Market-Based Coordination of Thermostatically Controlled Loads – Part I: A Mechanism Design Formulation

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***Abstract*—This paper focuses on the coordination of a population of thermostatically controlled loads (TCLs) with unknown parameters to achieve group objectives. The problem involves designing the device bidding and market clearing strategies to motivate self-interested users to realize efficient energy allocation subject to a peak energy constraint. This coordination problem is formulated as a mechanism design problem, and we propose a mechanism to implement the social choice function in dominant strategy equilibrium. The proposed mechanism consists of a novel bidding and clearing strategy that incorporates the internal dynamics of TCLs in the market mechanism design, and we show it can realize the team optimal solution. This paper is divided into two parts. Part I presents a mathematical formulation of the problem and develops a coordination framework using the mechanism design approach. Part II presents a learning scheme to account for the unknown load model parameters, and evaluates the proposed framework through realistic simulations.**

***Index Terms*—Mechanism design, demand response, market-based coordination, thermostatically controlled loads**

I. Introduction

Demand response has attracted considerable research attention in recent years, and is regarded as one of the most important means to improve the efficiency and reliability of the future smart grid. A natural way to achieve demand response is through various pricing schemes, such as Real Time Pricing (RTP), Time of Use (TOU) and Critical Peak Pricing (CPP) [1], [2]. Many validation projects [3] have been carried out to demonstrate the performance of these pricing schemes in terms of payment reduction, load shifting, and peak shaving. These price-based methods either directly pass the wholesale energy price to end-users [2] or design pricing strategies in heuristic ways [4]. It is thus hard to achieve predictable and reliable aggregated response, which is essential in various demand response applications, such as energy capping, load following, frequency regulation, among others.

To achieve accurate and reliable load response, aggregated load control has been extensively studied in the literature. A simple form of aggregated load control is the direct load control (DLC), where the aggregator can remotely control the operations of residential appliances based on the agreement between customers and the utility company. While traditional DLC is mainly concerned with peak load management [5], [6], recent research effort focuses more on the modeling and control of different kinds of aggregated loads, such as data center servers [7], [8], hybrid electrical vehicles [9], [10] and thermostatically controlled loads [11]–[14], to participate in various demand response programs. Some of these DLC methods require fast communications between the aggregator and individual loads. The communication overhead can be reduced using advanced state estimation algorithms [15], [16] that can accurately estimate load state information without frequently collecting measurements from the loads.

Another important paradigm of aggregated load control is the market-based coordination. It borrows ideas from economics [17] to coordinate a group of self-interested users to achieve desired aggregated load response [18], [19]. Different from DLC, the market-based coordination affects the load response indirectly via an internal price signal. The internal price can be dramatically different from the wholesale price due to specific group objectives. For instance, in [20] and [21], a market-based approach is proposed to efficiently allocate thermal resources among offices only based on local information. In [22] and [23], a multi-agent based control framework is proposed to integrate distributed energy resources for various coordination objectives. A distributed algorithm is developed in [24] and [25] for the utility company and users to jointly determine optimal prices and demand schedules via an iterative bidding and clearing process. In [26], a group of smart buildings are coordinated through an internal price signal to provide frequency regulation services to the ancillary market. In addition, the Pacific Northwest National Laboratory launched the GridWise® demonstration project to validate the market-based coordination strategies for residential loads [27]. The demonstration project involved 112 residential houses in Washington and Oregon, and showed that the market-based coordination strategies could reduce the utility demand and congestion at key times.

Although the aggregated dynamics of TCLs may significantly affect the performance of the control strategies, many existing market-based coordination strategies either neglect this internal dynamics or use a simplified model to characterize it. In this paper, we consider the coordination

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of a group of TCLs to maximize the social welfare subject to a peak energy constraint, where the internal dynamics of TCLs are taking into account. This coordination problem poses several challenges. First, the user utilities are private information, making it rather challenging for the coordinator to achieve group objectives with incomplete information. Second, many existing works adopt the Nash equilibrium concept [28], [29], which requires multiple iterations between the agents and the coordinator to achieve the optimal social outcome. The real time implementation of such coordination algorithms requires considerable communication resources. Third, a lot of existing literature assumes accurate load models with known parameters. However, the Gridwise® demonstration project [27] suggests this is not always the case. In practice, the information each user sends to the coordinator can only depend on local measurements, such as room temperature and "on/off" state. Therefore, an estimation scheme is needed for the users to compute their bids only based on online measurements.

The key contribution of this paper lies in the development of a market-based coordination framework for residential air conditioning loads with a systematic consideration of all the aforementioned challenges. In this paper, we formulate the coordination problem as a mechanism design problem [17], [30]. The price-responsive loads are modeled as individual utility maximizers, while the group objective is encoded in the social choice function, which is to maximize the social welfare subject to a peak energy constraint. We propose a mechanism and show it can implement the social choice function in dominant strategy equilibrium. Such solution concept does not require iterative information exchanges between the coordinator and the individual loads, and can be implemented with limited communication resources. The proposed mechanism contains a novel bidding and clearing strategy that incorporates the internal dynamics of the TCLs into the market mechanism design, and we show that it can realize the team optimal solution.

Different from many existing works [27], [25], the problem is addressed with a systematic consideration of various practical factors, such as heterogeneous load dynamics, private information of individual users, unknown parameters of the load model, communication resources for the information exchange, etc. All these factors are brought up based on the observations in the GridWise® demonstration project [27]. They are important not only for customer privacy protection and the end user engagement, but also for the cost-effective implementation of the real-time control strategies. Once our framework is properly implemented, it can accurately achieve the desired load responses, and improve the operational efficiency of the distribution system in an economically feasible way.

The rest of the paper proceeds as follows. A motivating example based on a real-world demonstration project is presented in Section II, followed by a problem formulation in Section III. A mechanism is constructed in Section IV to implement the optimal energy allocation. Simulation results and the joint state-parameter estimation framework are presented in the companion paper [31].

II. Motivating Example

The framework proposed in this paper is largely motivated by the Pacific Northwest GridWise® demonstration project [27], where a 5-minute double-auction market is created to coordinate a group of TCLs to cap the aggregated peak energy. Each device is equipped with a smart thermostat that can measure the room temperature and communicate with the coordinator. Before each market period, the device measures its room temperature, T_c, and submits a bid to the coordinator. The bid should consist of the load power and the bidding price. Since the rated power of the load is different from its actual power due to environmental disturbances, in practice each device is required to bid the measured average power of the most recent market period during which the load is on. The bidding price is determined by a bidding curve shown in Fig. 1, where P_{avg} is the average clearing price of certain price history (e.g., 24 hours), σ is the standard variation of the clearing prices during the given history, and T_{min}, $T_{desired}$ and T_{max} are user-specified minimum, desired, and maximum temperature, respectively. We denote the bidding power and price as Q_{bid} and P_{bid}, respectively. In addition, each user can specify energy use preferences through a smart thermostat interface (see Fig. 2). This user preference will affect the slope of the bidding curve.

The coordinator collects all the bids and orders the bids in a decreasing sequence, $P_{bid}^1, \ldots, P_{bid}^N$. With the associated power sequence, $Q_{bid}^1, \ldots, Q_{bid}^N$, a demand curve can be constructed to map the clearing price to aggregated power. Fig. 3 illustrates how the demand curve is constructed. This curve is then used to determine the market clearing price that respects the feeder capacity constraint: when the total demand is less than the feeder capacity, the market clearing price is equal to the base price, P_{base} (Fig. 4), which is the wholesale energy price plus a retail modifier as defined by the tariff of American Electric Power (AEP) [32]; otherwise the market price, P_c, is determined by the intersection of the demand curve and the feeder capacity constraint (Fig. 5).

After the market is cleared, each device receives the energy price and adjusts its setpoint, T_{set}, according to a response curve as shown in Fig. 6. This setpoint modifies the system dynamics and affects the temperature trace of the TCL, and therefore affects the bid of each user for the next market period. Notice that all the bidding and user response processes are executed by a programmable controller, and the user only needs to specify his/her preferences via the thermostat interface. To initialize the market process, the user needs to specify T_{min}, T_{max}, $T_{desired}$ and K, the device needs to measure the temperature and the power of the last "on" cycle, and the coordinator needs to collect all the bids, estimate the power of the unresponsive loads, Q_{uc}, and the feeder capacity constraint, D.

Apart from the GridWise® project, a similar demonstration project is also implemented in AEP, Ohio [33], which involves more households and more sophisticated market

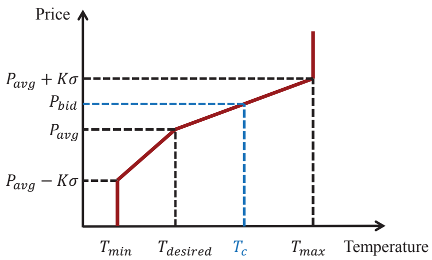

Figure 1. The controller measures its current temperature T_c and submits a bid P_{bid} to the coordinator using this curve.

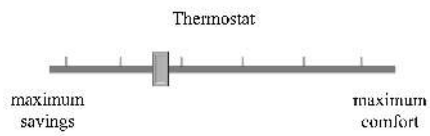

Figure 2. User interface used in the GridWise® demonstration project [27].

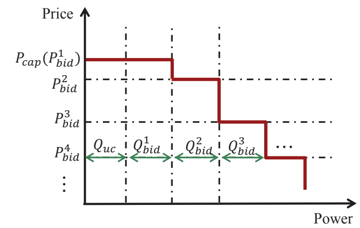

Figure 3. The demand curve based on the user bids, where P^i_{bid} is the bidding price sequence in decreasing order, and Q^i_{bid} is the power of the most recent on cycle.

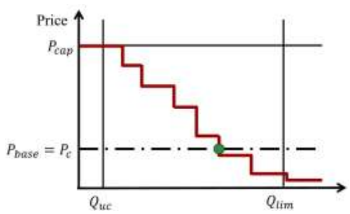

Figure 4. The demand curve constructed based on all the bids. If the total demand is less than the feeder capacity constraint, then the clearing price is equal to the base price.

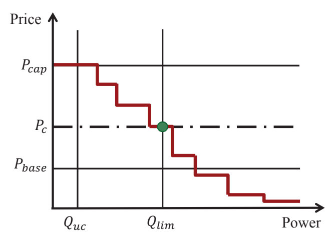

Figure 5. When the total demand is greater than the feeder power constraint, then the clearing price is determined by the intersection of demand curve and feeder capacity constraint.

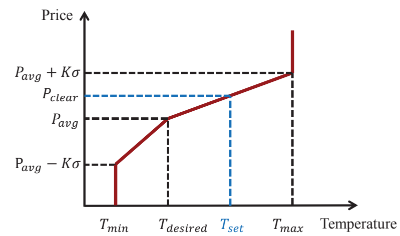

Figure 6. The user response to the price. For any given price, the devices determine the temperature setpoint according to this curve.

bidding design. These projects provide insights for the coordination of residential loads from the practical point of view. However, the bidding and pricing strategies are designed in a heuristic way, which may result in constraint violations and market inefficiencies. To address these challenges, there is a strong need to develop a general coordination framework that can serve as a theoretical foundation to improve the performance of the control scheme and help to design other similar market-based coordination strategies.

III. Problem Formulation

Consider a coordination problem for a group of TCLs, where the coordinator allocates energy to users to maximize the social welfare subject to a feeder capacity constraint. Each device is assumed to be equipped with a smart thermostat that has two main functions. First, it allows the user to specify energy use preferences via an interface such as the sliding bar shown in Fig. 2 to indicate one's trade-off between comfort and cost. Second, before each market period it submits a bid to the coordinator based on user's preference and local device measurement, such as power consumption, "on/off" states, and local temperature. The coordinator collects the user bids, determines the energy price, and broadcasts the price to all the devices. Each device will then adjust the temperature setpoint in response to the energy price to maximize the individual utility. This will modify the system dynamics and therefore affect the user bids for the next period. In the considered scenario, we assume that each user is a price taker, namely, an individual user's decision will not significantly affect the market price. This is a standard assumption when the market involves a large number of players [17, chap. 12.F], [34], [35].

The rest of this section provides formal mathematical descriptions of the main components of the proposed framework.

A. User Preferences and Utility

Assume that there are N self-interested users. Each user needs to determine the temperature setpoint to obtain an energy allocation that maximizes his individual utility (the user's comfort minus the electricity cost). In other words, each user is confronted with the trade-off between comfort and electricity cost: when the electricity price is high, the device will adjust the temperature setpoint to save electricity cost at the sacrifice of user comfort. Formally, a function $V_i : \mathbb{R} \to \mathbb{R}$ can be used to represent the comfort level for each user with energy allocation a_i. Assume that $V_i(a_i)$ is concave, continuously differentiable, $V_i(0) = 0$ and $V_i'(0) > 0$. Let $\theta_i(t_k)$ represent the private information of user i. Denote E_i^m as the energy consumption for the ith load if it is "on" during the entire period, which gives $a_i \leq E_i^m$. The individual utility maximization problem can be formulated as follows:

$$\begin{aligned} \max_{a_i} \quad & V_i(a_i;\theta_i(t_k)) - P_c a_i \\ & \text{subject to: } 0 \leq a_i \leq E_i^m, \end{aligned} \tag{1}$$

where P_c is the energy price. Let $h_i : \mathbb{R} \to \mathbb{R}$ be the optimal solution to the optimization problem (1), we have:

$$h_i(P_c;\theta_i(t_k)) = \underset{0\leq a_i \leq E_i^m}{\arg\max}\, V_i(a_i;\theta_i(t_k)) - P_c a_i. \tag{2}$$

We assume that h_i is continuous and non-increasing with respect to P_c for each $i = 1, \dots, N$. Notice that the user can not directly choose his optimal energy allocation. Instead, he can only determine the temperature setpoint, which affects the energy consumption through the load dynamics.

B. Individual Load Dynamics

Let $\eta_i(t) \in \mathbb{R}^n$ be the continuous state of the ith load. Denote $q_i(t)$ as the "on/off" state: $q_i(t) = 0$ when the TCL is off, and $q_i(t) = 1$ when it is on. For both "on" and "off" states, the thermal dynamics of a TCL system can be typically modeled as a linear system:

$$\dot{\eta}_i(t) = \begin{cases} A_i \eta_i(t) + B^i_{on} & \text{if } q_i(t) = 1 \\ A_i \eta_i(t) + B^i_{off} & \text{if } q_i(t) = 0. \end{cases} \tag{3}$$

Many existing works use a first-order linear system to capture the TCL dynamics [11], [15], [16], where $\eta_i(t)$ only consists of the room temperature. Although the first-order model is adequate for small TCLs such as refrigerators, it is not appropriate for residential air conditioning systems, which require a 2-dimensional linear system model incorporating both air and mass temperature dynamics [12]. Such a second-order model is typically referred to as the Equivalent Thermal Parameter (ETP) model [36]. In this paper we focus on the second-order ETP model, which includes the first-order model as a special case. Let $\varphi_i = [A_i, B^i_{on}, B^i_{off}]^T$ be the model parameters. Typical values of these parameters and the factors that affect these parameters can be found in [12].

The power state of the TCL is typically regulated by a hysteretic controller based on the control deadband $[u_i(t) - \delta/2, u_i(t) + \delta/2]$, where $u_i(t)$ is the temperature setpoint of the ith TCL and δ is the deadband. Let $T^i_c(t)$ denote the room temperature of the ith load. In the cooling mode, the load is turned off when $T^i_c(t) \le u_i(t) - \delta/2$, and it is turned on when $T^i_c(t) \ge u_i(t) + \delta/2$, and remains the same power state otherwise. This hysteretic control policy can be described as:

$$q_i(t^+) = \begin{cases} 1 & \text{if } T^i_c(t) \ge u_i(t) + \delta/2 \\ 0 & \text{if } T^i_c(t) \le u_i(t) - \delta/2 \\ q_i(t) & \text{otherwise .} \end{cases} \tag{4}$$

For notation convenience, we define a hybrid state $z_i(t) = [\eta_i(t), q_i(t)]^T$, which consists of both the temperature and the "on/off" state of the load. Let $[t_k, t_k + T]$ be the kth market period, then the energy consumption of each load during the kth period depends on the system state and setpoint control $u_i(t)$. In this case, the private information consists of system state and model parameters. Therefore, the energy consumption of each load can be represented as $e_i(u_i(t_k), z_i(t_k), \varphi_i)$. This energy consumption function can be derived by calculating the portion of time that the system is on over the entire market period (details of this calculation are presented in Section IV). An example is shown in Fig 7, where a second-order ETP model is used and the initial room temperature is 72.8°F. Let $\theta_i(t_k) = (z_i(t_k), \varphi_i)$ be the overall private information of load i, then the energy function can be written as $e_i(u_i(t_k), \theta_i(t_k))$. Notice that the private information for users is time varying, as it contains the system state.

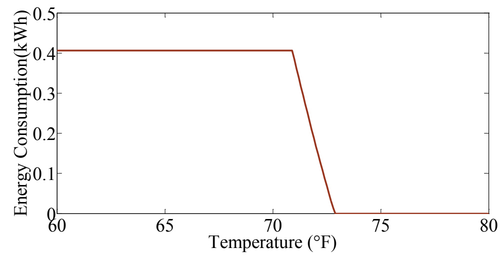

Figure 7. Energy consumption of the TCL during a market clearing cycle as a function of the temperature setpoint.

After the market is cleared, each user wants to determine the control action $u_i(t_k)$ such that the resulting energy consumption equals the optimal solution to (1). Since the optimal control depends on the energy price, we can define a user response function, $\Lambda_i : \mathbb{R} \to \mathbb{R}$ with $u_i(t_k) = \Lambda_i(P_c)$. Therefore, the optimal energy allocation function h_i as defined in (2) should satisfy the following:

$$h_i(\cdot; \theta_i(t_k)) = e_i(\Lambda_i(\cdot), \theta_i(t_k)). \tag{5}$$

The left-hand side of equation (5) represents the optimal energy allocation for a given price, while the right-hand side arises from the physical property of the individual loads, and indicates that the user can specify the control action u_i to match the actual energy consumption to the optimal allocation. An example of function h_i is shown in Fig. 8, where the response curve is piecewise linear (as shown in Fig. 1) and the initial room temperature is 72.8°F. To derive the function $h_i(\cdot; \theta_i(t_k))$, we first determine the control setpoint based on the market price using the response curve (Fig. 1), then calculate the corresponding energy consumption based on the energy function $e_i(\cdot, \theta_i(t_k))$. Since the energy function $e_i(\cdot, \theta_i(t_k))$ depends on the system dynamics (3) and the control policy (4), the load dynamics are incorporated in function h_i through this process.

C. Problem Statement

The coordinator obtains energy from the wholesale market at a cost denoted as $C\left(\sum_{i=1}^N a_i\right)$. We assume that $C(\cdot)$ is differentiable and convex. The energy is then allocated to users via a price signal to maximize the social welfare, which can be defined as $\sum_{i=1}^N V_i(a_i; \theta_i(t_k)) - C(\sum_{i=1}^N a_i)$. Therefore, the coordinator's optimization problem can be formulated as follows:

$$\max_{a_1, \dots, a_N} \sum_{i=1}^N V_i(a_i; \theta_i(t_k)) - C\left(\sum_{i=1}^N a_i\right) \tag{6}$$

$$\text{subject to: } \begin{cases} \sum_{i=1}^N a_i \le D \\ 0 \le a_i \le E^m_i, \forall i = 1, \dots, N \\ a_i = h_i(P_c; \theta_i(t_k)), \forall i = 1, \dots, N, \end{cases}$$

where D is the maximum energy for the aggregated loads. Without loss of generality, we assume that $D \le N E^m_i$.

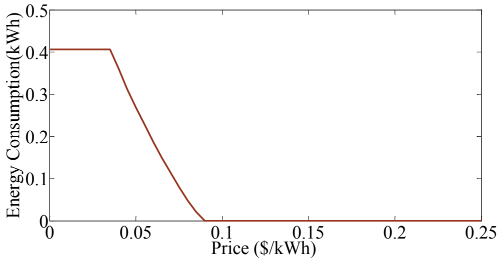

Figure 8. Energy consumption of the TCL during a market clearing cycle as a function of the energy price.

Note that the feeder capacity constraints considered in the GridWise® demonstration project can be represented by the total energy constraint. This is because the feeder capacity constraint is mainly due to the consideration of the thermal characteristics of the feeder. The instantaneous power can exceed the feeder power limit without causing damages to the grid, as long as the energy over a certain period is effectively capped to protect the feeder from overheating.

The optimization problem (6) defines a Stackelberg game [37], where the coordinator first makes control decisions to maximize the social welfare, then the individual users choose energy consumption to maximize individual utility based on the coordinator's control decisions. In such Stackelberg games, the upper bound on the social welfare can be typically characterized by the team optimal solution [37], which is the optimal solution to the following team problem:

$$\max_{a_1,\ldots,a_N} \sum_{i=1}^{N} V_i(a_i;\theta_i(t_k)) - C\left(\sum_{i=1}^{N} a_i\right) \tag{7}$$

$$\text{subject to: } \begin{cases} \sum_{i=1}^{N} a_i \leq D \\ 0 \leq a_i \leq E_i^m, \forall i = 1,\ldots,N, \end{cases}$$

In the above team problem, the coordinator and the users cooperatively maximize the social welfare subject to the peak energy constraint. In general, the team solution results in a higher social welfare than the solution to (6), since the coordinator's optimization problem (6) is more restrictive: one only needs to find an energy allocation to maximize the social welfare to solve the team problem, while in the coordinator's optimization problem, we also need to find a price to satisfy the additional constraint in (6). However, such a clearing price may not always exist for an arbitrarily given team optimal solution.

Example 1: As an example, consider two users with $V_1(a_1;\theta_1(t_k)) = a_1$, $V_2(a_2;\theta_2(t_k)) = 3a_2$. The energy cost for the coordinator is $C(a_1 + a_2) = 2a_1 + 2a_2$. The team problem is to maximize the social welfare subject to an energy constraint, i.e.:

$$\max_{a_1,a_2} \sum_{i=1}^{2} V_i(a_i;\theta_i(t_k)) - C(a_1 + a_2) \tag{8}$$

$$\text{subject to: } \begin{cases} a_1 + a_2 \leq 1 \\ 0 \leq a_i \leq 2, \text{ for } i = 1, 2 \end{cases}$$

The team optimal solution is $a_1 = 0$, $a_2 = 1$. However, according to (1), given any energy price, a_i is either 0 or 2. Therefore, the coordinator can not find a price to realize the team optimal solution.

To address this concern, we introduce the concept of realizable energy allocation:

Definition 1: The energy allocation vector, $a = (a_1,\ldots,a_N)$, can be realized by P_c, if $a_i = h_i(P_c;\theta_i(t_k))$ for all $i = 1,\ldots,N$.

It is clear that not all the energy allocations can be realized. In this paper, we have assumed that V_i is concave and continuously differentiable, and h_i is continuous and non-increasing. We will show in Section V that under these conditions, there is always a price to realize the team optimal solution. In other words, the upper bound given by the team optimal solution is tight. Therefore, the problem of the paper can be formulated as follows:

Problem 1: Design the bidding and clearing strategy, such that the cleared price realizes the team optimal solution a^*.

The coordinator's optimization problem (6) can not be directly addressed using standard optimization techniques, since the individual valuations are unknown to the coordinator. For this reason, to achieve the group objectives, the coordinator needs to design a bidding strategy to collect information from the individual users, and then determine the price based on the user bids.

Remark 1: The market design for many traditional assets is well-understood. For instance, in energy market, generators can be simply characterized by an output range depending on its ramp rate during each market period. However, the internal dynamics of TCLs are more complex and depend more on the environment, and thus cannot be handled in the same way. Therefore, an important contribution of this paper is to incorporate the dynamics of TCLs in the energy market design. In addition, although this paper only considers the load dynamics within one market period, it is an important step towards establishing a fully dynamic version of the problem where multiple market periods are taken into account.

IV. A Mechanism Design Framework

In this section, we adopt the mechanism design approach to solve Problem 1. First the problem is formulated as a mechanism design problem, then a mechanism is constructed to implement the desired social outcome. In addition, a realistic bidding strategy with a simplified message space is proposed to reduce the communication overhead.

A. The Mechanism Design Problem

Mechanism design studies how to aggregate the individual preferences into a social choice while the individual's actual preferences are not publicly observable. In a mechanism design problem, each user is assumed to selfishly take actions to maximize the individual utility, while the coordinator makes the collective choice that achieves various group objectives. Since the individual utility is unknown to the coordinator, he can require each user to submit a bid to collect information. In this case, the key problem for the coordinator is to align individual objectives with system-level objectives. In other words, a proper bidding and pricing strategy needs to be designed, such that when each user selfishly maximizes the individual utility, the resulting outcome also achieves the desired group objectives (for example, maximizes the social welfare). The rest of this subsection introduces basic concepts in mechanism design.

Let $x \in X$ be the outcome of the mechanism that consists of the energy allocation and the energy price, i.e., $x = (a_1, \dots, a_N, P_c)$. The utility of each user (comfort minus electricity cost) depends on the outcome. Moreover, we assume that at time t_k, each user can privately observe his utility, U_i, over different outcomes. In other words, we can model this by supposing that user i privately observes a parameter θ_i that determines his utility. Notice that we drop the dependence of θ_i on t_k throughout the rest of the paper for notation convenience. In mechanism design, $\theta_i \in \Theta_i$ is usually referred to as the user i's *type* [17, p. 858], where Θ_i denotes the set of all the possible *types*. In our problem, the user *type* contains the system state, $z_i(t_k)$, and the model parameter, φ_i, in particular:

$$U_i(x; \theta_i) = V_i(a_i; \theta_i) - P_c a_i, \tag{9}$$

where $\theta_i = [z_i(t_k), \varphi_i]$.

As the user preferences are private, to determine the optimal energy price, the coordinator also needs to require each user to submit a bid to reveal some information. Formally, this can be formulated as a message space $M = M_1 \times \cdots \times M_N$, where M_i denotes the space of messages (bids) the ith user can communicate to the coordinator. The structure of M_i depends on particular applications. For example, in the demonstration project, each device submits a price and a quantity, then we have $(P^i_{bid}, Q^i_{bid}) \in M_i$. In [24] each device submits the slope of the demand curve, β_i, in which case $\beta_i \in M_i$. After collecting all the user bids, the market is cleared with an energy price and a corresponding energy allocation. The clearing strategy can be represented by an outcome function, $g : M \to X$, that maps the user bids to an outcome, x. The message space and the outcome function together fully characterize the rules governing the procedure for making the collective choice. This is typically referred to as a *mechanism* [17], which can be denoted as $\Gamma = (M_1, \dots, M_N, g(\cdot))$.

Each user observes θ_i privately and determines what to bid to maximize his utility. This process can be represented by a bidding strategy $m_i : \Theta_i \to M_i$ that maps the user *type* to a message. There are many solution concepts for a mechanism, such as Nash equilibrium, Bayesian Nash equilibrium, etc. Of particular interest to our framework in this paper is the dominant strategy equilibrium. Denote m_{-i} as the collection of strategies of all the users other than i, then the dominant strategy equilibrium is defined as follows:

Definition 2 (Dominant Strategy Equilibrium [17]): The strategy profile $(m_1^*(\cdot), \dots, m_N^*(\cdot))$ is a dominant strategy equilibrium of mechanism $\Gamma = (M_1, \dots, M_N, g(\cdot))$ if for all i and all $\theta_i \in \Theta_i$, $U_i(g(m_i^*(\theta_i), m_{-i}), \theta_i) \ \geq \ U_i(g(m_i'(\theta_i), m_{-i}), \theta_i)$ for all $m_i'(\theta_i) \in M_i$ and all $m_{-i} \in M_{-i}$.

Remark 2: In a Nash equilibrium, each agent plays the equilibrium strategy only when he has correct forecast of the actions of other agents. When such knowledge is unavailable, it usually takes multiple iterations for the coordinator and the users to reach the equilibrium strategy of the game. In contrast, dominant strategy equilibrium is a very strong and robust solution concept, where a rational agent always follows the equilibrium strategy regardless of other agent's action. In other words, even when one does not know the actions of others, he still plays the equilibrium strategy. This enables each user to only bid once at each market period, which significantly reduces the communication overhead of the proposed framework.

The equilibrium strategy characterizes the individual's self-interested behavior: each user is an individual welfare maximizer. However, in the coordinator's point of view, a more interesting question is to find the best choice for the overall social welfare. For this reason, a social choice function $f : \Theta \to X$ can be defined to represent the desired social outcome of the coordinator. More specifically, $f(\cdot)$ determines what outcome will be chosen by the coordinator when he knows all the private information. In our problem, f consists of the optimal price to the optimization problem (6) and the resulting energy allocation. If we define $\theta = (\theta_1, \dots, \theta_N)$, the conflict between the personal interest and social interest can be captured by the concept of *implementation*:

Definition 3 (Implementation [17]): A mechanism $\Gamma = (M_1, \dots, M_N, g(\cdot))$ implements the social choice function $f(\cdot)$ in dominant strategies if there exists a dominant strategy equilibrium $m^*(\cdot)$ of Γ, such that $g(m_1^*(\theta_1), \dots, m_N^*(\theta_N)) = f(\theta)$ for all $\theta \in \Theta$.

In the above definition, $g(m_1^*(\theta_1), \dots, m_N^*(\theta_N))$ represents the resulting outcome of individual maximization, while $f(\theta)$ denotes the desired social outcome. The concept of implementation characterizes the social choice that can be realized when all the users take actions to selfishly maximize the individual utility. To this end, Problem 1 can be equivalently stated as follows:

Problem 2: Design a mechanism to implement the social choice function $f(\cdot)$ that maximizes the social welfare subject to a peak energy constraint, i.e., $f(\theta) = (h_1(P_c^*; \theta_i(t_k)), \dots, h_N(P_c^*; \theta_i(t_k)), P_c^*)$ and P_c^* is the solution to the optimization problem (6). Furthermore, P_c^* realizes the team optimal solution.

In the above mechanism design problem, the coordinator needs to design the message space and the market clearing rule such that the optimal social welfare can be implemented when each user selfishly maximizes the individual utility. In the meanwhile, the peak energy constraint needs to be respected.

B. Constructing the Mechanism

Let $f(\theta) = (a_1^*, \dots, a_N^*, P_c^*)$ be the social choice function that maximizes the social welfare subject to the peak energy constraint. Specifically, P_c^* is the optimal solution to (6), and $f(\theta)$ satisfies the following condition:

$$a_i^* = h_i(P_c^*; \theta_i), \forall i = 1, \ldots, N. \quad (10)$$

This subsection constructs a mechanism to implement $f(\cdot)$. Consider a mechanism Γ^*, where each device is asked to submit function $h_i(\cdot; \theta_i)$. Since we have assumed that $h_i(P_c; \theta_i)$ is continuous and non-increasing with respect to P_c, the message space is the function space of all non-increasing and continuous functions. Notice that the user's actual bids may deviate from function h_i, unless they are motivated to bid h_i. Let $b_i(\cdot; \theta_i)$ be a non-increasing and continuous function that represents the user's actual bid. The aggregated demand curve $b(\cdot; \theta)$ can be obtained by adding individual bidding functions, i.e., $b(\cdot; \theta) = \sum_{i=1}^{N} b_i(\cdot; \theta_i)$. In this mechanism, each user is required to submit a function, which requires considerable communication resources. This bidding strategy will be simplified in the next subsection to reduce the communication overhead.

Here we propose the following outcome function $g(b_1, \ldots, b_N) = (a_1^*, \ldots, a_N^*, P_c^*)$ to clear the market:

$$\begin{cases} a_i^* = b_i(P_c^*; \theta_i) \text{ for all } i = 1, \ldots, N & (11) \\ P_c^* = \max\{\bar{P}, P^*\} & (12) \\ P^* = C'\left(\sum_{i=1}^{N} a_i^*\right) & (13) \\ b(\bar{P}, \theta) = D, & (14) \end{cases}$$

where C' represents the derivative of the cost function $C(\cdot)$. According to (13) and (14), P^* is the marginal production cost of procuring $\sum_{i=1}^{N} a_i^*$ amount of energy, while $\bar{P}$ is the energy price at which the aggregated demand is equal to the maximum allowed amount. Since b_i is continuous and non-increasing, and we have assumed that $D \leq NE_i^m$, $\bar{P}$ exists. Intuitively, the social welfare is maximized when the market price equals the marginal production cost, i,e, $P_c^* = P^*$. However, in equation (14), the function b is non-increasing with respect to price, indicating that any feasible price that respects the feeder capacity constraint should be greater than $\bar{P}$. Therefore, in the proposed outcome function, the clearing price equals to P^* whenever $P^* > \bar{P}$, and equals to $\bar{P}$ otherwise. When the energy price is determined, the allocation exactly follows the user bids, i.e., $a_i^* = b_i(P_c; \theta_i)$. For illustrating purpose, we construct the following example to show how to derive the optimal solution from the proposed clearing strategy.

Example 2: Consider 100 users with $V_i = -\frac{1}{2}a_i^2 + (i - P_c)a_i$. Assume that after proper scaling, the maximum energy consumption for each user is 1. The individual utility maximization problem can be formulated as follows:

$$\max_{a_i} -\frac{1}{2}a_i^2 + (i - P_c)a_i \quad (15)$$
$$\text{subject to: } 0 \leq a_i \leq 1$$

The optimal solution to this problem is:

$$a_i^* = \begin{cases} 0 & \text{if } P_c \geq i \\ 1 & \text{if } P_c \leq i - 1 \\ i - P_c & \text{otherwise .} \end{cases} \quad (16)$$

In addition, let us assume that the real time price is 20, and the maximum 5-minute energy due to the feeder capacity constraint is 50, i.e., $P^* = 20$ and $D = 50$. According to (16), when $P_c = 99$, only the 100th user consumes 1 unit of energy, and the aggregated energy is 1. When $P_c = 98$, the 99th and the 100th user consume 1 unit of energy, respectively, and the corresponding aggregated energy is 2, and so forth. Therefore, the price that corresponds to the energy limit is 50, i.e., $\bar{P} = 50$. Since $\bar{P} > P^*$, we conclude that $P_c^* = \bar{P}$.

The rest of this subsection discusses some properties of the proposed mechanism.

Proposition 1: When each user is a price taker, the strategy profile $(h_1(\cdot; \theta_1), \ldots, h_N(\cdot; \theta_N))$ is a dominant strategy equilibrium of the proposed mechanism Γ^*.

This result follows easily from the price taker assumption. Its proof can be found in the technical report [38]. In the proposed mechanism, the optimal bid of each user does not depend on the bidding decisions of others. This is a very important property, since in our particular problem, each user does not know other user's preferences or actions. Therefore, if the bidding decision of one user has to depend on the action of another, then the equilibrium strategy can not be achieved unless all the users have accurate predictions on other user's action, which may not be a reasonable assumption. In addition, we also want to comment that Proposition 1 only holds when there are many users such that the influence of an individual user on the market price is negligible. In other cases (such as the oligopolistic market), the mechanism needs to be designed differently.

Now we can establish the following key property of the proposed mechanism:

Proposition 2: The proposed mechanism Γ^* implements the social choice function $f(\cdot)$. Furthermore, the resulting market clearing price realizes the team optimal solution.

The proof of this proposition can be found in the technical report [38].

C. Realistic Bidding Strategy

The proposed mechanism provides a general solution to the coordination problem formulated in this paper. In real-world applications, directly submitting function h_i requires considerable communication resources, and might impinge on the customer privacy. Therefore, in this subsection we explore the structure of function $e_i(\cdot; \theta_i)$ and $h_i(\cdot; \theta_i)$ to simplify the message space and reduce the communication overhead.

In this paper we assume that the TCL consumes a constant power when it is "on", and consumes no energy when it is "off". For this reason, the energy consumption function $e_i(\cdot, \theta_i(t_k))$ can be derived by calculating the portion of time that the system is on during the entire market period. For example, assume that the system is "on" at the end of the $(k-1)$th period. When the initial temperature $\eta_i(t_k)$ is given, the state trajectory of the linear dynamic model (3) can be derived as $\eta_i(t) = e^{A_i t}\eta_i(t_k) + A_i^{-1}(e^{A_i t} - I)B_{on}$, where $\eta_i(t_k) = [\eta_i^{(1)}(t_k), \eta_i^{(2)}(t_k)]^T$, $\eta_i^{(1)}(t_k) = T_c^i(t_k)$ and I is the identity matrix. When the trajectory hits the boundary of the control deadband defined in (4), the power state will switch and the system is off. Therefore, the trajectory of the system state $\eta_i(t)$ and the power state $q_i(t)$ for the entire period can be derived, and the portion of time that the system is "on" can be calculated based on $q_i(t)$. In particular, consider a system in cooling mode. If the load is "on" at the end of the $(k-1)$th period, i.e., $q_i(t_k^-) = 1$, we have the following (the case for $q_i(t_k^-) = 0$ can be derived similarly):

$$e_i(u_i(t_k), \theta_i(t_k)) = \begin{cases} E_i^m & \text{if } u_i(t_k) \le T_f^i(t_k) + \delta/2 \\ 0 & \text{if } u_i(t_k) \ge T_c^i(t_k) + \delta/2 \\ E_i^m \times \alpha & \text{otherwise }, \end{cases}$$

where $\alpha = \int_0^T q_i(t)dt = \frac{T'}{T}$ is the portion of time that the system is on, and T' satisfies the following:

$$\begin{cases} \eta_i(t_k + T') = e^{A_i T'}\eta_i(t_k) + A_i^{-1}(e^{A_i T'} - I)B_{on} \\ \eta_i^{(1)}(t_k + T') = u_i(t_k) - \delta/2 \\ \eta_i^{(1)}(t_k) = T_c^i(t_k). \end{cases}$$

$T_f^i(t_k)$ is the room temperature at $t_k + T$ given that the system is on during the entire period between t_k and $t_k + T$, which satisfies the following:

$$\begin{cases} \eta_i(t_k + T) = e^{A_i T}\eta_i(t_k) + A_i^{-1}(e^{A_i T} - I)B_{on} \\ \eta_i^{(1)}(t_k + T) = T_f^i(t_k) \\ \eta_i^{(1)}(t_k) = T_c^i(t_k). \end{cases} \tag{17}$$

T_f^i is defined in (17) to characterize the condition in which the load is "on" for the entire period and therefore consumes the maximum energy. Intuitively, if the room temperature at t_k is less than the lower bound of the control deadband ($T_c^i(t_k) \le u_i(t_k) - \delta/2$), the power state will be "off" until the room temperature hits the boundary of the deadband. On the other hand, if $u_i(t_k) \le T_f^i(t_k) + \delta/2$, it indicates that the load is always "on", and the room temperature does not hit the boundary for the entire period.

Due to the complicated nature of the hybrid system dynamics, directly submitting the function h_i may require considerable communication resources in the real time implementation. To simplify the message space, we approximate h_i with a step function as illustrated in Fig. 9, where c_1 and c_2 are computed based on the control setpoint and user *type*. For notation convenience, define $c_1 = e_i(u_1, \theta_i)$ and $c_2 = e_i(u_2, \theta_i)$, where u_1 and u_2 are the temperature control setpoints corresponding to c_1 and c_2, respectively. For example, using the second-order ETP model (3) and control policy (4), u_1 and u_2 for the ith device can be obtained as:

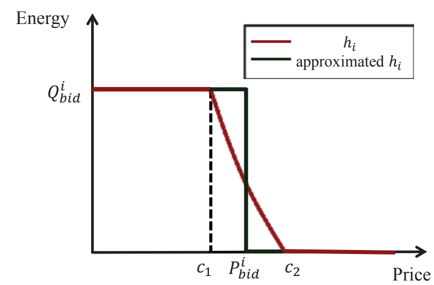

Figure 9. The energy response curve h_i and its approximation.

$$\begin{cases} u_1 & = T_c^i(t_k) + \delta/2 \\ u_2 & = LA_i^{-1}e^{A_i T}(A_i\eta_i(t_k) + B_{on}^i) - LA_i^{-1}B_{on}^i + \delta/2 \\ & = T_f^i(t_k) + \delta/2, \end{cases}$$

where $L = [1, 0]$, and the power state of the ith TCL is "on" at t_k^-.

The step function in Fig. 9 can be fully characterized by two scalars: P_{bid}^i and Q_{bid}^i, where P_{bid}^i is the middle point of c_1 and c_2, while Q_{bid}^i is the power consumption when the device is on during the market period. In this case, the message space of each user M_i is reduced from a function space to a space of $\mathbb{R}_+^2$, and each bid is of the form $[P_{bid}^i, Q_{bid}^i]$.

Remark 3: Bidding and pricing can be viewed as information exchange between the coordinator and the loads that is essential for optimal decision making. Many advanced DLC methods also have communication requirements [9]–[14], [39] and can also accomplish certain group objectives. Some DLC strategies may even learn the user responses through the input/output user behaviors. The main difference of the proposed market-based approach lies in its emphasis on the quantitative incorporation of user preferences, the economic interpretation of user bids and coordination signals, and the encoding of internal load dynamics and user preference information into the bids.

Remark 4: The proposed bidding strategy assumes the knowledge of ETP model parameters. In practice these parameters are difficult to derive, and the ETP model used in the framework may be inaccurate in terms of characterizing the real energy consumption of the TCLs. To address these challenges, we present a joint state and parameter estimation framework in our companion paper [31], which enables users to compute bidding prices only based on local measurements.

V. CONCLUSION

This paper presents a market mechanism for the coordination of thermostatically controlled loads, where a coordinator manages a group of TCLs using pricing incentives to maximize the social welfare subject to a peak energy constraint. In the paper, a mechanism is proposed to implement the desired social choice function in dominant strategy equilibrium. This mechanism consists of a novel bidding strategy that incorporates information on both the load dynamics and the time-varying user preferences. It is proven that under the proposed mechanism, the coordinator can not only maximize the social welfare but also realize the team optimal solution. Future work includes formulating the fully dynamic market-based coordination framework with multiple periods and extending the results to energy storage devices and deferrable loads such as plug-in electric vehicles, washers, dryers, among others.

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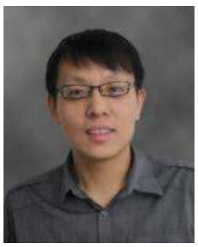

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