Stockholms universitet

Market Volatility's Relationship with Pairwise Correlation of Stocks and Portfolio Manager's Performance

Weigang Qie

Masteruppsats i matematisk statistik
Master Thesis in Mathematical Statistics

Masteruppsats 2011:6
Matematisk statistik
Juni 2011

www.math.su.se

Matematisk statistik
Matematiska institutionen
Stockholms universitet
106 91 Stockholm

Mathematical Statistics
Stockholm University
Master Thesis **2011:6**
http://www.math.su.se

Market Volatility's Relationship with Pairwise Correlation of Stocks and Portfolio Manager's Performance

Weigang Qie*

June 2011

Abstract

The objective of this article is to deal with two questions. First, what is the relationship between market volatility and pairwise correlations of stocks? Second, how portfolio managers' performances vary during turbulent periods and stable periods? Two parts are employed to answer those questions separately via empirical data. In Part I, a data set consisting of OMXS30 Index and five stocks is investigated and the relationship between market volatility and pairwise correlations of the stocks is quantified by a linear regression model. The slope of linear model represents the strength of market volatility's influence on the pairwise correlation of the stocks. Therefore, we conclude that there exists significantly positive relationship between the market volatility and pairwise correlations of the stocks. In part II, we investigate a data set consisting of OMXS30 Return Index and 69 funds. The excess return of the funds is measured by Á and Jensen's Á respectively. Four portfolios, Average Portfolio,T5, M5 and B5 which represent the average performance of all the 69 funds, top 5 funds, median 5 funds and bottom 5 funds respectively are set up for comparison. Two conclusions are derived. First, considering the magnitude of the excess return, Average Portfolio, M5 and B5 in times of high market volatility are inferior to those during periods with low market volatility, whereas T5 is superior. Second, in times of high market volatility T5 is superior to the other three portfolios while M5 performs better than B5. In times of low market volatility B5 is inferior to the other three portfolios. Besides, based on the other intercomparisons of the four portfolios, no significant difference is observed.

*Postal address: Mathematical Statistics, Stockholm University, SE-106 91, Sweden. E-mail: qiwe8333@student.su.se. Supervisor: Thomas Höglund.

Preface

The article constitutes a Master's thesis for the degree of Master in Financial Mathematics and Finance at Stockholm University.

Acknowledgement

I would like to give my sincere thanks to my supervisor at Stockholm University, Thomas Höglund for his great support and guidance.

I would also like to give my sincere thanks to my examiner at Stockholm University, Joanna Tyrcha for hers valuable comments.

Contents

Notation List

- $\sigma_{\mathbf{M}}$: Market volatility
- $\rho_{\mathbf{pmcc}}$: Pearson moment correlation coefficient
- $\rho_{\mathbf{s}}$: Spearman's rank correlation coefficient
- **OLS**: Ordinary Least Squares
- **WLS**: Weighted Least Squares
- $\beta_{\mathbf{1}}$: The slope of linear model represents the strength of market volatility's influence on the pairwise correlation of stocks.
- $\beta_{\mathbf{01}}$: The intercept of linear regression model based on Pearson moment correlation and market volatility
- $\beta_{\mathbf{11}}$: The slope of linear regression model based on Pearson moment correlation and market volatility
- $\beta_{\mathbf{02}}$: The intercept of linear regression model based on Spearman's rank correlation and market volatility
- $\beta_{\mathbf{12}}$: The slope of linear regression model based on Spearman's rank correlation and market volatility
- $\mathbf{t_W}$: The two samples Welch's t test statistics with unequal variances
- **Group 1**: The pairwise correlations or excess returns over the market when market volatility is higher than 30%
- **Group 2**: The pairwise correlations or excess returns over the market when market volatility is lower than 30%
- $\mathbf{r_M}$: The market return
- $\mathbf{r}_{P}$: The portfolio return
- $\mathbf{r_f}$: The risk free interest rate
- **CAPM**: Capital Asset Pricing Model
- **SML**: Security Market Line

- $\boldsymbol{\beta}$: The systematic risk of the fund
- $\boldsymbol{\alpha}$: Alpha, the measurement of excess return of fund over the market without considering the risk
- $\mathbf{J}_{\boldsymbol{\alpha}}$: Namely Jensen's alpha, the measurement of excess of return of fund over the market that estimated from a linear regression model in the entire period
- **Jensen's** $\boldsymbol{\alpha}$: The Jensen's alpha derived in weekly period
- **Average Portfolio**: The portfolio consists of 69 funds with the same weight
- **T5**: The portfolio consists of the top 5 funds with the same weight
- **M5**: The portfolio consists of the median 5 funds with the same weight
- **B5**: The portfolio consists of the bottom 5 funds with the same weight
- $\mathbf{t}_{\mathbf{D}}$: The dependent samples t test statistics

1 Introduction

1.1 Purpose

The market always goes up and down in response to the latest information and it creates uncertainty which is represented by market volatility. In our article, we assume that the market volatility higher than 30%(including) is the times of high market volatility and that lower than 30% is the times of low market volatility. The market volatility which is denoted by σ_M varies all the time and it is essential for the market practitioners to grasp that. The awareness of the connection between market volatility and correlations of different stocks is also important for portfolio managers, risk managers, financial firm conductors, and monetary policy makers. Financial market observers have noted that during the periods of high market volatility, the correlations of asset returns vary substantially in comparison with those in stable market. The performance of portfolio managers might be impacted by the increasing correlations of stocks in stressful market. Their trading strategy is formulated based on the researches on the market information. And their allocations are updated according to the market volatility, their risk aversion and target. They probably stay at long positions in bullish market and short positions in bearish market. Then the objective of our article is to answer two questions concerning the market volatility based on empirical data.

1.2 First Question

First, what is the relationship between market volatility and pairwise correlations of stocks?

In reality, the prices of stocks are decided by their fundamental values, whereas, if a market shock takes place, it might lead the prices to drop or rally and violate their fundamental values. It is high market volatility. In such a situation, investors are usually overreacting and they will all sell or buy the stocks at the same time. In response, the prices of stocks tend to change accordingly. So pairwise correlations of stocks increase when market volatility is high. Moreover, previous studies also suggest that the correlations between international stock markets tend to increase during turbulent market periods. They claim that there exists positive relationship between market volatility and correlations of stocks. Loretan and English (2000) employed a theoretical model derived by Boyer, Gibson and Loretan (1999) to illustrate the link between them. They proposed a critical assumption that the two series of returns are jointly bivariate normal distributed. Their empirical data consisting of market index, stocks, bonds and fix-change rates fits the theory well. Their conclusion is that the correlation of underlying assets and market volatility is positive. Besides, Boyer, Gibson, and Loretan (1999) proposed that the high market volatility tend to accompany the increase in pairwise correlations of stocks. They pointed out that if the market volatility is high, correlations play a nontrivial role to price and hedge derivatives which consist of more than one asset. They also observed that the correlations which computed separately in low and high market volatility periods change considerably. This situation is the so called "correlation breakdown." They suggested that the correlation breakdowns may reflect time varying volatility of financial markets. It is consistent with the results derived from Welch's t test in our article. So under the stressful market condition, the participants need to know the latest underlying correlations to conduct their decisions. In our article, through the application of Pearson and Spearman's rank correlation coefficient tests, we arrive at a conclusion that there exists significantly positive relationship between market volatility and pairwise correlations of the stocks. Then, we also adopted a linear regression model to quantify their relationship which is measured by the slope parameter in our article. It also represents the strength of the market volatility's influence on pairwise correlations of stocks.

1.3 Second Question

Second, how portfolio managers' performance vary during turbulent periods and stable periods?

The portfolio managers are professional investors with years of investing experience, comprehensive information and experienced trading techniques. Traditionally, a portfolio manager should meet two major requirements. One is the ability to attain excess returns over given risk classes. The other is the ability to diversify the portfolio to remove the unsystematic risk. (One way to measure portfolio diversification is to calculate the correlations of it with market portfolio. If the portfolio is perfectly diversified, the correlation equals 1.) Both of two requirements can be evaluated by the composite measurements, but they do not distinguish them. Thus, we introduce three portfolio performance measurements based on the capital asset pricing model (CAPM) which is an economic model to price securities and derive the expected return and also the basic model for performance measurements. Treynor (1965) developed the first composite measurements which combine the returns and risk in single value. It represents a reward-to-risk ratio in which the numerator is risk premium (average portfolio return − average risk free interest rate) and denominator is the risk of portfolio measured by standard deviation. Then Sharpe (1996) proposed another composite measurement which replaces the standard deviation with β. So, the only difference between Sharpe and Treynor's is the measure of portfolio risk. And then Jensen (1968) proposed Jensen's alpha to measure the portfolio performance. It is based on the Security Market Line (SML) and estimated from linear regression model where portfolio risk premium is the response variable while the market portfolio risk premium is the independent variable. Jensen's alpha is the intercept of this regression model. In our article, we focus on the evaluations of the performance of portfolio which is represented by excess return over the market. Two measurements is adopted to measure it. One denoted by α is computed by: portfolio return − market return. The other one is Jensen's alpha. We do not intend to compare the measurements of portfolio performances. Instead, we will manage to illustrate two issues. First, how the funds perform in times of high market volatility compared with those in times of low market volatility? Second, what are the differences between the performances of the funds both in times of high and low market volatility?

1.4 Outline

Therefore, the rest of our article consisting of Part I, Part II and Part III is organized as follows. Two parts will answer the two questions respectively with the help of empirical data. Part I focuses on the relationship between market volatility and pairwise correlations of stocks which contains section 2, section 3 and section 4. In section 2, the data consisting of OMXS30 Index and five stocks are introduced. In section 3, along with the methods to compute market volatility and correlation, Welch's t test and linear regression model are presented. Part II evaluates the performance of portfolio managers both in times of high and low market volatility which contains section 5 and section 6. In section 5, a data set consisting of the weekly returns of OMXS30 Return Index and 69 funds which invest in Swedish market is employed to deal with the second question. In section 5, The tests such as Welch's t test and dependent samples t test are described as well. We focus on the comparison of four portfolios, Average Portfolio, T5, M5 and B5 in section 6. Part III presents the conclusions of our article based on the results obtained from Part I and Part II.

Part I

The Relationship Between Market Volatility and Pairwise Correlations of Stocks

In Part I, we examine the relationship between market volatility and pairwise correlations of stocks by empirical studies in Sweden Stock Market. As pointed out by Pollet and Wilson (2008), the average pairwise correlations is suitable to forecast the market excess returns both in and out of samples. Moreover, since the stable and clear relationship between average pairwise correlations of the stocks and market volatility is preliminarily observed, we focus on the relationship between them in our article. Then, we adopt the traditional approach of standard deviation of market log-returns to measure the market volatility. Since it computed in the relatively short time interval tends to track the market's variation, it is approach to the market's reality. The longer the time interval is, the more smooth the market volatility is. However, if the time interval is too short, the results obtained are imprecise. If it is too long, some of the fluctuations to different directions will counterbalance each other in our calculations and we can not observe those fluctuations in the results. Therefore, in order to see how the relationship varies as the time interval increases, we investigate the market volatility by dividing the entire period by four different types of time span (10 days, 25 days, 75 days and 100 days) respectively and calculate the market volatility in them respectively by equation (2). The results show that the length of 100 days is sufficient to illustrated the relationship. The corresponding average pairwise correlations are computed as well.

2 Data Description

In Part I, the data set consisting of OMX Stockholm 30 Index(OMXS30) and five stocks is obtained from NASDQOMX. Comprising 30 most traded stocks in Stockholm Stock Exchange, the OMXS30 Index is a market value-weighted index and proxy for the market in Part I. It was quoted at 500.00 for the first time on 30 September 1986. A time span of about twenty years from 1 November 1990 to 1 November 2010 is selected to reflect the fluctuation of the market. Then we pick up five individual stocks from OMXS30 Index namely SEB, Ericsson, Volvo Group, Skanska and SSAB. The closing prices of them at the end of the day are viewed as the price for investigation. The date for the OMS30 index and the five stocks is exactly the same after deleting some missing days. So we have 5000 trading days. The standard deviation of daily log-returns, a traditional approach, is employed to compute the annualized market volatility. And then, based on the log-returns of stocks, the corresponding pairwise correlation is computed. Since the five stocks are in different industries(Finance, Communication, Steel, Architecture, Construction and Farm machinery), their pairwise correlations are expected to change substantially in the stressful market. There are 10 pairwise correlations for the five stocks. The dynamics of OMXS30 Index along with the five stocks are drawn by Figure 1.

Since the five stocks splitted several times during the entire period, the prices of them are adjusted to approach the reality. As the dynamics of OMXS30 Index shown by the Figure 1, there are two turbulent periods in the market. One is from the end of 2001 to 2002—following the *September 11 attacks*, and the other is the year of 2007-2009—the financial crisis initiated by American's sub-prime loan crisis. The price of OMXS30 started at 153.98 on 1 November 1990, peaked at 1539.00 on 7 March 2000 and ended at 1091.69 on 1 November 2010. The dynamics of SEB, Volvo, Skanska and SSAB declined intensely during the period of 2007-2009 and recovered for a while until October 2010. They are consistent with the market fluctuation. And the prices of Ericsson declined intensely during

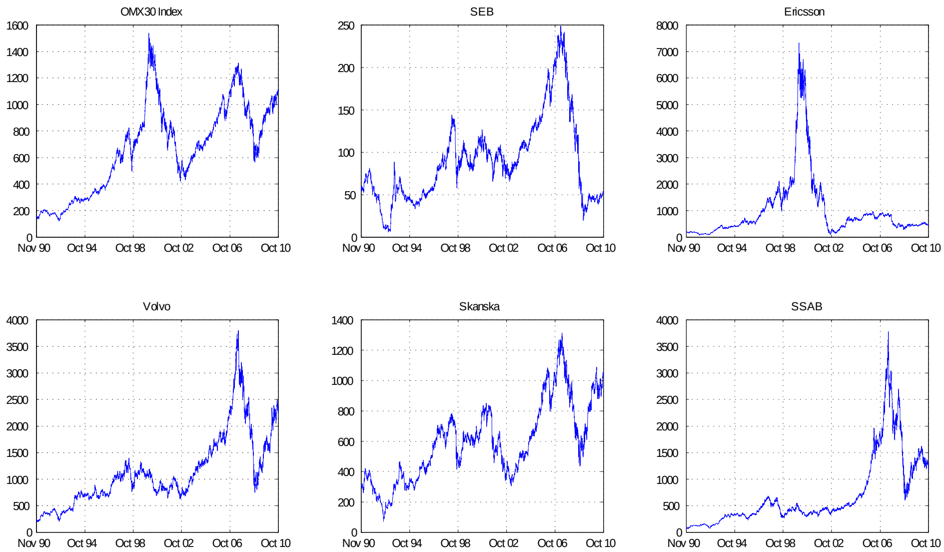

Figure 1: The dynamics of the adjusted daily prices for OMXS30 Index and the five stocks, SEB, Ericsson, Volvo Group, Skanska and SSAB.

the period 1999-2001 after achieving the highest level in the twenty years. The current financial crisis seems to have weak influence on it. Then Table 1 gives the basic statistics of daily log-returns for OMX30 index and the five stocks.

Table 1: Basic statistics for adjusted daily log−returns

Statistics	OMXS30	SEB	Ericsson	Volvo	Skanska	SSAB
Min.	-8.53%	-52.97%	-35.42%	-15.38%	-26.80%	-16.89%
Max.	11.02%	34.83%	22.31%	15.13%	26.14%	23.46%
Mean	0.03918%	-0.00247%	0.01550%	0.04378%	0.02379%	0.05408%
Std.	0.0154	0.0321	0.0314	0.0218	0.0218	0.0245
Kurtosis	6.8187	38.0032	13.4497	7.4486	17.0531	10.5259
Skewness	0.1712	-1.0460	-0.6100	0.0405	-0.0616	0.2826
Volatility	24.29%	50.71%	49.61%	34.43%	34.51%	38.67%

As reported by Table 1, SSAB and Volvo with high means of log-returns outperform the market. It is interesting to see that the mean of SEB is less than 0. The volatility of SEB is 50.71% which is the highest compared with the other stocks and market. The investor who is holding it in twenty years will get loss. All the volatility of five stocks are higher than 24.29%, the market volatility. In the sharp aspects of distribution, the values of kurtosis of OMXS30 Index and the five stocks are larger than 6. In particular, SEB is extremely large which exceeds

38. While the Skewness of SEB is excess 1, that of the others are relative low but above 0. Since the values of Kurtosis and Skewness for normal distribution are zero, these log-returns series are not normal distributed. The ordinary Pearson's correlations between the OMXS30 index and the five stocks in the entire period are given by Table 2. The correlations matrix for these log-returns are listed.

Table 2: Pearson correlation between the OMXS30 Index and the five stocks

	OMXS30	SEB	Ericsson	Volvo	Skanska	SSAB
OMXS30	1.0000	0.5344	0.7420	0.6547	0.5758	0.5373
SEB	0.5344	1.0000	0.2807	0.3857	0.3882	0.3315
Ericsson	0.7420	0.2807	1.0000	0.3848	0.3155	0.2912
Volvo	0.6547	0.3857	0.3848	1.0000	0.4457	0.4778
Skanska	0.5758	0.3882	0.3155	0.4457	1.0000	0.3950
SSAB	0.5373	0.3315	0.2912	0.4778	0.3950	1.0000

As demonstrated by Table 2, it is reasonable to find that the correlations of the OMXS30 and the five stocks are all higher than 0.5. In particular, Ericsson is 0.7420 and highly correlated with the market. According to Table 3 presented in section 3.2.1, the strength of relationship between OMXS30 Index and the five stocks is moderate. The pairwise correlations of the five stocks are all positively correlated. However, the strength of them are relatively low. It indicates that their log-returns tend to change to the same directions during this period.

3 Methodology

3.1 Market Volatility Estimation

Let σ_n denote the volatility on day n which is estimated at the end day of $n-1$. The traditional approach to compute it is introduced as follows. Suppose that the price of underlying asset at the end of day n is S_i. The variable of continuously log-return is denoted by u. The log-return between the end of day $i-1$ and the end of day i is computed by equation (1):

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \tag{1}$$

Given the length of m days, the unbiased estimation of volatility:

$$\sigma_n = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}\left(u_i - \bar{u}\right)^2} \tag{2}$$

where $\bar{u} = \frac{1}{m} \sum_{i=1}^{m} u_i$.

The 10 days, 25 days, 75 days and 100 days market volatility are computed respectively by equation (2). Besides that, we also refer to several other approaches such as Exponentially Weighted Moving Average Model and GARCH (1, 1) in Hull (2009).

3.2 Correlations

3.2.1 Pearson Moment correlation

Suppose there are two random variables: $X = (X_1, X_2 \cdots X_N)$ and $Y = (Y_1, Y_2 \cdots Y_N)$. Pearson moment correlation coefficient(pmcc) is widely used as the measure of correlation. It is developed by Karl Pearson and denoted by ρ_{pmcc}. Then the ρ_{pmcc} is computed by equation (3):

$$\rho_{pmcc} = \frac{cov\,(X, Y)}{\sigma_X \sigma_Y} \tag{3}$$

where $cov\,(X, Y)$ is the covariance of X and Y, σ_X and σ_Y are the standard deviance of X and Y respectively. Then the estimator of ρ_{pmcc} denoted by $\hat{\rho}_{pmcc}$ is calculated on the two samples $x = (x_1, x_2 \cdots x_n)$ and $y = (y_1, y_2 \cdots y_n)$ in which the sample size is n, the equation (4) is :

$$\hat{\rho}_{pmcc} = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} \tag{4}$$

where $S_{xx} = \frac{1}{n-1} \sum (x_i - \bar{x})^2$, $S_{xy} = \frac{1}{n-1} \sum (y_i - \bar{y})^2$ and $S_{xy} = \frac{1}{n-1} \sum (x_i - \bar{x})(y_i - \bar{y})$.

The correlation coefficient $\hat{\rho}_{pmcc}$ ranges in $[-1, 1]$, it reflects the linear relationship between the two variables. The strength of the correlations are listed in the Table 3.

Table 3: The Strength of Correlation

$\lvert\rho\rvert$	Interpretation
$[0.9, 1]$	Very high correlation
$[0.7, 89]$	High correlation
$[0.5, 0.69]$	Moderate correlation
$[0.3, 0.49]$	Low correlation
$[0.0, 0.30]$	Little if any correlation

The $\hat{\rho}_{pmcc}$ characterizes the joint distribution when two variables are bivariate normal distributed. This is not true for other joint distributions. However, it is very informative in cases of large sample size. The outlier affects the accuracy of $\hat{\rho}_{pmcc}$ which might be overcame by robust estimation.

The null hypothesis for pmcc test is that there is no correlations between the two variables. The underlying assumption is that the joint distribution of two variables is bivariate normal distribution. The null hypothesis of pmcc test is:

$$\begin{aligned} H_0 &: \textit{There is no correlation}, \rho_{pmcc} = 0 \\ H_1 &: \textit{There exists correlation}, \rho_{pmcc} \neq 0 \end{aligned}$$

The sampling distribution of $\hat{\rho}_{pmcc}$ approximately follows Student's t distribution with freedom degrees $\sqrt{n-2}$:

$$t_{pmcc} = \hat{\rho}_{pmcc}\sqrt{\frac{n-2}{1-\hat{\rho}^2_{pmcc}}} \tag{5}$$

Given the $\hat{\rho}_{pmcc}$ and sample size n, the t_{pmcc} test value is computed by equation (5). It is compared with the critical values of t for one tail or two tail test. We can not just apply the pmcc test when the joint distribution of two random variables are not bivariate Normal distribution. Then the nonparametric statistics Spearman's rank correlation coefficient is chosen as an alternative.

3.2.2 Spearman's Rank Correlation

There are two measures of rank correlation such as Spearman's and Kendall's. Spearman's rank correlations named after Charles Spearman is widely used. If the association of two random variable is non linear, their ranks of values transfer it to a linear relationship. Two new variables is set up by their ranks of values. Spearman's rank correlation coefficient (srcc) is denoted by ρ_s, which is calculated by the equation (6):

$$\rho_s = 1 - \frac{6\sum d_i^2}{n\,(n^2-1)} \tag{6}$$

where n is the sample size and d is the difference of the ranks of values in two variable denoted by $(rank_X - rank_Y)$. The estimator of it $\hat{\rho}_s$ is computed by the two samples $x = (x_1, x_2 \cdots x_n)$ and $y = (y_1, y_2 \cdots y_n)$.

If the two variables perfectly match each other, $\hat{\rho}_s$ is +1 or -1. The scatter points of their ranks of values match the diagonal line. The statistical significance of it is examined by Spearman's rank correlation coefficient test. Both the two correlation coefficients can not interpret the causality. The two variables with high correlation might impact each other.

Spearman's rank correlation coefficient test is presented as well. There is no underlying distribution assumption to implement it. However, there are difficulties associated with using Spearman's rank test with the data from very small samples or very large samples. We set up a null hypothesis and the corresponding alternative hypothesis:

$$\begin{aligned} H_0 &: \textit{There is no association between the variables in the underlying population}, \rho_s = 0 \\ H_1 &: \textit{There is association between the variables in the underlying population}, \ \rho_s \neq 0 \end{aligned}$$

.

The test statistics varies for different sample size.

1. If the sample size is smaller than 20, the critical values for it can be found from the table provided by Dudzic (2007).
2. If the sample size is about 20 upwards, Jerrold (1972) proposes that $t = \hat{\rho}_s \sqrt{\frac{n-2}{1-\hat{\rho}_s^2}}$ approximately follows Student's t distribution.
3. If the sample size is about 40 upwards, Dudzic (2007) presents that

$$Z = \hat{\rho}_s \sqrt{n-1} \tag{7}$$

approximately follows $N(0, 1)$

Since the sample size in our article is always larger than 40, we compute the Z test values by equation (7) after calculated $\hat{\rho}_s$ by equation (6). Then they are compared with the critical value represented by $Z_{\tau/2} = 1.96$, given significantly level $\tau = 0.05$. Thereafter, the results of whether the test values reject null hypothesis are obtained. If the test value rejects the null hypothesis, the coefficient is significantly larger or smaller than zero.

3.3 Welch's t test

In statistics, two samples t test and one way analysis of variance is widely used to inspect the equality of means between the different samples. One way analysis of variance (ANOVA) is the extension of two samples t test when there are more than two samples. The assumption to apply two sample t test is that the two samples are from normal distribution with the same variance. If the means violate normal distribution, the nonparametric methods such as Mann-Whitney U test and Wilcoxon signed-rank test are chosen as alternatives. According to the Central Limit Theorem, the sample mean is approximately normal distributed when the sample size n greater than 30. As a result, the means of the two samples are approximately normal distributed in our article. Then, the Welch's t test extends the two samples t test when the samples sizes and variances of two samples are unequal. Then the Welch's t test is adopted to testify whether the pairwise correlations significantly differ from each other in times of low and high market volatility. Suppose that we have two populations with expected means μ_1 and μ_2. Then the corresponding two samples x_1 and x_2 with sample sizes n_1 and n_2 respectively are observed. The null and alternative hypothesis are set up:

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_1 : \mu_1 - \mu_2 \neq 0$$

Then the equation (8) is to compute the t_W statistics:

$$t_W = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \tag{8}$$

where $\bar{x}_1$ and $\bar{x}_2$ are means of the two samples x_1 and x_2, S_1^2 and S_2^2 are the corresponding samples variances. Then the freedom degrees V_W associated with it is estimated by Welch-Satterthwaite equation, see Welch (1947) and Satterthwaite (1946):

$$V_W = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{S_1^4}{n_1^2(n_1-1)} + \frac{S_2^4}{n_2^2(n_2-1)}} \tag{9}$$

Given the significant level $\tau = 0.05$, the critical value for two tail test is $t_{W\frac{\tau}{2}(V_W)}$. If the test value rejects null hypothesis, we can conclude that the means of two samples are significantly different from each other.

3.4 Linear Regression Model

In statistics, the linear regression model is employed to quantify the relationship between two or more variables. Here we set up a simple linear regression model which is given, see Charles and Corrinne (2008):

$$y = \beta_0 + \beta_1 x + \epsilon \tag{10}$$

where $y = (y_1, y_2, \cdots, y_n)$ is the response variable with n observations;

$x = (x_1, x_2, \cdots, x_n)$ is the independent variable with n observations;

If $x = 0$, $y = \beta_0$ and it is the intercept parameter;

If x changes 1 unite, corresponding y changes β_1 unite. It is the slope parameter;

$\epsilon = (\epsilon_1, \epsilon_1, \cdots, \epsilon_n)$ is the error term not explained by the model.

The assumptions of linear regression model are given as follows:

- x is a independent variable and it is independent with the error term.
  - None autocorrelation.
  - None relationship with the error terms ϵ.
- $\epsilon = (\epsilon_1, \epsilon_1, \cdots, \epsilon_n)$ is the error term.
  - Independent to each other: $Cov\left(\epsilon_i, \epsilon_j\right) = 0$ if $i \neq j$.
  - Identical normal distribution $N\left(0, \sigma_\epsilon^2\right)$.
  - Zero mean: their expected $E\left(\epsilon_i\right) = 0$ are zero.
  - Homoscedasticity, $var\left(\epsilon_i\right) = \sigma_\epsilon^2$ where σ_ϵ^2 is a constant variance.

3.4.1 Ordinary Least Squares (OLS) Estimation

The sum of squared residuals denoted by SSE_{OLS} is to measure the estimation error:

$$\begin{aligned} SSE_{OLS} &= \sum e_i^2 \\ & \sum (y_i - \hat{y}_i)^2 \end{aligned} \tag{11}$$

where e_i is the residual, $e = (e_1, e_2, \cdots, e_n)$ and the fitted value for each i is $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$.

Then the parameters of β_0 and β_1 are estimated by Ordinary Least Squares (OLS)[1] which is a widely used method to achieve the criterion of the minimum SSE_{OLS}. The unbiased and consistent estimators are computed by the equations given below:

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \end{pmatrix} = \frac{1}{S_{xx}} \begin{pmatrix} \sum (y_i - \bar{y}) \\ S_{xy} \end{pmatrix} \tag{12}$$

where $\bar{y}$ and $\bar{x}$ are the sample means of Y and X respectively;
$S_{xx} = \sum (x_i - \bar{x})^2$;
and $S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y})$.

Based on the assumption mentioned earlier, the estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are approximate normal distribution. The one sample t test is employed to examine whether they are significantly unequal 0. The null and alternative hypothesis are:

$$\begin{aligned} H_0 &: \beta_0 = 0 \; or \; \beta_1 = 0 \\ H_1 &: \beta_0 \neq 0 \; or \; \beta_1 \neq 0 \end{aligned}$$

Then the t_{OLS} test statistical with freedom degrees $n-2$ is computed by:

$$t_{OLS} = \frac{\hat{\beta}_0 - \beta_0}{\sqrt{\frac{SSE_{OLS}}{n-2}} \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}}} \; or \; \frac{\hat{\beta}_1 - \beta_1}{\frac{\sqrt{SSE_{OLS}}}{\sqrt{S_{xx}(n-2)}}} \tag{13}$$

where S_{xx} and SSE_{OLS} are mentioned above.

Good mathematical properties such as unbiased, consistent and efficiency are contained by the OLS estimators.

So the 95% confidence intervals for fitted value $\hat{y}_i$ and the estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are given as follows:

[1]OLS repesents Ordinary Least Squares in the rest of our article

$$\hat{y}_i \pm t_{0.025(n-2)} \sqrt{SSE_{OLS}\left[\frac{1}{n} + \frac{(x_i - \bar{x})^2}{S_{xx}}\right]} \tag{14}$$

$$\hat{\beta}_0 \pm t_{0.025(n-2)} \sqrt{\frac{SSE_{OLS}}{n-2}} \sqrt{\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}} \tag{15}$$

$$\hat{\beta}_1 \pm t_{0.025(n-2)} \frac{\sqrt{SSE_{OLS}}}{\sqrt{S_{xx}(n-2)}} \tag{16}$$

In regarding to the goodness of fit, S_{yy} and SSR_{OLS} represents the total variance of y and the variance of linear model. They are derived from:

$$S_{yy} = SSR_{OLS} + SSE_{OLS}$$

where $S_{yy} = \sum (y_i - \bar{y})^2$;
$SSR_{OLS} = \sum (y_i - \hat{y}_i)^2$.

One measurement of goodness of fit is denoted by R^2_{OLS} to illustrate how the linear model interprets the variation of y.

$$R^2_{OLS} = 1 - \frac{SSE_{OLS}}{S_{yy}} \tag{17}$$

R^2_{OLS} ranges from 0 to 1. If the linear model perfectly match the relationship of y and x, $R^2 = 1$. The adjusted R^2_{OLS} is to remove the effects of freedom degrees which is calculated by:

$$\text{Adjusted } R^2_{OLS} = 1 - \frac{SSE_{OLS}}{S_{yy}} \frac{(n-1)}{n-d-1} \tag{18}$$

where d is the numbers of parameters.

3.4.2 Weighted Least Squares (WLS) Estimation

If we observe that there exists some patterns of relationship between the independent variable x and error term ϵ, the assumption of homoscedasticity is violated. Then ϵ have different variances: $var(\epsilon_i) = \sigma_i^2$. The consequence of heteroscedasticity is that the test statistics t_{OLS} might be overestimated and make the null hypothesis test to fall. The estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are still unbiased and consistent but no longer efficient. They may lead to incorrect conclusion. One way to handle the heteroscedasticity is to estimated the parameters by Weighted Least

Squares(WLS)[2]. The suitable weights denoted by $W = (w_1, w_2, \cdots w_n)$ are established to ensure that variance of the error terms derived from OLS is constant. A simple way is to let $w_i = \frac{1}{e_i^2}$ where e_i is the residuals derived from the OLS. So the sum of square error terms $\sum w_i e_i^2 = 1$ for Weighted Least Squares equals 1. We multiply the $\sqrt{W}$ to the two sides of linear model (10).

$$\begin{aligned} \sqrt{W}y &= \sqrt{W}\beta_0 + \beta_1\sqrt{W}x + \sqrt{W}\epsilon \\ y_{WLS} &= \beta_{0WLS} + \beta_{1WLS}x_{WLS} + \epsilon_{WLS} \end{aligned} \tag{19}$$

The sum of squared errors denoted by SSE_{WLS} is to measure the estimation error:

$$\begin{aligned} SSE_{WLS} &= \sum (y_{iWLS} - \hat{y}_{iWLS})^2 \\ &\sum w_i (y_i - \hat{y}_i)^2 \end{aligned} \tag{20}$$

It is the same as OLS, we estimate the parameters α_{WLS} and β_{WLS} by achieving the criterion minimum of SSE_{WLS}. The $\hat{\beta}_{0WLS}$ and $\hat{\beta}_{1WLS}$ is estimated by:

$$\begin{pmatrix} \hat{\beta}_{0WLS} \\ \hat{\beta}_{1WLS} \end{pmatrix} = \frac{1}{S_{xxWLS}} \begin{pmatrix} \sum \sqrt{w_i} (y_i - \bar{y}) \\ S_{xyWLS} \end{pmatrix} \tag{21}$$

Where $S_{xxWlS} = \sum w_i (x_i - \bar{x})^2$;
$S_{xyWLS} = \sum w_i (x_i - \bar{x})(y_i - \bar{y})$.

The null and alternative hypothesis for testing the estimators $\hat{\beta}_{0WLS}$ and $\hat{\beta}_{1WLS}$ are the same as OLS. We refer to the the equations to compute the corresponding test statistics t_{WLS}, the confidence levels and goodness of fit R^2_{WLS} to the equations of (13), (14), (15), (16), (17) and (18) by incorporating with w_i.

4 Results of Part I

The results and conclusions of Part I are demonstrated in this section. With the aim of acquiring stable results, we focus on the relationship between market volatility and average pairwise correlations of the stocks. Then there are 499,

[2]WLS repesents Weighted Least Squares in the rest of our article

199, 66 and 49 observations for the four time intervals respectively. We first measure the pairwise correlations of stocks by Pearson moment correlation. Then we categorize average pairwise correlations into two groups according to the market volatility. If the market volatility is higher than 30%(including), the corresponding pairwise correlations are categorized as Group 1. On the other hand when the market volatility is lower than 30%, they are categorized as Group 2. Welch's t test, a two samples t test, is employed to inspect the equality of means between Group 1 and Group 2. Then the association between average pairwise Pearson correlations and market volatility is measured by Pearson moment correlation and Spearman's rank correlation respectively. The corresponding correlation coefficient tests are employed to examine whether the coefficient equals zero. Furthermore, a linear regression model in which the market volatility is independent variable and Pearson moment correlations is the response variable is set up to quantify the relationship. The slope parameter β_1 is estimated by Ordinary Least Squares (OLS) first and then by Weighted Least Squares (WLS) to remove the heteroscedasticity of linear model. Thereafter, we also implement Spearman's rank correlation to measure pairwise correlation of stocks to create a linear model which is similar to the case of Pearson moment correlation. OLS and WLS are also applied to estimate the parameters. Based on the comparison of the two kinds of correlations, several similarities and differences between them are presented as well.

4.1 Pearson Moment Correlation

We first investigate the relationship between the market volatility and average pairwise Pearson correlations of the five stocks. The plots of dynamics for market volatility and average pairwise correlations of stocks are first drawn to illustrate the association by Figure 2.

As shown by Figure 2, it is reasonable to see that the market volatility fluctuates intensely in the case of 10 days. The market volatility is the closest to market reality compared with the other time intervals. The corresponding Person's correlations vary drastically as well. When the market volatility peaks, the corresponding correlation usually achieves the highest level at the same time. Then it is obvious to find that high pairwise correlations of stocks are accompanied by

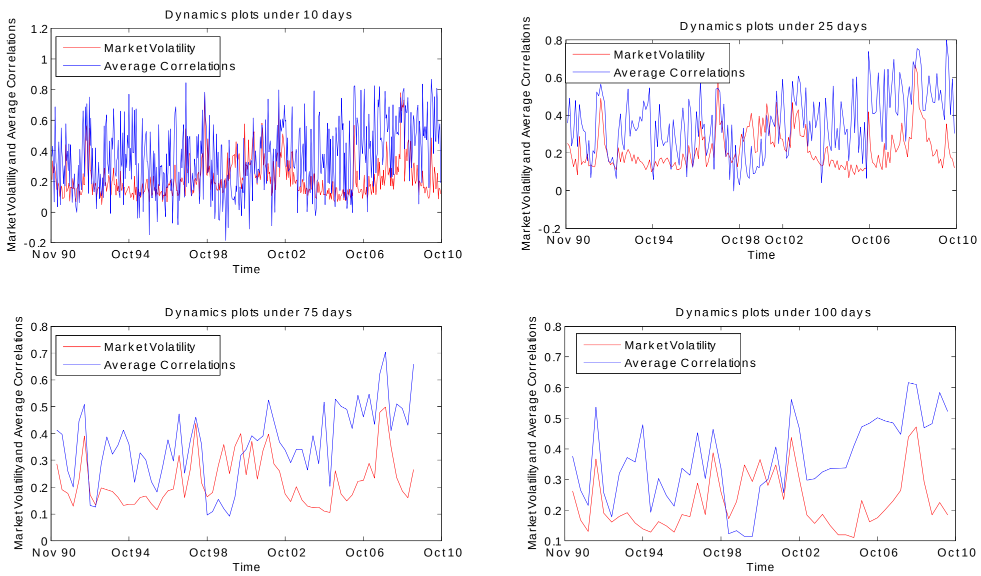

Figure 2: The dynamics for the four types of time intervals(10 days, 25 days, 75 days and 100 days) of market volatility and the corresponding average pairwise Pearson moment correlations of the five stocks.

high market volatility, especially when it exceeds 60%, whereas they remain at the relatively low level when market volatility is low. We also observe that the longer length of time interval, the more smooth and stable the market volatility is. Next, Group 1 and Group 2 are categorized according to the market volatility. If market volatility is higher than 30%(including), the corresponding pairwise correlations are categorized as Group 1. On the other hand when market volatility is lower than 30%, they are categorized as Group 2. Given significant $\tau = 0.05$, Welch's t right tail test is employed to examine whether the mean of average pairwise correlations in Group 1 is significantly higher than Group 2. The null and alternatives hypothesis are set up as follows:

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_1 : \mu_1 - \mu_2 > 0$$

where μ_1 and μ_2 are the means of average pairwise correlations for Group 1 and Group 2 respectively. The corresponding test statistics t_W are computed by equation (6) and the freedom degrees V_W by equation (7). The Welch's t test table for average pairwise Pearson correlations are listed by Table 4.

Table 4: *Welch's t test table for average pairwise Pearson correlations*

Time Interval	Groups	Sample	Mean	Variance	V_W	t_W	P_{t_W}
10 Days	1	91	0.4978	0.0554	121.7935	6.3656	0
	2	408	0.3281	0.0415			
25 Days	1	34	0.4577	0.0453	40.4693	3.0954	0.0018
	2	165	0.3388	0.0239			
75 Days	1	12	0.4241	0.0283	14.0595	1.5401	0.0729
	2	53	0.3445	0.0166			
100 Days	1	9	0.4503	0.0272	9.9262	1.8448	0.0475
	2	40	0.3431	0.0140			

As reported by Table 4, the comparisons of the means and variances of average pairwise correlations based on different days confirm the results observed in Figure 2. There are two other evaluations as well. First, P_{t_W} that represents the probability of that greater than t_W are lower than the significant level $\tau = 0.05$ for the four time intervals. Then the test statistics reject null hypothesis of means' equality and accept that the mean of average pairwise correlations in Group 1 is significantly higher than Group 2. We conclude that pairwise correlation of the five stocks in times of high market volatility is significantly different from it in times of low market volatility. It confirms the conclusion suggested by Boyer, Gibson and Loretan (1999). Second, The variances of Group 1 are larger than them in Group 2 for all the four time intervals as well. The average pairwise Pearson correlations tend to be unstable in high market volatility. It demonstrates that the pairwise correlations increases even if their relationship is weak in normal time. So the market volatility truly influences the pairwise correlations and their relationship is positive.

Furthermore, the Pearson's moment correlation is to measure the strength of the relationship. Pearson moment correlations coefficients are computed by equation (4). The test statistics t_{pmcc} mentioned by equation (5) is employed to examine whether it is positive. Then null and alternative hypothesis of right tail test are brought out:

$$H_0 : \rho_{pmcc} = 0$$
$$H_1 : \rho_{pmcc} > 0$$

where ρ_{pmcc} is the Pearson moment correlation coefficient. Given significant level $\tau = 0.05$, the critical values of two tail t_{pmcc} test are provided by Dudzic

(2007). Moreover, the association is also measured by Spearman's rank correlation. Then its coefficient is calculated by equation (6) and the corresponding Z test values by equation (7) are presented as well. The null and alternative hypothesis are same as the Pearson correlation coefficient test.

Table 5: Correlation Coefficients tests for all days

Time Interval	Sample	$\hat{\rho}_{pmcc}$	t_{pmcc}	P_t	$\hat{\rho}_s$	Z	P_Z
10 Days	499	0.4500	10.0301	0	0.4574	10.2082	0
25 Days	199	0.4376	6.3130	0	0.4228	5.9493	
75 Days	65	0.4164	3.5669	0.0003	0.4017	3.2140	0.0007
100 Days	49	0.4022	3.0798	0.0017	0.3299	2.2856	0.0111

As reported by Table 5, the P_t for all the four time intervals are lower than significant level $\tau = 0.05$. we reject the null hypothesis that $\rho_{pmcc} = 0$ and conclude that the relationship between the market volatility and average pairwise Pearson correlations of the five stocks is significantly positive for all the four time intervals. It is consistent with the results obtained from Spearman's rank correlation coefficient test. Besides, both the coefficients of Pearson and Spearman's correlations are around 0.4. According to the Table 3, the strength of them is low and it increases as the length of time interval decreases from 100 days to 10 days.

4.1.1 Linear Regression by Ordinary Least Squares (OLS)

Since the relatively independent time intervals are adopted to compute the market volatility, the dependence between its observations is relatively weak. Then it satisfies the assumption of linear regression model that the variable is independent. According to the tests of Pearson and Spearman's rank correlations, there exists significantly positive relationship between them. Therefore, we set up a simple linear regression model to quantify the positive relationship in which the market volatility is independent variable and corresponding Pearson correlation is the response variable. The slope parameter β_1 of the model represents the strength of the market volatility's influence on the pairwise correlation. So the standard error of its estimator, t tests and 95% confidence intervals of it are our mean concerns in this part. Together with the regression fitting lines and 95% confidence level estimated by OLS, the scatter plots of average pairwise correlations coefficients of stocks against market volatility for the 10 days, 25 days, 75 days and 100 days respectively are first drawn by Figure 3.

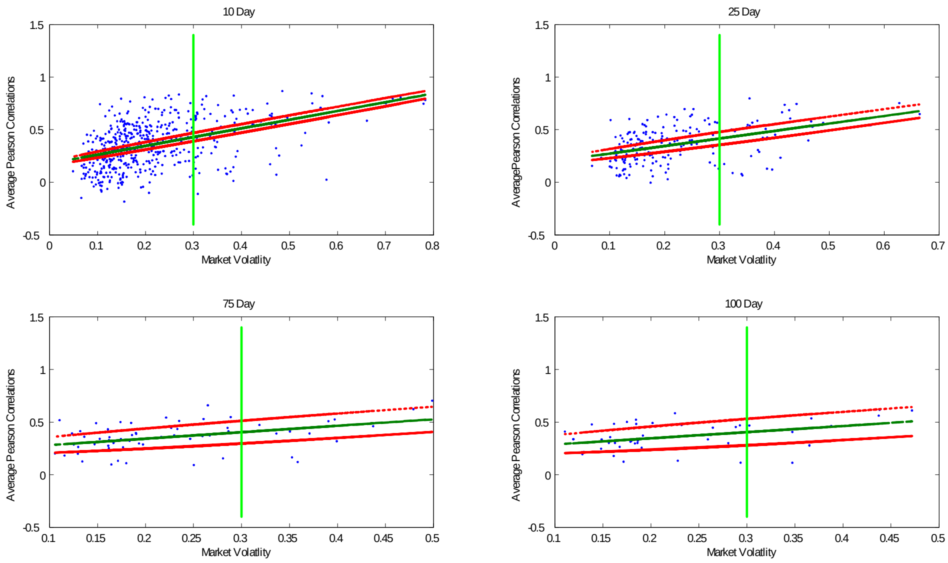

Figure 3: Scatter plots of average pairwise Pearson correlations against market volatility for the four time intervals. The slash green line in each plots represents the regression fit line where market volatility is the independent variable and corresponding Person correlation is response variable. The two red slash lines situate at both sides of green line in each plot. They are upper bound and lower bound of 95% confidence interval respectively for the fitted line. The vertical green line represents that market volatility is 30%.

The positive linear relationship is much more obviously illustrated by Figure 3. In the case of Pearson correlation, the slope parameter is denoted by β_{11}. The slope of regression line tends to steep as the time interval decreases from 100 days to 10 days. If market volatility is higher than 30%, the linear regression model fit the data much better than those less than 30% in cases of 10 and 25 days. The similar results are emerged from the cases of 75 and 100 days. But the differences between the higher and less than 30% are relatively slight. Then only a small amount of points are covered by the 95% confidence intervals of the fitted regression model in cases of all the four time intervals. We also observe that the length of confidence interval is longest in the case of 100 days and shortest in the case of 10 days. Moreover, we expect the slope is positive and larger than zero.

Hence, the unbiased and consistent estimators of the regression models for intercept $\hat{\beta}_{01}$ and slope $\hat{\beta}_{11}$ are estimated by OLS for the four time intervals. Given significant level $\tau = 0.05$, the null and alternative hypothesis are set up to examine whether the intercept β_{01} and slope β_{11} are larger than zero:

$$H_0 : \beta_{01} = 0 \; or \; \beta_{11} = 0$$
$$H_1 : \beta_{01} > 0 \; or \; \beta_{11} > 0$$

Their estimations and 95% confidence intervals for them are given by Table 6. The corresponding t_{OLS} statistics with freedom degrees $n-1$ computed by equation (13) are presented as well.

Table 6: The OLS estimations and test table for Pearson correlation

Time Interval		Estimate	std.	t_{OLS}	P	2.5%	97.5%	R^2_{OLS}	Adj. R^2_{OLS}
10 Days	$\hat{\beta}_{01}$	0.1810	0.0181	9.991	0	0.1454	0.2166	0.1962	0.2025
	$\hat{\beta}_{11}$	0.8310	0.0738	11.234	0	0.6856	0.9763		
25 Days	$\hat{\beta}_{01}$	0.2030	0.0253	8.010	0	0.1530	0.2529	0.1915	0.1874
	$\hat{\beta}_{11}$	0.7147	0.1046	6.831	0	0.5083	0.9210		
75 Days	$\hat{\beta}_{01}$	0.2214	0.0410	5.393	0	0.1394	0.3035	0.1734	0.1603
	$\hat{\beta}_{11}$	0.6115	0.1682	3.635	0	0.2753	0.9476		
100 Days	$\hat{\beta}_{01}$	0.2308	0.0472	4.887	0	0.1358	0.3258	0.1618	0.1439
	$\hat{\beta}_{11}$	0.5832	0.1936	3.012	0	0.1936	0.9728		

As illustrated by Table 6, several remarks are brought out. We reject the null hypothesis that the estimates equal 0 for the all time intervals and accept that both of the $\hat{\beta}_{01}$ and $\hat{\beta}_{11}$ are positive and larger than 0. Then the slope parameter $\hat{\beta}_{11}$ decreases as the time interval increases from 10 days (0.8301) to 100 days (0.5832), whereas, the standard deviation of it increases from 0.0738 to 0.1936. As a result, 95% confidence interval of $\hat{\beta}_{11}$ is the shortest in the case of 10 days. It also indicates that the maker volatility in a short time interval have larger influence on the pairwise Pearson correlations than those in a long time interval. The results we observed by Figure 3 can reconfirm it. Moreover, the goodness of fit of the linear regressions is measured by R^2_{OLS} and Adjusted R^2_{OLS}. Since the linear regression model have large estimation errors when market volatility is lower than 30%, it is reasonable to observe that the R^2_{OLS} and Adjusted R^2_{OLS} for all the four time intervals are lower than or close to 20%. The low R^2_{OLS} and Adjusted R^2_{OLS} probably indicate that some of other factors have neglected by the linear regression model. So, about 80% of variation of Pearson correlations are not interpreted by the market volatility. It is not suitable for us to apply the model to forecast the new observations by market volatility.

4.1.2 Linear Regression by Weighted Least Squares (WLS)

According to Welch's t test, the homoscedasticity assumption on average pairwise Pearson correlations is not met. Then there exists positive relationship between the Pearson correlations and the error terms obtained by OLS. As a consequence of this, the misleading results might derived by the underestimated or overestimated test statistics t_{OLS}. We turn to employ Weighted Least Squares (WLS) to deal with heteroscedasticity. The estimators of intercept $\hat{\beta}_{01WLS}$ and slope $\hat{\beta}_{11WLS}$ and their corresponding tests are presented by Table 7[3].

Table 7: The WLS estimations and test table for Pearson correlation

Days		Estimate	std.	t_{WLS}	P	2.5%	97.5%	R^2_{WLS}	Ad. R^2_{WLS}
10 Days	$\hat{\beta}_{01WLS}$	0.1815	0.0017	103.9	0	0.1780	0.1849	0.9760	0.9760
	$\hat{\beta}_{11WLS}$	0.8301	0.0058	142.2	0	0.8186	0.8416		
25 Days	$\hat{\beta}_{01WLS}$	0.2047	0.0035	58.66	0	0.1978	0.2116	0.8758	0.8751
	$\hat{\beta}_{11WLS}$	0.7041	0.0189	37.26	0	0.6668	0.7413		
75 Days	$\hat{\beta}_{01WLS}$	0.2207	0.0110	20.10	0	0.1987	0.2426	0.6834	0.6784
	$\hat{\beta}_{11WLS}$	0.6099	0.0523	11.66	0	0.5053	0.7143		
100 Days	$\hat{\beta}_{01WLS}$	0.2198	0.0071	30.84	0	0.2053	0.2340	0.8891	0.8867
	$\hat{\beta}_{11WLS}$	0.6244	0.0321	19.41	0	0.5596	0.689		

Compared with Table 6, the results reported by Table 7 indicate that WLS improve the OLS a lot. Although we both reject the null hypothesis by OLS and WLS for all the time intervals, the test statistics t_{OLS} computed by OLS are underestimated. We also observe that the slope estimators $\hat{\beta}_{11WLS}$ are slightly different from $\hat{\beta}_{11}$, but their corresponding standard deviations reduce strikingly by WLS. It is evidently illustrated by the 95% confidence intervals that the much more stable estimator $\hat{\beta}_{11WLS}$ are obtained. For instance, in the case of 10 days, it is [0.6856, 0.9763] for $\hat{\beta}_{11}$ while [0.8186, 0.8416] for $\hat{\beta}_{11WLS}$. And then the goodness of fit in terms of R^2_{WLS} and Adjusted R^2_{WLS} improve noticeably by WLS. For instance, 97.60% variation of Pearson correlations are explained by market volatility, whereas it is just 20.25% by OLS in the case of 10 days. So according to the stable estimator $\hat{\beta}_{11WLS}$ and high goodness of fit, it is sufficient for us to apply the linear model by WLS to predict the average correlations by market volatility.

[3]The null and alternative hypothesis are the same as those in Table 6

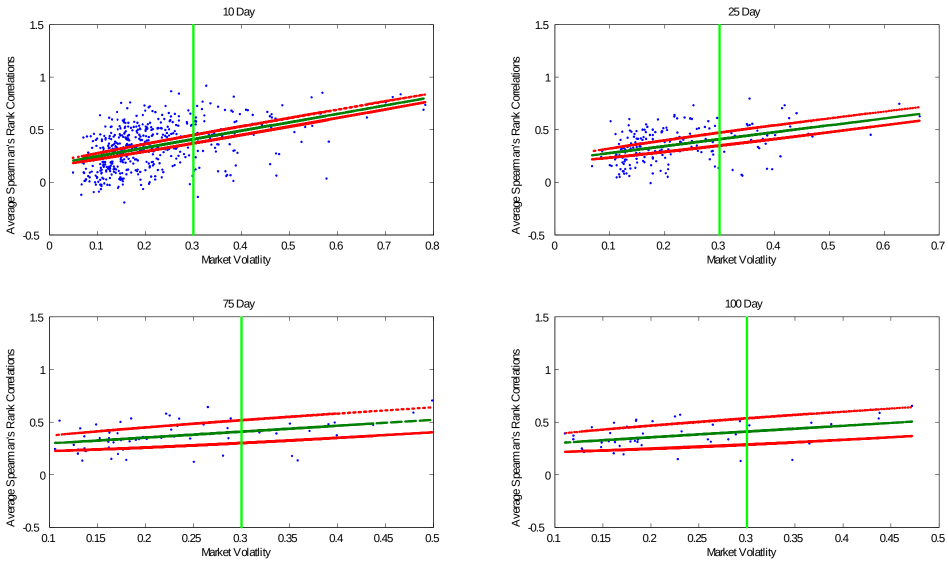

Figure 4: Scatter plots of average pairwise Spearman's Rank correlations against market volatility for the four time intervals. The slash green line in each plots represents the regression fit line where market volatility is the independent variable and corresponding Person correlation is response variable. The two red slash lines in each plot are upper bound and lower bound of 95% confidence interval. The vertical green line represents that market volatility is 30%.

4.2 Spearman's Rank Correlation

In this part, we turn to adopt Spearman's rank correlation to measure the pairwise correlations of stocks. The way to compute the market volatility is the same as in section 3.1. The four different time intervals of 10, 25, 75 and 100 days are also implemented for comparison. The linear regression model estimated by OLS is first applied to the average pairwise correlations against the market volatility. The slope of it denoted by β_{12} represents the strength of market volatility's influence on the pairwise Spearman's rank correlation of stocks. The scatter plots and corresponding fitting line with their 95% confidence interval by OLS are illustrated by Figure 4.

Since the coefficient of Spearman's rank correlation is close to that of Pearson moment correlation, similar results are illustrated by Figure 4 in comparison with Figure 3. The slope of linear model decreases as the length of time interval increases from 10 days to 100 days which is consistent with the case of Pearson correlation. Moreover, the slope in the case of 100 days is almost parallel to the horizontal line. The statistical significant of slopes will be examined as well. Then the corresponding results of linear regression model for Spearman's rank correlation are presented by Table 8[4].

Table 8: The OLS estimations and test table for Spearman's rank correlation

Day		Estimate	std.	t_{OLS}	$P_{t_{OLS}}$	2.5%	97.5%	R^2_{OLS}	Ad. R^2_{OLS}
10 Days	$\hat{\beta}_{02}$	0.1687	0.0176	9.613	0	0.1342	0.2032	0.2032	0.2016
	$\hat{\beta}_{12}$	0.8066	0.0717	11.257	0	0.6657	0.9473		
25 Days	$\hat{\beta}_{02}$	0.2129	0.0246	8.669	0	0.1645	0.3198	0.1769,	0.1727
	$\hat{\beta}_{12}$	0.6598	0.1014	6.507	0	0.2426	0.8789		
75 Days	$\hat{\beta}_{02}$	0.2421	0.0389	6.230	0	0.1645	0.3198	0.1645	0.1512
	$\hat{\beta}_{12}$	0.5607	0.1592	3.522	0	0.2426	0.8789		
100 Day	$\hat{\beta}_{02}$	0.2474	0.0442	5.588	0.0008	0.1583	0.3365	0.1616	0.1438
	$\hat{\beta}_{12}$	0.5466	0.1816	3.010	0.0041	0.1813	0.9118		

As reported by Table 8, the tests statistics t_{OLS} reject the null hypothesis for the four time intervals and we suggest that the slope estimator $\hat{\beta}_{12}$ is significantly larger than zero. The estimate of $\hat{\beta}_{12}$ is slightly lower than $\hat{\beta}_{11}$ in the cases of all time intervals compared with Table 6. It is interesting to observe that the 95% confidence intervals of $\hat{\beta}_{02}$ and $\hat{\beta}_{12}$ are exactly the same in cases of 25 and 75 days, although their estimates and standard deviances are different from each other. Besides, the R^2 and Adjusted R^2 are all under or close to 20% as well. Correspondingly, the parameters estimated by WLS are reported by Table 9.

Table 9: The WLS estimations and test table for Spearman's rank correlation

Day		Estimate	std.	t_{WLS}	$P_{t_{WLS}}$	2.5%	97.5%	R^2_{WLS}	Adj. R^2_{WLS}
10 Days	$\hat{\beta}_{02WLS}$	0.1679	0.0012	139.7	0	0.1656	0.1703	0.9541	0.954
	$\hat{\beta}_{12WLS}$	0.8106	0.0080	101.6	0	0.7949	0.8262		
25 Days	$\hat{\beta}_{02WLS}$	0.2047	0.0035	58.66	0	0.1978	0.2116	0.8758	0.8751
	$\hat{\beta}_{12WLS}$	0.7041	0.0189	37.26	0	0.6668	0.7414		
75 Days	$\hat{\beta}_{02WLS}$	0.2401	0.0038	63.86	0	0.2326	0.2477	0.92	0.9187
	$\hat{\beta}_{12WLS}$	0.5637	0.0210	26.91	0	0.5218	0.6055		
100 Days	$\hat{\beta}_{02WLS}$	0.2431	0.0041	58.83	0	0.2348	0.2514	0.9459	0.9448
	$\hat{\beta}_{12WLS}$	0.5713	0.0199	28.68	0	0.5312	0.6113		

The linear regression model estimated by WLS makes an enormous improvement compared with OLS given by Table 8 in the case of Spearman's rank correlation. Although the slope $\hat{\beta}_{12WLS}$ slightly differ from the corresponding $\hat{\beta}_{12}$ presented by Table 8 in the cases of all the four time intervals, $\hat{\beta}_{12WLS}$ are more

[4]The null hypothesis and t test statistics are similar to Table 6

stable and with unusual smaller standard deviation than $\hat{\beta}_{12}$. They all reject the null hypothesis and the test statistics t_{OLS} by OLS are underestimated as well. Moreover, in terms of R^2_{WLS} and Adjusted R^2_{WLS}, the linear model fit the data well by WLS. Based on comparison of Table 8 and Table 9, they confirms the conclusion arrived in the case of Pearson moment correlation.

Apart from this, we turn to compare the results in Table 7 with those in Table 9. The slopes $\hat{\beta}_{11WLS}$ are slightly larger than $\hat{\beta}_{12WLS}$ for all the four time intervals. It indicates that the influence of market volatility on Pearson correlations is higher than it on Spearman's rank correlations. Then since the R^2_{WLS} and Adjusted R^2_{WLS} are all higher than 90% except the case of 25 days for Spearman's rank correlation, the goodness of fit of this case performs better than Pearson correlations on a whole. So it is more suitable to investigate the relationship between market volatility and correlation of stocks which is measure by Spearman's rank correlation in terms of goodness of fit.

4.3 Conclusions

According to Welch's t test, the average pairwise Pearson correlations of the five stocks in times of high market volatility are significantly higher than those in times of low market volatility. The positive relationship between the average pairwise Pearson correlation of the five stocks and market volatility is verified by the Pearson moment and Spearman's rank correlation coefficient tests. Furthermore, we set up a linear regression model to quantify the market volatility's relationship with Pearson moment correlation and its relationship with Spearman's rank correlations respectively. We focus on the estimation and evaluation of slope parameters which represents the strength of market volatility's influence on the pairwise correlations. Then several similarities and differences based on the comparisons of the two cases are presented as follows. The slopes of linear regression model estimated by OLS reject the null hypothesis that it equals zero and conclude that it is significantly larger than zero. According to the comparisons of the results obtained from dividing the whole period by four different types of time intervals, 10, 25, 75 and 100 days respectively, the longer time interval, the more smooth the slopes are. Then we observe that the strength of the market volatility's influence on the correlations of stocks denoted by β_1 is less than 1. It tends to decrease as the time interval increases, whereas its standard deviation increases. Since the assumption of the homoscedasticity on error terms obtained by OLS is not met, then WLS is employed to remove the effects of heteroscedasticity. Compared with OLS, the estimation of parameter by WLS improve the linear model apparently in terms of goodness of fit. More stable and slightly smooth slopes are estimated by WLS as well. In regarding to the differences, the slope of Spearman's rank correlation is more smooth than Pearson correlation for all the four time intervals. Moreover, the linear model fit the data better in Spearman's correlation than Pearson moment correlation on a whole. So it is more suitable to reflect the relationship between market volatility and pairwise correlation of stocks which is measured by Spearman's rank correlation in terms of goodness of fit.

Part II
The Comparison of Performances of Portfolio Managers

Swedish mutual fund industry have gained more and more public interest in recent years. According to a survey commissioned by Swedish Investment Funds Association during February and March 2010,

"*Fund-based savings are by far the most popular savings format in Sweden. The percentage of people saving in funds over or above their fund-based premium pension savings has increased to 82 per cent in 2010 from 74 per cent in 2008. If premium pensions are included, 99 per cent of Swedes save in funds*".

So the question that how funds' performances vary during turbulent periods and stable periods is an important issue for investors to know. In Part II, two issues are brought out to deal with this question. First, how the funds perform in times of high market volatility compared with those in times of low market volatility? Second, what are the differences between the performances of the funds both in times of high and low market volatility? Then the funds which invest in Swedish markets are our main concerns. Since the funds usually replicate a benchmark index as their objective, their correlations with the market is close to 1 and they are well diversified. Then the requirement of diversification mentioned in section 1.3 is satisfied. So we turn to focus on the other requirement, the excess return over the market to illustrate their performances. Then two groups of them such as Group 1(market volatility higher than 30%) and Group 2(market volatility is less than 30%) which are similar to those in Part I are generated in Part II. Hence, in Part II, we will evaluate the performances of 69 funds where OMXS30 Return Index is proxy for the market. Then four portfolios namely Average Portfolio, T5, M5 and B5[5] which represent the average performances of all those 69 funds, the top 5 funds, the median 5 funds and the bottom 5 funds respectively are set up for further comparisons. Due to the obvious and typical results are obtained from the four portfolios, we foucs on the intercomparisons of them both in times of high and low market volatility. Therefore, we implement Welch's t test[6] to testify whether means of Group 1 for 69 funds and the four portfolios are higher than those of Group 2. Then the first issue is addressed. Moreover, the differences of reactions of the four portfolios in different market times conducted by dependent t test[7] are illustrated. Thus, the second issue is answered as well.

5 Funds and Methodology

5.1 Funds and Benchmark Information

In Part II, we have a data set consisting of 69 funds which all invest in Swedish market. Their weekly returns are obtained from the period from 31 Deceber 2005 to 20 September 2010. Then each fund contains 248 observations. The funds rebalance their portfolio allocations according to the market and they usually take an market index as their benchmark. For instance, Nordea Sverigefond choose the SIX Portfolio Return Index as its benchmark. So it is important to find out a suitable index proxy for the market, otherwise it might lead us to reach incorrect results and conclusions. We observe that the returns of OMXS30 Return Index and 69 funds are highly correlated in the entire period. It is appropriate to be proxy for the market. The high correlation which is close to 1 indicates that these funds are well diversified. Then the corresponding market volatility is computed

[5] See the details in section 6
[6] See the details in section 5.3.1
[7] See the details in section 5.3.2

by the standard deviation of 5 weeks history of OMS30 Return Index. As a result, 244 observations for each fund are left for investigation. Moreover, the risk free interest rate is used to adjusted the excess return over the market. We choose the 7-day STIBOR[8] as proxy for it. The same time span for the weekly average data of STIBOR is obtained from the Swedish Central Bank, Riksbank. Then the Average Portfolio is set up. There are two reasons to propose it. The first is that the stable returns which are crucial to compare them with market returns are obtained from it. The second reason is as a consequent that the portfolio allocations and trading strategy of the 69 funds are unknown.

Definition 1 ***Average Portfolio:*** *It is consisting of 69 funds with the same weights. The allocation for all the funds stays constantly in the entire period.*

Thus, the basic statistics for the returns of market, Average Portfolio and means of 69 funds are first given by Table 10.

Table 10: Basic Statistics for the reurns of market and Average Portfolio and means of funds

	Market Returns	Average Portfolio	Means of 69 Funds	Std. of 69 funds
Length	244	244	69	69
Min.	-20.1722%	-17.5466%	0.0857%	0.0282
Max	13.0689%	10.5969%	0.2263%	0.0389
Mean	0.1956%	0.1721%		
25% Quantile	-1.6789%	-1.4082%	0.1605%	0.0340
Median	0.4135%	0.2723%	0.1755%	0.0344
75% Quantile	2.1931%	1.9982%	0.1875%	0.0361
Std.	0.0358	0.0340		

As reported by Table 10, the mean of market returns is 0.1956% with standard deviation 0.0358. Then the Average portfolio can not outperform the market in terms of mean which is 17.21%. It confirms the results suggested by Malkiel (1995) that the funds managers can not outperform the market on a whole. The standard deviation of it is 0.0340 which is more stable compared with the market. Based on the quantiles of means and standard deviations of 69 funds, 50% of means of them are ranging from 16.05% to 18.75% and 50% of standard deviation are from 0.0340 to 0.0361. Only 11 funds outperform the market in terms of mean. At the mean time, the means of corresponding risk free rate is 0.0475%. In order to clearly show the comparison of these funds and market, the scatter plot

[8] Stockholm Interbank Offered Rate

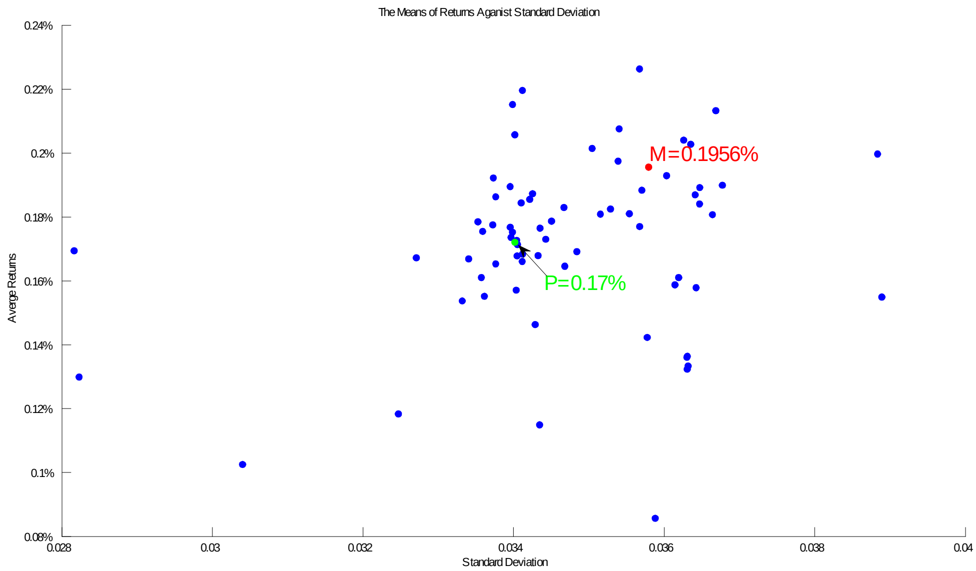

Figure 5: The means of weekly returns of the 69 funds against their corresponding standard deviations, Average portfolio and the market. The red "M" and green "P" represent the market and Average Portfolio respectively, the other blue points are 69 funds.

of means of returns of the 69 funds against with their standard deviation is drawn by Figure 5. The returns of Average Portfolio and market and their corresponding standard deviations are given as well.

As shown by Figure 5, there is a roughly positive relationship between the means and their standard deviations. If investors want to receive higher return, they usually need to take higher risk which is measured by standard deviation. However, the high risk does not necessarily lead to high return. For instance, the mean of Alfred Berg Sverige Plus is just 0.0857% and its standard deviation is 0.0359 which is higher than the market and Average Portfolio. And then Average Portfolio is the average performance of 69 funds both in terms of mean and standard deviation. It clearly reveals the fluctuation of these funds on a whole when market volatility is high. Thus, the dynamics for returns of Average Portfolio and the market are illustrated by Figure 6. The annualized 5 weeks history of market volatility of OMXS 30 Return index is drawn as well.

As shown by Figure 6, the returns of Average Portfolio denoted by dash line usually tracked the returns of the market. So the Pearson moment correlation coefficient between them is 0.9739 in the entire period. However, the returns of

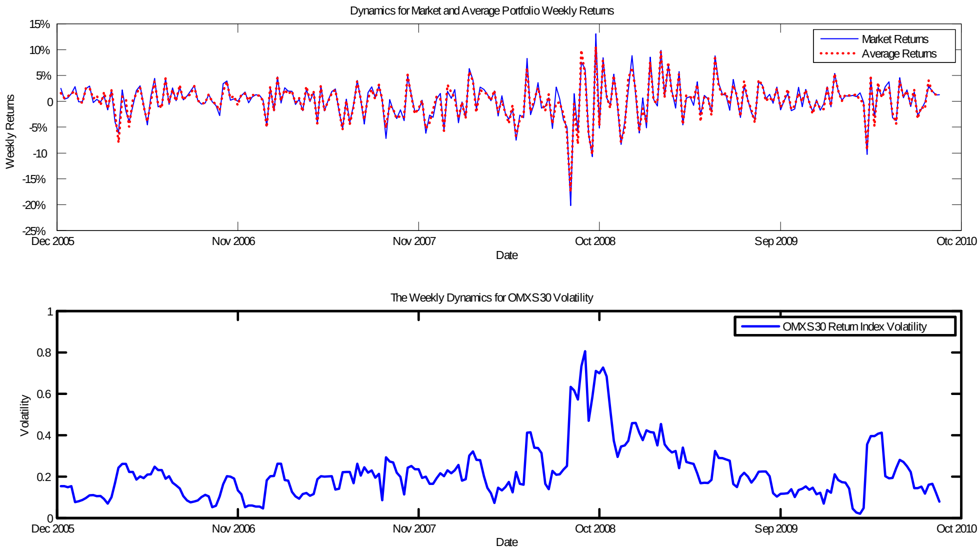

Figure 6: The dynamics for the returns of the market and Average Portfolio and the corresponding market volatility. In the upper plot, the dash line represents Average Portfolio while the solid line is the market.

Average Portfolio vary stably compared with the market. They usually resisted from drooping to the lower level than the market except in a few times. It is confirmed by the results reported by Table 10. Moreover, the corresponding relationships between the market volatility and two types of returns are obviously demonstrated by Figure 6. When the market volatility of OMXS30 Return index achieved the peaks or dropped to valley floor in a short time, the returns of market and Average Portfolio responded accordingly. During the turbulent period from September 2008 to the October 2009 where volatility was extremely high, both the market and Average Portfolio experienced the smallest and largest return separately. While the proportion of volatility lower than 30%(including) is 82.38%, the volatility higher than 30% is 17.62%. The preliminary results given above show that the high market volatility has influences on the performance of these funds. We will discuss it with more details in the section 6.

5.2 Excess Return of Fund

In Part II, two different measurements are adopted to measure the excess return over the market. The first one denoted by α is computed by

$$\alpha = r_p - r_M \tag{22}$$

where r_p is the returns of portfolio and r_M is the corresponding returns of market. In Part II, since there are 244 weekly returns for each fund and the market, a series of α consisting of 244 observations is obtained from computing by equation (22). The α is the regular way to measure the excess return over the market. However, it does not consider the risk exposure of the portfolios. Thus, a risk adjusted measure, the so called Jensen's alpha which is one of the widely used measures in previous studies is presented as below.

Since Jensen's alpha is obtained from CAPM, then a simple introduction is given. The Capital Asset Pricing Model (CAPM) is developed by Jack Treynor (1961), and it is also independently proposed by William Sharp (1964) and Lintner (1965). Two types of portfolio's risk such as systematic and unsystematic are decomposed by CAPM. Systematic risk denoted by β represents the market risk of portfolio and it can not be eliminated by diversifying. On the other hand, unsystematic risk is related to the individual stock risk which is allowed to remove by increasing the number of securities. The CAPM assumes that there is no unsystematic risk if investor enters into the market portfolio. According to CAPM, the expected excess return of a stock or portfolio over the risk free rate which is adjusted by systematic risk is equal to the market risk premium under the market equilibrium conditions. The equation (23) is given as:

$$E\left(r_i\right) - r_f = \beta_i \left(E\left(r_m\right) - r_f\right) \tag{23}$$

where $E\left(r_i\right)$ is expected return of portfolio i;

$E\left(r_m\right)$ is expected return market;

r_f is the given risk-free interest rate;

$E\left(r_m\right) - r_f$ is market premium;

and β_i is the systematic risk of portfolio i which incorporates the correlation between market and is computed by:

$$\beta_i = \frac{cov\left(r_i, r_m\right)}{var\left(r_m\right)} \tag{24}$$

where $var\left(r_m\right)$ is the variance of market returns.

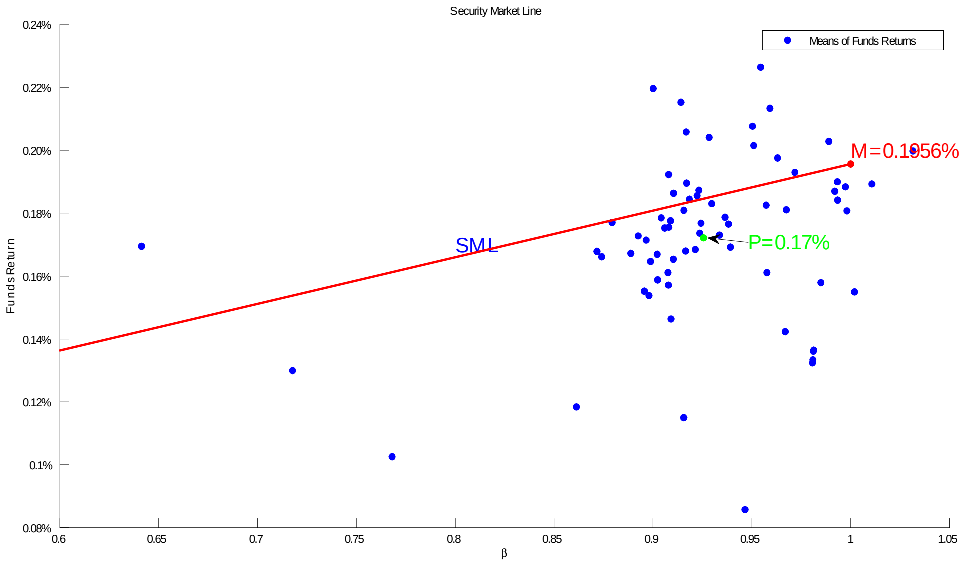

Figure 7: Security Market Line for 69 funds. The red and green points represent the market and Average Portfolio respectively.

Then the Security Market line(SML) based on CAPM is introduced here. We rewrite the formula of CAPM to get SML:

$$E\left(r_i\right) = r_f + \beta_i \left(E\left(r_m\right) - r_f\right) \tag{25}$$

The relationship between β_i and required return is plotted by SML. The x-axis is the systematic risk β_i, and the y-axis is the expected return. The slope of it is determined by the market risk premium $E\left(r_m\right) - r_f$. The intercept is the given risk-free interest rate. If the portfolio's return versus risk is above the SML, the asset price is undervalued since it yields a higher return for a given risk amount, whereas if the underlying asset's return versus risk is under the SML, the asset price is overvalued since it yields a lower return for a given risk amount. Then the SML for the 69 funds and Average Portfolio is drawn by Figure 7. Here, the y-axis is the means of returns for 69 funds and average portfolio.

The β for each fund is computed by equation (24) in the entire period. The larger β for a portfolio means the higher risk it exposes and it usually require higher expected return. However, the portfolio with high β does not always derive higher returns in reality. There is only 1 fund fell over the the security market line. 18 of 68 remaining funds stay above the line and 40 of them stay below it.

The $\beta_p = 0.9256$ represents the systematical risk of Average Portfolio and it is under the line. So, given the amount of risk assumed, it is overvalued and the investor would to accept lower return. The basic statistics of β for 69 funds are reported by Table 14.

Furthermore, one way to compute the Jensen's alpha in the entire period is to estimate the intercept from a linear regression model. The excess return of a portfolio over risk-free interest rate is the response variable and market risk premium is independent variable. The model assumes that the portfolio is well diversified. The SML is regarded as the benchmark. Jensen's alpha measures the excess return of underlying asset over the market required by portfolio, whereas CAPM suggests that there is no excess return of underlying asset over the market. It is viewed as the risk adjustments, and then riskier underlying assets are expected to achieve higher returns.

Then Jensen's alpha for a portfolio is obtained by a regression model given below:

$$\left(r_p - r_f\right) = J_\alpha + \beta\left(r_m - r_f\right) + \varepsilon_p \tag{26}$$

where r_p is the return of the portfolio;

r_f is risk-free interest rate

J_α is the Jensen's alpha which measure the performance of the portfolio in the entire period and $\hat{J}_\alpha$ is its estimator.

β is the systematic risk of the portfolio which is consistent with it computed by equation (24);

r_m is return of the market portfolio;

and ε_p is the random error term.

In Part II, the $\hat{J}_\alpha$ estimated by linear regression model (26) is to measure the performance of an underlying asset in the entire period and it is used to rank the 69 funds. If the Jensen's α is positive, we claim that the portfolio manager outperforms the market index during the specific period. The larger value of α, the higher return we expect. When the alpha is negative, the performance of the portfolio manager is inferior to the market.

Levy and Sarnat 1984 suggest that the average of error term ε_p is always zero. So the error term is removed by taking mean on both sides. Therefore, in Part II another way to compute Jensen's α for a single period is given:

$$\text{Jensen's } \alpha = \left(r_p - r_f\right) - \beta\left(r_m - r_f\right) \tag{27}$$

where Jensen's α consists of 244 observations which is the same as α computed by equation (22).

In section 6, according to results obtained from the empirical data, the mean of Jensen's α is very close to $\hat{J}_\alpha$ for each fund. Then in Part II, we first estimate the $\hat{J}_\alpha$ in the entire period and 69 funds are ranked according to it. And then α and Jensen's α are computed by equation (22) and (27) respectively for all the funds and the four portfolios as well.

5.3 Tests

The key assumption for the tests in this part is that the mean of excess return remains a constant in a specific period. Two samples of Welch's t test and dependent samples t test are described below.

5.3.1 Welch's t test

The Welch's t test is employed to examine whether the performance of portfolio managers differ significantly in times of high market volatility and the times of low market volatility. We refer to the details in the section 3.3. Since the autocorrelations exists in the series of weekly returns, it might affect the test statistics, the significant level $\tau = 0.1$ is adopted in Part II. We also assume that there are two populations with μ_1 and μ_2 which represent the mean of Group 1 when market volatility is higher than 30%, and mean of Group 2 when it is lower than 30% respectively. Then the corresponding two samples x_1 and x_2 with sample sizes n_1 and n_2 respectively are observed. Their means are $\bar{x}_1$ and $\bar{x}_2$. Hence the null hypothesis is given:

$$H_0 : \mu_1 - \mu_2 = 0$$

The equation (8) is used to compute the Welch's t statistics t_W. The freedom degree V is computed by equation (9) as well.

Since the difference of sample means of Group 1 and Group 2 might be positive or negative, we construct different alternative hypothesis according to it.

If the sample differences of Group 1 and Group 2 is positive $\bar{x}_1 - \bar{x}_2 > 0$, then $t_W > 0$ and we apply the right tail test:

$$H_1 : \mu_1 - \mu_2 > 0$$

If the difference is negative $\bar{x}_1 - \bar{x}_2 < 0$, then $t_W < 0$, we turn to implement the left tail test.

$$H_1 : \mu_1 - \mu_2 < 0$$

If the test statistics t_W is above(below) the critical value given by the significant level $\tau = 0.1$, we conclude that it rejects the null hypothesis and the mean of Group 1 is significantly larger(smaller) than Group 2. The probabilities that larger than $t_{W(V_W)}$ for right tail and lower than it for left tail denoted by P_{t_W} are presented as well. If it can not reject null hypothesis, we would evaluate the managers performance in terms of excess returns or its standard deviation.

5.3.2 Dependent Samples t test

If the two samples with equal sample size are highly correlated, we incorporate their correlation when the difference of their means is under tested. Since the market and the four portfolios namely Average Portfolio, T5, M5 and B5 are highly correlated, dependent samples t test is employed to intercompare the means of the four portfolios both in times of high and low market volatility separately. Therefore, the differences of reactions in different times of market volatility for the four portfolios are illustrated via intercomparisons. We assume that there are two populations with true expected means μ_1 and μ_2 which represent the means of two portfolios in the four portfolios for Group 1(Group 2) respectively. Then the corresponding two samples x_1 and x_2 with the same sample sizes N for Group 1(Group 2) are observed. Their means are $\bar{x}_1$ and $\bar{x}_2$. Then the null and alternative hypothesis are similar to those described in section 5.3.1. Given the significant level $\tau = 0.1$, the null hypothesis is set up:

$$H_0 : D_0 = 0$$

where $D_0 = \mu_1 - \mu_2$ is the difference of the means of two population.

Hence the t_D statistics:

$$t_D = \frac{\bar{D} - D_0}{\sqrt{\frac{S_{1,2}^2}{N-1}}} \tag{28}$$

where $\bar{D} = \bar{x}_1 - \bar{x}_2$ and $S_{1,2}^2 = S_1^2 + S_2^2 - 2\rho_{pmcc}S_1S_2$;

S_1^2 and S_2^2 are the sample variances of x_1 and x_2 respectively;

N is sample size.

ρ_{pmcc} is the Pearson correlation between the two samples x_1 and x_2.

The freedom degrees is $N-1$ in this case.

Similar to Welch's t test mentioned above, the two cases of alternative hypothesis are implemented here as well. If the difference of sample means is positive $\bar{x}_1 - \bar{x}_2 > 0$, then $t_D > 0$ and we choose the right tail test:

$$H_1 : D_0 > 0$$

If the difference of sample means is negative $\bar{x}_1 - \bar{x}_2 < 0$, then $t_D > 0$ we turn to the left tail test.

$$H_1 : D_0 < 0$$

If the test statistics t_D is above(below) the critical value given by the significant level $\tau = 0.1$, we conclude that it rejects the null hypothesis and the means of one portfolio is significantly larger(smaller) than another portfolio. The probabilities that larger than $t_{D(N-1)}$ for right tail and lower than it for left tail denoted by P_{t_D} are computed as well.

6 Results of Part II

6.1 Funds Ranks

We first rank the 69 funds by $\hat{J}_\alpha$ which is estimated by linear regression model (26) in the entire period. In order to get typical results which are convenient for inter-comparisons, three portfolios namely T5, M5 and B5 respectively are constructed. Their constituents are with same weights and proxy for the average performances of top 5 funds, median 5 funds and bottom 5 funds. The constituents of T5, M5 and B5 are listed by Table 12.

Table 12: *The constituents of T5 M5 and B5 portfolios with* $\hat{J}_{\alpha}$

Rank	Portfolios	Funds	$\hat{J}_{\alpha}$
1		Carlson Sverige Koncis	0.0388%
2		Enter Sverige Pro	0.0375%
3	T5	Carnegie Sverigefond	0.0327%
4		Handelsbanken AstraZeneca Allemans	0.0268%
5		Danske Invest Sverige	0.0273%
33		SPP Aktieindexfond Sverige	-0.0076%
34		SSgA Sweden Index Equity Fund P	-0.0080%
35	M5	Eldsjäl Gåvofond Inc.	-0.0088%
36		Banco Ideell Miljö	-0.0090%
37		Swedbank Robur Ethica Miljö Sverige	-0.0098%
65		Eldsjäl Biståndsfond	-0.5876%
66		Banco Samaritfonden	-0.5944%
67	B5	Humanfonden	-0.6035%
68		Nordea-1 Swedish Equity BP	-0.0682%
69		Alfred Berg Sverige Plus	-0.1020%

As illustrated by Table 12, the $\hat{J}_{\alpha}$ for first rank is 0.0388%, whereas for the bottom rank is -0.1020%. The difference between them is 0.1408%. The $\hat{J}_{\alpha}$ of funds for both M5 and B5 are negative. Then they failed to achieve the positive excess return as T5 did in the entire period. Hence, the basic statistics of returns for T5, M5 and B5 are presented by Table 13.

Table 13: *Basic Statistics for market and four portfolios and means of funds reurns*

	T5	M5	B5
Length	244	244	244
Min.	-17.7420%	-17.8934%	-13.5428%
Max	10.2593%	8.4436%	10.5969%
Mean	0.2088%	0.1772%	0.0763%
25% Quantile	-1.4476%	-1.4817%	-1.0266%
Median	0.5165%	0.3672%	0.2452%
75% Quantile	2.0797%	1.5181%	1.9982%
Std	0.0324	0.0342	0.0276

As reported by Table 13, it is reasonable to find that the mean of T5 is the highest and M5 is the median and B5 is the lowest. Compared with that of T5 and M5, the standard deviation of B5 is the smallest and it indicates that its returns are relatively stable.

6.2 The Means of α and Jensen's α

Then both the two ways of weekly excess returns of the 69 funds over the market are first computed. So there are 244 observations for each of them. The basic sta-

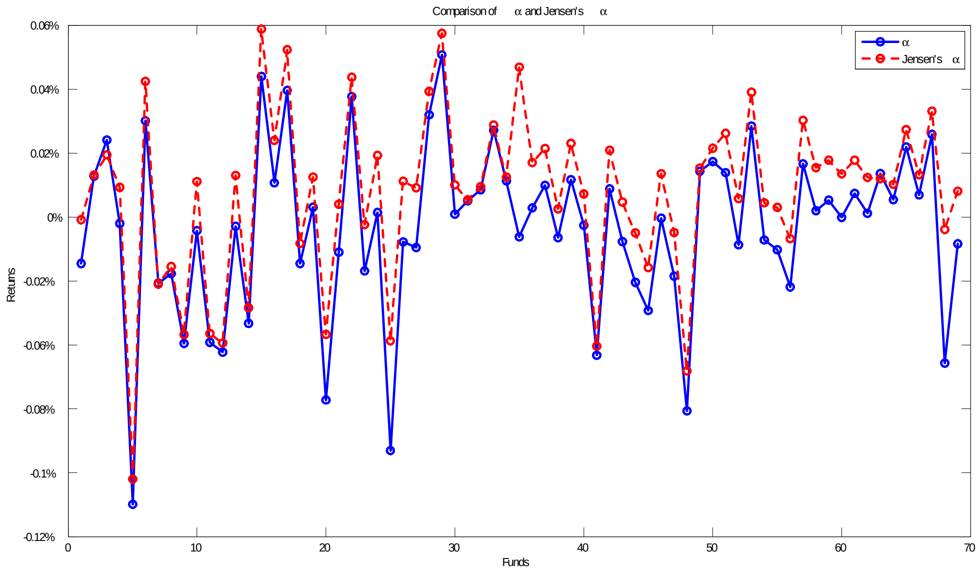

Figure 8: The Comparisons of means of α and Jensen's α for 69 funds

tistics for β and means of α and Jensen's α of the 69 funds computed by equations (24), (22) and (27) respectively are presented by Table 14.

Table 14: Basic Statistics for β and means of α and Jesnen's α of the 69 funds

	β	means of α	means of Jensen's α
Length	69	69	69
Min.	0.6414	-0.1099%	-0.1019%
Max	1.0315	0.0308%	0.0388%
25% Quantile	0.9055	-0.0351%	-0.0227%
Median	0.9224	-0.0201%	-0.0088%
75% Quantile	0.9640	-0.0080%	0.0010%

If β is 1 where the unsystematic risk is totally diversified, the term of risk-free interest r_f of Jensen's α in equation (27) is removed and Jensen's α equals α. Since the β values of the 69 funds are lower than 1 except 3 funds, it indicates that their returns vary relatively less than the market does. Then risk adjusted performance measurement Jensen's α tends to be higher than α for most funds. As the Spearman's rank correlation of them is 0.95, if we rank these funds by α and Jensen's α, their orders are almost the same. The comparisons of means of α and Jensen's α of the 69 funds are more clearly illustrated by Figure 8.

As obviously indicated by Figure 8, the means of Jensen's α are frequently higher than alpha. It is consistent with the results reported by Table 14. Furthermore, together with Average Portfolio, the basic statistics for β, α and Jensen's α for T5, M5 and B5 portfolios are given by Table 15 as well.

Table 15: β and means of α and Jesnen's α for the four portfolios

Porfolios	β	mean of α	mean of Jensen's α
Average Portfolio	0.9256	-0.0235%	-0.0125%
T5	0.8739	0.0132%	0.0319%
M5	0.9342	-0.0183%	-0.0086%
B5	0.7291	-0.1193%	-0.0792%

The excess return of T5 measured by α and Jensen's α are both positive. So the average performances of first 5 funds is superior to the market in terms them in the entire period, whereas, the other three portfolios, Average Portfolio, M5 and B5 are inferior to the market since the means of α and Jensen's α are negative.

6.3 Welch's t test

The first issue that how the funds perform in times of high market volatility compared with those in times of low market volatility is illustrated in this part.

As shown by Figure 6, the returns of average portfolio change considerably when market volatility is high. The volatility essentially influence the expected returns of the 69 funds. Group 1 and Group 2 are categorized accordingly for 69 funds and the four portfolios of Average portfolio, T5, M5 and B5. There are 43 observations when volatility is higher than 30% and 201 for volatility lower than 30%. Then the basic statistics for Group 1 and Group 2 of means of α and Jensen's α for the 69 funds are illustrated by Table 16.

Table 16: The Basic Statistics for two Groups of means of α and Jesnen's α for the 69 funds

	means of α		means of Jensen's α	
Groups	Group 1	Group 2	Group 1	Group 2
min	-0.0037%	-0.0009%	-0.3713%	-0.0818%
max	0.0020%	0.0003%	0.2832%	0.0377%
25% Quantile	-0.0012%	-0.0002%	-0.0847%	-0.0118%
Median	-0.0006%	-0.0001%	-0.0333%	-0.0033%
75% Quantile	-0.0003%	0.0001%	0.0007%	0.0097%

It is reasonable to observe that the minimum of the 69 funds in Group 1 are lower than those in Group 2 both in terms of α and Jensen's α, while the maximum of them are in Group 1 higher than those in Group 2. The further analysis is to testify whether the excess returns measured by α and Jensen's α are significantly different in times high and low market volatility. Then Welch's t test is employed

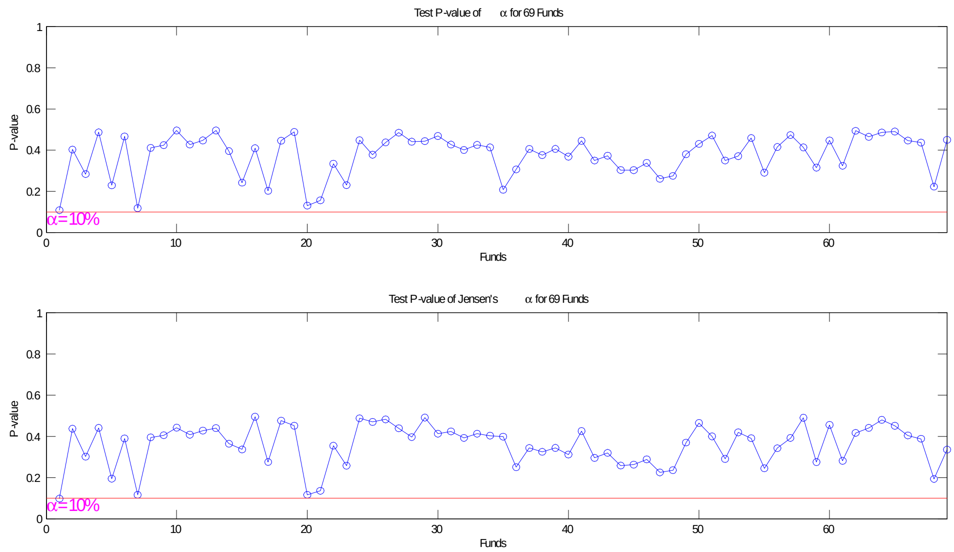

Figure 9: The Welch's t test's P_{t_W} of α and Jensen's α for the 69 funds. The upper plot is the case of α and the lower one is the case of Jensen's α. The horizontal lines in both of them represent significant level $\tau = 0.1$.

to illustrate it. We first conduct the Welch's t test on both α and Jensen's α for the 69 funds. Given the significant level $\tau = 0.1$, the corresponding test statistics t_W and freedom degrees V_W are computed by equations (8) and (9) respectively. The corresponding null and alternative hypothesis are demonstrated in section 5.3.1. The probabilities that larger than $t_{W(V_W)}$ for right tail and lower than it for left tail denoted by P_{t_W} for α and Jensen's α are presented by figure 9.

The P_{t_W} derived for α are similar to those for Jensen's α which is shown by Figure 9. Only the fund of Enter Sverige of P_{t_W} is smaller than significant level $\tau = 0.1$. It rejects the null hypothesis and we conclude that the mean of excess returns are lower in Group 1 than it in Group 2. Then the $t_{W(V_W)} = -1.3096$ and -1.02970 for α and Jensen's α respectively are negative, the left tail test is implemented here. The mean and standard deviation of it for Group 1 are -0.2372% and 0.0118 in terms of α. The corresponding of them for Group 2 are 0.0089% and 0.0084. The similar results are obtained by Jensen's α. So we claim that the managers of Enter Sverige perform poorly in times of high market volatility. Moreover, regarding to the remaining 68 funds, the P_{t_W} are higher than significant level $\tau = 0.1$. We can not reject the null hypothesis and accept that the means of excess returns stay constantly in times of high and low market volatility. The high market volatility does not cause these portfolio managers perform differently. However, based on the means and variances analysis of them both in terms of α and Jensen's α, we observe that only the means of Group 1 for 11 funds are higher than those in Group 2. And all the funds except SEB Ethical Sweden D have larger standard deviations in Group 1 than those in Group 2. So the high market volatility leads to high standard deviation of excess returns and does not necessarily result in high excess return. If we just focus on the magnitude of excess return, only 11 funds can handle the high market volatility and receive relatively higher returns than those in times of low market volatility.

Furthermore, with the aim of obtaining more stable results, the α and Jensen's α for the four portfolios are also examined by Welch's t test. The test statistics table for α and Jensen's α respectively are given by Table 17[9].

Table 17: *Welch's t test table for the four portfolios*

Portfolios		Group	Mean	Std.	df	t_w	P_w
Average Portfolio	α	1	-0.0816%	0.0122	48.1375	-0.3660	0.3580
		2	-0.0110%	0.0070			
	Jensen's α	1	-0.0396%	0.0111	48.9102	-0.1868	0.4263
		2	-0.0066%	0.0068			
T5	α	1	0.0929%	0.0146	47.8737	0.4204	0.3380
		2	-0.0039%	0.0082			
	Jensen's α	1	0.1640%	0.0123	48.8740	0.7294	0.2346
		2	0.0036%	0.0075			
M5	α	1	-0.0348%	0.0116	47.1502	-0.1098	0.4565
		2	-0.0148%	0.0061			
	Jensen's α	1	0.0023%	0.0105	47.1955	-0.1030	0.4592
		2	-0.0109%	0.0060			
B5	α	1	-0.1625%	0.0135	49.2717	-0.4557	0.3253
		2	-0.0645%	0.0085			
	Jensen's α	1	-0.1165%	0.0128	49.5501	-0.2801	0.3903
		2	-0.0597%	0.0081			

We first focus on the results reported by Table 17 in terms of α. In regrading to Average Portfolio, the mean of it in Group 1(-0.0816%) is smaller than Group 2 (-0.0110%). The similar results are emerged from the portfolios of M5 and B5. So the t_w are negative and left tail test is applied in the three portfolios. However, in the case of T5, the mean of α of it in Group 1 (0.0929%) is higher than Group 2 (-0.0039%) where t_w is positive and it is right tail test. The standard deviations

[9] Where the df represents the freedom degrees for Welch's t test

for the four portfolios are all larger in Group 1 than Group 2. The means of T5 in both Group 1 and Group 2 are higher than the other portfolios, whereas its standard deviations are the highest in Group 1. The lowest excess returns are received by B5 both in Group 1 and Group 2. And then the results derived from Jensen's α in Table 17 are consistent with those from α. Moreover, the values of P_{t_W} in Table 17 are larger than the given significant level $\tau = 0.1$ and it indicates that we can not reject the null hypothesis. By statistically, the performances of these four portfolios do not perform differently between in times of high and low market volatility. The two different alternative hypothesis are also implemented according to section 5.3.1. However, considering the magnitude of excess return, only the T5 handle the high market volatility well while the other three portfolios perform poorly.

6.4 Intercomparison of the Four Portfolios

In this part, the second issue that what are the differences between the performances of the funds both in times of high and low market volatility is presented. First, the Pearson moment correlations between the four Portfolios computed by equation (4) are presented by Table 18.

Table 18: The Pearson correlations between the four portfolios

	Average Portfolio	T5	M5	B5
Average Portfolio	1.0000	0.9922	0.9970	0.9956
T5	0.9922	1.0000	0.9866	0.9857
M5	0.9970	0.9866	1.0000	0.9927
B5	0.9956	0.9857	0.9927	1.0000

As clearly reported by Table 18, all the four portfolios are highly and positively correlated. Then we conduct the intercomparisons of the four portfolios that Average portfolio, T5, M5 and B5 both in times of high and low market volatility via dependent t test which incorporates their correlations. The test statistics t_D are computed by equation (28). Since the increases of Type I error caused by adopting the multivariate t test, we turn to use the multiple comparisons of the four portfolios. Then there are 6 pairs of comparisons in terms of α and Jensen's α. The null and alter hypothesis are following the rule presented in section 5.3.2. We first intercompare the means of Group 2 that is in times of low market volatility for the four portfolios. Given the significant level $\tau = 0.1$, the corresponding test results are illustrated by Table 19[10]. The T5 & M5 in Table 19 represents that the numerator of test statistic t_D is the mean of excess return of T5 minus the mean of excess return of M5. Then it is applied to all the intercomparisons.

Table 19: The Dependent t test table for portfolios intercomparisons of Group 2

Pairs of Comparison	Measurement	t_D	P_D
T5 & M5	α	0.2986	0.3828
	Jensen's α	0.4323	0.3330
T5 & B5	α	1.7124	0.0442*
	Jensen's α	1.8243	0.0348*
T5 & Average Portfolio	α	0.2627	0.3965
	Jensen's α	0.4125	0.3402
M5 & B5	α	1.7990	0.0368*
	Jensen's α	1.8118	0.0358*
M5 & Average Portfolio	α	-0.2295	0.4094
	Jensen's α	-0.2648	0.3957
B5 & Average Portfolio	α	-2.4998	0.0066*
	Jensen's α	-2.4925	0.0067*

As reported by Table 19, according to P_D, T5 significantly outperform the B5 both in terms of α and Jensen's α, whereas it performs as good as M5 and Average Portfolio in stable market. M5 is superior to B5 and performs no difference with T5 and Average Portfolio. Then the performance of B5 is significantly inferior to the other portfolios. We also pay attention to the sign of t_D for all the 6 pairs of comparisons. The mean of excess return in Group 2 for T5 are all higher than those for all the other portfolios, while it for M5 is higher than B5 and lower than Average Portfolios. And then for B5 it is lower than the other three portfolios. The results in terms of α are the same as those in terms of Jensen's α. Then with the aim of obviously illustrating their reactions, we present the intercomparisons of the means of Group 1 that is in times of high market volatility for the four portfolios. Then the test results for Group 1 are reported by Table 20 as well.

[10]The * represnts that we reject the null hypothesis.

Table 20: The Dependent t test table for portfolios intercomparisons of Group 1

Pairs of Comparison	Measurement	t_D	P_D
T5 & M5	α	1.0662	0.1462
	Jensen's α	1.4346	0.0794*
T5 & B5	α	1.8861	0.0331*
	Jensen's α	2.2299	0.0156*
T5 & Average Portfolio	α	1.7598	0.0429*
	Jensen's α	2.2922	0.0135*
M5 & B5	α	1.6298	0.0553*
	Jensen's α	1.4620	0.0756*
M5 & Average Portfolio	α	0.8120	0.2107
	Jensen's α	0.7187	0.2382
B5 & Average Portfolio	α	-1.3256	0.0961*
	Jensen's α	-1.2363	0.1116

Both in terms of α and Jensen's α, T5 significantly performs better than B5 and Average Portfolio when market volatility is high, while M5 is superior to B5 and performs as good as the average portfolio. Moreover, T5 is superior to M5 in terms of Jensen's α, whereas there is no significant difference between them in terms of α. B5 is inferior to the Average Portfolio in terms of α and no difference for Jensen's α. We turn to concern the sign of t_D for all the 6 pairs of comparisons in Group 1 again. The mean of excess return of T5 are higher than those of all the other portfolios as well, while M5 is higher than B5 and Average Portfolios. And then that of B5 is lower than all the other portfolios. The results based on α are consistent with those derived from Jensen's α.

Furthermore, several conclusions based on the comparison of Table 19 in times of low market volatility and Table 20 in times high market volatility respectively are described as follows.

1. T5 performs better than B5 and as good as M5 and Average Portfolio in times of low market volatility, whereas it outperforms all the other portfolios in terms of Jensen's α in times of high market volatility. The average performance of T5 measured by Jensen's α indicates that they handle the high market volatility well and their excess returns are significantly higher than the other portfolios. By excepting a slight difference, the comparisons of T5 with the other portfolios brought out in terms of α are consistent with thoes in Jensen's α. The difference is that in times of high market volatility, the performance of T5 is superior to B5 and Average Portfolio and there is no difference with M5 in terms of α.

2. No matter in times of high and low market volatility, M5 outperforms the B5 and it performs as good as Average Portfolio both in terms of α and Jensen's α respectively. Whereas we observe that t_D for M5 & Average Portfolio is negative in Group 1, it is positive in Group 2. The mean of excess return yielded by M5 is higher than Average Portfolio in times of high market volatility. It indicates that the Average Portfolio is influenced by the high market volatility, whereas M5 can handle it by certain degrees in terms of magnitude of excess return.

3. If the market volatility is lower than 30%, B5 is inferior to the other portfolios both in terms of α and Jensen's α.The similar results are illustrated in terms of α in times of high market volatility except that the M5 and Average Portfolios perform no significant difference with it. Moreover, if we focus on the magnitude of t_D, then that of Group 1 for M5 & B5 and Average Portfolios & B5 are larger than those of Group 2 both in terms of α and Jensen's α. So in times of high market volatility, some improvements of the performance of B5 relative to M5 and Average Portfolio are observed.

6.5 Conclusions

In Part II, Average Portfolio, M5 and B5 can not outperform the market, whereas only T5 performs better than the market in terms of weekly returns. In regarding to the first issue, according to Welch's t test conducted on the 69 funds and the four portfolios respectively, only one fund rejects the null hypothesis and we conclude that the manager of it can not handle the high market volatility. We also observed that the standard variations of most funds and the four portfolios are larger in times of market volatility than those in times of low market volatility. However, if we just focus on the magnitude of the excess return, the means of only 11 funds and T5 in Group 1 are higher than those in Group 2. Then we also conclude that the performances of remaining 58 funds and the other three portfolios in Group 1 are inferior to those in Group 2. It is harder for them to handle the high market volatility and perform well when market volatility is high. Furthermore, in regarding to the second issue, we focus on intercomparisons of the four portfolios derived from the dependent t test. The differences of their performances both in times of high and low market volatility are presented as follows. In times of high market volatility, T5 is superior to the other three portfolios while M5 performs better than B5. Then in times of low market volatility, B5 is inferior to the other three portfolios. Besides, in the other cases of intercomparisons, no significant difference in them is observed. Moreover, if we turn to only concern the magnitude of t_D, by compared with Average Portfolio, M5 can handle the market volatility by certain degrees. And then some improvements of the performance of B5 relative to M5 and Average Portfolio are observed in times of high market volatility. Based on results given by both the first and second issue, T5 performs relatively well in times of high market volatility in comparison with that in times of market volatility and it is also superior to the other three portfolios.

Part III
Conclusion and Discussion

In Part I, we provide the evidence that there exists positive relationship between the market volatility and pairwise correlations of stocks. High market volatility usually leads to high correlations of stocks and the outliers of them are usually derived by extremely high market volatility. By quantifying the relationship as a linear model, we observe that the strength of the market volatility's influence on the correlations of stocks is less than 1 and tends to decrease as the time interval increases. The correlations of stocks also have effects on the market volatility. The conclusions revealed above are crucial to portfolio managers. Suppose there is a portfolio comprising the five stocks mentioned in Part I. In the stable market, the allocations for them are optimized to receive a relatively high portfolio return. If a market shock happens and the market volatility is high, it induces that the pairwise correlations of the stocks in portfolio increase accordingly. The allocations might not work any more. Thus, with the aim of ensuring the portfolio returns remain stable, the pairwise correlations of stocks are to be considered to rebalance the portfolio's allocation.

In part II, we investigate the reactions of funds in times of high market volatility and compare them during turbulent and stable periods by two types of tests which incorporate the mean and standard deviation of the two performance measurements. Then, four portfolios namely, Average Portfolio, T5, M5 and B5, are set up for comparison. Based on comparisons of these 69 funds and the four portfolios both in times of high and low market volatility, we arrive at two conclusions. First, if the magnitude of excess returns is concerned, along with the 58 funds, Average Portfolio, M5 and B5 do not perform well as the remaining 11 funds and T5 do in times of high market volatility in comparison with those in times of low market volatility. Second, T5 are superior to the other three portfolios in times of high market volatility and M5 performs better than B5, whereas T5 performs no significant difference with M5 and Average Portfolio in times of low market volatility. However, there are something needed further development and examination in future researches, for example, taking into consideration of size, fee structure, trading activity and net flows in these funds, as well as trading strategies or looking into their techniques of arranging the portfolios' allocations.

References

[1] Boyer, Brian H, Michael S Gibson and Mico Loretan (1999): "*Pitfalls in Tests for Changes in Correlations*". Federal Reserve Board, IFS Discussion Paper, No. 597R, March.

[2] Charles H. B., Corrinne P. B. (2008), "*Understanding Basic Statistics*", Cengage Learning, ISBN: 0547132492.

[3] Huafeng (Jason) Chen, Shaojun (Jenny) Chen and Feng Li(2010), "*Empirical Investigation of an Equity Pairs Trading Strategy*", Working Paper Series

[4] Jerrold H. Zar (1972), "*Significance Testing of the Spearman Rank Correlation Coefficient*", Journal of the American Statistical Association, Vol. 67, No. 339 pp. 578-580

[5] Jörg D. Wicharda Christian Merkwirthb and Maciej Ogorzaleka(2004), "*Detecting correlation in stock market*", Physica A 344 (2004) 308–311

[6] John C. Hull (2009), "*Options, Futures, and Other Derivatives*", Seven Edition, Pearson Eduction, Inc. ISBN-13:978-0-13-500994-9

[7] Joshua M. Pollet and Mungo Wilson(2008), "*Average Correlation and Stock Market Returns*", Working Paper Series

[8] Lintner, John (1965). "*The Valuation of Risk Assets and the Selection of Risky investments in Stock Portfolios and Capital Budgets*", Review of Economics and Statistics, 47, 13-37.

[9] Loannis A. Venetis, David Peel (2005), "*Non-linearity in stock index returns: the volatility and serial correlation relationship*", Economic Modelling 22 (2005) 1–19

[10] Mico Loretan and William B English (2000), "*Evaluating "correlation breakdowns" during periods of market volatility*", Working Paper Series

[11] Malkiel, B. G. (1995), "*Returns from Investing in Equity Mutual Funds 1971 to 1991*." Journal of Finance, 50 549-572.

[12] Satterthwaite, F. E. (1946), "*An Approximate Distribution of Estimates of Variance Components*.", Biometrics Bulletin 2: 110–114, doi:10.2307/3002019

[13] Stella Dudzic (2007), "*Companion to Advanced Mathematics & Statistics*", Hodder Murray, ISBN-13: 978-0340959237

[14] Sharpe, William F. (1964), "*Capital asset prices: A theory of market equilibrium under conditions of risk*", Journal of Finance, 19 (3), 425-442.

[15] Sharpe, William F (1966), "*Mutual fund performance*", Journal of business, January 1966

[16] SHARPE, WILLIAM F. (1964), "*CAPITAL ASSET PRICES: A THEORY OF MARKET EQUILIBRIUM UNDER CONDITIONS OF RISK*", Journal of Finance, Sep64, Vol. 19 Issue 3, p425-442, 18p

[17] Tobin, James (1958). "*Liquidity Preference as Behavior Towards Risk*", The Review of Economic Studies, 25, 65-86.

[18] Welch, B. L. (1947), "*The generalization of "student's" problem when several different population variances are involved*.", Biometrika 34, pp. 28–35

[19] Zhiwu Chen and Peter J. Knez (1996), "*Portfolio Performance Measurement: Theory and Applications*", The Review of Financial Studies Summer 1996 Vol. 9, No. 2, pp. 511–555