

Mapping relativistic to ultra/non-relativistic conformal symmetries in 2D and finite $\sqrt{T\bar{T}}$ deformations

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ABSTRACT: The conformal symmetry algebra in 2D ($\text{Diff}(S^1) \oplus \text{Diff}(S^1)$) is shown to be related to its ultra/non-relativistic version ($\text{BMS}_3 \approx \text{GCA}_2$) through a nonlinear map of the generators, without any sort of limiting process. For a generic classical CFT_2 , the BMS_3 generators then emerge as composites built out from the chiral (holomorphic) components of the stress-energy tensor, T and \bar{T} , closing in the Poisson brackets at equal time slices. Nevertheless, supertranslation generators do not span Noetherian symmetries. BMS_3 becomes a bona fide symmetry once the CFT_2 is marginally deformed by the addition of a $\sqrt{T\bar{T}}$ term to the Hamiltonian. The generic deformed theory is manifestly invariant under diffeomorphisms and local scalings, but it is no longer a CFT_2 because its energy and momentum densities fulfill the BMS_3 algebra. The deformation can also be described through the original CFT_2 on a curved metric whose Beltrami differentials are determined by the variation of the deformed Hamiltonian with respect to T and \bar{T} . BMS_3 symmetries then arise from *deformed conformal Killing equations*, corresponding to diffeomorphisms that preserve the deformed metric and stress-energy tensor up to local scalings. As an example, we briefly address the deformation of N free bosons, which coincides with ultra-relativistic limits only for $N = 1$. Furthermore, Cardy formula and the S-modular transformation of the torus become mapped to their corresponding BMS_3 (or flat) versions.

KEYWORDS: Conformal and W Symmetry, Space-Time Symmetries

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1 Introduction

Conformal symmetries enhance those of special relativity, and become pivotal in the description of generic relativistic systems enjoying scale invariance. Conformal field theories (CFT's), built in terms of these extended symmetries, are well-known to play a fundamental role in a broad variety of subjects. Their power turns out to be particularly impressive in two spacetime dimensions, as a direct consequence of the fact that the conformal group exceptionally becomes infinite-dimensional. The conformal algebra in 2D is described by two copies of the Witt or centerless Virasoro algebra, being isomorphic to two copies of the algebra of diffeomorphisms on the circle ($\text{Diff}(S^1) \oplus \text{Diff}(S^1)$), spanned by

$$[L_m, L_n] = (m - n) L_{m+n} \quad , \quad [\bar{L}_m, \bar{L}_n] = (m - n) \bar{L}_{m+n} \quad , \quad (1.1)$$

with $[L_m, \bar{L}_n] = 0$ and $m, n \in \mathbb{Z}$.

Another interesting accident that occurs in 2D is that the ultra and non-relativistic limits of the conformal algebra are isomorphic (see e.g., [1]). Intuitively, ultra/non-relativistic limits are such that the light cone tends to shrink towards the vertical/horizontal axis, and so one limit can be attained from the other by swapping the role of time and space coordinates. As an additional curiosity, the ultra/non-relativistic algebra also becomes isomorphic to the Bondi-Metzner-Sachs one in 3D without central extensions [2–4]. In other words, the so-called Galilean Conformal and Conformal Carrollian algebras in 2D turn out to be isomorphic to the BMS₃ algebra ($\text{GCA}_2 \approx \text{CCA}_2 \approx \text{BMS}_3$), given by the semidirect sum of the Witt algebra and supertranslations:

$$[J_m, J_n] = (m - n) J_{m+n} \quad , \quad [J_m, P_n] = (m - n) P_{m+n} \quad , \quad (1.2)$$

where $[P_m, P_n] = 0$. The algebra (1.2) and its centrally-extended version appeared long ago in the context of the tensionless limit of string theory [5–7], and more recently in the “flat” analog of Liouville theory [8, 9] as well as in fluid dynamics and integrable systems in 2D [10–12]. It also plays a leading role for nonrelativistic and flat holography [13, 14], and it emerges from the spacetime structure near generic horizons [15–19]. Induced, coadjoint and unitary representations have also been developed in [20–22].¹

It is worth emphasizing that the conformal algebra in 2D (1.1) and the BMS_3 algebra (1.2) are not isomorphic. Nevertheless, the latter can be obtained from the former through suitable Inönü-Wigner contractions. Indeed, changing the basis of the conformal algebra (1.1) according to

$$P_m = \ell^{-1} (L_m + \bar{L}_{-m}) \quad , \quad J_m = L_m - \bar{L}_{-m} \quad , \quad (1.3)$$

one recovers the BMS_3 algebra (1.2) in the limit $\ell \rightarrow \infty$ (see e.g., [9, 40]). Alternatively, the following change of basis: $P_m = \ell (L_m - \bar{L}_m)$, $J_m = L_m + \bar{L}_m$ yields the same result provided that $\ell \rightarrow 0$ [13, 41]. The parameter ℓ can then be naturally identified with the inverse of the speed of light.

2 Map between relativistic and ultra/non-relativistic conformal algebras in 2D

Intriguingly, the conformal symmetry algebra in 2D (1.1) can be shown to be related to its ultra/non-relativistic version (1.2) by means of a precise nonlinear map of the generators, without the need of performing any sort of limiting process.

In order to explicitly see the map it is useful to work in the continuum, so that the generators of the conformal algebra (1.1) can be traded by two arbitrary periodic functions defined on the circle, according to $L_m = \int d\phi \bar{T}(\phi) e^{-im\phi}$, $\bar{L}_m = \int d\phi T(\phi) e^{im\phi}$. Thus, the conformal algebra is equivalently expressed as

$$\begin{aligned} \{T(\varphi), T(\theta)\} &= (2T(\varphi) \partial_\varphi + \partial_\varphi T(\varphi)) \delta(\varphi - \theta) \quad , \\ \{\bar{T}(\varphi), \bar{T}(\theta)\} &= -\left(2\bar{T}(\varphi) \partial_\varphi + \partial_\varphi \bar{T}(\varphi)\right) \delta(\varphi - \theta) \quad , \end{aligned} \quad (2.1)$$

with $\{T(\varphi), \bar{T}(\theta)\} = 0$, and $[\cdot, \cdot] = i\{\cdot, \cdot\}$. Note that the continuous version of the conformal algebra (2.1) can be naturally interpreted as a Poisson structure.

The searched for mapping is then defined as follows

$$P = T + \bar{T} + 2\sqrt{T\bar{T}} \quad , \quad J = T - \bar{T} \quad , \quad (2.2)$$

so that the corresponding brackets involving J and P can be readily found by virtue of the “fundamental” ones in (2.1), which exactly reproduce the continuous version of the BMS_3 algebra, given by

$$\begin{aligned} \{J(\varphi), J(\theta)\} &= (2J(\varphi) \partial_\varphi + \partial_\varphi J(\varphi)) \delta(\varphi - \theta) \quad , \\ \{J(\varphi), P(\theta)\} &= (2P(\varphi) \partial_\varphi + \partial_\varphi P(\varphi)) \delta(\varphi - \theta) \quad , \end{aligned} \quad (2.3)$$

¹The algebra (1.2) also manifests as nonlocal symmetries of a massless Klein-Gordon field in 3D [23]. Different extensions of the BMS_3 algebra have been constructed in [24–39].

with $\{P(\varphi), P(\theta)\} = 0$. In Fourier modes, $J_m = \int d\phi J(\phi) e^{im\phi}$, $P_m = \int d\phi P(\phi) e^{im\phi}$, the algebra (2.3) then reduces to (1.2).

Bearing in mind that supertranslation generators are defined up to a global scale factor, making $P \rightarrow \alpha P$ with constant α in the map (2.2), yields the same result. Thus, for simplicity and later convenience, we keep assuming $\alpha = 1$ afterwards.

In sum, the nonisomorphic conformal and BMS_3 algebras, in (2.1) and (2.3) respectively, are nonlinearly related by virtue of the map defined through (2.2), and it is worth highlighting that no limiting process is involved in the mapping.

3 BMS_3 generators within CFT_2

The mapping in (2.2) naturally makes one wondering about how precisely the BMS_3 algebra manifests itself for a generic (nonanomalous) classical CFT_2 . Indeed, the mapping directly prescribes a way in which BMS_3 generators emerge as composites of those of the conformal symmetries. Nonetheless, it can be shown that the composite generators do not span a Noetherian symmetry of the CFT_2 .

In order to see that, let us consider a generic CFT_2 on a cylinder. In the conformal gauge, using null coordinates $x = t + \phi$ and $\bar{x} = t - \phi$, the canonical generators of the conformal symmetries are given by

$$\mathcal{Q}_{\text{CFT}}[\epsilon, \bar{\epsilon}] = \int d\phi \left(\epsilon T + \bar{\epsilon} \bar{T} \right), \quad (3.1)$$

being conserved ($\dot{\mathcal{Q}}_{\text{CFT}} = 0$) by virtue of the (anti-)chirality of the components of the stress-energy tensor and the parameters ($\partial \bar{T} = \bar{\partial} T = \partial \bar{\epsilon} = \bar{\partial} \epsilon = 0$). The transformation laws of T and \bar{T} then read from the conformal algebra (2.1), since $\delta_{\eta_1} \mathcal{Q}_{\text{CFT}}[\eta_2] = \{\mathcal{Q}_{\text{CFT}}[\eta_2], \mathcal{Q}_{\text{CFT}}[\eta_1]\}$ with $\eta_i = (\epsilon_i, \bar{\epsilon}_i)$, so that

$$\delta T = 2T\partial\epsilon + \partial T\epsilon, \quad \delta \bar{T} = 2\bar{T}\bar{\partial}\bar{\epsilon} + \bar{\partial}\bar{T}\bar{\epsilon}. \quad (3.2)$$

The nonlinear map (2.2) implies that the generators (3.1) and transformation laws (3.2), can be expressed as

$$\mathcal{Q}_{\text{CFT}}[\epsilon, \bar{\epsilon}] = \int d\phi \left(\epsilon_J J + \epsilon_P P \right), \quad (3.3)$$

$$\delta P = 2P\epsilon'_J + P'\epsilon_J, \quad \delta J = 2P\epsilon'_P + P'\epsilon_P + 2J\epsilon'_J + J'\epsilon_J, \quad (3.4)$$

where prime stands for ∂_ϕ , while the parameters ϵ_J, ϵ_P , relate to ϵ and $\bar{\epsilon}$ through

$$\epsilon = \epsilon_J + \left(1 + \sqrt{\frac{\bar{T}}{T}} \right) \epsilon_P, \quad \bar{\epsilon} = -\epsilon_J + \left(1 + \sqrt{\frac{T}{\bar{T}}} \right) \epsilon_P. \quad (3.5)$$

Thus, the generators and transformation laws in (3.3), (3.4), acquire the expected form of those for the BMS_3 algebra (see e.g., [8, 11]).²

²A warning note is in order: if the parameters ϵ and $\bar{\epsilon}$ were still assumed to be chiral, this would be just a mirage; because in that case, the new ones, ϵ_J, ϵ_P , would become state-dependent, and hence, this would only amount to an alternative way of expressing the original conformal algebra generators and transformation laws in (3.1), (3.2), in terms of different variables.

Legitimate BMS₃ generators are obtained when the parameters ϵ , $\bar{\epsilon}$ are no longer chiral, but instead, being manifestly state-dependent according to (3.5). Hence, at fixed time slices, the parameters ϵ_J , ϵ_P can be consistently assumed to be state-independent arbitrary functions, so that the Poisson brackets of the generators

$$\tilde{Q}[\epsilon_J, \epsilon_P] = \int d\phi (\epsilon_J J + \epsilon_P P) , \quad (3.6)$$

clearly close according to the BMS₃ algebra by virtue of (2.3).

It is worth emphasizing that since J stands for the momentum density, superrotation generators yield the corresponding conserved charges. Nevertheless, supertranslation generators are not conserved, as it can be seen from the time evolution of P , that can be obtained from that of the (anti-)chiral T and \bar{T} by virtue of the map (2.2), given by

$$\dot{P} = 2J' - J(\log P)' . \quad (3.7)$$

Therefore, supertranslations do not correspond to Noetherian symmetries of the CFT₂.

4 BMS₃ symmetries from $\sqrt{T\bar{T}}$ deformations

According to the map (2.2), the supertranslation density P can be seen as a finite nontrivial marginal deformation of the CFT₂ energy density $H = T + \bar{T}$. Hence, a simple way to achieve conservation of supertranslations consists in deforming the original Hamiltonian of the CFT₂ to coincide with the supertranslation generator. Thus, starting from the CFT₂ in the conformal gauge, the simplest deformation is implemented through the Hamiltonian density $\tilde{H} = H + 2\sqrt{T\bar{T}} = P$, so that the deformed action reads

$$\tilde{I} = I_{\text{CFT}} - \int dx d\bar{x} \sqrt{T\bar{T}} . \quad (4.1)$$

Note that since only the Hamiltonian was deformed, the Poisson brackets remain the same as those of the original CFT₂ in (2.1). Hence, the time evolution of supertranslation and superrotation densities can be readily obtained from (2.3)

$$\dot{P} = \{P, \tilde{\mathcal{H}}\} = 0 , \quad \dot{J} = \{J, \tilde{\mathcal{H}}\} = P' ,$$

with $\tilde{\mathcal{H}} = \int d\phi P$; so that the canonical BMS₃ generators (3.6) are now manifestly conserved ($\dot{\tilde{Q}} = 0$) provided that the parameters fulfill $\dot{\epsilon}_P = \epsilon'_J$ and $\dot{\epsilon}_J = 0$, being apparently state independent.

Therefore, the BMS₃ generators (3.6) span a bona fide Noetherian symmetry of the deformed action (4.1).

It is also worth pointing out that the deformed theory (4.1) retains the integrability properties of the original CFT₂, since the universal enveloping algebra of BMS₃ also contains an infinite number of independent commuting (KdV-like) charges [11].³

³This last property resembles that of the $T\bar{T}$ deformation [42–44] (being widely studied in e.g., [45–50]); nonetheless, some differences must be stressed. Indeed, in that case the conformal weight of the deformation implies that it is an irrelevant one, also depending on a single continuous parameter, and where T and \bar{T} stand for those of the deformed theory; while in our case, they correspond to those of the original CFT₂ and yield to a rigid finite marginal deformation.

For a generic gauge choice, the deformation (4.1) can be written as

$$\tilde{I} = I_{\text{CFT}} - \int d^2x \sqrt{\det T_{\mu\nu}} . \quad (4.2)$$

where it is implicitly assumed that I_{CFT} is written in Hamiltonian form, and $T_{\mu\nu}$ stands for the stress-energy tensor of the undeformed CFT_2 . Remarkably, the action (4.2) keeps being invariant under diffeomorphisms and local scalings, but it is no longer a CFT_2 because the energy and momentum densities of the deformed theory yield to generators that fulfill the BMS_3 algebra (2.3) instead of the conformal one in (2.1).

In order to see that, let us consider the original CFT_2 in a generic (non-conformal) gauge, so that in a local patch, the two-dimensional metric can be brought to the same conformal class as the following one, cf. [51]

$$ds^2 = -N^2 dt^2 + \left(d\phi + N^\phi dt\right)^2 , \quad (4.3)$$

where N and N^ϕ stand for the lapse and shift functions, respectively.⁴ The total Hamiltonian of the CFT_2 then reads

$$\mathcal{H}_{\text{CFT}} = \int d\phi \left[N \left(T + \bar{T} \right) + N^\phi \left(T - \bar{T} \right) \right] = \int d\phi \left(NH + N^\phi J \right) . \quad (4.4)$$

The deformation in (4.2) has the net effect of deforming the energy density of the CFT_2 to be that of a supertranslation, i.e., $H \rightarrow P$, so that the total Hamiltonian deforms as $\mathcal{H}_{\text{CFT}} \rightarrow \tilde{\mathcal{H}}$, with

$$\tilde{\mathcal{H}} = \int d\phi \left(NP + N^\phi J \right) . \quad (4.5)$$

Supertranslation and superrotation densities evolution is then spanned by the deformed Hamiltonian $\tilde{\mathcal{H}}$, which by virtue of (2.3) reads

$$\begin{aligned} \dot{P} &= \{P, \tilde{\mathcal{H}}\} = 2PN^{\phi'} + P'N^\phi , \\ \dot{J} &= \{J, \tilde{\mathcal{H}}\} = 2PN' + P'N + 2JN^{\phi'} + J'N^\phi . \end{aligned} \quad (4.6)$$

In absence of global obstructions, the canonical generators become expressed as an integral over the spatial circle precisely as in (3.6), but now being conserved provided that the state-independent parameters fulfill

$$\dot{\epsilon}_P = N\epsilon'_J - N'\epsilon_J + N^\phi\epsilon'_P - N^{\phi'}\epsilon_P , \quad \dot{\epsilon}_J = N^\phi\epsilon'_J - N^{\phi'}\epsilon_J . \quad (4.7)$$

Thus, the transformation law of supertranslation and superrotation densities is given by (3.4), corresponding to Noetherian BMS_3 symmetries.

5 Geometric aspects

Since the deformed action is manifestly invariant under diffeomorphisms $\xi = \xi^\mu \partial_\mu$, it is reassuring to verify that the Noether current $j^\mu = \tilde{T}^\mu_\nu \xi^\nu$, with

$$\tilde{T}^\mu_\nu = \begin{pmatrix} NP + N^\phi J & J \\ -N^\phi (N^{\phi'} J + 2NP) & -(NP + N^\phi J) \end{pmatrix} , \quad (5.1)$$

⁴In null (holomorphic) coordinates, this would amount to switch on the Beltrami differentials.

is conserved ($\partial_\mu j^\mu = 0$) provided that the evolution equations of the energy and momentum densities (4.6), as well as those of the parameters in (4.7) hold. The precise form of the diffeomorphisms is then identified as

$$\xi^\mu = N^{-1} \left(\epsilon_P, N\epsilon_J - N^\phi \epsilon_P \right), \quad (5.2)$$

which close in the Lie brackets, $[\xi_1, \xi_2] = \xi_3$, with

$$\epsilon_P^3 = \epsilon_J^1 (\epsilon_P^2)' + \epsilon_P^1 (\epsilon_J^2)' - (1 \leftrightarrow 2), \quad \epsilon_J^3 = \epsilon_J^1 (\epsilon_J^2)' - (1 \leftrightarrow 2), \quad (5.3)$$

according to the BMS₃ algebra when the parameters $\epsilon_P^i, \epsilon_J^i$ obey (4.7).

Note that one might be tempted to extract an stress-energy tensor $\Theta^\mu{}_\nu$ from the corresponding density in (5.1) by making use of the metric of the undeformed theory $g_{\mu\nu}$ in (4.3), according to $\tilde{T}^\mu{}_\nu = \sqrt{-g} \Theta^\mu{}_\nu$. However, this tensor is not conserved ($\nabla_\mu \Theta^\mu{}_\nu \neq 0$), reflecting the fact that the metric of CFT₂ is not preserved under BMS₃ diffeomorphisms ξ^μ up to a local scaling, i.e.,

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - \lambda g_{\mu\nu} \neq 0. \quad (5.4)$$

Hence, the metric of the undeformed CFT₂ is not a suitable object to describe the geometric properties of the deformed theory.

An appropriate Riemannian metric for the geometric description of the deformation is obtained as follows. Note that the total deformed Hamiltonian (4.5) is a homogeneous functional of T and \bar{T} of degree one, so that it fulfills the following identity

$$\tilde{\mathcal{H}} = \int d\phi \left(\frac{\delta \tilde{\mathcal{H}}}{\delta T} T + \frac{\delta \tilde{\mathcal{H}}}{\delta \bar{T}} \bar{T} \right) = \int d\phi \left(\frac{\delta \tilde{\mathcal{H}}}{\delta H} H + \frac{\delta \tilde{\mathcal{H}}}{\delta J} J \right). \quad (5.5)$$

Therefore, the deformed theory can be equivalently described by placing the original CFT₂ on a state-dependent curved metric, whose lapse and shift functions, \tilde{N} and \tilde{N}^ϕ , are respectively given by the variation of the deformed Hamiltonian with respect to the energy and momentum densities of the undeformed theory, i.e.,⁵

$$d\tilde{s}^2 = - \left(\frac{\delta \tilde{\mathcal{H}}}{\delta H} \right)^2 dt^2 + \left(d\phi + \frac{\delta \tilde{\mathcal{H}}}{\delta J} dt \right)^2. \quad (5.6)$$

The mapping (2.2) allows to express the deformed metric (5.6) in terms of the supertranslation and superrotation densities, so that it reads

$$d\tilde{s}^2 = - N^2 \left(\frac{2P^2}{J^2 - P^2} \right)^2 dt^2 + \left(d\phi + \left(N^\phi + N \frac{2JP}{J^2 - P^2} \right) dt \right)^2, \quad (5.7)$$

where N and N^ϕ correspond to the (state-independent) lapse and shift functions of the original undeformed metric in (4.3), respectively.⁶

⁵Beltrami differentials are determined by the variation of the deformed Hamiltonian with respect to T and \bar{T} .

⁶Note that the Ricci scalar of the deformed metric $\tilde{g}_{\mu\nu}$ differs from that of the undeformed one $g_{\mu\nu}$ ($\tilde{R} \neq R$). In contradistinction, the corresponding metrics in the geometric interpretation of the $T\bar{T}$ deformation [52–55] are related through state-dependent diffeomorphisms.

It must be emphasized that the manifest state dependence of the lapse and shift functions (or Beltrami differentials) of the deformed metric (5.7) provides a local obstruction to gauge them away, preventing the possibility of choosing the standard conformal gauge once the theory is deformed.

A proper stress-energy tensor $\tilde{\Theta}^\mu{}_\nu$, consistent with invariance under diffeomorphisms and local scalings of the deformed action (4.2), is then readily obtained from $\tilde{\mathcal{T}}^\mu{}_\nu = \sqrt{-\tilde{g}}\tilde{\Theta}^\mu{}_\nu$, where $\tilde{g}_{\mu\nu}$ stands for the state-dependent metric in (5.7). Indeed, the deformed stress-energy tensor fulfills

$$\tilde{\Theta}_{\mu\nu} = \tilde{\Theta}_{\nu\mu} \ , \quad \tilde{\Theta}^\mu{}_\mu = 0 \ , \quad \tilde{\nabla}_\mu \tilde{\Theta}^\mu{}_\nu = 0 \ , \quad (5.8)$$

being automatically symmetric and traceless, while its conservation implies the evolution equations of supertranslation and superrotation densities (4.6). Therefore, the canonical BMS₃ generators (3.6) can be written in manifestly covariant way as

$$\tilde{\mathcal{Q}}[\epsilon_J, \epsilon_P] = \int d\phi \sqrt{\tilde{\gamma}} \tilde{n}_\mu \tilde{\Theta}^\mu{}_\nu \xi^\nu \ , \quad (5.9)$$

with ξ^μ given by (5.2), and according to the deformed metric $\tilde{g}_{\mu\nu}$ in (5.7), the unit timelike normal is given by $\tilde{n}_\mu = (\tilde{N}, 0)$, and $\tilde{\gamma} = 1$.

The geometric description of the deformed theory is then suitably carried out in terms of the two relevant structures, $\tilde{g}_{\mu\nu}$ and $\tilde{\Theta}^\mu{}_\nu$, being inextricably intertwined. In fact, since both objects are state dependent, they acquire nontrivial functional variations when acting on them under diffeomorphisms, given by

$$\delta_\xi \tilde{g}_{\mu\nu} = \frac{\delta \tilde{g}_{\mu\nu}}{\delta P} \delta_\xi P + \frac{\delta \tilde{g}_{\mu\nu}}{\delta J} \delta_\xi J \ , \quad \delta_\xi \tilde{\Theta}_{\mu\nu} = \frac{\delta \tilde{\Theta}_{\mu\nu}}{\delta P} \delta_\xi P + \frac{\delta \tilde{\Theta}_{\mu\nu}}{\delta J} \delta_\xi J \ . \quad (5.10)$$

Therefore, since the functional variations (5.10) must be taken into account, BMS₃ symmetries geometrically arise from diffeomorphisms ξ that preserve the form of both relevant structures up to a local scaling, i.e., from the solutions of the following *deformed conformal Killing equations*

$$\begin{aligned} \tilde{\nabla}_\mu \xi_\nu + \tilde{\nabla}_\nu \xi_\mu - \lambda \tilde{g}_{\mu\nu} &= \delta_\xi \tilde{g}_{\mu\nu} \ , \\ \mathcal{L}_\xi \tilde{\Theta}_{\mu\nu} &= \delta_\xi \tilde{\Theta}_{\mu\nu} \ , \end{aligned} \quad (5.11)$$

where \mathcal{L}_ξ stands for the Lie derivative.

It is amusing to verify that starting from scratch with the deformed metric and stress-energy tensor, $\tilde{g}_{\mu\nu}$ and $\tilde{\Theta}^\mu{}_\nu$, the deformed conformal Killing equations (5.11) can be exactly solved. Indeed, the solution is precisely given by the BMS₃ diffeomorphisms ξ^μ in (5.2) with parameters ϵ_P, ϵ_J fulfilling (4.7), where the transformation law of supertranslation and superrotation densities is also found to be given by (3.4).

Note that the geometric interpretation also allows to find the transformation law of the fields in the deformed theory from those of the original undeformed (primary) fields, collectively denoted by χ , by writing them in a manifestly covariant way, and then acting with the Lie derivative along BMS₃ symmetries spanned by ξ , i.e., $\delta_\xi \chi = \mathcal{L}_\xi \chi$.

6 Deformed free bosons

Let us see how the deformation works in a simple and concrete example, given by the action of N free bosons with flat target metric,

$$I[\Phi^I] = -\frac{1}{2} \int d^2x \sqrt{-g} \delta_{IK} \partial_\mu \Phi^I \partial^\mu \Phi^K. \quad (6.1)$$

Before implementing the generic deformation (4.2), it is useful to express the background metric $g_{\mu\nu}$ in the gauge choice (4.3), so that the Hamiltonian action reads

$$I[\Phi^I, \Pi_J] = \int d^2x \left(\Pi_I \dot{\Phi}^I - NH - N^\phi J \right), \quad (6.2)$$

where $\Pi_I = \frac{\delta L}{\delta \dot{\Phi}^I}$, and $H = \frac{1}{2} (\Pi^I \Pi_I + \Phi'^I \Phi'_I)$, $J = \Pi_I \Phi'^I$. The deformed Hamiltonian action is then given by

$$\tilde{I}[\Phi^I, \Pi_J] = \int d^2x \left(\Pi_I \dot{\Phi}^I - NP - N^\phi J \right), \quad (6.3)$$

with $P = H + \sqrt{H^2 - J^2}$. The transformation law of the fields and their momenta under BMS_3 symmetries spanned by ξ in (5.2) are then found to be

$$\begin{aligned} \delta_\xi \Phi^I &= \{\Phi^I, \tilde{\mathcal{Q}}\} = \epsilon_J \Phi'^I + \epsilon_P \left(\Pi^I + \frac{H \Pi^I - J \Phi'^I}{\sqrt{H^2 - J^2}} \right), \\ \delta_\xi \Pi_I &= \{\Pi_I, \tilde{\mathcal{Q}}\} = \left[\epsilon_J \Pi_I + \epsilon_P \left(\Phi'_I + \frac{H \Phi'_I - J \Pi_I}{\sqrt{H^2 - J^2}} \right) \right]', \end{aligned} \quad (6.4)$$

where $\tilde{\mathcal{Q}}$ reads as in (3.6). The field equations $\dot{\Phi}^I = \{\Phi^I, \tilde{\mathcal{H}}\}$, $\dot{\Pi}_I = \{\Pi_I, \tilde{\mathcal{H}}\}$, with $\tilde{\mathcal{H}}$ given by (4.5), then follow from (6.4) by the replacement $\epsilon_P \rightarrow N$ and $\epsilon_J \rightarrow N^\phi$. Note that the transformation law of Φ^I in the deformed theory also reads from $\delta_\xi \Phi^I = \mathcal{L}_\xi \Phi^I$. The transformation of supertranslation and superrotation densities in (3.4) is then recovered from those in (6.4), which goes hand in hand with the fact that P and J now fulfill the BMS_3 algebra (2.3) by virtue of the canonical Poisson bracket $\{\Phi^I(\phi), \Pi_K(\varphi)\} = \delta^I_K \delta(\phi - \varphi)$. Moreover, the stress-energy tensor of the deformed theory is obtained from $\tilde{\mathcal{T}}^\mu_\nu = \sqrt{-\tilde{g}} \tilde{\Theta}^\mu_\nu$ with $\tilde{\mathcal{T}}^\mu_\nu$ and $\tilde{g}_{\mu\nu}$ respectively given by (5.1) and (5.7).

It is worth highlighting that the deformed action (6.3) clearly cannot be obtained from any standard limiting process of the undeformed one for $N > 1$. The peculiarity of the deformed single free boson ($N = 1$) stems from the fact that the supertranslation density simplifies as $P = \Pi^2$, so that the momentum can be eliminated from its own field equation, and the deformed action (6.3) can be written in Lagrangian form as

$$\tilde{I}[\Phi] = \frac{1}{4} \int d^2x (\mathcal{V}^\mu \partial_\mu \Phi)^2, \quad (6.5)$$

where $\mathcal{V}^\mu = (\sqrt{-g})^{1/2} n^\mu$ stands for a vector density of weight 1/2, constructed out from the metric $g_{\mu\nu}$ in (4.3) of the undeformed theory. Noteworthy, this vector density is invariant under the BMS_3 symmetries spanned by ξ in (5.2), since $\mathcal{L}_\xi \mathcal{V}^\mu = 0$. Therefore, the

deformed action of a single free boson (6.5) coincides with the ultra-relativistic limit of the undeformed theory (6.1) for $N = 1$, when the Carrollian limit is taken in a similar way as for the tensionless string [56–59].

Additionally, the vector density can be reexpressed as $\mathcal{V}^\mu = \frac{1}{\sqrt{2}}e^{1/2}\tau^\mu$, where e and τ^μ correspond to the einbein and the dual of the “clock one-form” of a Carrollian geometry [60], respectively; so that action of the deformed free boson agrees with the Carrollian one found in [61].

Remarkably, the action (6.5) can be understood in terms of two inequivalent geometric structures. One of them is Riemannian and described through the state-dependent metric $\tilde{g}_{\mu\nu}$ in (5.7), while the remaining structure stands for a Carrollian manifold.

7 Ending remarks

Since the map (2.2) possesses a square root, our results also carry out for its negative branch, i.e., when the supertranslation density is given by

$$P_{(-)} = T + \bar{T} - 2\sqrt{T\bar{T}}. \quad (7.1)$$

In particular, the deformed action of a single free boson for the negative branch reads

$$\tilde{I}_{(-)}[\Phi, \Pi] = \int dx^2 \left(\Pi\dot{\Phi} - NP_{(-)} - N^\phi J \right), \quad (7.2)$$

with $P_{(-)} = \Phi'^2$. Curiously, the deformed action $\tilde{I}_{(-)}$ agrees with an inequivalent ultra-relativistic limit of the single free boson defined by $\Phi \rightarrow \Phi/c$, $\Pi \rightarrow c\Pi$, when $c \rightarrow 0$. This limit coincides with that needed to pass from the standard Liouville theory to its “flat” version [40]. Indeed, starting from a single free boson in the conformal gauge ($N = 1$, $N^\phi = 0$) the deformed free boson in the negative branch corresponds to the kinetic term of the flat Liouville theory.

It is also worth to pointing out that the centrally extended conformal algebra (given by two copies of the Virasoro algebra) can be shown to be related to BMS_3 with central extensions, in terms of a map that is necessarily nonlocal. Nevertheless, if only zero modes are involved, the local nonlinear map in (2.2) still holds. Thus, blindly applying the map (2.2) for the zero modes, the Cardy formula once expressed in terms of left and right groundstate energies ($\mathcal{L}_0, \bar{\mathcal{L}}_0$), given by

$$S = 4\pi\sqrt{-\mathcal{L}_0\bar{\mathcal{L}}} + 4\pi\sqrt{-\bar{\mathcal{L}}_0\mathcal{L}}, \quad (7.3)$$

reduces to its BMS_3 (or flat) version

$$\tilde{S} = 2\pi \frac{1}{\sqrt{-\mathcal{P}_0\bar{\mathcal{P}}}} [\mathcal{P}\mathcal{J}_0 + \mathcal{P}_0\mathcal{J}], \quad (7.4)$$

when the deformed energy and momentum of the groundstate \mathcal{P}_0 and \mathcal{J}_0 are expressed in terms of the BMS_3 central charges [62–65].

Noteworthy, the hypotheses that ensure positivity of the Cardy formula (7.3) ($\mathcal{L}_0 < 0$, $\bar{\mathcal{L}}_0 < 0$, $\mathcal{L} > 0$, $\bar{\mathcal{L}} > 0$), by virtue of both branches of the map, imply that the deformed entropy (7.4) is also positive ($\tilde{S} > 0$).

Furthermore, the map between the chemical potentials follows the same rule as that of the parameters in (3.5), with $(\epsilon, \bar{\epsilon}) \rightarrow (\beta, \bar{\beta})$ and $(\epsilon_P, \epsilon_J) \rightarrow (\tilde{\beta}, \tilde{\theta})$, where left and right temperatures relate to the modular parameter of the torus as $\tau = \beta/2\pi$, and $\tilde{\beta}, \tilde{\theta}$ stand for the temperature and chemical potential of the deformed theory. Therefore, around equilibrium, the S-modular transformation $\tau \rightarrow -1/\tau$ precisely maps into its BMS₃ (flat) version [62, 63].

$$\tilde{\beta} \rightarrow \frac{4\pi^2 \tilde{\beta}}{\tilde{\theta}^2}, \quad \tilde{\theta} \rightarrow -\frac{4\pi^2}{\tilde{\theta}}. \quad (7.5)$$

The fact that the standard Cardy formula and S-modular transformations map to their corresponding “flat versions” naturally suggests that a holographic realization of the mapping could be carried out through a suitable $\sqrt{T\bar{T}}$ -like deformation of the standard boundary conditions in three-dimensional gravity. Thus, asymptotically AdS₃ spacetimes would curiously enjoy asymptotic BMS₃ symmetries, or conversely, asymptotically flat three-dimensional spacetimes would possess those of the conformal group. Indeed, a remarkable fact that provides strong support to the latter assertion is the following. Once the map is applied for the zero modes, one readily verifies that the entropy of asymptotically AdS₃ black holes is exactly reproduced from the BMS₃ (flat) version of the Cardy formula in (7.4); and analogously, the entropy of flat cosmological spacetimes [62, 63] or asymptotically locally flat black holes [66, 67] is precisely recovered from the standard Cardy formula (7.3). Furthermore, since standard and flat S-modular transformations are also mapped between themselves around equilibrium, the corresponding Hawking temperatures once expressed in terms of the global charges also do. Note that the intriguing fact that black hole and flat cosmology thermodynamics can be successfully reproduced by both (standard and flat) microscopic countings in each case, stems from the fact that the counting is actually performed in different thermodynamics ensembles being connected by the map.

As a closing remark, it would be worth exploring whether a suitable uplift of the deformed theories to higher dimensions might be invariant under the conformal Carrollian algebra, which is known to be isomorphic to BMS_{D+1} [4].

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