



Market-Share Contracts as Facilitating Practices

Roman Inderst*

*University of Frankfurt and Imperial College London

Greg Shaffer**

**University of Rochester and University of East Anglia

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Contact Details:

Roman Inderst, University of Frankfurt and Imperial College London, E-mail:
inderst@finance.uni-frankfurt.de

Greg Shaffer, University of Rochester and University of East Anglia, E-mail:
shaffer@simon.rochester.ed

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Roman Inderst[†]

Greg Shaffer[‡]

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Abstract

This article investigates how the use of contracts that condition discounts on the share a supplier receives of a retailer's total purchases (market-share contracts) may affect market outcomes. The case of a dominant supplier that distributes its product through retailers that also sell substitute products is considered. It is found that when the supplier's contracts can only depend on how much a retailer purchases of its product (own-supplier contracts), both intra- and inter-brand competition cannot simultaneously be dampened. However, competition on all goods can simultaneously be dampened when market-share contracts are feasible. Compared to own-supplier contracts, the use of market-share contracts increases the dominant supplier's profit and, if demand is linear, lowers consumer surplus and welfare.

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[†]University of Frankfurt and Imperial College London; E-mail: inderst@finance.uni-frankfurt.de.

[‡]University of Rochester and University of East Anglia; E-mail: shaffer@simon.rochester.edu.

1 Introduction

Recent antitrust decisions on both sides of the Atlantic have sparked interest in the appropriate treatment of rebates and discounts that do not reflect a firm’s cost structure. Of particular interest and concern are rebates and discounts that apply to all units purchased by a retailer once the retailer’s purchases reach some threshold. These discounts and rebates, which feature a discrete jump down in the retailer’s outlay when the threshold is reached, can take many forms. They may be based solely on how much is purchased from the supplier, or they may, in addition, be linked expressly or implicitly to the supplier’s share of the retailer’s overall purchases. Discounts that are based on the supplier’s share of purchases are known as market-share discounts, and the contracts that give rise to them are known as market-share contracts.

A concern among policy makers is that dominant suppliers might use market-share contracts to foreclose their competitors (either by denying them scale economies or by exploiting their lack of financial resources).¹ A similar concern also applies to the use of exclusive-dealing arrangements, an extreme form of market-share contract, about which much has been written.² As with any foreclosure concern, however, these views are subject to the “Chicago-school critique:” if the exclusion of competitors reduces industry profits, why can the firms not do better?

This paper considers an alternative rationale for the use of market-share contracts. When the dominant supplier sells to competing retailers, each of whom can sell both its good and an imperfect, competitively-supplied substitute, competition between retailers can lead to prices that are lower than what a fully (horizontally and vertically) integrated firm would charge. Although the supplier can try to mitigate this effect by using its wholesale price to dampen competition on its own good, doing so gives retailers an incentive to divert sales to the substitute product, thereby increasing competition between goods. By using market-share contracts, the supplier can both dampen competition on its own good and prevent the diversion of its sales.

Intuitively, contracts in which a retailer’s payment to the supplier depends solely on how much the retailer purchases of the supplier’s product (henceforth referred to as ‘own-supplier’

¹See, for example, a summary of the issues raised in *Michelin II*, *Virgin Atlantic v. British Airways*, and *LePape’s v. 3M*, as discussed in Majumdar et al. (2005), chapter 5, Annexes B and C, respectively. See also the EC Commission’s finding against Intel for offering share-based discriminatory rebates, discounts and marketing incentives in the market for microprocessors (<http://www.nytimes.com/2009/05/14/business/global/14compete.html>).

²One can think of an exclusive-dealing arrangement as a special case of a market-share contract in which the supplier offers a discount for exclusivity and charges a prohibitively high price for market shares less than 100%.

contracts) do not suffice to maximize industry profits. The reason is that there is a conflict between two goals: dampening intra-brand competition, which would require a high per-unit wholesale price, and inducing a profit-maximizing ratio between the retail price of the dominant supplier's good and that of the competitively-supplied good, which would require a low per-unit wholesale price. This conflict can be overcome when using market-share contracts, which can be chosen to induce retailers to set relative prices at the industry-profit-maximizing level, in spite of the dominant supplier's wholesale price being high enough to dampen intra-brand competition.

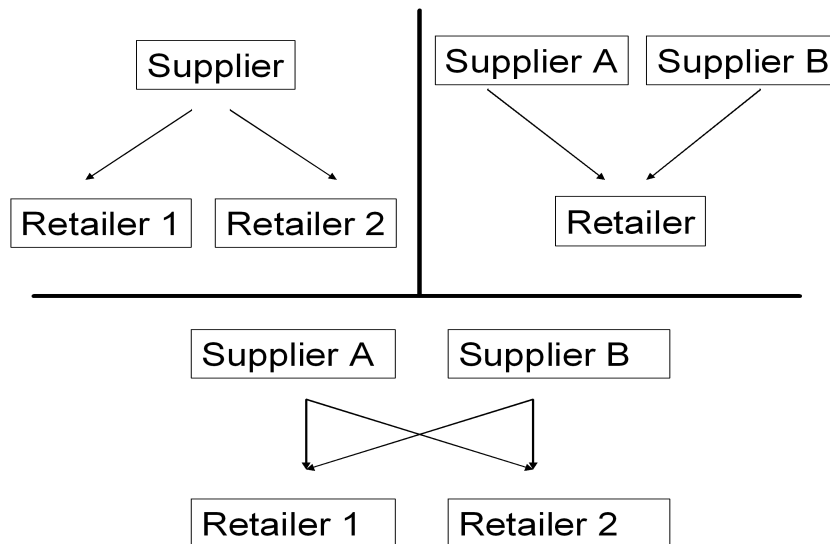
Market-share contracts are more effective than own-supplier contracts in increasing industry profit because they can be used to simultaneously dampen competition (both intra-brand and inter-brand) on both goods, whereas the latter cannot. This dampening-of-competition motive for market-share contracts does not appear in the existing literature, which focuses instead on the case of a dominant supplier selling to a downstream monopolist. In this literature, intra-brand competition on the supplier's product is absent, and other motivations for market-share contracts are considered, such as the use of market-share contracts to induce more service provision (Mills, 2004), act as a rent-shifting device (Marx and Shaffer, 2004), mitigate risk (Chioveanu and Akgun, 2007), or facilitate price discrimination (Majumdar and Shaffer, 2009).

Although the welfare effects of market-share contracts in the model are in general ambiguous, they can be signed when demands are linear. In this case, market-share contracts unambiguously lead to higher retail prices and hence lower consumer surplus and welfare. The latter are lower even though none of the ingredients usually considered necessary for engendering such harm, such as a first-mover advantage for the dominant supplier, economies of scale, discriminatory offers, non-coincident markets, liquidated damages and/or financial constraints, need be present.³ Moreover, the adverse welfare effects are directly attributable to the dominant supplier's use of market-share contracts, as opposed to contracts that may feature 'non-incremental' discounts (e.g., all-units discounts) but that are not tied to the supplier's share of the retailers' purchases. When the dominant supplier can only offer own-supplier contracts, for example, restrictions that rule out the use of non-incremental discounts have no effect on equilibrium prices and quantities.

The rest of this article is organized as follows. Section 2 provides an overview of the argument. Section 3 describes the model and defines notation. Section 4 compares own-supplier contracts

³See the articles by Rasmusen et al, 1991; Bernheim and Whinston, 1998; Segal and Whinston, 2000; Fumagalli and Motta, 2006; Simpson and Wickelgren, 2007; and Ordovery and Shaffer, 2008. See also the discussion in the recent symposium on the effects of market-share contracts (Spector, 2005; Kobayashi, 2005; and Heimler, 2005).

Figure 1: Variants of the 2x2 model



and market-share contracts under quantity competition. Section 5 considers several extensions. Section 6 looks at the case of price competition. Section 7 summarizes the article’s findings.

2 Overview of the argument

Before proceeding to the formal model, it may be helpful to begin by providing an overview of the argument. The case we consider is that of a dominant supplier selling its good through downstream firms that also carry competing products. For tractability reasons, we restrict attention to competing products that are produced by a competitive fringe (alternatively, one can think of the competing products as private label brands that are imperfect substitutes for the dominant supplier’s name brand.) There is both intra- and inter-brand competition in this setting: each good is sold by multiple competing retailers, and each retailer sells multiple competing goods. The dominant supplier’s problem then is how to choose its contract terms to maximize its profit given that these two types of competition present very different challenges.

The challenge posed by intra-brand competition is best seen in the canonical case of an upstream monopolist contracting with competing (differentiated) downstream firms under complete information (see the top left-hand diagram in Figure 1). In this case, some degree of

double marginalization is necessary to maximize industry profits. If the product were sold to the retailers at upstream marginal cost, downstream competition would result in a dissipation of profits to consumers. To prevent the corresponding loss of profits, the supplier can specify a two-part tariff with a fixed fee that is set to extract all the surplus and a wholesale price that is chosen to induce the downstream firms to choose the industry-profit-maximizing retail prices.

The challenge posed by inter-brand competition differs. This is best seen in the canonical case of two upstream firms contracting with a downstream monopolist (see the top right-hand diagram in Figure 1). In this case, if an upstream firm were to raise its wholesale price above marginal cost, the downstream monopolist would cut back on its sales of the now less desirable product and increase its sales of the now more desirable product, thus hurting the firm that raised its wholesale price. As such, it is optimal for each upstream firm to transfer its good at cost to the downstream firm (with fixed fees chosen to extract surplus), which then internalizes the demand externalities between products and sets the industry-profit-maximizing retail prices.

These opposing forces create tension for a supplier when, as in our setting, both intra-brand and inter-brand competition are present (see the bottom middle diagram in Figure 1). The supplier needs to charge a wholesale price above marginal cost in order to dampen downstream competition on its product. However, a mark-up on the supplier's product induces the retailers to substitute their purchases away from the supplier's product to the competitively-supplied brand. In order to replicate the industry-profit maximizing outcome, therefore, the supplier needs a contract that can eliminate completely the horizontal externality introduced by downstream competition. But, inter-brand competition deprives contracts that only depend on the quantities purchased of the supplier's product (own-supplier contracts) of their ability to do so. Market-share contracts on the other hand work well to solve the horizontal externality. The upstream supplier charges a discounted unit price higher than marginal cost to weaken competition on its own product, and at the same time, by making the rebated price conditional on a target market share, can restrict competition on the competitively-supplied brand as well. Were the retailers to substitute away from the supplier's product in favor of the substitute product, they would fail to reach the target market share and thus would not qualify for the supplier's discount.

Downstream competition among retailers is key in our setting because if each retailer were a local monopolist, the dominant supplier could induce the monopoly outcome by offering each retailer a two-part tariff with a wholesale price equal to marginal cost. Inter-brand competition is also key because if there were only one upstream firm, two-part tariff contracts could again induce

the monopoly outcome. In contrast, when each retailer sells multiple competing products, no own-supplier contract (whether a two-part tariff or otherwise) can induce the monopoly outcome.

3 The model

We analyze market-share contracts in a supplier-retailer framework. Two differentiated retailers compete in the downstream market. Each retailer carries good A and good B , which are also differentiated.⁴ The goods can be produced at constant marginal costs $c^A \geq 0$ and $c^B \geq 0$, respectively. Good A is produced by supplier A , whose contracts with the retailers will be the focus of our analysis. Good B is produced by one or more upstream firms and supplied competitively downstream at a per-unit cost of c^B to each retailer.⁵ For simplicity, we assume the retailers are symmetrically differentiated, so that if both retailers charge the same price for good A and the same price for good B , then the demands facing both retailers will be the same.

There is complete information about demand and contract offers. In particular, both retailers know that the other retailer can obtain good B at cost, and both retailers can observe supplier A 's contract offers before acceptance decisions are made. Thus, we implicitly assume that supplier A has no incentive to engage in opportunism, either because it wishes to uphold its reputation or because doing so might risk violating competition laws against price discrimination. The effect of this assumption, and the assumption of complete information about demand, will be discussed in more detail, namely in Section 5, after our main results and intuition are presented.

The game proceeds as follows. Supplier A simultaneously makes take-it-or-leave-it offers to both retailers. The offers specify the retailers' payment terms. We first consider contracts whose terms can depend only on how much the retailer buys of good A . We then consider contracts whose terms can depend, in addition, on the share supplier A receives of the retailer's overall purchases. We will refer to the former as "own-supplier contracts" and to the latter as "market-share contracts." Each will be formally introduced and analyzed in the next section.

Retailers simultaneously and independently decide whether to accept or reject their offer. If both retailers reject their offer, the game continues but competition takes place on good B only. If both retailers accept their offer, there is no further contracting and competition takes place on both goods. If only one retailer accepts supplier A 's offer, we assume that supplier A and

⁴Cases involving more than two goods and more than two retailers are considered in Section 5.

⁵Our focus is on the behavior of a single dominant firm. The case of more than one strategic supplier is discussed in Section 5.

the retailer that accepted its offer can renegotiate their contract and agree on new supply terms (it will be in their joint interest to do so if the rival retailer's rejection was unexpected). After the renegotiation phase, if any, the game proceeds with competition on both goods (under the initial contract if there is no renegotiation, or under the new contract if there is renegotiation).

Given our assumptions on the contracting game, in particular the assumption that offers to both retailers are made simultaneously, and the assumption that renegotiation is possible in the out-of-equilibrium event that one retailer rejects supplier A 's contract, it is straightforward to show that each retailer's outside option (which is what each retailer could earn by selling only good B) is independent of supplier A 's initial contract offers. This observation also has an equally straightforward implication. Given that each retailer's outside option does not depend on the simultaneous offer made to the other retailer, supplier A 's choice of contracts is not distorted by any attempt to use its contract with one retailer to shift rent from the other retailer. Hence, it follows that supplier A 's equilibrium contract offers will be chosen to maximize overall joint profits subject to each retailer earning its fixed outside option. We will return to discuss the specification of the retailers' outside options as well as the issue of renegotiations in Section 5.

4 The case of quantity competition

We assume for now that retailers compete in quantities. Let $s \in S = \{A, B\}$ and $r \in R = \{1, 2\}$. Denote the quantity of good s that is supplied to retailer r by q_r^s . Denote also the vector of quantities by $\mathbf{q} = (q_1^A, q_2^A, q_1^B, q_2^B)$ and the inverse demand for good s at retailer r by $P_r^s(\mathbf{q})$. Then, the joint profit of supplier A and the two retailers, i.e., industry profit, can be written as

$$\sum_{r \in R, s \in S} (P_r^s(\mathbf{q}) - c^s) q_r^s. \quad (1)$$

We assume that $P_r^s(\mathbf{q})$ is continuously differentiable and downward sloping in all quantities whenever $P_r^s(\mathbf{q}) > 0$. We also assume that industry profits are strictly concave and maximized by strictly positive values. Let \bar{q}_r^s denote the "monopoly" quantity of good s at retailer r , and let the corresponding vector of monopoly quantities be denoted by $\bar{\mathbf{q}} = (\bar{q}_1^A, \bar{q}_2^A, \bar{q}_1^B, \bar{q}_2^B)$. Then, for any monopoly quantity \bar{q}_r^s and corresponding monopoly price $\bar{p}_r^s = P_r^s(\bar{\mathbf{q}})$, it follows that

$$\bar{p}_r^s - c^s + \sum_{r' \in R, s' \in S} \bar{q}_{r'}^{s'} \left. \frac{\partial P_{r'}^{s'}}{\partial q_r^s} \right|_{\mathbf{q}=\bar{\mathbf{q}}} = 0. \quad (2)$$

Own-supplier contracts

The set-up described above is relatively simple, and yet, as we discussed in the overview in Section 2, and as we now show, contracts that can only condition a retailer's payment on how much is purchased of good A cannot induce the monopoly outcome. To see this, let contract $T_r(q)$ denote retailer r 's payment to supplier A when the retailer purchases q units of good A .

We start by restricting attention to two-part tariff contracts. These contracts are easy to work with and allow the maximization of industry profit to be disentangled from how it is shared. More importantly, as we will show, this restriction turns out to be without loss of generality.

Two-part tariff contracts A two-part tariff contract specifies for each retailer r a fixed payment F_r and a per-unit wholesale price w_r , such that $T_r(q) = F_r + w_r q$. In this case, retailer r 's profit is given by

$$\pi_r = (P_r^A(\mathbf{q}) - w_r) q_r^A + (P_r^B(\mathbf{q}) - c^B) q_r^B - F_r.$$

Since F_r serves only to distribute realized industry profits without affecting quantities and prices, we will focus in what follows on supplier A 's choice of w_r . Moreover, although it is not needed for our key results, we will henceforth assume that when retailers have constant marginal costs (as in the present case in which supplier A 's offers consist of two-part tariff contracts), retail profits are submodular in addition to being strictly concave in the retailer's quantity choices.⁶

At an interior solution, the quantities q_r^s that maximize π_r solve the first-order conditions

$$(p_r^A - w_r) + q_r^A \frac{\partial P_r^A}{\partial q_r^A} + q_r^B \frac{\partial P_r^B}{\partial q_r^A} = 0 \quad (3)$$

and

$$(p_r^B - c^B) + q_r^B \frac{\partial P_r^B}{\partial q_r^B} + q_r^A \frac{\partial P_r^A}{\partial q_r^B} = 0, \quad (4)$$

where $p_r^s = P_r^s(\mathbf{q})$ is the price of good s at retailer r when the retailers sell \mathbf{q} to consumers. Holding the rival retailer's quantities constant, it follows that w_r affects retailer r 's choice of q_r^B only *indirectly*, through its choice of q_r^A . Thus, if the other retailer's quantity choices as well as q_r^A were fixed, retailer r 's choice of q_r^B would be independent of supplier A 's contract. As we will show later, this constitutes an important difference from the case of price competition, where the retailer's choice of p_r^B is *directly* influenced by the size of the markup it earns on good A .

⁶For positive demand, submodularity implies $\frac{\partial^2 \pi_r}{\partial q_r^s \partial q_{r'}^{s'}} \leq 0$ for all \mathbf{q} and $(r, s) \neq (r', s')$. See Vives (1999) p.25.

The preceding observation implies that supplier A will not be able to induce the monopoly outcome with a two-part tariff contract. To see this formally, suppose that retailer r' were to choose the monopoly quantities $q_{r'}^A = \bar{q}_{r'}^A$ and $q_{r'}^B = \bar{q}_{r'}^B$, and that $q_r^A = \bar{q}_r^A$. Then the remaining quantity q_r^B would be determined by solving retailer r 's first-order condition in (4). Evaluating the left-hand side of condition (4) at $q_r^B = \bar{q}_r^B$, and after substituting from (2), it follows that

$$- \sum_{r' \in R/r, s' \in S} \bar{q}_{r'}^{s'} \left. \frac{\partial P_{r'}^{s'}}{\partial q_r^B} \right|_{\mathbf{q}=\bar{\mathbf{q}}} > 0. \quad (5)$$

By the strict concavity of π_r , this implies that retailer r would thus choose $q_r^B > \bar{q}_r^B$. Or, in other words, retailer r would choose a quantity q_r^B that is too large to maximize industry profits.

A few remarks are in order. First, note that in the *absence* of good B , a two-part tariff contract would be sufficient to induce the monopoly outcome if w_r is specified appropriately. For example, it follows from condition (3) that in the absence of good B , choosing the monopoly quantity $q_r^A = \bar{q}_r^A$ is optimal for retailer r when its rival chooses quantity $q_{r'}^A = \bar{q}_{r'}^A$ if and only if

$$(\bar{p}_r^A - w_r) + \bar{q}_r^A \left. \frac{\partial P_r^A}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} = (\bar{p}_r^A - c^A) + \sum_{r' \in R} \bar{q}_{r'}^A \left. \frac{\partial P_{r'}^A}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}},$$

which supplier A can satisfy by setting

$$w_r - c^A = - \sum_{r' \in R/r} \bar{q}_{r'}^A \left. \frac{\partial P_{r'}^A}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} > 0.$$

Second, note that supplier A can use its two-part tariff to affect the equilibrium quantities of good B even though this occurs only indirectly through its affect on q_1^A and q_2^A . To see how a change in q_r^A that is induced by a change in w_r will affect retailer r 's choice of q_r^B , recall that the assumption of submodularity implies that all four goods are strategic substitutes. It follows that if supplier A wants to induce retailers to decrease their sales of good B , it must lower its wholesale price to them, which simultaneously increases sales of its own good. Thus, it will not be possible for supplier A to dampen competition on both goods simultaneously. Put differently, supplier A can dampen competition on good B but only by increasing competition on good A .

General own-supplier contracts We now show that the restriction to two-part tariff contracts is without loss of generality. This follows from two observations. The first observation is that for any choice of quantities q_r^A , the corresponding choice of quantities q_r^B in any interior equilibrium must satisfy the respective first-order conditions (4) and thus does not depend directly on supplier A 's choice of contracts $T_r(q)$. The second observation is that the restriction to

two-part tariff contracts already allows supplier A to implement *all* incentive compatible allocations of q_r^A that can be obtained with own-supplier contracts.⁷ To obtain the *unique* incentive compatible outcome that is associated with a pair of quantities q_1^A and q_2^A , it suffices for supplier A to choose its per-unit price w_1 (and likewise for w_2) to satisfy

$$w_1 = p_1^A + q_1^A \frac{\partial P_1^A}{\partial q_1^A} + q_1^B \frac{\partial P_1^B}{\partial q_1^A}, \quad (6)$$

where (6) is obtained simply by rearranging the first-order condition in (3) and setting $r = 1$.⁸

This result has policy implications because it implies that any restriction on own-supplier contracts that still allows for two-part tariffs would leave the final outcome unchanged.⁹ For instance, a ban on contracts that offer non-incremental discounts (recall that such contracts entail a discrete jump down in the retailer’s outlay when the target threshold is reached), or, more generally, in which $T_r(q)$ or $T_r(q)/q$ change discontinuously, would have no effect. This applies, in particular, to “quantity-forcing contracts” that specify high punishment prices for quantities other than the target quantity, as well as to the contracts considered by Kolay et. al. (2004) in which the discounts apply to all units purchased once the target quantity is reached.

Proposition 1 *It is not possible to induce the monopoly outcome with own-supplier contracts when retailers compete in quantities. Moreover, a ban on own-supplier contracts other than two-part tariffs (e.g., a ban on non-incremental discounts) has no effect on the equilibrium outcome.*

The finding that two-part tariffs are without loss of generality has also been shown in other vertical settings. In the case of two suppliers and one retailer, O’Brien and Shaffer (1997) and Bernheim and Whinston (1998) have shown that the monopoly outcome can be induced by two-part tariff contracts when marginal costs are weakly convex. And Mathewson and Winter (1984) have shown a similar result when there is one supplier and two or more retailers. In contrast to our article, however, a restriction to two-part tariff contracts in these settings is without loss

⁷Supplier A can also attempt to force a corner solution by inducing each retailer to purchase sufficiently large quantities of good A . In this case, condition (4) need not hold with equality. However, even in this case, it is still the case that a retailer’s decision to set $q_r^B = 0$ does not depend directly on its contract with supplier A , and that *all* incentive-compatible allocations of q_r^A that exclude good B can be implemented with two-part tariffs.

⁸Note that the per-unit price w_1 that arises in (6) could, in principle, be positive or negative. In the latter case, we assume that supplier A can limit its sales and stipulate that each retailer must sell all that it purchases.

⁹This holds as long as the set of feasible contracts is sufficiently rich to allow the maximization of industry profit to be separated from how it is shared (which would not be true if only linear contracts were feasible).

of generality *because* these contracts can be used to achieve the monopoly outcome. Here, the restriction is without loss of generality even though the monopoly outcome cannot be achieved.

Market-share contracts

Own-supplier contracts cannot induce the monopoly outcome because they cannot dampen competition on both goods simultaneously. One way for supplier A to solve the problem is to offer a market-share contract. Market-share contracts typically take the following form:

$$T_r(q, \hat{\rho}_r) = \begin{cases} F_r + w_r^+ q & \text{if } \rho_r \geq \hat{\rho}_r \\ F_r + w_r^- q & \text{if } \rho_r < \hat{\rho}_r \end{cases}, \quad (7)$$

where w_r^+ and w_r^- are per-unit prices, $\rho_r := q_r^A / (q_r^A + q_r^B)$ is the share of retailer r 's purchases that are sourced from supplier A , and $\hat{\rho}_r$ is the minimum threshold at which retailer r receives the per-unit price of w_r^+ . Note that F_r serves only to distribute surplus between the supplier and retailer r , and that w_r^+ and w_r^- apply to *all* units purchased. For $w_r^+ < w_r^-$ this means a rebate of $(w_r^- - w_r^+)q_r^A$ if the retailer's purchases meet the minimum target threshold of $\rho_r \geq \hat{\rho}_r$.

That market-share contracts can simultaneously dampen competition on both goods and in the desired amounts may not be obvious given that they only reward retailers for meeting or exceeding a particular sales target relative to their own total sales and do not also condition the retailers' payments on how much they purchase of good B . However, the intuition is straightforward. Note first that supplier A can always specify $\hat{\rho}_r = \bar{\rho}_r := \bar{q}_r^A / (\bar{q}_r^A + \bar{q}_r^B)$ and make the difference between w_r^- and w_r^+ be sufficiently large that deviations that give it less than $\hat{\rho}_r$ in market share would be unprofitable for retailer r . Thus, supplier A can always choose its contract terms to ensure that it receives at least the same market share that it would receive in the monopoly outcome. Next, note that supplier A can always adjust w_r^+ to induce each retailer to choose the monopoly quantities. This is possible because, by holding the ratio $\bar{\rho}_r$ constant, supplier A can induce lower quantities of *both* q_r^A and q_r^B by increasing w_r . Lastly, note that supplier A can make its rebate conditional on it obtaining a minimum share of the retailer's purchases as opposed to a fixed share of the retailer's purchases because at the w_r^+ that induces the monopoly quantities, the retailer would prefer to sell more of good B and less of good A .

It follows that market-share contracts can indeed be used to induce the monopoly outcome. And, since contracts maximize industry profits in equilibrium, it also follows that in any equilibrium of the game, the monopoly outcome will be obtained if market-share contracts are feasible.

Proposition 2 *Market-share contracts can be used to induce the monopoly outcome when the downstream firms compete in quantities. In any equilibrium of the game with quantity competition, therefore, the monopoly outcome will be obtained if market-share contracts are feasible.*

Proof. See Appendix.

The proof of Proposition 2 formalizes the intuition above and shows that the monopoly outcome can indeed be obtained with market-share contracts when $\widehat{\rho}_r$, w_r^- , and w_r^+ are appropriately chosen—with F_r being chosen to ensure that retailer r earns no more than its outside option. Own-supplier contracts *fail* to achieve the monopoly outcome in our setting because, however complex they may be, they can only affect the retailers’ quantity choices of good B indirectly, through their effect on the retailers’ quantity choices of good A . In contrast, market-share contracts *can* induce the monopoly outcome because they can affect the retailers’ quantity choices of good B directly. As such, although both can dampen intra-brand competition, only market-share contracts can restrict competition on the competitively-supplied brand as well.

Policy makers often express concern that the discounts in market-share contracts may be predatory in the sense that the after-discount per-unit price (i.e., the lower of the two wholesale prices in (7)) may be below supplier A ’s marginal cost. It is worth noting, however, that the w_r^+ that induces the monopoly quantities in supplier A ’s optimal market-share contract always exceeds supplier A ’s marginal cost. This follows because, on subtracting c^A from both sides of the expression in (A.7) in the proof of Proposition 2, and substituting in from (2), one obtains

$$w_1^+ - c^A = - \left(\sum_{s \in S} \bar{q}_2^s \left[\left. \frac{\partial P_2^s}{\partial q_1^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} + \frac{1 - \bar{p}_1}{\bar{p}_1} \left. \frac{\partial P_2^s}{\partial q_1^B} \right|_{\mathbf{q}=\bar{\mathbf{q}}} \right] \right) > 0 \quad (8)$$

as the markup for retailer 1, and an analogous expression $w_2^+ - c^A$ as the markup for retailer 2.

Note also that market-share contracts are not the only means supplier A has of monopolizing the retail market for goods A and B in our setting.¹⁰ Supplier A could, in principle, also obtain the monopoly outcome by specifying the exact quantity of good B to be sold by each retailer. To the extent that this might make the facilitating purpose of the dominant supplier’s contract more evident, and since these contracts cannot strictly do better, it may be that market-share contracts will be more preferred by firms as a ‘camouflaged’ way of dampening competition.¹¹

¹⁰For example, supplier A may be able to obtain the monopoly outcome by imposing min RPM on its own good and setting its wholesale price to induce retailers to charge the monopoly price on good B (Shaffer, 1990).

¹¹It may also be possible for supplier A to monopolize the downstream market by inducing each retailer to

Welfare and consumer surplus

In comparing welfare and consumer surplus when only own-supplier contracts are feasible and welfare and consumer surplus when market-share contracts are feasible, note first that with own-supplier contracts, a symmetric increase in each retailer's wholesale price leads to a reduction in the quantity sold of good A and an increase in the quantity sold of good B . Moreover, at the symmetric wholesale price that induces retailers to purchase the monopoly quantity of good A , retailers purchase more than the monopoly quantity of good B . It follows immediately from these observations that when only own-supplier contracts are feasible, sales of at least one good (possibly both) will be higher than their corresponding monopoly quantities. In contrast, when market-share contracts are feasible, the monopoly quantities of both goods are obtained in equilibrium. Hence, it follows that the sales of at least one good (possibly both) will be higher when only own-supplier contracts are feasible than when market-share contracts are feasible.

To obtain further results, consider the following symmetric linear demand system

$$P_r^s = v^s - [q_r^s + \gamma q_r^{s'}] - \beta [q_{r'}^s + \gamma q_{r'}^{s'}], \quad (9)$$

for all $r \neq r' \in R$ and $s \neq s' \in S$. Note that $\gamma \in [0, 1]$ measures the substitution between goods A and B , and $\beta \in [0, 1]$ measures the substitution between retailers. For example, the goods are independent when $\gamma = 0$, and perfectly substitutable when $\gamma = 1$. Similarly, the retailers are local monopolists when $\beta = 0$, and perfectly substitutable when $\beta = 1$. To ensure that all quantities are positive, we assume that $(v^A - c^A) > \gamma (v^B - c^B)$ and $(v^B - c^B) > \gamma (v^A - c^A)$.

Using this specification, it is straightforward to show that when only own-supplier contracts are feasible, it is optimal for supplier A to induce the monopoly quantity on its own good and accept the loss in industry profit that comes with higher-than-monopoly sales on good B . In contrast, when market-share contracts are feasible, it is optimal for supplier A to choose its contract terms to induce the monopoly quantities on both goods. It follows that good B 's market share will be higher when only own-supplier contracts are feasible, and, since both goods' prices are lower when more is sold of good B , consumer surplus and welfare will also be higher.

Proposition 3 *Given the demand specification in (9), the induced retail prices on both goods will be higher, and thus consumer surplus and welfare will be lower, when market-share contracts*

demand an up-front payment from the competitive fringe firms in exchange for paying them a higher per-unit price. See Innes and Hamilton (2006) for an elaboration of this point. See also Shaffer (1991) and Marx and Shaffer (2007) for models in which slotting allowances are demanded by retailers in order to dampen competition.

are feasible than when only own-supplier contracts are feasible. Moreover, good B's sales and share of the market—although not zero—will be lower when market-share contracts are feasible.

Proof. See Appendix.

Proposition 3 implies that the use of market-share contracts can decrease consumer surplus and welfare even though industry profit increases, and even though supplier *A*'s competitors are not driven from the market. Although there is some foreclosure of good *B* at the margin, supplier *A*'s intent is not to drive sales of good *B* to zero (as this would decrease industry profit) but rather to dampen competition in order to induce higher retail prices. The higher prices lead to higher industry profit, of which supplier *A* captures its share with infra-marginal transfers.

This mechanism for harm calls into question how market-share contracts are evaluated in antitrust law, which currently bases a rule-of-reason case on whether or not a substantial enough fraction of the market has been foreclosed, and thus on whether supplier *A* has monopolized or has a 'dangerous probability' of monopolizing this market. In contrast, market-share contracts decrease consumer surplus and welfare in our setting by dampening—but not eliminating—competition in the retail market. They allow supplier *A* to focus on implementing the joint-profit maximizing outcome rather than attempting to destroy surplus (as would be the case if supplier *A* were to resort to an exclusive-dealing arrangement) by trying to foreclose good *B*. Put differently, the reason why market-share contracts that involve less extreme demands than sheer exclusivity are in general more profitable is simply that market-share contracts allow supplier *A* to maximize the joint profit of both retailers and itself, and joint-profit maximization does in general involve the sale of some quantity of good *B* (given that goods *A* and *B* are differentiated).

Note finally that the adverse effect of market-share contracts on welfare and consumer surplus is immune to the critique of the "Chicago School," which argues that exclusion (and implicitly anti-competitive outcomes) will not be sustainable if industry profits are compromised.¹² In our setting, industry profits increase, and the partial foreclosure of good *B* that occurs is sustainable.

¹²According to the Chicago school, this holds as excluded firms should be able to "bribe themselves" back into the market while it should likewise be too costly for any one firm to ensure that its competitors are excluded.

5 Extensions

General Market Structure

We have thus far considered the case of two goods and two retailers. For reasons analogous to those discussed above, the latter restriction is without loss of generality. This follows from two observations. First, it is not possible to induce the monopoly outcome with only own-supplier contracts regardless of the number of competing retailers. Second, with market-share contracts, supplier A can implement any pair of quantities of goods A and good B at any given retailer, holding all other retailers' quantities fixed. Hence, the optimal market-share contract can implement the monopoly outcome and is independent of the number of competing retailers.¹³

We now consider the extension to more than two goods and begin by giving conditions under which market-share contracts can achieve the monopoly outcome. For this purpose, let $s \in S = \{A, B, C, \dots, \bar{S}\}$ denote the goods, and, as before, let \bar{q}_r^s denote the monopoly quantity of good s at retailer r and \bar{q}_r denote the sum of the monopoly quantities at retailer r . Let $\bar{\rho}_r^A := \bar{q}_r^A / \bar{q}_r$ denote the market share of good A at the monopoly quantities, and let $\bar{\rho}_r^s := \bar{q}_r^s / \bar{q}_r$ denote the corresponding market shares of all other goods $s \in S / \{A\}$. Then, to induce the monopoly outcome, two conditions must be satisfied. First, the respective market shares must satisfy $\rho_r^s = \bar{\rho}_r^s$, and second, the sales of good A must be at the monopoly levels, with $q_r^A = \bar{q}_r^A$.

Proposition 4 *The monopoly outcome can be implemented with a market-share contract if the demands of the competitively-supplied goods at each retailer are symmetrically differentiated.*

Proof. See Appendix.

As shown in the proof of Proposition 4, when the competitively-supplied goods are symmetrically differentiated, supplier A can implement the monopoly outcome by specifying a market-share threshold of $\hat{\rho}_r^A = \bar{\rho}_r^A$ and setting its per-unit price w_r^+ to induce each retailer to sell \bar{q}_r^A . The market shares and quantities sold of the remaining goods are then given by $\frac{\bar{q}_r - \bar{q}_r^A}{(N-1)\bar{q}_r}$ and $\frac{\bar{q}_r - \bar{q}_r^A}{N-1}$, respectively, where N denotes the total number of goods sold by each retailer. In essence, the N dimensional choices of each retailer reduces under symmetry to two choices, the quantity to sell of good A and the aggregate quantity to sell of all other (competitively supplied) goods.

The single per-unit price, w_r^+ , and the single market-share threshold, $\hat{\rho}_r^A$, will in general no longer sufficient to implement the monopoly quantities of all goods when the demands of

¹³Note that by this argument the restriction to symmetric retailers is also inconsequential for the key results.

the competitively-supplied goods at each retailer are not symmetric. In this case, however, it is straightforward to show that market-share contracts that can condition retailer r 's payment terms on all pairwise ratios q_r^s/q_r^A would again be sufficient to induce the monopoly outcome.

To obtain further results, consider now the following linear-demand system with any number of competitively-supplied products:

$$P_r^s = v^s - \left[q_r^s + \gamma \sum_{s' \neq s} q_r^{s'} \right] - \beta \left[q_r^{s'} + \gamma \sum_{s' \neq s} q_r^{s'} \right]. \quad (10)$$

Setting $c^s = 0$ for convenience, and defining $\bar{v} := \sum_{s \in S} v^s$, industry profits are maximized at

$$\bar{q}_r^s = \frac{v^s (1 + \gamma(N - 1)) - \gamma \bar{v}}{2(1 + \beta)(1 + \gamma(N - 2) - \gamma^2(N - 1))}. \quad (11)$$

Given these monopoly quantities, the market share of good s at each retailer r is given by

$$\bar{p}_r^s = \frac{\bar{q}_r^s}{q_r^s} = \left(\frac{1 + (N - 1)\gamma}{1 - \gamma} \right) \frac{v^s}{\bar{v}} - \frac{\gamma}{1 - \gamma}.$$

Proposition 4 implies that market-share contracts can induce the monopoly outcome in this setting when v^s is the same for all competitively-supplied goods. If v^s is not the same for all competitively-supplied goods, however, then the monopoly outcome cannot be induced with a single market-share threshold. Nevertheless, the main insights from Section 4 continue to hold.

Proposition 5 *Given the linear demand system in (10), the following results hold. The optimal own-supplier contract and the optimal market-share contract lead to the same quantity sold of good A , namely the monopoly quantity $q_r^A = \bar{q}_r^A$, while the quantities sold of all competitively-supplied goods are lower under the optimal market-share contract. Consumer surplus and welfare are thus lower and the market share of good A is higher under the optimal market-share contract.*

Proof. See Appendix.

The discussion thus far assumes that all goods other than good A are competitively supplied. When this is not the case, a new motivation for the use of market-share contracts may arise.

We illustrate this for the case of two goods. Suppose first that only own-supplier contracts are feasible, and that supplier B anticipates that supplier A will be offering a two-part tariff contract. Then, in setting its optimal wholesale price, supplier B will anticipate that, at the margin, a decrease in w_r^B would induce retailer r to divert sales in its favor by selling relatively more of good B . Now suppose that market-share contracts are feasible, and that supplier B

anticipates that supplier A will be offering a market-share contract with a threshold that will be binding. Then, in setting its optimal wholesale price, supplier B will anticipate that, at the margin, a decrease in w_r^B would *not* cause retailer r to divert sales in its favor (assuming that retailer r 's penalty for not satisfying supplier A 's market-share requirement is sufficiently large). Hence, there is little incentive for supplier B to charge a low wholesale price in the latter case.

All else being equal, therefore, one might expect both firms to have higher wholesale prices in equilibrium when market-share contracts are feasible than when only own-supplier contracts are feasible. Higher wholesale prices will in general lead to higher retail prices and industry profits (unless the firms are already at the joint-profit maximum), but also lower consumer surplus and welfare. A formalization of this argument is, however, beyond the scope of this article, as it would require changing the game to allow players to bargain over their contract terms or, alternatively, changing the game to force suppliers to make their contract offers sequentially.¹⁴

Distribution of profits

Market-share contracts may offer another advantage over own-supplier contracts in that they may be better at reducing the value of each retailer's outside option. Although these values are independent of supplier A 's initial contract offers, they are not independent of the renegotiation that would take place with the other retailer in the out-of-equilibrium event that one of the retailers rejects its offer. For example, the value of retailer i 's outside option depends on the contract terms that would be negotiated with retailer j after retailer i rejects supplier A 's offer.

To fix the ideas, suppose that—out-of-equilibrium—only retailer 1 accepts supplier A 's offer. Then the quantity of good A sold by retailer 2 would be zero and the inverse-demand functions, P_1^A , P_1^B , and P_2^B , would be evaluated at $q_2^A = 0$. In this case, our assumption that retailer 1's contract can be renegotiated, and that the outcome of this renegotiation can be observed by retailer 2, bestows on supplier A and retailer 1 a “first-mover” advantage vis-à-vis retailer 2. One might expect this advantage to be greater the more control supplier A has over retailer 1's subsequent quantity choices (i.e., the less flexibility retailer 1 has in making its quantity choices, the more commitment value supplier A 's first-mover advantage has). Since market-share contracts allow for greater control of retailer 1's quantity choices (and similarly for retailer

¹⁴These changes would be necessary because it is known that when competing suppliers make simultaneous take-it-or-leave-it offers to competing retailers, and only own-supplier contracts are feasible, there does not exist an equilibrium in pure strategies in which all suppliers trade with all retailers (see Marx and Shaffer, 2006).

2's choices when only retailer 2 accepts supplier A 's contract), one might expect the retailers' profits to be lower when they are feasible than when only own-supplier contracts are feasible.¹⁵

We now prove this conjecture and characterize the outcome for the case of linear demand.

Proposition 6 *Suppose the dominant supplier and its retailer can renegotiate their contract in the out-of-equilibrium event that the other retailer rejects its offer, and suppose the terms of this renegotiation are observable. Then, under the demand system in (9), the retailers are worse off when market-share contracts are feasible than when only own-supplier contracts are feasible.*

Proof. See Appendix.

Relative to own-supplier contracts, market-share contracts can thus benefit supplier A in two ways. First, they can more effectively dampen competition along the equilibrium path, allowing industry profits to be maximized, and second, as shown in Proposition 6, they are better at reducing the value of the retailers' outside options in the event supplier A 's contract is rejected.

It can be shown that the value of each retailer's outside option is lower when the retailers are closer substitutes (when β is higher), and that when the retailers are local monopolists (when $\beta = 0$), the two values coincide. In this latter case, supplier A can no longer use its renegotiated contract with the other retailer to affect the profits of a retailer that does not sell its good.¹⁶

Supplier opportunism

We have shown that supplier A can dampen competition on its own good (and, with market-share contracts, also on good B) by increasing its wholesale price to each retailer. This assumes that contracts are observable. If contracts are not observable, then retailer 1's quantities will be independent of the wholesale price that supplier A charges retailer 2 and vice versa. This can distort supplier's A 's incentives and lead to an opportunism problem (Hart and Tirole, 1990).¹⁷

¹⁵In fact, one can show that when demand is linear there is no first-mover advantage in the case of own-supplier contracts, implying that the renegotiated wholesale price will be $w_r^A = c^A$ (cf. the proof of Proposition 6).

¹⁶The values of the retailers' outside options are displayed in equations (A.19) and (A.20) in the Appendix. The comparative-static implications follow straightforwardly. As part of the proof of Proposition 6, the fixed payments F_1 and F_2 under both own-supplier contracts and market-share contracts are also derived, thereby completing the characterization of the equilibrium contracts. The sign of the fixed payments can be negative or positive.

¹⁷See also the discussion in O'Brien and Shaffer (1992), McAfee and Schwartz (1994) and Rey and Verge (2004).

To consider the opportunism problem in our setting, suppose that supplier A 's contracts and the retailers' acceptance decisions are not observable (and that there are no antitrust consequences or loss of reputation in offering different retailers different contracts). Consider some candidate equilibrium outcome in which retailer 2 chooses quantities \widehat{q}_2^A and \widehat{q}_2^B . Note that since retailer 2 cannot observe retailer 1's contract offer, these quantities will not be affected if supplier A *secretly* offers retailer 1 a different (deviating) contract. Suppose furthermore that retailer 1 has passive beliefs, implying that it will accept any deviation contract as long as it expects to break even under the assumption that retailer 2 will choose $q_2^A = \widehat{q}_2^A$ and $q_2^B = \widehat{q}_2^B$. Then, because supplier A can extract the incremental surplus from its deviation simply by raising its fixed fee to retailer 1, it follows that in equilibrium it will choose its contract terms to maximize the pair's bilateral surplus. This is accomplished by setting $w_1 = c^A$ and not offering a market-share discount. Similarly, supplier A will have no incentive to distort retailer 2's incentives in equilibrium. Market-share contracts in this case offer no advantage over own-supplier contracts.

Since the anticompetitive mechanism highlighted in this article is possible only if contract offers are at least partially observable, it is important to assess the likelihood of observability. One way to do this is to check whether suppliers often resist retailers' requests for discounts. For instance, casual evidence from grocery retailing suggests that in circumstances in which contractual terms are frequently adjusted, suppliers often resist requests for special discounts (and requests for additional promotional campaigns) on the grounds that this would trigger similar requests by competing retailers.¹⁸ When this is the case, it is plausible that market-share contracts may be used as a facilitating practice, and thus competition authorities may have reason to be especially concerned by market-share contracts that lack an efficiency rationale.¹⁹

Retailer private information

We have assumed complete information about demand and costs. Suppose now that one retailer has private information about its marginal costs (e.g., its costs of 'handling' each sold unit).²⁰

¹⁸Similar observations are made, for instance, in McAfee and Schwartz (1994) regarding franchise contracts.

¹⁹With respect to grocery retailing our model may also throw some light on the common practice of retailers to designate certain of their suppliers as category captains, who advise retailers on the 'optimal' shelf placements of their own and rivals' brands within product categories. Retailers share demand and cost information with their captains who in turn may have an incentive to allocate shelf space in a way that dampens competition, similar to the use of market-share contracts, not only on intra-brand competition but also on inter-brand competition.

²⁰The discussion also holds when the private information is with respect to the retailer's local demand.

When this retailer learns its private information only after signing its contract, it is well-known for the *single-agency* case that, absent other contractual constraints, such as limited liability, the outcome will maximize joint surplus. Thus, for the design of the respective menu of contracts, there is no trade-off between surplus maximization and (information) rent extraction. It is straightforward to show that this insight also holds in our setting. Hence, for any subsequent realization of the retailer's own cost, the respective set of contracts that would be chosen will, in terms of final prices and quantities, lead to the same outcome as that under complete information.

The case where the retailer has *ex-ante* private information at the time of contracting is not as straightforward. Suppose first that, contrary to our model, there is only one supplier and one retailer. Then, the supplier faces a standard problem of screening, resulting in a trade-off between surplus maximization and surplus extraction. Majumdar and Shaffer (2009) show in this setting that when the monopolist retailer also carries a competing brand, market-share contracts can be used to relax the retailer's incentive-compatibility constraint and thus allow the supplier to extract (weakly) higher profits. The key feature in our model, in contrast to that of the earlier literature, is the presence of competing retailers and thus of intra-brand competition. Given our previously derived results, and given the insights from Majumdar and Shaffer (2009), however, it is immediate that even when retailers have *ex-ante* private information, contracts that condition payment terms on market shares can be strictly more profitable for the supplier.²¹

6 The case of price competition

In this section, we assume that retailers compete in prices. Denote the price of good s at retailer r by p_r^s and the vector of chosen prices by $\mathbf{p} = (p_1^A, p_1^B, p_2^A, p_2^B)$. Denote also the demand for good s at retailer r by $Q_r^s(\mathbf{p})$. Then, industry profits can be written as $\sum_{r \in R, s \in S} (p_r^s - c^s) Q_r^s(\mathbf{p})$.

We assume that $Q_r^s(\mathbf{p})$ is continuously differentiable and downward sloping with $\frac{\partial Q_r^s}{\partial p_r^s} < 0$ whenever $Q_r^s(\mathbf{p}) > 0$. We also assume that $\frac{\partial Q_r^s}{\partial p_r^{s'}} > 0$ for all $r' \in R$ and $s' \in S$ such that $(r, s) \neq (r', s')$, and that the monopoly quantities \bar{q}_r^s and prices \bar{p}_r^s are unique and positive. Let $\bar{\mathbf{p}}$ be the vector of monopoly prices. Then, the first-order condition for good s at retailer r is

$$\bar{q}_r^s + \sum_{r' \in R, s' \in S} (\bar{p}_{r'}^{s'} - c^{s'}) \left. \frac{\partial Q_{r'}^{s'}}{\partial p_r^s} \right|_{\mathbf{p}=\bar{\mathbf{p}}} = 0. \quad (12)$$

²¹In an earlier version of this paper, we showed with linear demand that when the products are sufficiently close substitutes (high γ) and the retailers are in sufficiently close competition (high β), the monopoly outcome can be obtained with market-share contracts for all realizations of the retailer's privately observed marginal cost.

Own-supplier contracts

Suppose supplier A offers two-part tariff contracts, $T_r(q) = F_r + w_r q$, and assume that the retailers' profits π_r for this case are strictly concave. Then, at an interior solution, profit maximization yields the following two conditions for retailer r :

$$q_r^A + (p_r^A - w_r) \frac{\partial Q_r^A}{\partial p_r^A} + (p_r^B - c^B) \frac{\partial Q_r^B}{\partial p_r^A} = 0 \quad (13)$$

and

$$q_r^B + (p_r^A - w_r) \frac{\partial Q_r^A}{\partial p_r^B} + (p_r^B - c^B) \frac{\partial Q_r^B}{\partial p_r^B} = 0. \quad (14)$$

Evaluating (13) and (14) at the monopoly prices and comparing them with condition (12), it follows that if the monopoly outcome is to be induced, the following two conditions must hold:

$$w_r - c^A = - \frac{\sum_{s \in S} (\bar{p}_{r'}^s - c^s) \left. \frac{\partial Q_{r'}^s}{\partial p_r^A} \right|_{\mathbf{p}=\bar{\mathbf{p}}}}{\left. \frac{\partial Q_r^A}{\partial p_r^A} \right|_{\mathbf{p}=\bar{\mathbf{p}}}} \quad (15)$$

and

$$w_r - c^A = - \frac{\sum_{s \in S} (\bar{p}_{r'}^s - c^s) \left. \frac{\partial Q_{r'}^s}{\partial p_r^B} \right|_{\mathbf{p}=\bar{\mathbf{p}}}}{\left. \frac{\partial Q_r^A}{\partial p_r^B} \right|_{\mathbf{p}=\bar{\mathbf{p}}}}. \quad (16)$$

But (15) and (16) cannot both hold because for (15) to hold supplier A must have a positive markup ($w_r - c^A > 0$), whereas for (16) to hold its markup must be negative ($w_r - c^A < 0$).²² Intuitively, to induce the monopoly price on good A , supplier A must dampen competition, for which it is necessary to charge $w_r > c^A$. However, to induce the monopoly price on good B , supplier A must dampen competition on good B , which can only be achieved by increasing the margin that retailers earn on good A , thereby making it less attractive for retailers to choose low retail prices on good B . According to (16), this requires that $w_r < c^A$. Since $w_r - c^A$ cannot be both positive and negative simultaneously, conditions (15) and (16) cannot jointly be satisfied.

Market-share contracts

Market-share contracts differ from own-supplier contracts in the case of downstream price competition in that they give the dominant firm a greater ability to control the retail prices of both goods. In particular, the dominant firm can use market-share contracts to force retailers to raise

²²This follows from the assumptions on demand and the fact that $\bar{p}_{r'}^s - c^s > 0$ for all $r' \in R$, $s \in S$. See also the discussion in Shaffer (1989), who first derived this result in the context of using RPM as a facilitating practice.

their price on the competitively-supplied good whenever they raise their price on good A , as the retailers would otherwise fail to meet the dominant firm’s minimum market-share threshold.

Assuming that retailers’ constrained profits are strictly quasi-concave under market-share contracts, this ability to ‘force’ each retailer’s hand is enough to be able to induce the monopoly outcome. By choosing w_r^- sufficiently high (cf. the definition of market-share contracts in (7)), supplier A can ensure that each retailer will give it the market-share it desires, whereas by choosing w_r^+ appropriately, it can effectively ensure that retailers set the monopoly price on good A . All that remains is for it to ensure that retail prices on good B are chosen to maximize industry profit, which it can do by specifying a minimum market-share threshold of $\rho_r = \bar{\rho}_r$.

This result, along with the findings in the previous subsection, has the following implication.

Proposition 7 *In any equilibrium of the game with price competition, the monopoly outcome will be obtained if market-share contracts are feasible. If instead the dominant supplier must choose from among only own-supplier contracts, then the monopoly outcome will not be obtained.*

Proof. See Appendix.

Proposition 7 thus establishes the robustness of our results to downstream price competition.²³

7 Conclusion

The emerging literature on market-share contracts has thus far focused attention on service provision and price discrimination explanations, and suggested that the welfare effects of these contracts are either benign or ambiguous. On the other hand, competition authorities and courts have been concerned that market-share contracts may be used by dominant firms to foreclose competitors. In contrast, this article offers an alternative rationale for their use. By conditioning discounts on the share it receives of each retailer’s overall purchases, a dominant firm can dampen both intra- and inter-brand downstream competition on its product. As a result, consumer surplus and welfare may decrease even though rivals have not been foreclosed.

The idea is that market-share contracts give the dominant firm the ability to influence not only the quantity sold of its own product, but also the quantity sold of its rivals’ products. The former is achieved by judiciously choosing the discounted per-unit price of its product. The latter

²³Strictly speaking, we have shown in (13) - (16) only that the monopoly outcome cannot be obtained with two-part tariff contracts. However, it is straightforward to show that the result holds for all own-supplier contracts.

is achieved by imposing a minimum market-share requirement on its retailers and inducing their compliance with a sufficiently large rebate or all-units discount. Since the rebate is applied to the retailers' inframarginal units, the dominant supplier can induce their participation by setting an artificially high pre-rebate per-unit price. As a result, it is effectively able to achieve the monopoly outcome even though there is both intra- and inter-brand competition downstream.

Our results imply that market-share contracts that stop short of requiring full exclusivity are not simply weaker versions of exclusive-dealing arrangements and, as a result, less worrisome from a policy perspective. To the contrary, they may be more worrisome in some settings. For example, an exclusive-dealing arrangement is suboptimal in our setting (unless industry profits are maximized when only the dominant firm's good is sold downstream), and so a ban on the use of exclusive-dealing arrangements by a dominant firm in this instance, while leaving intact the feasibility of more general market-share contracts, would not solve the problem that we consider.

Our results also imply that a policy of banning the use of own-supplier contracts that offer non-incremental discounts would have no effect in our setting, unless the prohibition were also to extend to contracts that condition the retailers' payment terms on its purchases of the competitively-supplied goods. The reason is that the latter allow the dominant firm to fully dampen downstream competition, whereas all outcomes obtainable under the former can, as was shown, alternatively be achieved with two-tariff contracts when retailers compete in quantities.

8 Appendix: Omitted Proofs

Proof of Proposition 2. Consider supplier A 's optimization problem under the market-share contract in (7), and let $\hat{\rho}_r = \bar{\rho}_r$. Setting w_r^- sufficiently high ensures that only deviations with $\rho_r \geq \bar{\rho}_r$ have to be considered. The problem for retailer r is then to maximize

$$\pi_r = (P_r^A(\mathbf{q}) - w_r^+) q_r^A + (P_r^B(\mathbf{q}) - c^B) q_r^B - F_r, \quad (\text{A.1})$$

subject to the constraint that $\rho_r \geq \bar{\rho}_r$, where \mathbf{q} is evaluated at $q_r^s = \bar{q}_r^s$. In what follows, it is convenient to transform the constraint $\rho_r \geq \bar{\rho}_r$ into

$$h(q_r^A, q_r^B) := q_r^B \bar{\rho}_r - q_r^A (1 - \bar{\rho}_r) \leq 0.$$

Define $L(\cdot) := \pi_r(\cdot) - \lambda h(\cdot)$. Then the (necessary) Kuhn-Tucker conditions are

$$\lambda \geq 0, \quad (\text{A.2})$$

$$\lambda h(\cdot) = 0, \quad (\text{A.3})$$

$$\frac{\partial \pi_r}{\partial q_r^A} = -\lambda(1 - \bar{\rho}_r), \quad (\text{A.4})$$

and

$$\frac{\partial \pi_r}{\partial q_r^B} = \lambda \bar{\rho}_r. \quad (\text{A.5})$$

Since the constraint $h(\cdot) \leq 0$ is linear in q_r^A and q_r^B , and since $\pi_r(\cdot)$ is strictly concave, these conditions are also sufficient for a global maximum. To obtain the monopoly outcome, it thus remains to specify w_r^+ such that (A.4) and (A.5) hold for some $\lambda > 0$ when evaluated at $q_r^s = \bar{q}_r^s$ (recall $h(\cdot) = 0$ given that $\rho_r = \bar{\rho}_r$). For this, define from (A.5)

$$\lambda = \frac{(\bar{p}_r^B - c^B) + \bar{q}_r^B \left. \frac{\partial P_r^B}{\partial q_r^B} \right|_{\mathbf{q}=\bar{\mathbf{q}}} + \bar{q}_r^A \left. \frac{\partial P_r^A}{\partial q_r^B} \right|_{\mathbf{q}=\bar{\mathbf{q}}}}{\bar{\rho}_r} > 0. \quad (\text{A.6})$$

(Note that the strict inequality follows immediately from the discussion in the text about why own-supplier contracts cannot obtain the monopoly outcome.) Thus it follows from (A.4) that

$$w_r^+ = \bar{p}_r^A + \bar{q}_r^A \left. \frac{\partial P_r^A}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} + \bar{q}_r^B \left. \frac{\partial P_r^B}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} + \lambda(1 - \bar{\rho}_r). \quad (\text{A.7})$$

Using the first-order condition for the monopoly quantities, we finally obtain that

$$\begin{aligned} w_r^+ - c^A &= - \left(\bar{q}_{r'}^A \left. \frac{\partial P_{r'}^A}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} + \bar{q}_{r'}^B \left. \frac{\partial P_{r'}^B}{\partial q_r^A} \right|_{\mathbf{q}=\bar{\mathbf{q}}} \right) \\ &\quad - \left[\left(\bar{q}_{r'}^A \left. \frac{\partial P_{r'}^A}{\partial q_r^B} \right|_{\mathbf{q}=\bar{\mathbf{q}}} + \bar{q}_{r'}^B \left. \frac{\partial P_{r'}^B}{\partial q_r^B} \right|_{\mathbf{q}=\bar{\mathbf{q}}} \right) \right] \frac{1 - \bar{\rho}_r}{\bar{\rho}_r}. \end{aligned}$$

Q.E.D.

Proof of Proposition 3. The quantities that would be sold if market-share contracts were feasible are obtained by maximizing industry profits as given in (1). Solving yields

$$\bar{q}_r^A = \frac{(v^A - c^A) - \gamma(v^B - c^B)}{2(1 + \beta)(1 - \gamma^2)} \quad \text{and} \quad \bar{q}_r^B = \frac{(v^B - c^B) - \gamma(v^A - c^A)}{2(1 + \beta)(1 - \gamma^2)}. \quad (\text{A.8})$$

Now suppose that only own-supplier contracts are feasible. Then, from the first-order conditions in (3) and (4) along with the analogous expressions for the rival retailer r' , it is straightforward to obtain the equilibrium quantities (as a function of each retailer's wholesale price)

$$\begin{aligned} q_r^A &= \frac{(2 - \beta)v^A + \gamma(2 - \beta)(v^B - c^B) - 2w_r + \beta w_{r'}}{(2 - \beta)(2 + \beta)(1 - \gamma^2)}, \\ q_r^B &= \frac{(2 - \beta)v^B + \gamma(2 - \beta)(v^A - c^A) + \gamma(2w_r - \beta w_{r'})}{(2 - \beta)(2 + \beta)(1 - \gamma^2)}. \end{aligned} \quad (\text{A.9})$$

Using (A.9), industry profits are maximized at the symmetric wholesale price

$$w = c^A + \frac{\beta}{2(1+\beta)} [(v^A - c^A) - \gamma(v^B - c^B)]. \quad (\text{A.10})$$

Substituting the optimal wholesale price in (A.10) into the quantities in (A.9), yields the equilibrium quantities q_r^A and q_r^B that would arise when only own-supplier contracts are feasible

$$q_r^A = \bar{q}^A \quad \text{and} \quad q_r^B = \bar{q}^B + \frac{\beta}{2(1+\beta)(2+\beta)} (v^B - c^B). \quad (\text{A.11})$$

Comparing the monopoly quantities \bar{q}^A and \bar{q}^B with the quantities in (A.11), it is easily seen that the quantity sold of good A is the same in both cases, whereas the quantity sold of good B is higher for all $\beta > 0$ when only own-supplier contracts are feasible. It follows that good B 's market-share will be higher when only own-supplier contracts are feasible than when market-share contracts are feasible. And since both goods' prices are lower when more is sold of good B , and there is less deadweight loss in this case, consumer surplus and welfare will also be higher.

Q.E.D.

Proof of Proposition 4. Suppose that the market-share contract specifies $\hat{\rho}_r^A = \bar{\rho}_r^A$ together with the pair of wholesale prices w_r^- and w_r^+ , where w_r^- is once again chosen sufficiently high to ensure that only deviations with $\rho_r^A \geq \bar{\rho}_r^A$ have to be considered. Retailer r thus maximizes

$$\pi_r = (P_r^A(\mathbf{q}) - w_r^+) q_r^A + \sum_{s \in S/\{A\}} (P_r^s(\mathbf{q}) - c^s) q_r^s - F_r,$$

subject to the set of constraints

$$h(\mathbf{q}_r) := \bar{\rho}_r^A \sum_{s \in S/\{A\}} q_r^s - (1 - \bar{\rho}_r^A) q_r^A \leq 0.$$

To see whether the monopoly outcome can be obtained, \mathbf{q} is evaluated at $q_{r'}^s = \bar{q}_{r'}^s$. With $L(\cdot) := \pi_r(\cdot) - \lambda h(\cdot)$, the Kuhn-Tucker conditions are again $\lambda \geq 0$, $\lambda h(\cdot) = 0$,

$$\frac{\partial \pi_r}{\partial q_r^A} = -\lambda(1 - \bar{\rho}_r^A), \quad (\text{A.12})$$

and, for $s \in S/\{A\}$,

$$\frac{\partial \pi_r}{\partial q_r^s} = \lambda \bar{\rho}_r^A. \quad (\text{A.13})$$

The monopoly outcome can thus be implemented by retailer r only if

$$(\bar{p}_r^s - c^s) + \sum_{s' \in S} \bar{q}_r^{s'} \left. \frac{\partial P_r^{s'}}{\partial q_r^s} \right|_{\mathbf{q}=\bar{\mathbf{q}}} \quad (\text{A.14})$$

is a constant, i.e., only if (A.14) is independent of $s \in S/\{A\}$. Substituting the monopoly quantities into (A.14), it must thus hold that

$$-\sum_{s' \in S} \bar{q}_r^{s'} \left. \frac{\partial P_r^{s'}}{\partial q_r^s} \right|_{\mathbf{q}=\bar{\mathbf{q}}} \quad (\text{A.15})$$

is independent of $s \in S/\{A\}$. That is, at $\mathbf{q}=\bar{\mathbf{q}}$, the marginal effect of a change in q_r^s on the other retailer's profit must be independent of s . A sufficient condition for this is that the inverse demand for all goods $s \in S/\{A\}$ be identical, as then *all* terms in (A.15) are independent of s .

Q.E.D.

Proof of Proposition 5. Define $\bar{v} := \sum_{s \in S} v^s$. The first-order conditions to maximize industry profits, which can be written as $\sum_{r \in R, s \in S} (P_r^s(\mathbf{q}) - c^s) q_r^s$, are

$$\bar{q}_r^s = \frac{v^s - c^s}{2(1 + \beta)} - \gamma \sum_{s' \in S/\{s\}} \bar{q}_r^{s'}.$$

Solving this system of equations and setting $c^s = 0$ yields

$$\bar{q}_r^s = \frac{v^s (1 + \gamma(N - 1)) - \gamma \bar{v}}{2(1 + \beta) (1 + \gamma(N - 2) - \gamma^2(N - 1))}. \quad (\text{A.16})$$

Given these monopoly quantities, the market share for good s at each retailer r is given by

$$\bar{p}_r^s = \frac{\bar{q}_r^s}{\bar{q}_r} = \left(\frac{1 + (N - 1)\gamma}{1 - \gamma} \right) \frac{v^s}{\bar{v}} - \frac{\gamma}{1 - \gamma}.$$

With two-part tariffs, retailer $r = 1$'s objective function is

$$\pi_1 = (P_1^A - w_1) q_1^A + \sum_{s \in S/\{A\}} (P_1^s - c^s) q_1^s,$$

and similarly for retailer 2. At $c^s = 0$, the respective first-order conditions are

$$2q_1^A + \beta q_2^A = \frac{(v^A - w_1) (1 + \gamma(N - 2)) - \gamma \sum_{s \in S/\{A\}} v^s}{(1 + \gamma(N - 2) - \gamma^2(N - 1))}$$

and, for $s \neq A$,

$$2q_1^s + \beta q_2^s = \frac{v^s (1 + \gamma(N - 2)) - \gamma \left[\sum_{s' \in S/\{A, s\}} v^{s'} + (v^A - w_1) \right]}{(1 + \gamma(N - 2) - \gamma^2(N - 1))}.$$

This can be solved to obtain

$$q_1^A = \frac{(1 + \gamma(N - 2)) [(2 - \beta) v^A - 2w_1 + \beta w_2] - (2 - \beta) \gamma \sum_{s \in S/\{A\}} v^s}{(1 + \gamma(N - 2) - \gamma^2(N - 1)) (4 - \beta^2)} \quad (\text{A.17})$$

and, for $s \neq A$,

$$q_1^s = \frac{(1 + \gamma(N - 1))(2 - \beta)v^s + \gamma[(2w_1 - \beta w_2) - (2 - \beta)\bar{v}]}{(1 + \gamma(N - 2) - \gamma^2(N - 1))(4 - \beta^2)}. \quad (\text{A.18})$$

Substituting (A.17) and (A.18) into industry profits, the optimal wholesale price is

$$w_r = w = \frac{\beta}{2(1 + \beta)} \frac{(1 + \gamma(N - 1))v^A - \gamma\bar{v}}{1 + \gamma(N - 2)}.$$

Substitution back into (A.17) confirms that $q^A = \bar{q}^A$ and $q^s > \bar{q}^s$ for all $s \in S/\{A\}$.

Consider now the case with market-share contracts. In this case, retailer 1 maximizes

$$\pi_1 = (p_1^A - w_1^+)q_1^A + \sum_{s \in S/\{A\}} (p_1^s - c^s)q_1^s$$

subject to the constraint

$$\rho_1^A := \frac{q_1^A}{\sum_{s \in S} q_1^s} = \hat{\rho}_1^A.$$

Substituting the retailer's profit-maximizing quantities into industry profits, and then maximizing these profits with respect to $\hat{\rho}^A$ and w^+ , yields $\hat{\rho}^A = \bar{\rho}^A$ and, after several transformations,

$$w^+ = \frac{\beta}{\beta + 1} \frac{(2\gamma(N - 2)! + (N - 1))(v^A)^2 - 2\gamma(N - 1)(\bar{v} - v^A)v^A + (\bar{v} - v^A)^2}{2(N - 2)((1 + (N - 2)\gamma)v^A - \gamma(\bar{v} - v^A))}.$$

Substituting this back into retailers' first-order conditions yields $q^A = \bar{q}^A$ and

$$q_1^s = \frac{1}{2(N - 1)} \frac{4\gamma(\beta + 1)(N - 2)! + (2(N - 2) + 1)\beta + 2(N - 1)}{(1 + \gamma(N - 2) - \gamma^2(N - 1))(\beta + 2)(\beta + 1)} v^s \\ - \frac{1}{2(N - 1)} \frac{2(N - 1)(\beta + 1)\gamma + \beta}{(1 + \gamma(N - 2) - \gamma^2(N - 1))(\beta + 2)(\beta + 1)} (\bar{v} - v^A - v^s) \\ - \frac{1}{2(N - 1)} \frac{((N - 1)\beta + 2(N - 1))\gamma}{(1 + \gamma(N - 2) - \gamma^2(N - 1))(\beta + 2)(\beta + 1)} v^A$$

for $s \in S/\{A\}$. Comparing these quantities to the quantities that would arise with two-part tariffs, given in (A.18), it follows that the latter quantities q^s for $s \in S/\{A\}$ differ by the amount

$$\frac{\beta(\bar{v} - v^A)}{2(N - 1)(1 + \beta)(2 + \beta)(1 + (N - 2)\gamma)} > 0.$$

Q.E.D.

Proof of Proposition 6. Suppose that only retailer 1 accepts supplier A 's offer, and note that by the same reasoning as in Section 4, a restriction to two-part tariff contracts is without loss of generality when only own-supplier contracts are feasible. Then retailer 2 maximizes

$$\pi_2^{Off} := q_2^B (v^B - c^B - q_2^B - \beta(\gamma q_1^A + q_1^B)).$$

Given some renegotiated wholesale price w_1 , retailer 1 chooses q_1^A and q_1^B to maximize

$$\widehat{\pi}_1^{Off} := q_1^A(v^A - w_1 - [q_1^A + \gamma q_1^B] - \beta\gamma q_2^B) + q_1^B(v^B - c^B - [q_1^B + \gamma q_1^A] - \beta q_2^B).$$

The system of first-order conditions yields

$$\begin{aligned} q_1^A &= \frac{v^A - \gamma(v^B - c^B) - w_1}{2(1 - \gamma^2)}, \\ q_1^B &= \frac{(2 + \beta\gamma^2)(v^B - c^B) - (2 + \beta)\gamma(v^A - w_1)}{2(1 - \gamma^2)(2 + \beta)}, \\ q_2^B &= \frac{v^B - c^B}{2 + \beta}. \end{aligned}$$

Note that the off-equilibrium value of q_2^B is *independent* of w_1 . It follows immediately that the joint profits of supplier A and retailer 1, $\pi_1^{Off} + w_1 q_1^A$, are maximized at $w_1 = 0$ (there is no distortion). Note also that, from the first-order condition for q_2^B , we thus have that

$$\pi_2^{Off} = (q_2^B)^2 = \left(\frac{v^B - c^B}{2 + \beta}\right)^2. \quad (\text{A.19})$$

Now suppose that market-share contracts are feasible. Recall first that through an appropriate specification of the market share, as well as w^+ , supplier A can ultimately force the remaining retailer, $r = 1$, to choose any pair of quantities (q_1^A, q_1^B) . Consequently, once retailer 2 has rejected its offer, the (out-of-equilibrium) game becomes one of Stackelberg leadership. Once again using $\pi_2^{Off} = (q_2^B)^2$ and substituting the derived quantity

$$q_2^B = \frac{(4 - \beta^2 - 2\beta)(v^B - c^B)}{4(2 - \beta^2)},$$

we obtain, for this case,

$$\pi_2^{Off} = \left(\frac{(4 - \beta^2 - 2\beta)(v^B - c^B)}{4(2 - \beta^2)}\right)^2. \quad (\text{A.20})$$

To complete the characterization of equilibrium contracts, note that with two-part tariffs,

$$F_r = [q^A(p^A - w) + q^B(p^B - c^B)] - \pi_r^{Off}.$$

After substituting from the retailers' first-order conditions, and using symmetry, this becomes

$$F_r = [(q^A)^2 + (q^B)^2 + 2q^A q^B \gamma] - \pi_r^{Off},$$

and thus

$$F = \frac{[v^A - c^A - \gamma(v^B - c^B)]^2}{4(1 + \beta)^2(1 - \gamma^2)}.$$

With market-share contracts, the equilibrium fixed fee satisfies

$$F = [q^A(p^A - w^+) + q^B(p^B - c^B)] - \pi^{Off}.$$

Using (8), it can be shown that

$$w^+ = \frac{1}{1 + \beta} c^A + \frac{\beta [(v^A)^2 - (c^A)^2 + (v^B - c^B)(v^B - c^B - 2\gamma v^A)]}{2(1 + \beta)(v^A - c^A - \gamma(v^B - c^B))}.$$

Substituting w^+ , the monopoly quantities, and π^{Off} into F , we thus obtain

$$F = \frac{(v^A - c^A)^2 + (v^B - c^B)^2 + 2\gamma(v^A - c^A)(v^B - c^B)}{4(1 - \gamma^2)(1 + \beta)^2} - \left(\frac{(\beta^2 + 2\beta - 4)(v^B - c^B)}{4(2 - \beta^2)} \right)^2.$$

Q.E.D.

Proof of Proposition 7. As was argued in the proof of Proposition 2, by setting w_r^- sufficiently high, only deviations leading to shares $\rho_r \geq \bar{\rho}_r$ must be considered. Retailer r thus maximizes

$$\pi_r = (p - w_r^+) Q_r^A(\mathbf{p}) + (p_r^B - c^B) Q_r^B(\mathbf{p}) - F_r,$$

subject to the constraint that $\rho_r \geq \bar{\rho}_r$, where \mathbf{p} is evaluated at $p_r^s = \bar{p}_r^s$. Suppose first that retailer r only considers deviations such that $\rho_r = \bar{\rho}_r$. Transforming this requirement into

$$q_r^B \bar{\rho}_r - q_r^A (1 - \bar{\rho}_r) = 0, \tag{A.21}$$

retailer r has only one variable under its control, say p_r^A , such that from implicit differentiation of (A.21) the respective necessary adjustment to p_r^B is pinned down by

$$\frac{dp_r^B}{dp_r^A} = - \frac{\frac{\partial Q_r^B}{\partial p_r^A} \bar{\rho}_r - \frac{\partial Q_r^A}{\partial p_r^A} (1 - \bar{\rho}_r)}{\frac{\partial Q_r^B}{\partial p_r^B} \bar{\rho}_r - \frac{\partial Q_r^A}{\partial p_r^B} (1 - \bar{\rho}_r)} > 0. \tag{A.22}$$

With this, the respective first-order condition under the constraint $\rho_r = \bar{\rho}_r$ is

$$\frac{\partial \pi_r}{\partial p_r^A} + \frac{dp_r^B}{dp_r^A} \frac{\partial \pi_r}{\partial p_r^B} = 0. \tag{A.23}$$

Substituting the first-order conditions in (12), (13) and (14), into (A.23), it follows that the monopoly outcome is obtained if

$$w_r^+ - c^A = - \frac{(\bar{p}_r^A - c^A) \left(\frac{\partial Q_r^A}{\partial p_r^A} + \frac{\partial Q_r^A}{\partial p_r^B} \frac{dp_r^B}{dp_r^A} \right) + (\bar{p}_r^B - c^B) \left(\frac{\partial Q_r^B}{\partial p_r^A} + \frac{\partial Q_r^B}{\partial p_r^B} \frac{dp_r^B}{dp_r^A} \right)}{\frac{\partial Q_r^A}{\partial p_r^A} + \frac{\partial Q_r^A}{\partial p_r^B} \frac{dp_r^B}{dp_r^A}}.$$

To sign this expression, note that the denominator is negative given that p_r^B adjusts following an increase in p_r^A so that the market share of good A at retailer r stays constant (that is, both q_r^A and q_r^B decrease accordingly), and note that the numerator, which one can write as

$$\begin{aligned} & (\bar{p}_r^A - c^A) \frac{\partial Q_{r'}^A}{\partial p_r^A} + (\bar{p}_r^B - c^B) \frac{\partial Q_{r'}^B}{\partial p_r^A} \\ & + \frac{dp_r^B}{dp_r^A} \left[(\bar{p}_r^B - c^B) \frac{\partial Q_{r'}^B}{\partial p_r^B} + \bar{p}_r^A - c^A \right] \frac{\partial Q_{r'}^A}{\partial p_r^B}, \end{aligned}$$

is strictly positive given our assumptions on demand and $\frac{dp_r^B}{dp_r^A} > 0$. Thus, $w_r^+ - c^A > 0$.

We show next that $\frac{\partial \pi_r}{\partial p_r^B} < 0$, which is used in (A.23), when evaluated at $q_r^s = \bar{q}_r^s$ and $p_r^s = \bar{p}_r^s$ together with $w_r^+ - c^A > 0$. Substituting from (12) into (14) it holds that

$$\left. \frac{\partial \pi_r}{\partial p_r^B} \right|_{\mathbf{p}=\bar{\mathbf{p}}} = (c^A - w_r^+) \frac{\partial Q_r^A}{\partial p_r^B} - \sum_{r' \in R/r, s' \in S} (\bar{p}_{r'}^{s'} - c^s) \frac{\partial Q_{r'}^{s'}}{\partial p_r^B},$$

which is strictly negative given our assumptions on demand and using $c^A - w_r^+ < 0$.

Given that $\frac{\partial \pi_r}{\partial p_r^B} < 0$, so as to satisfy (A.23) it must hold that $\frac{\partial \pi_r}{\partial p_r^A} > 0$. As a consequence, any other local deviation from $p_r^s = \bar{p}_r^s$ where $\frac{dp_r^B}{dp_r^A}$ is lower than the value in (A.22), implying that $\rho_r > \bar{\rho}_r$, is also not profitable. This, together with the assumed quasiconcavity of the retailer's program, confirms that the characterized market-share contract implements the monopoly prices.

Q.E.D.

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