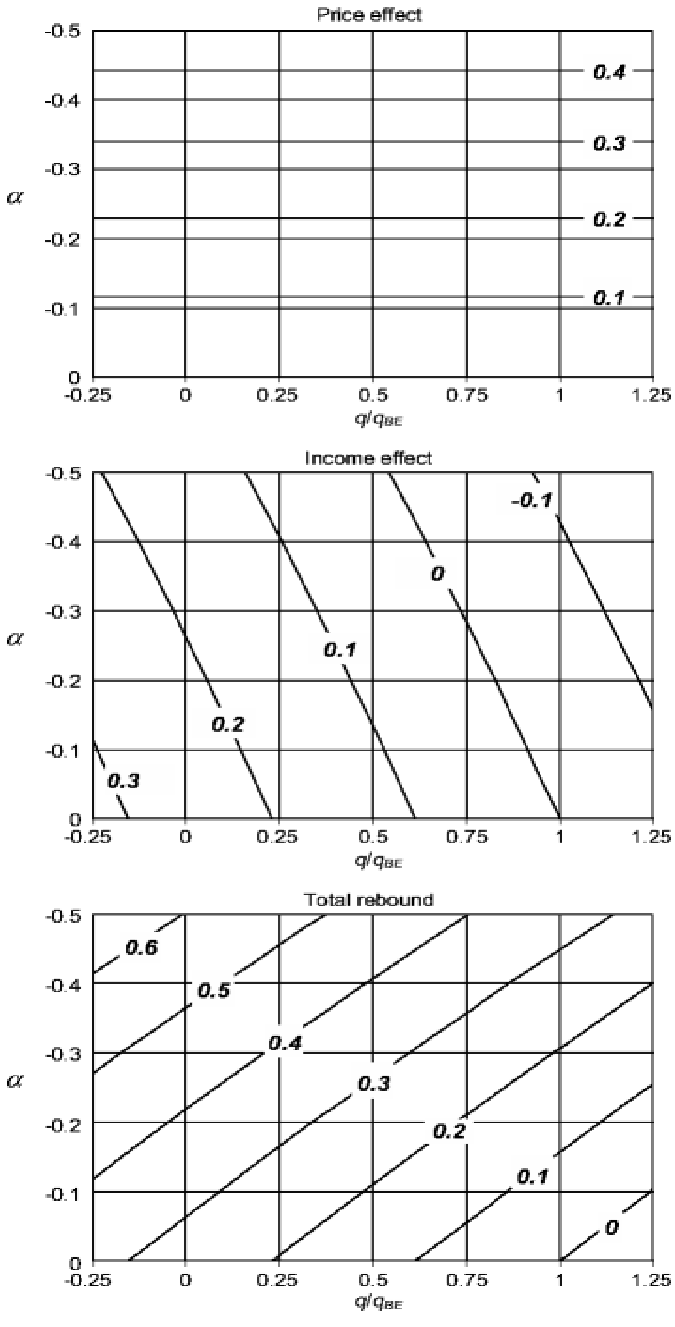
Using Swedish cross-sectional data on the household consumption and energy intensity of 42 commodities, Nassen and Holmberg estimate total rebound effects for three 'energy services' (e_1), namely electricity, heating and transport fuels. However, these are actually commodities (x_i) rather than services (s_i), so the relevant marginal budget shares ($\partial w_i / \partial x$) refer to commodities while the exogenous estimates of α refer to services (e.g. heating). So again, combining exogenous estimates for some variables with endogenous estimates for others creates difficulties.

Nassen and Holmberg estimate rebound effects for various combinations of α, β and q / q_{BE}. They find the results to be insensitive to β and therefore set this to 30% for their simulations, the results of which are summarised in Figure 10. They find rebound effects to be higher for price-elastic energy services (high R_P), and from investments that are either highly cost effective or have a negative capital cost (high R_I). Rebound effects are especially high when these two factors coincide. As an illustration, for $\alpha = -0.4$, the total rebound effect is estimated to be less than 10% for a relatively expensive efficiency improvement ($(q / q_{BE}) > 1$) and more than 50% for a negative cost improvement ($(q / q_{BE}) > 0$). An example of the former could be the purchase of a hybrid car, while an example of the latter could be shifting to a smaller car with lower fuel, capital and maintenance costs.

The rebound effect for energy conserving behaviour is estimated to be 9% for electricity, 14% for heating and 20% for transport fuels. These figures are relatively low since these energy services are relatively energy intensive and much of the re-spending is on goods and services with a lower energy intensity. But the converse also applies: reductions in the consumption of goods and services with a low energy intensity could lead to much larger rebound effects.

Overall, while Nassen and Holmberg's methodology has some important weaknesses, their exploration of the relative importance of different variables is helpful and the sensitivities and trade-offs they highlight (Figure 10) could be usefully explored within other models.

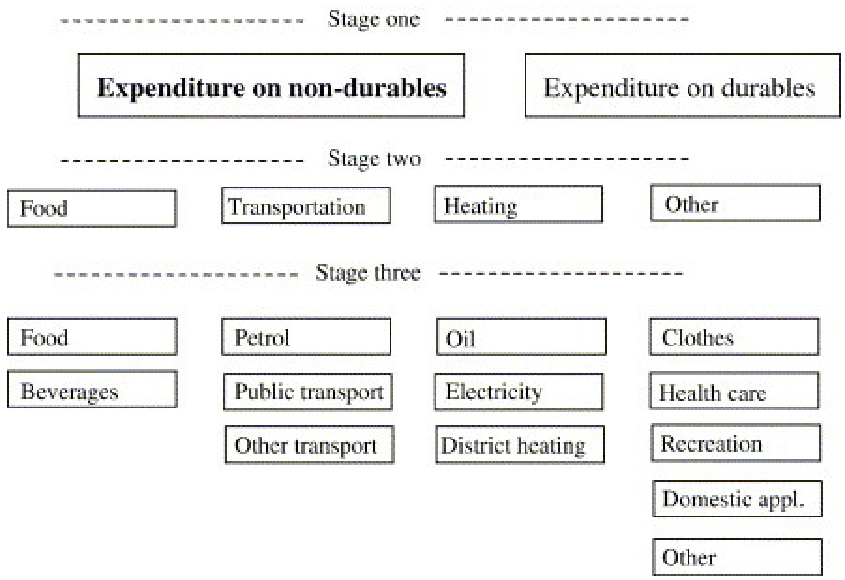
Figure 10 Sensitivity of the rebound effect to variations in energy service price elasticity (α) and cost effectiveness (q/q_{BE})



4.6 Brännlund *et al.*

Brännlund *et al.* [18] examine the effect on carbon emissions of a 20% improvement in the energy efficiency of personal transport (all modes) and space heating in Sweden. In contrast to the studies reviewed above, they use pooled cross-sectional data on Swedish household consumption covering the period 1980-1997 and are therefore able to estimate both expenditure and cross-price elasticities for aggregate commodity groups. They estimate an Almost Ideal Demand System for aggregate household expenditure (see Section 3.8) assuming the three stage budgeting process illustrated in Figure 11. In the first stage, total expenditure is allocated between durables and non-durables; in the second stage, non-durables expenditure is allocated between food, transportation, heating and other goods; and in the third stage, expenditure is allocated between a total of 13 individual categories of commodities distributed between these four groups. The seperability assumptions required to justify this approach are not discussed and the source of the emission coefficients for each commodity group is not mentioned.

Figure 11 Brännlund et al.'s three-stage budgeting model



Note: The text of Brannlund et al. refers to 'car transport' rather that 'petrol' and indicates that petrol is only one component of the expenditure in this category.

Using the notation of Section 3, the AIDS equation for the share of commodity group *r* in non-durable expenditure for year *t* (w_t^r) can be written as:

$$w_t^r = \alpha^r + \sum_j \gamma^{rs} \ln p_t^s + \beta^r \ln(x_t - \ln P_t) = 0 \qquad (174)$$

Where *r=1,..4* and $\ln P_t = \sum_r w_t^r \ln P_t^r$ is Stone's price index for the aggregate groups.

Similarly, the share of commodity *i* within the budget for group *r* in year *t* can be written:

$$w_{it}^r = \alpha_i^r + \sum_{j \in r} \gamma_{ij}^r \ln p_{jt}^r + \beta_i^r \ln(x_t^r / P_t^r) = 0 \qquad (175)$$

Where: $\ln P_t^r = \sum_r w_{it}^r \ln p_{it}^r$:

The AIDS model allows the expenditure share for the thirteen categories of non-durable goods and services to be expressed as a function of total expenditure, the price of each good or service and an overall price index. The adding-up, homogeneity and symmetry conditions are imposed and the expenditure and cross-price elasticities are calculated using the empirical formula summarised in Section

3.7.2 [29]. Brannlund *et al*. also test a quadratic AIDS model (QAIDS) [63], but find the (simpler) linear model performs comparably well.

Brannlund *et al*. use the model to estimate the effect of a 20% improvement in the 'energy efficiency' of each commodity within the transport and heating groups (namely car transport, public transport, other transport, electricity, oil and district heating). The price of each commodity is reduced in accordance with assumptions about the proportion of fuel costs in total costs. For example, since petrol forms approximately half the costs of car travel, the price of car transport is reduced by 10%. The use of the term 'heating' is misleading, since this category actually represents total direct energy consumption and therefore includes non-heating end-uses. Also, Brannlund *et al*. model a reduction in the price of the energy commodity groups rather than the corresponding energy services and in the case of 'heating' this amounts to a reduction in the unit price of the relevant energy carriers. Furthermore, these price reductions are assumed to be costless.

Brannlund *et al*. then recalculate the expenditure share of each commodity by inserting the adjusted prices into the estimated AIDS equations, together with the adjusted price index. As an illustration, the reduction in the price of the transport commodities leads to a smaller expenditure share for the transport group, an increase in transport demand and a shift towards car transport. Total carbon emissions are then calculated and compared with those that would have occurred in the absence of any rebound effects.

The results of the simulations are rather confusingly presented, but the most notable result is that both scenarios lead to an *increase* in emissions (i.e. backfire). Rebound effects are not presented in the conventional way, but instead as the percentage difference in carbon emissions between the two scenarios. So for example, the transport scenario is estimated to reduce carbon emissions by 6.2% in the absence of rebound effects but to increase carbon emissions by 1.3% once rebound effects are accounted for. Brannlund *et al*. report this as a rebound effect of 7.5%, whereas in fact it corresponds to a rebound effect of 121% (i.e. backfire).

In the absence of rebound effects, the heating scenario is estimated to reduce carbon emissions by 4.1%, but allowing for the latter leads to a 3.1% increase in emissions –corresponding to a rebound effect of 175%. Combining the transport and heating efficiency improvements in a single scenario leads to a 4.7% increase in carbon emissions and a rebound effect of 140%. These estimates are remarkably high given that economy-wide adjustments are not considered. Indeed, the direct rebound effects *within* the transport and heating sectors are estimated to be greater than 100%. Brannlund *et al*. do not report the rebound effects for energy consumption, so it is not clear whether comparably large estimates would be obtained here. If they were, these results would not only contradict the results of a large body of work estimating direct rebound effects for household heating and transport [4], but also the results of an even larger body of work estimating the own-price elasticity of energy consumption within these sectors [64].

This suggests a flaw in Brannlund *et al*.'s methodology, but the source of the difficulties is unclear. Possibilities include the modelling of commodities rather than energy services, the relatively high level of commodity aggregation (implying, for example, simultaneous improvements in the energy efficiency of public and private transport), the neglect of the capital and other costs associated with improving energy efficiency and the difficulties with the seperability and other assumptions that underpin the AIDS model. Also, the source of the carbon emission coefficients is not made explicit and comparable results are not presented for energy consumption.

Nevertheless, Brannlund *et al.*'s paper provides an important (and widely cited) contribution and their approach could usefully be repeated using other data sources, differing levels of commodity aggregation and other relevant variations.

4.7 Mizobuchi

The study by Mizobuchi [19] is very similar to Brännlund *et al.* [18] and is intended to address some of the weaknesses of the latter - in particular, the neglect of capital costs. Mizobuchi [19] estimates a two-stage linear approximate AIDS model for total expenditure by Japanese households using monthly time series data for the period January 1990 to December 1998. In the first stage, total expenditure is allocated between four aggregate groups (*r*), namely food, fuel and light, transport and other; while in the second stage, expenditure is allocated between a total of 13 individual commodity categories (*j*) distributed between these four groups (Figure 12). A key difference from the Brännlund *et al* study is that the expenditures within each of these subcategories include both durables and non-durables and hence include expenditure on relevant capital equipment, such as electrical appliances, boilers, and vehicles. The prices for each of these subcategories (p_{jt}^{r}) is formed as follows:

$$p_{jt}^{r} = w_{jt}^{r}(N) * p_{jt}^{r}(N) + w_{jt}^{r}(K) * p_{jt}^{r}(K) \qquad (176)$$

Where *K* and *D* refer to the durable and non-durable components of expenditure for commodity *j* in group *r*. As with Brännlund *et al.*, the seperability assumptions required to justify this approach are not discussed.

Figure 12 Mizobuchi's two-stage budgeting model

************************** Main Group **************************

FOOD	FUEL & LIGHT	TRANSPORT	OTHER

************************** Sub Group **************************

Food	Electricity	Car transport	Clothing
Eating-out	Gas	Public transport	Medical care
	Heating oil	Other transport	Recreation
			Communication
			Miscellaneous

Mizobuchi uses a Bayesian estimation method for estimating the parameters of the AIDS model and calculates the within-group and total elasticities using the empirical formula summarised in Section 3.7.2 [29]. He also tests log-linear and translog specifications, but finds the AIDS model performs best. He then uses the model to explore the effect of *simultaneous* energy efficiency improvements in all of the 'fuel and light' subcategories (electricity, gas and oil) and in the car transportation subcategory. These improvements are modelled as a reduction in the price of the *non-durable* component of those commodities. In contrast to Brännlund *et al.*, the price of other transportation modes is left unchanged and the assumed improvement in energy efficiency is assumed to vary from one commodity to another - namely 20% for electricity and car transport, 10% for gas and 3% for oil. Also, the results are only presented for the combination of efficiency improvements, and not for each in isolation. These features make Mizobuchi's results difficult to compare with those of Brannlund *et al.*.

Mizobuchi models a percentage improvement in energy efficiency for commodity *j* (ψ_j)as a comparable reduction in the price of non-durable expenditures within that commodity group (i.e. $p^r_{jt}(N)*(1-\psi_j/100)$). But this improvement is assumed to require additional capital expenditure ($p^r_{jt}(K)*(1+\theta_j/100)$). Mizobuchi estimates the percentage increase in capital expenditure for commodity *j* (θ_j) that is required to achieve a percentage improvement in energy efficiency of commodity *j* (ψ_j) by comparing the energy efficiency and capital cost of different models of relevant capital equipment in Japan (e.g. vehicles, air conditioners, TVs) and taking a simple average of the difference. For example, for *n* models of a particular durable that forms part of commodity *j*:

$$\theta_j = \frac{1}{n}\sum_{i=1,n}\left[(increase\_price_i)/increase\_efficiency_i)\right]*\psi_j \qquad (177)$$

This somewhat crude procedure leads to the conclusion that the efficiency improvements indicated above require a 22% increase in expenditure on durables for electricity, 35% for gas, 12% for heating oil and 28% for vehicles. The new price of each of these commodities is then estimated as follows:

$$p^r_{jt} = w^r_{jt}(N)*p^r_{jt}(N)*(1-\psi_j/100)+w^r_{jt}(K)*p^r_{jt}(K)*(1+\theta_j/100) \qquad (178)$$

In other words, the reduction in energy (non-durable) costs within each category is offset by an increase in capital (durable) costs – although in each case the overall cost is estimated to fall.

The revised prices (p^r_{jt}) are then used to recalculate the Stone's price index and both the revised prices and the revised index are substituted into the budget share equations using the estimated values for the parameters. This allows the new expenditure shares to be calculated for each of the 13 commodity groups (i.e. allowing for the efficiency improvements). However, since these expenditure shares are used in calculating the Stone's price index, the latter will also change. Hence, Mizobuchi uses an *iterative* procedure to obtain the final values of the expenditure shares for each commodity group – and argues that the failure of Brännlund *et al.'s* to do this is an important weakness.

Finally, Mizobuchi uses the revised expenditure shares to estimate the associated carbon emissions and compares these with the emissions that would have occurred in the absence of any rebound effects. This allows the percentage rebound effect for the combined set of energy efficiency improvements to be estimated. He concludes that the rebound effect is 27%[9] when the additional capital costs are allowed for, compared to 115% (i.e. backfire) when they are ignored. This result is used to support Henly *et al.*'s conclusion [16] that studies that neglect capital costs lead to biased results.

Despite the sophistication of the econometric techniques employed, Mizobuchi's study has some important flaws. The method of calculating additional capital costs is crude and leads, for example, to the odd result that fuel-efficient cars are more expensive than inefficient cars. This is because newer and more fuel-efficient cars of

[9] Mizobuchi quotes this result to three decimal places!

a particular model type are more expensive than older and less efficient cars of the same type, and Mizobuchi takes a simple average of these differences. But this neglects the differences in cost and fuel efficiency *between* model types in the same year and in particular between different sizes of vehicle. Also, the capital costs are modelled in a static sense, with accounting in any way for the dynamics of stock adjustment.

More importantly, the method of allowing for capital costs simply amounts to adjusting the assumed percentage reduction in the total cost of a particular commodity category. Once capital costs are allowed for, the percentage reduction is less than would otherwise be the case. That being the case, it seems odd that the estimated rebound effects are larger when the assumed cost reduction is smaller (absent non-linear responses, the percentage rebound effect should be independent of the percentage cost savings). The answer appears to lie in the way in which the cost reductions are calculated. Not only does the assumed percentage improvement in energy efficiency (i.e .the percentage reduction in non-durable costs) vary between the four commodity categories, but so does the assumed percentage increase in capital costs. So the scenario *without* accounting for capital costs amounts to assuming an *a*% change in the price of electricity, *b*% for gas, *c*% for heating oil and *d*% for vehicles, while the scenario accounting for capital costs amounts to assuming an *a'*% change in the price of electricity, *b'*% for gas, *c'*% for heating oil and *d'*% for vehicles. The ratio of *a* to *a'* is different from the ratio of *b* to *b'* and so on. Hence, the difference in results between these two scenarios is most likely attributable to the differences in the assumed cost reductions in each category. Since only the total rebound effect is reported and not the rebound effects for each individual efficiency improvement, this point is obscured.

In summary, Mizobuchi's methodology appears to be flawed and does not fully justify his conclusions. While he is correct highlight the importance of capital costs for rebound effects, a more sophisticated model is likely to be required to accurately quantify their importance.

4.8 Kratena and Wuger

Kratena and Wueger [61] is one of a number of working papers from the Austrian Institute of Economic Research [e.g. 60, 61, 65] that use systems of consumer demand equations to examine household energy consumption. This particular study is notable in that: first, it attempts to include energy services rather than energy commodities within a consumer demand model; second, it links this to a second model of the energy efficiency of the capital stock; and third, it shows how the capital expenditure required to improve energy efficiency can significantly modify the rebound effect – and hence how models that neglect capital costs may lead to biased results.

In contrast to the other studies discussed above, Kratena and Wuger do *not* use estimates of the embodied energy of different categories of goods and services and hence neglect many indirect rebound effects. However, they *do* model the effect of improving the energy efficiency of one type of energy service (e.g. car travel) on the energy consumption for another energy service (e.g. heating). Hence, their model does capture some important indirect rebound effects - namely those associated with the consumption of other energy commodities. Since these are significantly more energy intensive than other goods and services, they could account for a significant proportion of the total indirect rebound effect. The general approach appears promising therefore, and could be extended to include all relevant indirect effects.

However, the paper provides insufficient explanation of several key steps and has yet to be published in a peer-reviewed journal.

Kratena and Wueger [61] use data for US households over the period 1972 to 2005 and assume that consumer demand for durables is separable from that for non-durables. For the latter, they use a high level of aggregation, distinguishing between two non-energy commodities (food and clothing) and three energy commodities (motor fuels, heating fuels and electricity). But rather than using commodity prices (P_E) for the latter, they develop indices of the corresponding 'energy service' prices (P_S). These are derived using indices of the energy efficiency (π_S) of the relevant capital stock for each of the energy commodities. Hence, their model effectively represents 'motor fuel services', 'heating services' and 'electricity services', with each corresponding to the *collective* use of the relevant energy commodities.

The energy efficiency indices rely on data from a number of sources, including the US Department of Transport and the Lawrence Berkeley Laboratory. Separate indices are developed for passenger cars, oil heaters, gas heaters, air conditioning equipment, refrigerators, freezers, clothes washers and dishwashers and these are transformed into an aggregate efficiency indices for the three energy services (π_S where S=1,3) using data on the fuel mix for each appliance 'for some base years'. The results suggest that energy service prices have grown more slowly than energy prices as a result of continuing improvements in energy efficiency, especially for motor vehicles. However, the authors provide insufficient detail on these calculations and it is not clear whether and how key variables such as the pattern of ownership of different appliances (e.g. the number of households with air-conditioners) and the capacity of individual appliances (e.g. the average size of refrigerators) are taken into account. In addition, some important types of energy-using capital equipment are ignored altogether - most notably lighting which is a dominant user of electricity. Hence, the accuracy of the efficiency indices and hence the corresponding service prices appears questionable.

Demand for the five categories of non-durables (i.e. food, clothing and the energy services associated with gasoline, heating fuels and electricity) is estimated with a quadratic version of the AIDS model (QUAIDS) [63], leading to to estimates for the compensated own-price elasticities for the three energy services (ε_{ii}). These can be interpreted as 'partial' estimates of the direct rebound effects for these energy services, since they neglect *both* the effect of price changes on the total expenditure for non-durables *and* the capital costs associated with achieving the energy efficiency improvements. The results suggest a (partial) direct rebound effect of 13% for motor fuels, 19% for heating fuels and 18% for electricity. Kratena and Wueger state that these estimates "...are within the range found in the literature", but since they are partial estimates the comparison is invalid.

The results suggest that heating fuels and motor fuels are inferior goods (i.e. they have a negative expenditure elasticity) while electricity is a highly elastic normal good ($\eta_i = 4.47$). However, the *total* expenditure elasticity of these services is the product of the estimated (within-group) elasticities and the income elasticity of total non-durable consumption – which is not derived. Exogenous estimates suggest that the latter is around 0.5 which implies that the total expenditure elasticities are around half of those estimated. Kratena and Wueger provide little comment on these results, but the conclusion that 'motor fuel services' is an inferior good appears particularly odd.

The results also suggest that heating services are gross substitute for motor fuel services and a gross complement to electricity services. This implies that an increase in heating efficiency would *reduce* the demand for motor fuel services and *increase* the demand for electricity services, so the indirect rebound effect for motor fuel would be negative while that for electricity would be positive (Table 2). Electricity services and motor fuel services are found to be gross complements which implies that an increase in electricity efficiency would increase the demand for motor fuel services (and vice versa). Kratena and Wueger do not use budget shares to estimate the resulting total rebound effect at this stage, but the magnitude of the cross-price effects between the energy services suggests that the associated indirect rebound effects should be significant.

Improvements in energy efficiency generally require investment in new capital equipment. This could reduce the money available for expenditure on non-durable commodities and thereby have an income effect on the consumption of energy services. The sign of the estimated expenditure elasticities above suggests that this income effect should *reduce* the rebound effect for improvements in electricity efficiency, but *increase* it for improvements in motor fuel and heating efficiency. The former possibility was first highlighted by Henly *et al.* [16] while the latter (counterintuitive) possibility results from the fact that motor fuel services and heating services are found to be inferior goods. Hence, estimates of the 'full' rebound effect need to take this into account.

To explore this effect, Kratena and Wueger first estimate equations for the energy efficiency of the capital stock for each energy service (π_S) using an 'autoregressive distributed lag' (ADL) model. This specifies energy efficiency as a function of the lagged values of the dependent variable, the capital stock (K_s) and the price of the relevant energy commodity (P_E). This formulation is intended to reflect both embodied and price-induced technical change.

$$\ln \pi_{S,t} = \kappa_0 + \sum_{\tau=1,l} \psi_\tau \ln \pi_{S,t-\tau} + \sum_{\tau=0,n} \theta_\tau \ln K_{S,t-\tau} + \sum_{\tau=1,m} \phi_\tau \ln P_{E,t-\tau} \tag{179}$$

The ADL model allows the long-run elasticities of energy efficiency with respect to the capital stock ($\varepsilon_{\pi_S,K}$) and energy prices (ε_{π_S,P_E}) to be estimated. The former are found to be significant for each energy service, but energy price is only found to be significant for motor fuel services. However, Kratena and Wueger do not clarify how the capital stock variable (K) is defined and measured and again provide insufficient information on how the time series' for energy efficiency is estimated. For example, they note that "...the efficiency of electricity is related to stock of audio and video goods/computers", but these only account for a relatively small proportion of total electricity demand.

Kratena and Wueger then use model simulations to estimate the 'full' impact of energy efficiency improvements. The simulations assume that the energy efficiency of both motor fuel services and electricity services increases by 10% more than was observed over the period 1990 to 2005, while that of heating services increases by 5% more. The procedure is then to: a) use the ADL model to calculate the additional capital accumulation that would be required to achieve this efficiency improvement; b) calculate the additional capital expenditure required and the corresponding reduction in non-durable expenditure; d) use the results of the QUAIDS model to estimate the resulting change in energy service and total energy consumption compared to the baseline; and e) estimate the resulting direct and total rebound effects. Once again, insufficient detail is provided on each step of this process and important points such as how capital expenditure is estimated remain very unclear.

The results of the simulation and are summarised in
Table 7. They suggest that the additional capital expenditure required to improve motor fuel and electricity efficiency by 10% by 2005 would reduce total expenditure on non-durables by nearly 10% - although for heating the effect is much lower. The differences result from the different long-run elasticities of energy efficiency with respect to the capital stock changes, the different magnitude of investment required for each energy service compared to total nondurable expenditure and the different assumptions for the percentage increase in energy efficiency.

The results suggest that the indirect effects of the energy efficiency improvements on other energy commodities can be both significant and counterintuitive. For example, a 10% increase in electricity efficiency leads to a 2.5% increase in consumption of all energy services (measured in kWh). The method by which rebound effects are calculated is not clear, but the results suggest a total rebound effect of **86%** following a 10% efficiency improvements in motor fuel services, **37%** following a 5% efficiency improvement in heating services and **-39%** following a 10% efficiency improvement in electricity services. The latter result indicates an additional reduction in energy consumption over above that caused by the 'engineering' improvement in electricity efficiency ('super conservation'). This derives from indirect effects, since the direct rebound effect from these improvements is estimated to be positive (+18%). The primary source of the additional reduction in energy consumption is the capital expenditure required to improve electricity efficiency, which reduces overall expenditure on nondurable commodities by ~10%. Improving the efficiency of motor fuel services leads to approximately the same reduction in non-durable expenditure, but in this case the indirect rebound effect is positive and contributes to a total rebound effect of 86%. The primary reason for this is a 12.3% increase in heating service consumption.

Table 7 Kratena and Wueger's estimates of the 'full' impact of energy efficiency improvements

Difference from baseline	Motor fuel services (+10% efficiency)	Heating services (+5% efficiency)	Electricity services (+10% efficiency)
Total nondurable consumption	-9.3	-0.3	-9.7
Motor fuel service consumption	-1.4	0.0	9.3
Heating service consumption	12.3	-3.2	19.6
Electricity service consumption	-1.9	-1.1	-13.9
Total energy service consumption	0.5	-0.8	3.5
Direct rebound effect	0.135	0.192	0.179
Total rebound effect	0.857	0.365	-0.385

As Kratena and Wueger observe:

> "....indirect effects can only be measured via model simulations and lead to results that are unpredictable from the estimates of elasticities. The most important result might be that an isolated increase in efficiency in one energy using category leads to considerable changes in the demand of other energy uses. Taking into account the necessary capital expenditure, efficiency improvements might decrease the rebound effect... or increase it" [61 p. 24]

Overall, Kratena and Wueger's approach appears promising and leads to some surprising results. It is unfortunate, therefore, that the paper includes insufficient explanation of both the methodology and the results.

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