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Research Article

Mappings on Fuzzy Soft Classes

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We define the concept of a mapping on classes of fuzzy soft sets and study the properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets, and support them with examples and counterexamples.

1. Introduction

To solve complicated problems in economics, engineering and environment, we cannot successfully use classical methods because of different kinds of incomplete knowledge, typical for those problems. There are four theories: Theory of Probablity, Fuzzy Set Theory (FST) [1], Interval Mathematics and Rough Set Theory (RST) [2], which we can consider as mathematical tools for dealing with imperfect knowledge. All these tools require the pre-specification of some parameter to start with, for example, probablity density function in Probability Theory, membership function in FST and an equivalence relation in RST. Such a requirement, seen in the backdrop of imperfect or incomplete knowledge, raises many problems. At the same time, incomplete knowledge remains the most glaring characteristic of humanistic systems—systems exemplified by biological systems, economic systems, social systems, political systems, information systems and more generally man-machine systems of various types.

Noting problems in parameter specification, Molodtsov [3] introduced the notion of soft set to deal with problems of incomplete information. Soft Set Theory (SST) does not require the specification of a parameter, instead it accommodates approximate descriptions of an object as its starting point. This makes SST a natural mathematical formalism for approximate reasoning. We can use any parametrization we prefer: with the help of words, sentences, real numbers, functions, mappings, and so on. This means that the problem of setting the membership function or any similar problem does not arise in SST.

Applications of SST in other disciplines and real life problems are now catching momentum. Molodtsov [3] successfully applied the SST into several directions, such as smoothness of functions, Riemann integration, Perron integration, Theory of Probability, Theory of Measurement and so on. Kovkov et al. [4] have found promising results by applying soft sets to Optimization Theory, Game Theory and Operations Research. Maji et al. [5] gave practical application of soft sets in decision making problems. It is based on the notion of knowledge reduction of rough sets. Zou and Xiao [6] have exploited the link between soft sets and data analysis in incomplete information systems.

In [7], Yang et al. emphasized that soft sets needed to be expanded to improve its potential ability in practical engineering applications. Fuzzy soft sets combine the strengths of both soft sets and fuzzy sets. Maji et al. [8] introduced the notion of fuzzy soft set and discussed its several properties. He proposed it as an attractive extension of soft sets, with extra features to represent uncertainty and vagueness, on top of incompleteness. Recent investigations [7–10] have shown how both theories can be combined into a more flexible, more expressive framework for modelling and processing incomplete information in information systems.

The main purpose of this paper is to continue investigating fuzzy soft sets. In [11], Kharal and Ahmad introduced the notions of a mapping on the classes of soft sets and studied the properties of soft images and soft inverse images. In this paper, we define the notion of a mapping on classes of fuzzy soft sets. We also define and study the properties of fuzzy soft images and fuzzy soft inverse images of fuzzy soft sets, and support them with examples and counterexamples.

2. Preliminaries

First we recall basic definitions and results.

Molodtsov defined a soft set in the following manner.

Definition 2.1 (see [3]). A pair (F, A) is called a soft set over a universe X and with a set A of attribues from E, where $F : A \to P(X)$ is a mapping.

In other words, a soft set over X is a parameterized family of subsets of the universe X. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the soft set (F, A), or as the set of ε-approximate elements of the soft set.

Maji et al. defined a fuzzy soft set in the following manner.

Definition 2.2 (see [8]). A pair (Λ, Σ) is called a fuzzy soft set over X, where $\Lambda : \Sigma \to \widetilde{P}(X)$ is a mapping, $\widetilde{P}(X)$ being the set of all fuzzy sets of X.

Definition 2.3 (see [8]). A fuzzy soft set (Λ, Σ) over X is said to be null fuzzy soft set denoted by $\widetilde{\Phi}$, if for all $\varepsilon \in \Sigma$, $\Lambda(\varepsilon) = \widetilde{0}$, where $\widetilde{0}$ denotes null fuzzy set over X.

Definition 2.4 (see [8]). A fuzzy soft set (Λ, Σ) is said to be absolute fuzzy soft set denoted by $\widetilde{\Sigma}$, if for all $\varepsilon \in \Sigma$, $\Lambda(\varepsilon) = \widetilde{1}$, where $\widetilde{1}$ denotes absolute fuzzy set over X.

Definition 2.5 (see [8]). For two fuzzy soft sets (Λ, Σ) and (Δ, Ω) over X, we say that (Λ, Σ) is a fuzzy soft subset of (Δ, Ω), if

(i) $\Sigma \subseteq \Omega$,

(ii) for all $\varepsilon \in \Sigma$, $\Lambda(\varepsilon) \leq \Delta(\varepsilon)$,

and is written as $(\Lambda, \Sigma)\widetilde{\subseteq}(\Delta, \Omega)$.

Maji et al. defined the intersection of two fuzzy soft sets as follows.

Definition 2.6 (see [8]). Intersection of two fuzzy soft sets (Λ, Σ) and (Δ, Ω) over X is a fuzzy soft set (Θ, Ξ), where $\Xi = \Sigma \cap \Omega$, and for all $\varepsilon \in \Xi$, $\Theta(\varepsilon) = \Lambda(\varepsilon)$ or $\Delta(\varepsilon)$, (as both are same fuzzy set), and is written as $(\Lambda, \Sigma)\widetilde{\bigwedge}(\Delta, \Omega) = (\Theta, \Xi)$.

We point out that generally $\Lambda(\varepsilon)$ and $\Delta(\varepsilon)$ may not be identical. Moreover, $\Sigma \cap \Omega$ must be nonempty to avoid the degenerate case. Thus we revise Definition 2.6 as follows.

Definition 2.7. Let (Λ, Σ) and (Δ, Ω) be two fuzzy soft sets over X with $\Sigma \cap \Omega \neq \phi$. Then intersection of two fuzzy soft sets (Λ, Σ) and (Δ, Ω) is a fuzzy soft set (Θ, Ξ), where $\Xi = \Sigma \cap \Omega$, and for all $\varepsilon \in \Xi$, $\Theta(\varepsilon) = \Lambda(\varepsilon) \wedge \Delta(\varepsilon)$. We write

$$(\Lambda, \Sigma)\widetilde{\bigwedge}(\Delta, \Omega) = (\Theta, \Xi). \tag{1}$$

Definition 2.8 (see [8]). Union of two fuzzy soft sets (Λ, Σ) and (Δ, Ω) over X is a fuzzy soft set (Θ, Ξ), where $\Xi = \Sigma \cup \Omega$, and for all $\varepsilon \in \Xi$,

$$\Theta(\varepsilon) = \begin{cases} \Lambda(\varepsilon), & \text{if } \varepsilon \in \Sigma - \Omega, \\ \Delta(\varepsilon), & \text{if } \varepsilon \in \Omega - \Sigma, \\ \Lambda(\varepsilon) \vee \Delta(\varepsilon), & \text{if } \varepsilon \in \Sigma \cap \Omega, \end{cases} \tag{2}$$

and is written as $(\Lambda, \Sigma)\widetilde{\bigvee}(\Delta, \Omega) = (\Theta, \Xi)$.

3. Mappings on Classes of Fuzzy Soft Sets

Definition 3.1. Let X be an universe and E a set of attributes. Then the collection of all fuzzy soft sets over X with attributes from E is called a fuzzy soft class and is denoted as $\widetilde{(X, E)}$.

Definition 3.2. Let $\widetilde{(X, E)}$ and $\widetilde{(Y, E')}$ be classes of fuzzy soft sets over X and Y with attributes from E and E', respectively. Let $u : X \to Y$ and $p : E \to E'$ be mappings. Then a mapping $f = (u, p) : \widetilde{(X, E)} \to \widetilde{(Y, E')}$ is defined as follows: for a fuzzy soft set (Λ, Σ) in $\widetilde{(X, E)}$, $f(\Lambda, \Sigma)$ is a fuzzy soft set in $\widetilde{(Y, E')}$ obtained as follows: for $\beta \in p(E) \subseteq E'$ and $y \in Y$,

$$f(\Lambda, \Sigma)(\beta)(y) = \begin{cases} \displaystyle\bigvee_{x \in u^{-1}(y)} \left(\bigvee_{\alpha \in p^{-1}(\beta) \cap \Sigma} \Lambda(\alpha) \right)(x), & \\ \qquad \text{if } u^{-1}(y) \neq \phi,\ p^{-1}(\beta) \cap \Sigma \neq \phi, & \\ 0, \qquad \text{otherwise}. & \end{cases} \tag{3}$$

$f(\Lambda, \Sigma)$ is called a fuzzy soft image of a fuzzy soft set (Λ, Σ).

Definition 3.3. Let $u : X \to Y$ and $p : E \to E'$ be mappings. Let $f : \widetilde{(X, E)} \to \widetilde{(X, E)}$ be a mapping and (Δ, Ω), a fuzzy soft set in $\widetilde{(Y, E')}$, where $\Omega \subseteq E'$. Then $f^{-1}(\Delta, \Omega)$, is a fuzzy soft set in $\widetilde{(X, E)}$, defined as follows: for $\alpha \in p^{-1}(\Omega) \subseteq E$, and $x \in X$,

$$f^{-1}(\Delta, \Omega)(\alpha)(x) = \begin{cases} \Delta(p(\alpha))(u(x)), & \text{for } p(\alpha) \in \Omega, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

$f^{-1}(\Delta, \Omega)$ is called a fuzzy soft inverse image of (Δ, Ω).

Above Definitions 3.2 and 3.3 are illustrated as follows.

Example 3.4. Let $X = \{a, b, c\}$, $Y = \{x, y, z\}$, $E = \{e_1, e_2, e_3, e_4\}$, $E' = \{e'_1, e'_2, e'_3\}$ and $\widetilde{(X, E)}$, $\widetilde{(Y, E')}$, classes of fuzzy soft sets. Let $u : X \to Y$ and $p : E \to E'$ be mappings defined as

$$\begin{gathered} u(a) = z, \qquad u(b) = y, \qquad u(c) = y, \\ p(e_1) = e_1', \qquad p(e_2) = e_1', \qquad p(e_3) = e_3', \qquad p(e_4) = e_2'. \end{gathered} \tag{5}$$

Choose two fuzzy soft sets in $\widetilde{(X,E)}$ and $\widetilde{(Y,E')}$, respectively, as

$$\begin{aligned} (\Lambda,\Sigma) &= \{e_1 = \{a_{0.5}, b_0, c_{0.8}\}, e_2 = \{a_{0.1}, b_{0.9}, c_{0.5}\}, \\ &\quad e_4 = \{a_{0.4}, b_{0.3}, c_{0.6}\}\}, \\ (\Delta,\Omega) &= \{e_1' = \{x_{0.3}, y_{0.5}, z_{0.1}\}, e_2' = \{x_{0.9}, y_{0.1}, z_{0.5}\}, \\ &\quad e_3' = \{x_{0.7}, y_{0.5}, z_{0.6}\}\}. \end{aligned} \tag{6}$$

Then the fuzzy soft image of (Λ,Σ) under $f : \widetilde{(X,E)} \to \widetilde{(Y,E')}$ is obtained as

$$\begin{aligned} f(\Lambda,\Sigma)(e_1')(x) &= \bigvee_{s\in u^{-1}(x)} \left(\bigvee_{\alpha\in p^{-1}(e_1')\cap\Sigma} \Lambda(\alpha) \right)(s) \\ &= 0, \quad (\text{as } u^{-1}(x) = \phi), \\ f(\Lambda,\Sigma)(e_1')(y) &= \bigvee_{s\in u^{-1}(y)} \left(\bigvee_{\alpha\in p^{-1}(e_1')\cap\Sigma} \Lambda(\alpha) \right)(s) \\ &= \bigvee_{s\in\{b,c\}} \left(\bigvee_{\alpha\in\{e_1,e_2\}} \Lambda(\alpha) \right)(s) \\ &= \bigvee_{s\in\{b,c\}} (\Lambda(e_1) \vee \Lambda(e_2))(s) \\ &= \bigvee_{s\in\{b,c\}} (\{a_{0.5}, b_{0.9}, c_{0.8}\})(s) \\ &= \bigvee(0.9, 0.8) = 0.9, \\ f(\Lambda,\Sigma)(e_1')(z) &= 0.5. \end{aligned} \tag{7}$$

By similar calculations, consequently, we get

$$\begin{aligned} (f(\Lambda,\Sigma),\Omega) &= \{e_1' = \{x_0, y_{0.9}, z_{0.5}\}, e_2' = \{x_0, y_{0.6}, z_{0.4}\}, \\ &\quad e_3' = \{x_0, y_0, z_0\}\}. \end{aligned} \tag{8}$$

Next, for $p(e_i) \in \Omega$, $i = 1, 2, 4$, we calculate

$$\begin{aligned} f^{-1}(\Delta,\Omega)(e_1)(a) &= \Delta(p(e_1))(u(a)) = \Delta(e_1')(z) \\ &= (\{x_{0.3}, y_{0.5}, z_{0.1}\})(z) = 0.1, \\ f^{-1}(\Delta,\Omega)(e_1)(b) &= \Delta(p(e_1))(u(b)) = \Delta(e_1')(y) \\ &= (\{x_{0.3}, y_{0.5}, z_{0.1}\})(y) = 0.5, \\ f^{-1}(\Delta,\Omega)(e_1)(c) &= 0.5. \end{aligned} \tag{9}$$

By similar calculations, consequently, we get

$$\begin{aligned} f^{-1}(\Delta,\Omega) &= \{e_1 = \{a_{0.1}, b_{0.5}, c_{0.5}\}, e_2 = \{a_{0.1}, b_{0.5}, c_{0.5}\}, \\ &\quad e_3 = \{a_{0.6}, b_{0.5}, c_{0.5}\}, e_4 = \{a_{0.5}, b_{0.1}, c_{0.1}\}\}. \end{aligned} \tag{10}$$

Remark 3.5. Note that the null (resp., absolute) fuzzy soft set as defined by Maji et al. [8], is not unique in a fuzzy soft class $\widetilde{(X,E)}$, rather it depends upon $\Sigma \subseteq E$. Therefore, we denote it by $\widetilde{\Phi}_\Sigma$ (resp., $\widetilde{X}_\Sigma$). If $\Sigma = E$, then we denote it simply by $\widetilde{\Phi}$ (resp., $\widetilde{X}$), which is unique null (resp., absolute) fuzzy soft set, called full null (resp., full absolute) fuzzy soft set.

Definition 3.6. Let $f : \widetilde{(X,E)} \to \widetilde{(Y,E')}$ be a mapping and (Λ,Σ), (Δ,Ω) fuzzy soft sets in $\widetilde{(X,E)}$. Then for $\beta \in E'$, $y \in Y$, the fuzzy soft union and intersection of fuzzy soft images $f(\Lambda,\Sigma)$ and $f(\Delta,\Omega)$ in $\widetilde{(Y,E')}$ are defined as

$$\begin{aligned} &\left(f(\Lambda,\Sigma) \widetilde{\bigvee} f(\Delta,\Omega) \right)(\beta)(y) \\ &\quad = f(\Lambda,\Sigma)(\beta)(y) \bigvee f(\Delta,\Omega)(\beta)(y), \\ &\left(f(\Lambda,\Sigma) \widetilde{\bigwedge} f(\Delta,\Omega) \right)(\beta)(y) \\ &\quad = f(\Lambda,\Sigma)(\beta)(y) \bigwedge f(\Delta,\Omega)(\beta)(y), \end{aligned} \tag{11}$$

where $\widetilde{\bigvee}$ and $\widetilde{\bigwedge}$ denote fuzzy soft union and intersection of fuzzy soft images in $\widetilde{(Y,E')}$.

Definition 3.7. Let $f : \widetilde{(X,E)} \to \widetilde{(Y,E')}$ be a mapping and (Λ,Σ), (Δ,Ω) fuzzy soft sets in $\widetilde{(Y,E')}$. Then for $\alpha \in E$, $x \in X$, the fuzzy soft union and intersection of fuzzy soft inverse images $f^{-1}(\Lambda,\Sigma)$ and $f^{-1}(\Delta,\Omega)$ in $\widetilde{(X,E)}$ are defined as

$$\begin{aligned} &\left(f^{-1}(\Lambda,\Sigma) \widetilde{\bigvee} f^{-1}(\Delta,\Omega) \right)(\alpha)(x) \\ &\quad = f^{-1}(\Lambda,\Sigma)(\alpha)(x) \bigvee f^{-1}((\Delta,\Omega))(\alpha)(x), \\ &\left(f^{-1}(\Lambda,\Sigma) \widetilde{\bigwedge} f^{-1}(\Delta,\Omega) \right)(\alpha)(x) \\ &\quad = f^{-1}(\Lambda,\Sigma)(\alpha)(x) \bigwedge f^{-1}((\Delta,\Omega))(\alpha)(x). \end{aligned} \tag{12}$$

Theorem 3.8. *Let $f : \widetilde{(X,E)} \to \widetilde{(Y,E')}$ and $u : X \to Y$ and $p : E \to E'$ be mappings. For fuzzy soft sets (Λ,Σ), (Δ,Ω) and a family of fuzzy soft sets (Λ_i,Σ_i) in $\widetilde{(X,E)}$, we have*

(1) $f(\widetilde{\Phi}) = \widetilde{\Phi}$,

(2) $f(\widetilde{X}) \widetilde{\leq} \widetilde{Y}$,

(3) $f((\Lambda,\Sigma) \widetilde{\bigvee} (\Delta,\Omega)) = f(\Lambda,\Sigma) \widetilde{\bigvee} f(\Delta,\Omega)$.
In general, $f(\widetilde{\bigvee}_i(\Lambda_i,\Sigma_i)) = \widetilde{\bigvee}_i f(\Lambda_i,\Sigma_i)$,

(4) $f((\Lambda,\Sigma) \widetilde{\bigwedge} (\Delta,\Omega)) \widetilde{\leq} f((\Lambda,\Sigma)) \widetilde{\bigwedge} f((\Delta,\Omega))$.
In general, $f(\widetilde{\bigwedge}_i(\Lambda_i,\Sigma_i)) \widetilde{\leq} \widetilde{\bigwedge}_i f(\Lambda_i,\Sigma_i)$,

(5) *if* $(\Lambda,\Sigma) \widetilde{\leq} (\Delta,\Omega)$, *then* $f(\Lambda,\Sigma) \widetilde{\leq} f(\Delta,\Omega)$.

Proof. We only prove (3)–(5).

(3) For $\beta \in E'$ and $y \in Y$, we show that

$$f\left((\Lambda, \Sigma)\widetilde{\bigvee}(\Delta, \Omega)\right)(\beta)(y) = f(\Lambda, \Sigma)(\beta)(y) \bigvee f(\Delta, \Omega)(\beta)(y). \tag{13}$$

Consider

$$f\left((\Lambda, \Sigma)\widetilde{\bigvee}(\Delta, \Omega)\right)(\beta)(y) = f(\Theta, \Sigma \cup \Omega)(\beta)(y) \quad \text{(say)}$$
$$= \begin{cases} \bigvee_{x \in u^{-1}(y)} \left(\bigvee_{\alpha \in p^{-1}(\beta) \cap (\Sigma \cup \Omega)} \Theta(\alpha) \right)(x), \\ \qquad \text{if } u^{-1}(y) \neq \phi,\ p^{-1}(\beta) \cap (\Sigma \cup \Omega) \neq \phi, \\ 0, \qquad \text{otherwise}, \end{cases} \tag{14}$$

where

$$\Theta(\alpha) = \begin{cases} \Lambda(\alpha), & \alpha \in \Sigma - \Omega \cap p^{-1}(\beta), \\ \Delta(\alpha), & \alpha \in \Omega - \Sigma \cap p^{-1}(\beta), \\ \Lambda(\alpha) \bigvee \Delta(\alpha), & \alpha \in \Sigma \cap \Omega \cap p^{-1}(\beta), \end{cases} \tag{15}$$

for $\alpha \in (\Sigma \cup \Omega) \cup p^{-1}(\beta)$.

Considering only the non-trivial case, we have

$$f\left((\Lambda, \Sigma)\widetilde{\bigvee}(\Delta, \Omega)\right)(\beta)(y) = \bigvee_{x \in u^{-1}(y)} \left(\bigvee \begin{cases} \Lambda(\alpha)(x), & \alpha \in \Sigma - \Omega \cap p^{-1}(\beta) \\ \Delta(\alpha)(x), & \alpha \in \Omega - \Sigma \cap p^{-1}(\beta) \\ \left(\Lambda(\alpha) \bigvee \Delta(\alpha)\right)(x), & \alpha \in \Sigma \cap \Omega \cap p^{-1}(\beta) \end{cases} \right). \tag{I}$$

Next, by Definition 3.6, we have

$$\left(f(\Lambda, \Sigma)\widetilde{\bigvee} f(\Delta, \Omega)\right)(\beta)(y)$$
$$= f(\Lambda, \Sigma)(\beta)(y) \bigvee f(\Delta, \Omega)(\beta)(y)$$
$$= \left(\bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Sigma} \Lambda(\alpha)(x) \right) \bigvee \left(\bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Omega} \Delta(\alpha)(x) \right)$$
$$= \bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap (\Sigma \cup \Omega)} \left(\Lambda(\alpha) \bigvee \Delta(\alpha)\right)(x)$$
$$= \bigvee_{x \in u^{-1}(y)} \left(\bigvee \begin{cases} \Lambda(\alpha)(x), & \alpha \in \Sigma - \Omega \cap p^{-1}(\beta) \\ \Delta(\alpha)(x), & \alpha \in \Omega - \Sigma \cap p^{-1}(\beta) \\ \left(\Lambda(\alpha) \bigvee \Delta(\alpha)\right)(x), & \alpha \in \Sigma \cap \Omega \cap p^{-1}(\beta) \end{cases} \right). \tag{II}$$

By (I) and (II), we have (3).

(4) For $\beta \in E'$ and $y \in Y$, and using Definition 3.6, we have

$$f\left((\Lambda, \Sigma)\widetilde{\bigwedge}(\Delta, \Omega)\right)(\beta)(y)$$
$$= f(\Theta, \Sigma \cap \Omega)(\beta)(y), \quad \text{(say)}$$
$$= \bigvee_{x \in u^{-1}(y)} \left(\bigvee_{\alpha \in p^{-1}(\beta) \cap (\Sigma \cap \Omega)} \Theta(\alpha) \right)(x)$$
$$= \bigvee_{x \in u^{-1}(y)} \left(\bigvee_{\alpha \in p^{-1}(\beta) \cap (\Sigma \cap \Omega)} \left[\Lambda(\alpha) \bigwedge \Delta(\alpha)\right] \right)(x)$$
$$= \bigvee_{x \in u^{-1}(y)} \left(\bigvee_{\alpha \in p^{-1}(\beta) \cap (\Sigma \cap \Omega)} \left[\Lambda(\alpha)(x) \bigwedge \Delta(\alpha)(x)\right] \right)$$
$$\leq \left(\bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Sigma} \Lambda(\alpha)(x) \right) \bigwedge \left(\bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Omega} \Delta(\alpha)(x) \right)$$
$$= f((\Lambda, \Sigma))(\beta)(y) \bigwedge f((\Delta, \Omega))(\beta)(y), \quad \text{for } \beta = p(\alpha)$$
$$= \left(f(\Lambda, \Sigma)\widetilde{\bigwedge} f(\Delta, \Omega)\right)(\beta)(y). \tag{16}$$

This gives (4).

(5) Considering only the non-trivial case, for $\beta \in E'$ and $y \in Y$, and since $(\Lambda, \Sigma)\widetilde{\leq}(\Delta, \Omega)$, we have

$$f((\Lambda, \Sigma))\beta(y) = \bigvee_{x \in u^{-1}(y)} \left(\bigvee_{\alpha \in p^{-1}(\beta) \cap \Sigma} \Lambda(\alpha) \right)(x)$$
$$= \bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Sigma} \Lambda(\alpha)(x)$$
$$\leq \bigvee_{x \in u^{-1}(y)} \bigvee_{\alpha \in p^{-1}(\beta) \cap \Omega} \Delta(\alpha)(x)$$
$$= f(\Delta, \Omega)(\beta)(y). \tag{17}$$

This gives (5). □

In Theorem 3.8, inequalities (2), (4) and implication (5) cannot be reversed, in general, as is shown in the following.

Example 3.9. Let $\widetilde{(X, E)}$, $\widetilde{(Y, E')}$ be classes of fuzzy soft sets and $f : \widetilde{(X, E)} \to \widetilde{(Y, E')}$ as defined in Example 3.4. For (2), we define mappings $u : X \to Y$ and $p : E \to E'$ as

$$u(a) = x, \qquad u(b) = z, \qquad u(c) = x,$$
$$p(e_1) = e_2', \qquad p(e_2) = e_1', \qquad p(e_3) = e_2', \qquad p(e_4) = e_2'. \tag{18}$$

Then calculations give

$$\begin{aligned}&\widetilde{Y}\widetilde{\ngeqq}\{e_2' = \{x_1, y_0, z_1\}, e_3' = \{x_0, y_0, z_0\}, e_1' = \{x_1, y_0, z_1\}\}\\&\quad = f\left(\widetilde{X}\right).\end{aligned} \tag{19}$$

For (4) and (5), define mappings $u : X \to Y$ and $p : E \to E'$ as

$$\begin{gathered}u(a) = y, \qquad u(b) = y, \qquad u(c) = y,\\ p(e_1) = e_3', \qquad p(e_2) = e_2', \qquad p(e_3) = e_2', \qquad p(e_4) = e_1'.\end{gathered} \tag{20}$$

Choose two fuzzy soft sets in $\widetilde{(X,E)}$ as

$$\begin{aligned}(\Lambda, \Sigma) &= \{e_3 = \{a_{0.4}, b_{0.8}, c_{0.6}\}\},\\ (\Delta, \Omega) &= \{e_3 = \{a_{0.6}, b_{0.1}, c_1\}\}.\end{aligned} \tag{21}$$

Then calculations give

$$\begin{aligned}&f(\Lambda, \Sigma)\widetilde{\bigwedge} f(\Delta, \Omega)\\&\quad = \{e_1' = \{x_0, y_0, z_0\}, e_2' = \{x_0, y_{0.8}, z_0\}, e_3' = \{x_0, y_0, z_0\}\}\\&\quad \widetilde{\ngeqq}\{e_1' = \{x_0, y_0, z_0\}, e_2' = \{x_0, y_{0.6}, z_0\}, e_3' = \{x_0, y_0, z_0\}\}\\&\quad = f\left((\Lambda, \Sigma)\widetilde{\bigwedge}(\Delta, \Omega)\right)\end{aligned} \tag{22}$$

also we have

$$\begin{aligned}&f(\Lambda, \Sigma)\\&\quad = \{e_1' = \{x_0, y_0, z_0\}, e_2' = \{x_0, y_{0.8}, z_0\}, e_3' = \{x_0, y_0, z_0\}\}\\&\quad \widetilde{\leqq}\{e_1' = \{x_0, y_0, z_0\}, e_2' = \{x_0, y_1, z_0\}, e_3' = \{x_0, y_0, z_0\}\}\\&\quad = f(\Delta, \Omega)\end{aligned} \tag{23}$$

but $(\Lambda, \Sigma)\widetilde{\nleqq}(\Delta, \Omega)$.

Theorem 3.10. *Let $f : \widetilde{(X,E)} \to \widetilde{(Y,E')}$, $u : X \to Y$ and $p : E \to E'$ be mappings. For fuzzy soft sets (Λ, Σ), (Δ, Ω) and a family of fuzzy soft sets (Λ_i, Σ_i) in $\widetilde{(Y,E')}$, we have*

(1) $f^{-1}(\widetilde{\Phi}) = \widetilde{\Phi}$,

(2) $f^{-1}(\widetilde{Y}) = \widetilde{X}$,

(3) $f^{-1}((\Lambda, \Sigma)\widetilde{\bigvee}(\Delta, \Omega)) = f^{-1}(\Lambda, \Sigma)\widetilde{\bigvee} f^{-1}(\Delta, \Omega)$. *In general,* $f^{-1}(\widetilde{\bigvee}_i(\Lambda_i, \Sigma_i)) = \widetilde{\bigvee}_i f^{-1}(\Lambda_i, \Sigma_i)$,

(4) $f^{-1}((\Lambda, \Sigma)\widetilde{\bigwedge}(\Delta, \Omega)) = f^{-1}(\Lambda, \Sigma)\widetilde{\bigwedge} f^{-1}(\Delta, \Omega)'$. *In general,* $f^{-1}(\widetilde{\bigwedge}_i(\Lambda_i, \Sigma_i)) = \widetilde{\bigwedge}_i f^{-1}((\Lambda_i, \Sigma_i))$,

(5) *If* $(\Lambda, \Sigma)\widetilde{\leqq}(\Delta, \Omega)$, *then* $f^{-1}(\Lambda, \Sigma)\widetilde{\leqq} f^{-1}(\Delta, \Omega)$.

Proof. We only prove (3)–(5).

(3) For $\alpha \in E$ and $x \in X$, we have

$$\begin{aligned}&f^{-1}\left((\Lambda, \Sigma)\widetilde{\bigvee}(\Delta, \Omega)\right)(\alpha)(x)\\&\quad = f^{-1}((\Theta, \Sigma \cup \Omega))(\alpha)(x)\\&\quad = \Theta(p(\alpha))(u(x)), \quad (p(\alpha) \in \Sigma \cup \Omega,\ u(x) \in Y)\\&\quad = \Theta(\beta)(u(x)), \quad \text{where } \beta = p(\alpha)\\&\quad = \left(\begin{cases}\Lambda(\beta)(u(x)), & \beta \in \Sigma - \Omega\\ \Delta(\beta)(u(x)), & \beta \in \Omega - \Sigma\\ (\Lambda(\beta) \cup \Delta(\beta))(u(x)), & \beta \in \Sigma \cap \Omega\end{cases}\right).\end{aligned} \tag{I}$$

Next, we use Definition 3.7 and get

$$\begin{aligned}&\left[f^{-1}(\Lambda, \Sigma) \bigvee f^{-1}(\Delta, \Omega)\right](\alpha)(x)\\&\quad = f^{-1}(\Lambda, \Sigma)(\alpha)(x) \bigvee f^{-1}((\Delta, \Omega))(\alpha)(x)\\&\quad = \Lambda(p(\alpha))(u(x)) \bigvee \Delta(p(\alpha))(u(x)), \quad \beta = p(\alpha) \in \Sigma \cup \Omega\\&\quad = \left(\begin{cases}\Lambda(\beta)(u(x)), & \beta \in \Sigma - \Omega\\ \Delta(\beta)(u(x)), & \beta \in \Omega - \Sigma\\ (\Lambda(\beta) \cup \Delta(\beta))(u(x)), & \beta \in \Sigma \cap \Omega\end{cases}\right).\end{aligned} \tag{II}$$

From (I) and (II), we get (3).

(4) For $\alpha \in E$, $x \in X$ and using Definition 3.7, we have

$$\begin{aligned}&f^{-1}\left((\Lambda, \Sigma)\widetilde{\bigwedge}(\Delta, \Omega)\right)(\alpha)(x)\\&\quad = f^{-1}((\Theta, \Sigma \cap \Omega))(\alpha)(x)\\&\quad = \Theta(p(\alpha))(u(x)), \quad p(\alpha) \in \Sigma \cup \Omega\\&\quad = \Theta(\beta)(u(x)), \quad \beta = p(\alpha)\\&\quad = (\Lambda(\beta) \wedge \Delta(\beta))(u(x))\\&\quad = \Lambda(p(\alpha))(u(x)) \wedge \Delta(p(\alpha))(u(x))\\&\quad = f^{-1}(\Lambda, \Sigma)(\alpha)(x) \wedge f^{-1}(\Delta, \Omega)(\alpha)(x)\\&\quad = \left(f^{-1}(\Lambda, \Sigma)\widetilde{\bigwedge} f^{-1}(\Delta, \Omega)\right)(\alpha)(x).\end{aligned} \tag{24}$$

This gives (4).

(5) Since $(\Lambda, \Sigma)\widetilde{\leqq}(\Delta, \Omega)$, we have

$$\begin{aligned}&f^{-1}((\Lambda, \Sigma))(\alpha)(x)\\&\quad = \Lambda(p(\alpha))(u(x)) = \Lambda(\beta)(u(x)), \quad p(\alpha) = \beta\\&\quad \leq \Delta(\beta)(u(x)) = \Delta(p(\alpha))(u(x))\\&\quad = f^{-1}((\Delta, \Omega))(\alpha)(x).\end{aligned} \tag{25}$$

□

The implication in (5) is not reversible, in general, as is shown in the following.

Example 3.11. Let $\widetilde{(X,E)}$, $\widetilde{(Y,E')}$ be classes of fuzzy soft sets and $f : \widetilde{(X,E)} \to \widetilde{(Y,E')}$ as defined in Example 3.4. For (5) define mappings $u : X \to Y$ and $p : E \to E'$ as

$$\begin{gathered} u(a) = y, \qquad u(b) = y, \qquad u(c) = z, \\ p(e_1) = e_1', \qquad p(e_2) = e_2', \qquad p(e_3) = e_2', \qquad p(e_4) = e_1'. \end{gathered} \tag{26}$$

Choose two fuzzy soft sets in $\widetilde{(Y,E')}$ as

$$\begin{aligned} (\Lambda,\Sigma) &= \{e_3' = \{x_{0.8}, y_0, z_0\}\}, \\ (\Delta,\Omega) &= \{e_3' = \{x_{0.3}, y_{0.1}, z_{0.5}\}\}. \end{aligned} \tag{27}$$

Then calculations give

$$f^{-1}(\Lambda,\Sigma) = \widetilde{\Phi} \,\tilde{\leq}\, \widetilde{\Phi} = f^{-1}(\Delta,\Omega), \tag{28}$$

but $(\Lambda,\Sigma) \,\tilde{\nleq}\, (\Delta,\Omega)$.

4. Conclusion

Fuzzy sets and soft sets complement each other to represent vague and incomplete knowledge, respectively. Synergy of these approaches has been proposed through the notion of fuzzy soft sets in [8]. In this paper, we have defined the notion of a mapping on the classes of fuzzy soft sets which is a pivotal notion for advanced development of any new area of mathematical sciences. We have studied the properies of fuzzy soft images and inverse images which have been supported by examples and counterexamples. We hope these fundamental results will help the researchers to enhance and promote the research on Fuzzy Soft Set Theory.

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