

Economics Working Paper 39

**Market Capture by Two Competitors: The  
Pre-Emptive Location Problem\***

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## Abstract

We consider a location and allocation game for two competitor firms, A and B, that each seek to locate  $p$  facilities in a network. A market is captured by a particular firm if that market's closest facility belongs to that firm rather than a competitor. The question is as follows: Firm A wants to locate its  $p$  facilities so that B, which enters also with  $p$  facilities *after* Firm A has located its facilities, will capture the minimum market value possible. That is, Firm A wishes to pre-empt Firm B in its bid to capture market share to the maximum extent possible. A model is presented that addresses this issue, together with solution methods and computing times.

# 1 Introduction

Spatial competition between firms has been a mainstream topic in the last decade in the field of location-allocation modelling. In general, these models focus on the location, pricing and production decisions faced by firms when entering and/or operating in a spatial market. The main objective of these firms is to maximize their profits by being more competitive in prices and/or location than their competitors. The literature is extensive and good general reviews can be found in Friez et al. (1988) and Eiselt and Laporte (1988).

This paper presents a model of competition between two profit-maximizing firms, (from now on, Firm A and Firm B) which want to enter a spatial market by locating several retail facilities each, in order to sell a single homogeneous product to the customers in that market. Both firms compete for clients on the basis of distance. A firm will capture a client if it locates a facility closer to him than its competitor. The space will be represented by a network where customers as well as potential facility sites for both firms are represented as discrete points. The question addressed in this study is as follows: Firm A wants to locate its  $p$  facilities so that B, which enters also with  $p$  facilities *after* Firm A has located its facilities, will capture the minimum market value possible. Firm A wants to choose its servers' locations taking into account the reaction of the future entrant in that market. That is, Firm A wishes to *pre-empt* Firm B in its bid to capture market share to the maximum extent possible. In this sense, it is similar to a leader-follower Stackelberg game: a firm acts as a leader (Firm A) since it takes action first by locating its servers, and the follower (Firm B), knowing Firm A servers' locations, decides where to locate its facilities. Since Firm A knows the strategy (but not the location) adopted by its competitor, it will try to locate its servers so as to *pre-empt* the market capture by Firm B. In other words, Firm A wants to locate its facilities so as to minimize the future capture by Firm B.

The preemption of entering firms in a market is becoming a mainstream topic in the field of industrial organization. Several models have been developed to study different strategies that a firm can take to pre-empt potential competitors entering a market. In general these strategies focuss on finding the necessary output level to be produced and/or the investment needed by (a) existing firm(s) to make the entry of a new firm ot profitable in that market (Dixit (1980), Fudenberg and Tirole (1981), Gilbert and Harris (1981; 1983)

among others). In general, these models are aspatial. They do not consider the effects of location on market preemption. If distance is considered as a factor in the determination of the level of output and/or investment needed to preempt competitors from entering a regional market, then considerations on the location of these firms have to be examined.

A similar model to the pre-emptive problem that studied competition within a locational context was developed by Miller et al. (1992). In their model, an entering Firm seeks the location in a discrete network where there are already several other oligopolistic competitors so as to maximize profits by producing and shipping a homogeneous good. While this firm has knowledge of the reaction of the competitors to its location, these do not have this information and assume that their competitors in the market will hold their production and shipping activities at existing levels: while the entering firm acts as a Stackelberg firm, the rest take the decisions using Cournot-type strategies. A heuristic solution algorithm was used to solve the problem.

ReVelle and Serra (1991) also used a leader-follower game between two firms competing for market share in discrete space. Both firms wanted to relocate some of their servers in order to capture the maximum market share. The strategies used by them were similar to those developed by Miller et al. The model was iterative and in each step one of the firms relocated a given number of servers. Independently of the strategies used by the firms, no clear locational equilibrium was observed at the end of several iterations.

## 2 The Pre-Emptive Location Model

The foundation of this study relies on the MAXimum CAPture (MAXCAP) Problem formulated by ReVelle (1986). This model, based on the classical Maximal Location Covering Problem of Church and ReVelle (1974), consists of the location of servers by an entering firm so as to maximize its market share capture in a market in which competitor servers are already in position. Its integer linear formulation together with the maximum client capture objective has been the starting point for several location problems. Eiselt and Laporte (1989) modified the MAXCAP formulation to include attraction parameters based on gravity models and Voronoi diagrams. ReVelle and Serra (1991) extended the formulation to allow relocation of existing servers as well as the location of new servers. The MAXCAP model has also been

adapted to consider facilities that are hierarchical in nature and where there is competition at each level of the hierarchy (Serra et al. (1992)).

Consider two firms, A and B, that wish to enter in a market by locating the same number of servers each. For simplicity, each firm bears the same unit costs and produces a homogeneous product. The price faced by consumers is uniform and fixed for both firms across the region of interest. Both firms face the same demand function and product differentiation is horizontal: consumers/users will travel to their closest server to obtain the desired product. A consumer node is captured by a firm if it has a server closer to it than any of its competitor's servers. If some population has two or more servers at the same distance (for example, if two servers locate on top of each other as in Hotelling's game) they will divide in equal share the captured population.

Suppose that Firm A knows that its competitor B will locate its servers *after* it has located its own servers. Therefore, Firm A acts as a Stackelberg leader since it locates its  $p$  servers first. Firm B acts as a follower since it will determine the location of its  $p$  servers with the knowledge of Firm A's siting decision.

The mathematical formulation of the model is as follows:

$$\text{Max } Z^A = \sum_{i=1}^n a_i y_i^A + \sum_{i=1}^n \frac{a_i}{2} z_i$$

subject to:

$$y_i^A \leq \sum_{j \in N_i(b_i)} x_j^A \quad \forall i \in I \quad (1)$$

$$z_i \leq \sum_{j \in O_i(b_i)} x_j^A \quad \forall i \in I \quad (2)$$

$$y_i^A + z_i \leq 1 \quad \forall i \in I \quad (3)$$

$$\sum_{j=1}^n x_j^A = p \quad (4)$$

$$y_i^A, z_i, x_j^A = (0, 1) \quad \forall i \in I, \forall j \in J$$

where:

$i, I$  = index and set of demand areas

$j, J$  = index and set of potential facilities

$$y_i^A = \begin{cases} 1, & \text{if node } i \text{ is captured by Firm A} \\ 0, & \text{otherwise} \end{cases}$$

$$z_i = \begin{cases} 1, & \text{if node } i \text{ is divided between A and B} \\ 0, & \text{otherwise} \end{cases}$$

$$x_j^A = \begin{cases} 1, & \text{if Firm A locates a server at node } j \\ 0, & \text{otherwise} \end{cases}$$

$a_i$  = Population at node  $i$

$b_i$  = closest Firm B server to node  $i$

$$N_i(b_i) = \{\forall j \in J, d_{ij} < d_{ib_i}\}$$

$$O_i(b_i) = \{\forall j \in J, d_{ij} = d_{ib_i}\}$$

$d_{ij}$  = distance between node  $i$  and node  $j$

$d_{ib_i}$  = distance between node  $i$  and the closest Firm B server to  $i$

This formulation is very similar to the mathematical form of the MAXCAP problem. The first set of constraints allows the capture of node  $i$  by Firm A if and only if Firm A has a server located closer to  $i$  than the closest Firm B server to node  $i$ . The second set of constraints examines the situation where there is a tie in the capture of a node. The variable  $z_i$  will be allowed to be 1 if and only if the distance from  $i$  to the closest Firm A server and to the closest Firm B server is equal. Therefore, the capture of node  $i$  will be divided between both firms, as stated in the objective function. Observe that for any node  $i \in I$  can be captured, or half captured, or lost to the competitor.

Constraints in group (3) will enforce one of these three states. Finally, the number of servers to be located by Firm A is determined by constraint (4).

The basic difference between the MAXCAP problem and the Preemptive Formulation presented relies on the sets  $N_i$  and  $O_i$  in restrictions (1) and (2) respectively. The  $N_i$  set contains all candidate nodes that are closer to  $i$  than the closest competitor server  $b_i$ , while  $O_i$  is the set of all candidate nodes that are located at the same distance from the closest competitor server  $b_i$ . In the MAXCAP problem these sets were known a priori because the locations of the competitor servers in the network were known. Therefore, it was possible to know all candidate nodes included in these sets. Now, however Firm A does not have this knowledge, since Firm B has not located its servers yet. On the other hand, Firm B *will have this class of information*. Since Firm B will locate its servers *after* Firm A has sited its A, in contrast, it will be able to locate optimally its servers based on the location of the servers of Firm A, so as to obtain the maximum capture possible by using the MAXCAP formulation, since sets  $N_i$  and  $O_i$  are known to Firm B. Firm A will try to find the location that will minimize the maximum capture that Firm B can achieve.

A key feature in the strategic location of Firm A is that it will never be able to capture more than 50% of the market after Firm B's locational reaction. Since Firm B will locate its servers after Firm A, independently of the location of Firm A's facilities, Firm B can achieve at least a 50% capture by locating its facilities on top of A facilities, and thus, dividing the market. Therefore, the best strategy for Firm A is to obtain a set of locations such that B will have no other option than locating its servers on top of Firm A servers, or at positions which yield the same 50% capture. The best objective value of the solution to the problem faced by A is already known: 50% of the market share. Then, the question is, is it possible to find an optimal siting strategy for Firm A that it will *always* prevent Firm B from obtaining more than half of the market share? In general, the answer to this problem is negative. This can be seen in the following simple example. Consider the 4-node network in Figure 1. The total population in the network is 11. Suppose now that both Firm A and Firm B want to locate one server each at the nodes of the network so as to maximize their market share. Firm A acts as the leader and Firm B as the follower. It is easy now to examine all the siting possibilities. If, for example, Firm A locates its server at node A, Firm B will capture 5.5 if locating at A (and therefore sharing the market), 3



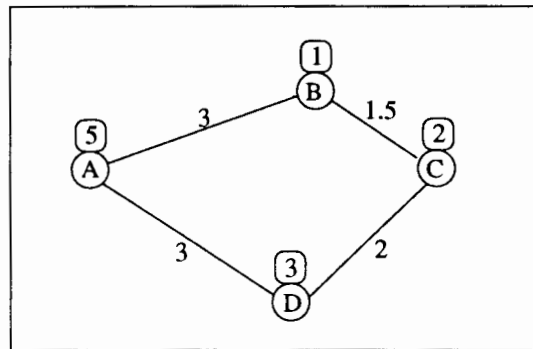


Figure 1: 4-node network

if the location occurs at B, 6 if at C and 5 at D. Thus, Firm B will maximize its market capture by locating at node C, since the other locations for its server would lead to a smaller market share. If the same process is repeated for all Firm A's server possible locations, Firm A will have perfect knowledge of the reaction of Firm B to its location. Table 1 shows all possible sites for A and the locational response of Firm B. After checking all the possible locations for A and its corresponding response by Firm B, the final result will be that Firm A will choose to locate in node A (or D) and hence capture 5 and Firm B will capture 6 by locating in C (or A). Firm B will always have more than 50% of the market share since there is no situation were A can locate its facility so as to force Firm B to obtain half.

From this example, it can be seen that in the general case Firm A may be unable to prevent Firm B from capturing more than 50% of demand. That is, the follower can always obtain at least 50% demand capture and sometimes more.

### 3 The Pre-Emptive Capture Heuristic Algorithm

A solution to the pre-emptive capture problem can be obtained by enumerating all locational possibilities for the servers of Firm A. For each locational situation, ReVelle's MAXCAP formulation could be used to see what Firm B's reaction to Firm A's locations is. After all possibilities are computed, Firm A

Table 1: 4-node Capture

Location of Firm A		Answer of Firm B	
Node	Capture	Node	Capture
A	5	C	6
B	5	A	8
C	3	D	8
D	5	A	6

would choose those sites for its locational choice that minimize Firm B's capture response since that choice maximizes A's own market capture. This method is computationally possible if the network is very small but it becomes intractable as the network gets larger. Therefore, there is a need to develop some other more efficient solution method.

Two heuristic algorithms have been built to obtain solutions to the Pre-Emptive Capture Problem. The PRE-EMptive heuristic ALgorithm 1 (PREMAL1) combines the well-known Teitz and Bart one-opt heuristic with the MAXCAP integer programming formulation. The second heuristic algorithm, PREMAL2, is fully based on a one opt heuristic.

The PREMAL1 heuristic procedure is iterative, and the first iteration has two phases. In the first phase, Firm A locates  $p$  servers using any heuristic or exact procedure (e.g., Teitz and Bart or a Covering or  $P$ -Median formulation) without regard to any response of Firm B. Once Firm A's servers are located, Firm B sites its  $p$  servers with the MAXCAP integer program using linear relaxation and branch and bound when necessary. Thus, Firm B will obtain the optimal maximum capture given the location of the  $p$  servers of Firm A.

In the second phase, Firm A, knowing that the location of its  $p$  servers is the determinant of the final market share, will try to find a better solution by relocating one or more of them using a one-opt procedure. At each iteration, Firm A will relocate one of its servers and then use the MAXCAP problem to obtain the maximum capture that B can achieve after the relocation of Firm A. If the relocation has provided a set of positions that is better than before the one-opt trade, i.e., A's market share after Firm B's response has improved, it will keep the new set of locations as the best so far. Otherwise, Firm A will ignore the relocation and will restore the previous solution. The

one-opt trade will be done for all nodes and Firm A servers. A step-by-step description of the PREMAL1 follows:

1. Locate Firm A's  $p$  facilities using any method.
2. Locate Firm B's  $p$  facilities using the MAXCAP integer program.
3. Compute the market share for each firm and store the locations for both firms.
4. Trade the location of one of the  $p$  servers of Firm A.
5. Locate the  $p$  servers of Firm B using the MAXCAP integer program.
6. Compute the new market share for both firms. If Firm A's market share has improved, store new solution. If not, restore old solution. Repeat steps 4 to 6 until all of Firm A's  $p$  facilities and nodes have been traded.
7. If Firm A has improved the market share obtained before steps 4-6, go to step 4 and restart the procedure. When no improvement is achieved on a complete set of one-at-a-time trades, stop.

If the number of nodes in the network is very large or if the availability of computing time is scarce, the heuristic can become computationally expensive, since at each iteration an integer program has to be solved. As the number of servers to be located and the number of nodes increase, the amount of integer programs needed to obtain the location of Firm B's servers grows very fast.

In order to reduce computing time, the MAXCAP problem used to locate Firm B's servers could be replaced by a modified Teitz and Bart heuristic, leading to a new heuristic PREMAL2. While the iterative procedure for the location of Firm A's servers in the modified PREMAL2 heuristic remains the same as in PREMAL1, the response location of the servers of Firm B can be obtained in a similar fashion. Step 5 of PREMAL1 can be replaced by a one-opt heuristic, where at each location trade of a Firm B server an objective function that computes the capture obtained by Firm B is computed, in order to evaluate if the realization (or not) of B's one-opt trade improves its capture objective. Only trades that lead to an improvement of Firm B's capture are

accepted. In the following section, computational experience of the Maximum Capture Heuristic Algorithm using both the MAXCAP integer program and the Teitz and Bart Heuristic for the response siting of B's servers is presented.

## 4 Computational Experience

PREMAL1 and PREMAL2 were tested on the well-known 55-node Swain network. The number of customers in each node and the nodes' coordinates are shown in the Appendix. The network is depicted in Figure A1. Standard fortran code and MINT, an adapted version of MINOS version 5.1 to solve integer linear programs using the branch and bound method, was used to solve the problems.

The total number of customers in the system is 3750. Firm A and Firm B want to enter in this market by locating each  $p$  servers to capture the maximum share. As mentioned before, whatever clients Firm A gains, Firm B loses. Several numbers of servers (from 1 to 9 servers for each firm) were used using the two heuristics PREMAL1 and PREMAL2 presented in the last section. For each number of servers  $p$  10 runs were made using different starting locations for Firm A generated randomly. The same starting locations for Firm A were used for PREMAL1 and PREMAL2. Table 2 and Table 3 present the results for PREMAL1 and PREMAL2 respectively. In both tables, the first column represents the number of servers located by each firm. The average initial capture by A after B has entered, both in absolute values and in percentages relative to the total population in the network (3570 people in 55 nodes) are indicated in the second and third columns respectively. Results for the final capture are presented in the last six columns. The first three indicate the lowest, average and final capture in absolute figures obtained by Firm A after the 10 runs. The figures in the last three columns represent the same final results but relative to total population.

Observe that no solutions where Firm B's optimal capture was 50% were obtained when PREMAL1 was used. Ninety different runs using PREMAL1 were done and Firm A's final capture was always below 50%. The capture values for Firm A concentrated approximately between 44% and 47.7% except when three servers were located for each firm. In this case, the lowest value obtained after the ten corresponding runs was 41.9%. The same solution was obtained for all runs when one and two servers for each firm are located:

Table 2: Results, PREMAL1 55-Node Network

number of facilities	Initial Capture		Final Capture					
	Avge		Low	Avge	High	Low	Avge	High
1	674	18.9%	1697	1697	1697	47.5%	47.5%	47.5%
2	777	21.7%	1705	1705	1705	47.7%	47.7%	47.7%
3	840	23.5%	1498	1591	1630	41.9%	44.5%	45.6%
4	847	23.7%	1623	1651	1677	46.0%	46.2%	46.9%
5	867	24.8%	1591	1627	1648	44.5%	45.5%	46.1%
6	872	24.4%	1594	1612	1627	44.6%	45.1%	45.5%
7	965	27.0%	1555	1594	1616	43.5%	44.6%	45.2%
8	1015	28.4%	1627	1637	1644	45.5%	45.8%	46.0%
9	901	25.2%	1652	1662	1673	46.2%	46.5%	46.8%

Table 3: Results, PREMAL2, 55-Node Network

number of facilities	Initial Capture		Final Capture					
	Avge		Low	Avge	High	Low	Avge	High
1	654	18.3%	1697	1697	1697	47.5%	47.5%	47.5%
2	777	21.7%	1705	1705	1705	47.7%	47.7%	47.7%
3	921	25.8%	1469	1674	1767	41.1%	46.8%	49.4%
4	892	25.0%	1622	1704	1818	45.4%	47.7%	50.9%
5	922	25.8%	1582	1670	1791	44.3%	46.7%	50.1%
6	976	27.3%	1578	1632	1692	44.1%	45.6%	47.3%
7	979	27.4%	1635	1666	1725	45.7%	46.6%	48.3%
8	954	26.7%	1624	1652	1706	45.4%	46.2%	47.7%
9	923	25.8%	1622	1689	1752	45.4%	47.3%	49.0%

47.5% if only one server is located and 47.7% when two servers are located. On the other hand, no relation was observed between initial capture locations and final capture results.

Recall that the locational reaction of Firm B's servers when PREMAL1 is used given the location of Firm A servers is always optimal since the MAXCAP problem is solved using linear relaxation and branch and bound when necessary. This is not true if PREMAL2 is employed, since a one-opt heuristic procedure described in the last section replaces the optimal MAXCAP problem when locating the servers of Firm B in the iteration process. The results using PREMAL2 presented in Table 3 show the capture obtained by Firm A if PREMAL2 were used. Observe that when 4 and 5 servers were located, at least one final solution to the problem gave Firm A more than 50% of the market share (50.9% and 50.1%). But Firm B's servers can locate on top of A's servers reducing A's capture to 50%.

Figure 2 presents an example of the the final locations for Firm A and Firm B obtained when 5 servers are located. The nodes within the dashed line are the ones where Firm A was able to pre-empt Firm B from entering in these markets. The nodes within the dotted line are divided by both firms, since in node 13 each firm has located a server. Therefore, all nodes whose closest server is node 13 are served by both competitors. Finally, the rest of the nodes are fully captured by Firm B. In this example, the final capture by Firm A is 1627.5 and 1947.5 for Firm B.

In most cases, the final capture obtained by Firm A using any number of servers is greater than the capture achieved when PREMAL1 is used. These results can lead to the wrong conclusion that PREMAL2 is more efficient in obtaining final solutions to the location of Firm A's servers. To see the extent of the error induced when PREMAL2 was used, for each final solution obtained for Firm A with PREMAL2 when locating 1 to 9 servers of both firms, the MAXCAP problem was applied to see what the real optimal location and capture of Firm B would be. For each number of servers and each one of the ten runs, the optimal final solution was computed for Firm B. Table 4 presents the mean error and its standard deviation when 1 to 9 servers are located. PREMAL2 usually leads to a 5% average error in the capture of Firm B. That is, if Firm A locates its servers using PREMAL2, it will overestimate the final capture by 5% before Firm B locates its servers.

Finally, real CPU user run times for each heuristic are presented in Table 5.

Table 4: Differences in final Capture for Firm A using PREMAL1 and PREMAL2

	number of servers located by each Firm								
error	1	2	3	4	5	6	7	8	9
avge	0.0%	0.0%	7.2%	6.5%	5.0%	6.3%	7.1%	3.9%	4.0%
$\sigma$	0.0%	0.0%	3.4%	4.2%	3.3%	5.5%	5.0%	2.6%	2.2%

For each number of servers the average run time for the 10 different runs is presented. The heuristics were solved in a Hewlett Packard 710 workstation with a Risc processor. As expected, PREMAL2 is far more efficient in run time than PREMAL1, since the adapted Teitz and Bart heuristic is much faster than the integer MAXCAP program. For example, when four servers were located for each firm, PREMAL1 took an average of 39.9 minutes for each run while PREMAL2 took only 1.8 minutes to find a solution, but with an error of 5% in the final capture of Firm A.

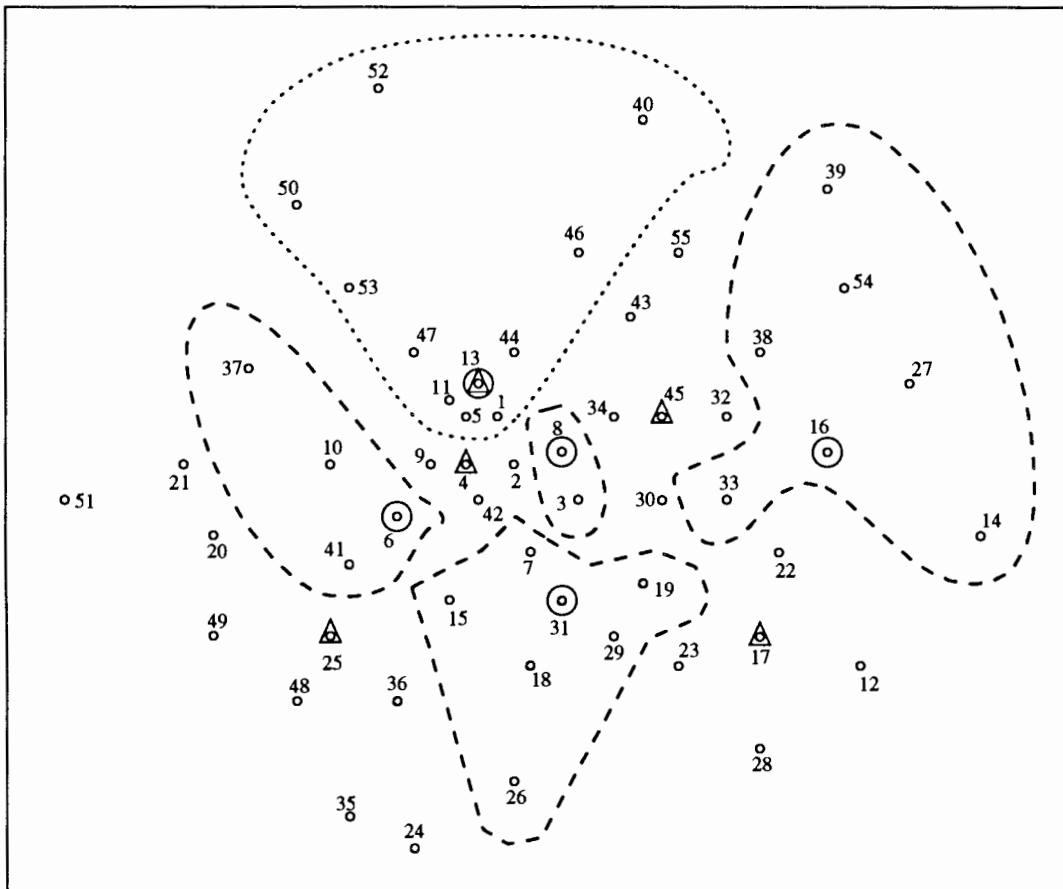
Recall that in PREMAL1 for each one-opt Firm A's server trade an integer program with 165 variables, all binary integer, and 166 restrictions was solved to find the response in the location of Firm B's servers. Even though the MAXCAP is very efficient and little or no branch and bound is needed when solved, it can become computationally very expensive if the network used is very large. Since the PREMAL heuristic needs to solve a MAXCAP problem for each one-opt trade, as the number of nodes increase, the amount of MAXCAP problems increases exponentially. If instead of an  $N$ -node network an  $(N+1)$ -node network is used, the number of MAXCAP problems to be solved increase *at least* by  $(N+1)$ .

## 5 Conclusions and Further Research

In this study, market preemption by a firm has been studied within a locational framework. In a system where two firms that offer the same good or service seek to enter a market, the location of their servers play a dominant role in the final profits that can be achieved. If a Stackelberg leader-follower

Table 5: Average Running Time using PREMAL1 and PREMAL2

Heuristic	number of servers located by each Firm								
	1	2	3	4	5	6	7	8	9
PREMAL1	0	13.7	19.8	26.9	39.9	47.9	51.7	62.4	73.5
PREMAL2	0.0	0.1	0.4	1.0	1.8	2.6	4.4	5.9	8.3
CPU time in minutes									



- - - Nodes captured by Firm A
- ..... Nodes divided by Firm A and Firm B
- Firm A's server
- △ Firm B's server

Figure 2: Example of Final Capture by Firm A



game is considered, the leader in the location of its servers will never be able to capture more than fifty percent of the market share, and in many cases the final capture will be less than half of the market. A model that seeks to locate servers to maximize the market capture by a firm that acts as a leader has been presented. The model, based on the MAXCAP problem, is quite efficient in obtaining solutions, even though no exact algorithm has been found to obtain optimal locations.

The condition of each firm locating the same number of servers is considered a first step in analyzing these problems. Followers might site fewer or more servers than the leader in this locational competition. Such problems, where the follower's numerical response is uncertain, deserve attention in the future.

## 6 Appendix

Table 6: 55-Node Network demand and coordinates

node	pop	coord		node	pop	coord		node	pop	coord	
		x	y			x	y			x	y
1	120	32	31	20	77	25	14	39	47	46	51
2	114	29	32	21	76	29	12	40	44	50	40
3	110	27	36	22	74	24	48	41	43	23	22
4	108	29	29	23	72	17	42	42	42	27	30
5	105	32	29	24	70	6	26	43	41	38	39
6	103	26	25	25	69	19	21	44	40	36	32
7	100	24	33	26	69	10	32	45	39	32	41
8	94	30	35	27	64	34	56	46	37	42	36
9	91	29	27	28	63	12	47	47	35	36	26
10	90	29	21	29	62	19	38	48	34	15	19
11	88	33	28	30	61	27	41	49	33	19	14
12	87	17	53	31	60	21	35	50	33	45	19
13	87	34	30	32	58	32	45	51	32	27	5
14	85	25	60	33	57	27	45	52	26	52	24
15	83	21	28	34	55	32	38	53	25	40	22
16	82	30	51	35	54	8	22	54	24	40	52
17	80	19	47	36	53	15	25	55	21	42	42
18	79	17	33	37	51	35	16				
19	79	22	40	38	49	36	47				

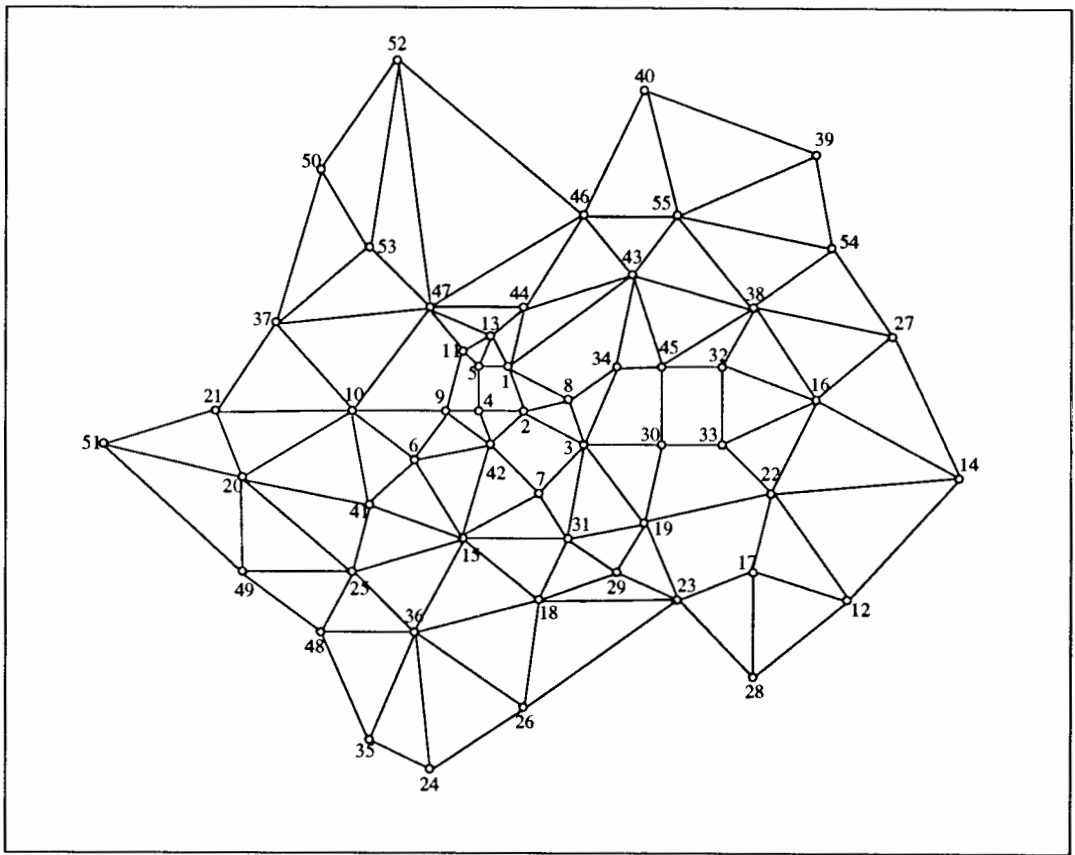


Figure 3: 55-node network

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