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Abstract

Using a newly defined notion of aversion to income risk, the behavior of the marketed-surplus producer under price risk is characterized. Unlike the familiar case first examined by Sandmo, output depends on both ordinal preferences for goods and on risk attitudes. Conditions are found that yield an output level under risk that is smaller than under certainty. If these conditions do not hold, both risk and risk-aversion may have a *positive* effect on output. Implications for econometric studies of risk attitudes are considered and illustrated with an example. Finally, we examine the effect of uncertainty on the peasant's long-run equilibrium.

Key words: marketed surplus, production under multivariate risk, peasant households.

Marketed Surplus Under Risk: Do Peasants Agree with Sandmo?

Agricultural economists have devoted considerable effort to studying production decisions by farmers in developing countries. Essential to this effort is a thorough understanding of the role of risk in peasants' production decisions. Important insights into this role of risk are provided by the literature on firms' behavior under risk. Perhaps the most influential paper in this area is by Sandmo, who established that a risk-averse firm facing output-price risk produces less output than a risk-neutral firm. Sandmo and the numerous studies that followed made use of the expected utility hypothesis, combined with an assumption that utility depends on only one random variable—the level of final wealth. Other risks affecting utility must be presumed absent or to have no effect on the behavior under study.

An important characteristic of the agricultural peasant household is the consumption of a portion of its own farm product (e.g. Haessel; Renkow; Toquero, Duff, Anden-Lacsina, and Hayami). Unlike the familiar case studied by Sandmo, when subject to output price risk, such a "marketed-surplus" household faces multivariate risk because the price of one of its consumption goods is also random.[1] Thus, the profits risk affecting wealth cannot be modeled in isolation from other risks. As a consequence, a risk-averse household may no longer choose to produce below the profit-maximizing level of output when the price is random.

The literature (*e.g.* Roe and Graham-Tomasi) has emphasized that maximization of utility defined on wealth alone is not adequate to describe choices under risk in agricultural households. Anderson, Dillon and Hardaker (p. 76) asserted that "money is not everything; and the consequences of many decision problems are....not well represented in terms of only a single attribute such as monetary gain or loss". Wolf (p. 2) observed that "peasants run a household, not a business concern." Similarly, Ellis (p. 102) proposed that "the interaction of consumption and production within the household causes a unique form of decision making which sets peasants apart from any other kind of production unit...". Restrictions can be found that imply separability between production and consumption decisions. However, Fabella, Pope, and Finkelshtain each offer examples showing that strong restrictions must be imposed on preferences, in two-period models or in cases of multiattribute objective functions. Therefore, one must take explicit account of the process by which profits produce utility, through household consumption decisions, in modeling peasants' production decisions.[2]

These aspects of household behavior are examined below, by combining Sandmo's model with the marketed-surplus literature, and utilizing an alternative to the univariate notion of aversion to risk. Following that, the implications for empirical studies of risk attitudes are explored and the long-run entry/exit decision of the household is described. Finally, we examine the potential for expanding the results to include additional sources of risk.

Peasant Behavior Under Price Risk

Virtually all studies of behavior under risk assume that the producer maximizes the expected value of a utility function defined only over income or final wealth. Aversion to risk in this single argument is measured by the curvature of the utility function—the Arrow-Pratt measure of risk-aversion. To model households facing risks in other arguments of the utility function requires a more general objective function and an alternative definition of aversion to risk. These tasks are taken up in this section.

The Household's Objective Function

We assume that the household is engaged in the production of a food crop (x). Consumption decisions concern leisure (l), a portion of the farm output (m), and an aggregate market good (z), representing the consumption of all other goods. Output and leisure are chosen prior to the realization of prices, while consumption goods are chosen in the harvest period, when prices are known. The prices of m and z are denoted by p and q and the wage rate is w.

We assume that the household makes consumption and production choices to maximize the expected value of a utility function U defined over z, m, and l:

$$\max_{m,z,x,l} E\,[U(z,m,l)]$$

subject to

$$pm + qz + wl = y(x),$$

where y denotes full income. Because the optimal consumption plans may be revised *ex post*, the *ex ante* decision involves only l and x. Substituting the *ex post* optimal plans for m and z into U leads to the variable indirect utility function $V(y, l, p, q)$. Epstein established its duality to U. Hence, the above problem is equivalent to

$$\max_{x \geq 0, l \geq 0} E[V(y, l, p, q)].$$

In the univariate case where only y is random, V can be reduced to the familiar objective function defined on income or final wealth alone. However, if goods' prices are unknown at the time output is chosen, even under flexibility of consumption choices, their joint distribution is likely to affect output.

Epstein showed that the variable indirect utility function is increasing in y and homogeneous of degree zero in y, p and q. Accordingly, the market good will represent a numeraire, so q is normalized to be 1 and other monetary variables are measured relative to q. The real wage rate and the prices of other inputs are assumed to be known when the production and labor supply decisions are made. Thus, we concentrate on risks in the relative price of the food crop and in real income, both of which depend on the random variable p.

Using the notion of full income, total income is the sum of initial wealth, labor income $(T - l)w$, and farm profits given by

$$\pi = p \cdot x - c(x).$$

T the is total time endowment of the household and c is the cost function. Any fixed costs are assumed to be contained in c. Output is assumed non-stochastic.[3] By the additional assumption that the input prices are known, $c(x)$ is deterministic.

An Alternative Risk Premium

Pratt's risk premium measures the maximum amount that an individual would pay to avoid income risk when nothing else is random. In terms of the current model, it is defined as $S_{\bar{p}}$

in

$$EV(l,y,\bar{p}) = V(l,\bar{y}-S_{\bar{p}},\bar{p}),$$

where the $\bar{p}$ subscript indicates that the price of m is fixed at its mean. If the price of the portion of output consumed at home remained random, with only income stabilized, Pratt's risk premium would not necessarily measure the willingness to pay for income insurance. The risk premium appropriate for such a situation must be defined as the maximum amount that the individual would pay to avoid income risk when p remains random:[4]

$$EV(l,y,p) = EV(l,\bar{y}-S,p).$$

The interpretation of the modified risk premium S is analogous to that of the regular Pratt risk premium in the traditional univariate model, which appears as a special case, when the consumption price p is fixed at its mean.

This can be illustrated for small risks by a Taylor approximation of the equation that defines S, a derivation of which is given in an appendix available upon request. Assume that the producer faces uncertainty about income and the price of one consumption good. For small risks, the risk premium is given by

$$S = -\frac{1}{2}\sigma_{yy}\frac{V_{yy}}{V_y} - \sigma_{yp}\frac{V_{yp}}{V_y},$$

where subscripts on V denote partial derivatives, σ_{yy} is the variance of total income, σ_{yp} is the covariance between total income and p, and the partial derivatives are evaluated at $(l,\bar{y},\bar{p})$. The first term in the above expression is Pratt's risk premium, the maximum amount that the producer is willing to pay to stabilize income when prices are fixed. The second term captures the value (or cost) associated with the stochastic interaction between the consumption price and his income. If this covariation did not affect utility (because $V_{yp}=0$, implying that V is additively separable) or did not exist (due to independence of y and p, or fixed p), the risk premium would reduce to the univariate measure.[5] Otherwise, S differs from the Pratt risk premium in magnitude and possibly in sign.

In the case of a marketed-surplus good, this expression can be simplified somewhat. If the price of output is the only random variable affecting profits and consumption, the above expression reduces to

$$S = -\sigma_{pp}\, x \left[\frac{1}{2} x \frac{V_{yy}}{V_y} + \frac{V_{yp}}{V_y}\right],$$

since the variance of income, σ_{yy}, is simply the variance of revenues from production, $x^2\sigma_{pp}$, where σ_{pp} denotes the variance of the output price, and the covariance of income and price, similarly, is $x\,\sigma_{pp}$. Since the sign of S is determined solely by the sign of the term in brackets, this expression suggests that it might be possible to find conditions on utility functions whose owners have a risk premium of a particular sign, regardless of the probability distribution of p. These new conditions are needed since $V_{yy} < 0$, the univariate condition, no longer is sufficient for $S > 0$. This is accomplished in Proposition 1, which illustrates the relationships between the sign of S and various parameters of the peasant's preferences, including the marketed surplus (the difference between x and m).

Proposition 1: Let η be the income elasticity of the household's demand for home-consumption (m) of the farm crop. A necessary condition for a positive risk premium S is

$$\eta > r\left[1 - \frac{1}{2\rho}\right],$$

where ρ is the ratio of home-consumption to total production (m/x) and r is the Arrow-Pratt measure of relative risk-aversion. A proof of the proposition is available from the authors upon request.

The expression derived for S using a Taylor series approximation suggests that, for small risks, this condition is both necessary and sufficient.[6] Also, if the household consumes less (more) than half of its output, a necessary condition for a positive (negative) risk premium is that the farm-produced good is a normal (inferior) one ($\eta >(<) 0$). If the household consumes exactly half of its output, a necessary condition for a positive risk premium is that the farm-produced good is a normal one ($\eta > 0$). Assuming that $r > 0$, as the proportion of output consumed at home increases, the lower bound on η necessary for $S > 0$ increases as well.

We turn now to the main results of the paper, concerning the level of output for the "marketed surplus" producer. To establish these results, the objective function and risk premium defined above are used.

Optimal Output in the Short-Run

The first-order conditions for the household's maximization problem are

$$(i) \quad E[V_l - V_y w] = 0$$

and

$$(ii) \quad E\{V_y[p - c'(x)]\} = 0.$$

Condition (i) requires that the optimal allocation of time equates the expected marginal utility from leisure and the expected marginal utility from the wage payment saved by substituting an additional unit of the owner's labor for hired labor. Condition (ii) states that the expected marginal utility from an additional unit of production vanishes. This condition will be used below to derive qualitative results regarding the level of production. The sufficient conditions for a local maximum are assumed to hold and are given by:

$$a_{11} \equiv E[V_{yy}(w)^2 - 2 \cdot V_{yl} w + V_{ll}] < 0$$

$$a_{22} \equiv E\{V_{yy}[p - c'(x)]^2 - c''(x) V_y\} < 0$$

$$a_{11} \cdot a_{22} - (a_{12})^2 > 0$$

where

$$a_{12} \equiv E\{V_{yy}[p - c'(x)]w - V_{yl}[p - c'(x)]\}.$$

The second necessary condition can be rewritten as

$$E[V_y p] = E[V_y c'(y)],$$

or

$$E[V_y(p - \bar{p})] = E\{V_y[c'(x) - \bar{p}]\}.$$

The left-hand side is the covariance between the marginal utility of income and the output price. In Sandmo's model, when the producer is risk-averse ($V_{yy} < 0$), this covariance is clearly negative. This is because V_y is decreasing in income and hence in p. This implies that the expected price of output exceeds marginal cost ($\bar{p} > c'(x)$). This observation leads to the conclusion that, in the classic case, output for risk-averse producers is strictly less than the expected profit-maximizing level.

However, in the case of a peasant producer, p affects V_y not only through the income argument (as captured by V_{yy}), but also through the cross derivative V_{yp}. As a result, $V_{yy} < 0$ is not sufficient to ensure Sandmo's result, without more information about the peasant's tastes, production technology, and the probability distribution of prices. Hence, the level of production may exceed the optimal level under certainty. Below we derive plausible conditions under which Sandmo's result is preserved and discuss the consequences of the failure of these conditions for eliciting risk-aversion or characterizing behavior. Proposition 2 establishes the conditions under which the level of output is adversely affected by risk.

Proposition 2: Let the optimal quantities produced under certainty and uncertainty be x^c and x^u, respectively. The relationship between x^c and x^u is given by

$$x^u \overset{<}{\underset{>}{-}} x^c \text{ for all risks } \iff \eta \overset{>}{\underset{<}{-}} r\left[1 - \frac{x^u}{m}\right].$$

Proof: The proof is based on the equivalence of statements (i)–(iv), which holds when conditions (i)–(iii) are required to hold for all price risks:

(i) $x^u \overset{>}{\underset{<}{-}} x^c$

(ii) $\bar{p} \overset{<}{\underset{>}{-}} c'(x)$

(iii) $Cov(V_y, p) \overset{>}{\underset{<}{-}} 0$

$$(iv)\quad \frac{dV_y}{dp} \gtrless 0.$$

Using Roy's identity, condition (iv) becomes

$$\frac{dV_y}{dp} \lessgtr 0 \iff \eta \gtrless r\left[\frac{s_m - \beta}{s_m}\right] \iff \eta \gtrless r\left[\frac{m - x^u}{m}\right] \iff \eta \gtrless r\left[1 - \frac{x^u}{m}\right]$$

where s_m denotes the share of the marketed-surplus good in total consumption expenditures and β denotes the share of the risky income in total wealth. □

The intuition behind the awkward looking expression is clearest using the following relationship involving the consumption parameters, derived by rearranging the first condition on η.

$$x^u \lessgtr x^c \iff r\,\beta \gtrless s_m\,(r - \eta) = s_m\, r\,(1 - \mu)$$

or

$$x^u \lessgtr x^c \iff \beta \gtrless s_m\,(1 - \mu),$$

where $\mu = \eta / r$. Besley made use of a "profit" function Φ for the consumer, to represent the expenditure required at any price vector to maintain a particular marginal utility of income. Among other things, this function satisfies

$$\frac{\partial \log \Phi}{\partial \log p} = s_m\,(1 - \mu)$$

which gives the percentage change in expenditures required to maintain marginal utility following a one percent change in p.

The left-hand side of the above inequality, β, has a similar interpretation—it is the percentage increase in wealth from a one percent increase in the price of output:

$$\frac{dy}{dp} \cdot \frac{p}{y} = x \cdot \frac{p}{y} = \beta.$$

Thus, uncertainty about p has no effect on the "marketed surplus" firm (*i.e.* $x^u = x^c$) when the change in wealth resulting from a price change equals the change in consumption expenditures needed to maintain a constant marginal utility of income:

$$\beta = s_m \,(1 - \mu).$$

Effectively, the producer is made risk-neutral with respect to price risk, since marginal utility of income is constant with respect to changes in p. When the wealth effect dominates, Sandmo's qualitative result is preserved. His result is reversed when the consumption effect dominates.

Proposition 2 also suggests that a crucial factor in the relationship between output levels under certainty and uncertainty is the degree of self-sufficiency of the farm—in other words, whether or not there is a marketed surplus. Empirical evidence regarding this parameter of peasants' behavior is provided by Ahmed and Bernard (Table 1, p. 17). They reported that 40% of the farms in Bangaladesh are deficit farms—net buyers of rice.[7] Only 30% of the rice farms in this country always have a positive marketed surplus.

Returning to the proposition, a sufficient condition for Sandmo's qualitative result to hold for the latter type of farms is that the good produced be a normal good ($\eta > 0$); the lower bound on η given in the Proposition will be negative, ensuring a Sandmo-like response for risk. On the other hand, for the other 40% of the farms that are net buyers of rice, a positive income elasticity becomes a necessary condition for Sandmo's result to hold while a negative one is sufficient for a reversal of that result.

If the producer does not consume any of his farm product ($s_m = 0$), the level of output is separable from preferences for goods and, just as in Sandmo's model, risk-aversion is sufficient for uncertainty about the price of output to affect production adversely. Alternatively, this last result is obtained if the variable indirect utility function is additively separable in income and the price p for consumption of m. However, this condition implies rather restrictive preferences, since it requires that $\eta = r > 0$.

Examination of Propositions 1 and 2 illustrates an important difference between the cases of univariate and multivariate risk. The condition for a positive risk premium, in the Sandmo framework, is identical to the one for a level of output that is less than under certainty. However, in the current framework, the condition that ensures a smaller level of output under uncertainty differs from the condition for a positive risk premium. Thus, a producer may prefer a stable income and at the same time, produce more than under certainty.

Figure 1 illustrates the above results. In each of the four graphs, the relationship between the optimal level of output (x) and the coefficient of variation of price (ψ) is given for three levels of aversion to risk, as measured by the Arrow-Pratt measure of relative risk-aversion ($r = 0, 1/2, 1$). The cost function is $1/2x^2$ and the mean price is assumed, for simplicity, to be 1. The four graphs combined show the effects of η, the income elasticity of demand for the marketed-surplus good, and ρ, the ratio of home consumption m to total output x.

In Figure 1a, the optimal level of output is shown to decrease with respect to risk for three levels of r. The Sandmo results hold, in the sense that increases in r or in the uncertainty about the output price (as measured by ψ) reduce the optimal output. However, note that output is affected by increasing risk even when the producer is risk-neutral. Such a producer is indifferent to the income risk associated with p but not the consumption effect. Returning to the condition

$$\eta \gtrless r \cdot \left[1 - \frac{1}{\rho}\right]$$

the risk-neutral producer will produce less than under certainty if $\eta > 0$, more if $\eta < 0$, and will be unaffected only if $\eta = 0$.

In Figure 1b, everything is the same except the producer is a net buyer of the product; here ρ is 1.5. The risk-neutral and slightly risk-averse producer's behavior is qualitatively the same as before, while the more risk-averse producer *increases* output with an increase in risk. Also, note that increases in r are associated with *increased* output for a given level of risk.

Figure 1*c* shows that a risk-neutral producer may also increase output as risk increases. In this case, the producers are net sellers, but the good is an inferior good. The Sandmo result concerning increases in r thus holds, but the negative value for η causes the increase in output, as risk increases, to be lessened for risk-averse producers. The risk-neutral or only slightly risk-averse producer actually produces more as risk increases; how risk-averse one could be and still exhibit this behavior would vary with the levels of η, s_m, and ρ.

Finally, the combined effects of a negative value for η and being a net buyer are shown in Figure 1*d*. Here, both aspects of the usual responses to risk are reversed. Increases in risk cause all producers to increase output, and the level of output is increasing in r, for any level of risk.

Implications for Econometric Studies

Econometric studies of peasants' risk attitudes (*e.g.* Antle) are usually based on the observed gap between expected revenue and marginal cost. In the traditional model, this gap equals the derivative of the Pratt risk premium with respect to output (*e.g.* Antle; Flacco and Larson). This last quantity, termed the marginal risk premium, is a function of the observed moments of the probability distribution and the agent's risk attitudes.[8] This relationship permits the agent's degree of risk-aversion to be inferred by a comparison of the above gap to the variance of profits.

However, this relationship is misspecified if the agent's objective function is defined over other random variables as well. The wrong risk premium will have been used. Instead, the derivative of the risk premium S defined earlier is appropriate. For a correct elicitation of aversion to risk in such cases, one needs information about the moments of the joint distribution of the different risks in the objective function and about the agent's attitudes toward the other risks. These may depend, as was shown in the preceding section, on the agent's ordinal preferences.

Suppose that the small risk assumption is valid. A first-order Taylor approximation of the producer's first-order condition yields

$$\frac{\bar{p} - c'(x)}{\bar{p}} = \psi^2 \left[r\beta + s_m(\eta - r) \right],$$

where ψ is the coefficient of variation of the output price. Data on $\bar{p}$, ψ, production inputs, and output would facilitate the estimation of the coefficient of ψ in the above equation. Previous studies have interpreted the coefficient on ψ^2 or on variance as the Arrow-Pratt measure of risk-aversion (e.g. Antle (p. 774)). However, in the marketed-surplus case, this coefficient clearly cannot be interpreted as a coefficient of risk aversion, because it also includes parameters that describe ordinal preferences for home consumption. Even if it is known that β is around one, so that safe income is negligible compared to the farm income, such an inference is not possible. When β=1 one obtains a linear combination of r and η, with the weights determined by the size of s_m—the larger is s_m, the more the estimate includes the effects of η. And, if the sufficient condition for Sandmo's result holds—that η is larger than r and both are positive—the result is that the estimated degree of risk-aversion is too large.

When does this quantity reduce to the Arrow-Pratt measure of risk aversion? Two alternatives exist. First, $s_m = 0$, so the producer does not consume any of his farm product. Second is the case where $\eta = r$, which was discussed in preceding section. As in the traditional model, if these conditions hold, estimation of an individual's attitude toward risk is possible using the observed gap between expected price and marginal cost. However, if $s_m > 0$, so that the peasant consumes a significant portion of the product grown on his farm, and his variable indirect utility function is not additively separable, the traditional estimation procedure yields biased estimates.

In some extremes, the result may even be estimates with the wrong signs, indicating risk-seeking behavior although the agent is, in fact, risk-averse. The above expression for the first order condition reveals that if $s_m > \beta$ and $\eta \leq 0$, the level of output under uncertainty is *increasing* in the Arrow-Pratt measure of relative risk-aversion. Thus, if the producer is a net buyer of the farm product,[9] and the good is inferior, greater risk-aversion is associated with a smaller gap between expected price and marginal cost. Hence, in this case if one simply regresses the gap between expected price and marginal cost against variance, the estimated coefficient, mistakenly believed to equal r, will be negative even though the the producer is risk-averse.

Figure 2 illustrates these findings for some plausible values of consumption parameters. On the vertical axis is the expected value of the estimated coefficient in the simple regression where the dependent variable is the percentage gap between expected price and marginal cost and the explanatory variable is the coefficient of variation of the output price. The horizontal axis gives values of the Arrow-Pratt measure of relative risk-aversion.

Ideally, the relationship between these two would be given by a 45 degree line from the origin. The figure illustrates that this is not the case when the estimation procedure is based on a simple univariate Sandmo-type model. Ignoring multivariate risk causes behavior that is partly due to uncertainty about a consumption price to be ascribed solely to aversion to income risk alone.

If $\rho = 1$, the Arrow-Pratt coefficient of risk-aversion has no effect on the estimate. In fact, the regression coefficient is an unbiased estimate of the product of the income elasticity and the budget share. For deficit farms ($\rho > 1$), one is likely to obtain negative relationships, as the figure shows: increases in the value of r lead to decreases in its estimated value. Only for farms with a positive marketed surplus will greater risk-aversion tend to result in larger estimates.

Long-Run Entry and Exit Decisions of the Risk Averse Peasant

The discussion thus far has assumed that the household produces a positive level of output and hence ignores the possibility of quitting farming. However, the effects of price risk may be severe enough to induce farmers to quit the farm and to look for alternative income sources. For example, peasants might choose to migrate from rural areas to urban areas to sell their labor services. This is the topic of the current section, which examines the long-run equilibrium and serves to illustrate how the modified risk premium can lead to different conclusions than the usual risk premium.

Proposition 3 generalizes the results of Sandmo, Paris, and Flacco and Larson, establishing the relationship between the peasant's attitude toward risk and the difference between expected price and average cost which induces entry/exit. Since the cost of production includes the opportunity cost of the peasant's labor endowment (that opportunity cost being the urban wage), the proposition yields the conditions under which it would be optimal to quit farming and migrate to the city.

Proposition 3: Exit from farming occurs where average cost is smaller (larger) than expected price if and only if the peasant is averse to (seeks) income risk, in the sense that $S > (<) \, 0$.

Proof: We prove the result for aversion to income risk; the case of risk seeking can be proved similarly. Let the initial level of wealth be W_0 and profits from farming π. The producer is indifferent between his two alternatives if and only if expected utility is the same whether he produces or not:

$$E[V(l, \pi + W_0, p)] = E[V(l, \bar{\pi} + W_0 - S, p)] = E[V(l, W_0, p)]$$

or

$$\bar{p} = \frac{C(x)}{x} + \frac{S}{x} = 0. \ \square$$

Various researchers (cited in Katz and Stark, for instance) have presented surprising evidence that migration has occurred even though "in many cases the expected income in the urban area is not larger than the expected income in the rural area" (Katz and Stark, p. 135). Risk-aversion in the Arrow-Pratt sense can explain such migration if income sources in the city are safer. However, urban job markets in developing countries are often characterized by a high level of uncertainty about employment.

Katz and Stark suggested imperfections in the capital market as a possible explanation. Another alternative is provided by aversion to income risk, as captured by a positive value for our modified risk premium S. Recall that S consists of the Arrow-Pratt premium plus an additional term. This additional term may be positive, providing an additional reason to quit farming. It may turn out, then, that peasants will be willing to bear additional income risks in the urban area—to escape a positive correlation between income and a consumption price—thus reconciling migration with aversion to income risk.

Turning to the industry, in long-run equilibrium each participant in the industry must be indifferent between operating or quitting farming. This requirement implies the following characteristics of the long-run equilibrium, stated as Corollary 1, an immediate result of Proposition 3.

Corollary 1: If the industry consists of peasants who are averse to income risk, the long-run equilibrium is characterized by positive expected profits.

The above condition replaces the traditional "zero profits" characterization of the long-run equilibrium of the industry under certainty. Thus, the average risk premium drives a wedge between expected price and average cost. As is true in the case of univariate risk (Flacco and Larson), production under risk, if producers are risk-averse, does not take place where long-run average cost is at a minimum.

Production may also take place even though expected profits are negative. While this could occur in univariate framework, it would be possible only with risk-preferring producers. Here, it could occur even with risk-aversion in the Arrow-Pratt sense ($V_{yy} < 0$), as long as the consumption effect of randomness in p was large enough to yield $S < 0$. To summarize, aversion to income risk, as defined by a positive modified risk premium, is both necessary and sufficient for the peasant to produce only when expected profits are strictly positive, while the Arrow-Pratt measure is neither necessary nor sufficient.[10] An empirical implication of this, once more, is that even if positive expected profits are observed in the long-run, one cannot conclude that producers are risk-averse in the sense of Arrow and Pratt. Similarly, negative expected profits (or apparently sub-optimal exits from farming) need not imply risk-preferring behavior.

Discussion of Possible Extensions

Uncertainty about prices is indeed one of the important risks with which peasants must contend. For example, Newbery and Stiglitz and Ahmed and Bernard present evidence that in developing countries, the coefficient of variation of the price of a typical agricultural commodity is around 0.5. However, additional risks affecting profits or other arguments of the utility function are typically present.

Uncertainty about the relative prices of other commodities consumed is another risk that may be present in the utility function. It is unlikely that the producer faces output price risk while all other prices in the economy are known or move together (pure inflation), as essentially was assumed in our model. It is straightforward to generalize our results, however. The risk premium S is defined as before, but p now denotes a vector of goods' prices. For small risks, it can be shown that

$$S = -\frac{1}{2}\,\sigma^2_{p_x}\,x^2\,\frac{V_{yy}}{V_y} - \sum_{i=1}^{N}\sigma_{p_x p_i}\,x\,\frac{V_{yp_i}}{V_y}$$

This is a generalization of the risk premium of the second section of the paper, which was defined for the case of uncertainty about the price of output and the price of one consumption good. This expanded risk premium consists of the univariate income-risk premium and a term capturing the individual's aversion to the covariation of income (through the output price p_x) with all other prices p_i $(i=1,..,n)$. The latter could be positive or negative.

Both components of S have a derivative with respect to output; as a result the marginal risk premium, which represents the difference between price and marginal cost, could have a sign opposite to that of the univariate risk premium. However, unlike the special case of the earlier section, where only the price of the marketed surplus good is random, knowing the sign of the marginal risk premium now requires knowledge of the covariances between prices and of preferences (each V_{yp_i} term). Our analysis that considers only the price of the marketed surplus good is thus a special case of the more general problem in which all prices are random. As long as other goods tend to have small budget shares and the risks in their relative prices have small degrees of correlation with the output price, it might be reasonable to assume that the overall effect on production of these other risks is negligible. Otherwise, at least for small risks, the expression above replaces the risk premium we used. It complicates the interpretation of the conditions for Sandmo's result or for a positive risk premium, since information about all goods is required, but it is still S, and not the univariate risk premium, that describes behavior. In fact, the multivariate risk premium S replaces the univariate (Arrow-Pratt) risk premium any time the situation is one of multivariate risks (Karni; Finkelshtain).

Other risks also affect profits. Certainly, peasants are exposed to uncertainty about yields as well as prices. To accommodate this additional uncertainty, the model must be extended in two directions. First, it must be modified to allow the choice variables to be inputs, rather than the level of output. Second, the risk premium is more complicated. Assuming small risks and returning to the case that other goods prices are known, the willingness to pay to stabilize income risk, when it consists of both yield risk and output price risk for the marketed surplus good, is given by

$$S = -\frac{1}{2}\,\sigma_{yy}\,\frac{V_{yy}}{V_y} - \sigma_{yp}\,\frac{V_{yp}}{V_y},$$

where σ_{yy}, the variance of revenues, now reflects variation in price and output, as well as their covariance. In addition, the covariance between income and the price of the good, σ_{yp}, is no longer the same as $\sigma_{pp} \cdot x$, hence this expression cannot be simplified any further. Both the covariance term and the derivative V_{yp} must be signed before the effect of risk can be determined. Since σ_{yp} depends on the level of production, the marginal risk premium is once more affected by the added risk.

This causes some difficulty in establishing results analogous to Propositions 1 and 2. These propositions were based on the assumption that only price risk exists. The risk premium S still may be interpreted as before, but because profits now depend on additional risks, the characterization in Proposition 1 of preferences that imply $S > 0$ is no longer valid. Moreover, a proposition similar to Proposition 2 is hard to derive, since except for extreme cases, such as where profits and the output price are perfectly correlated, it is impossible to find an expression for $\frac{dV_y}{dp_i}$ of condition (*iv*).

The conditions under which a producer will *always* behave as a risk averter with respect to output or will always be willing to pay a positive premium to stabilize income are thus much stronger. While more difficult, the problem is not intractable. Either the problem must be attacked for small risks only, with the Taylor approximation of S, or restrictions must be placed on preferences to obtain large-risk characterizations of behavior.

The main conclusions remain valid—the level of output may well be greater than under certainty and ordinal preferences affect the level of production. In fact, it can be shown that, for any preferences (except the logarithmic utility function), there is always some probability distribution that implies a higher output level under uncertainty. Also, if additional restrictions are imposed on the joint probability distribution of wealth and the price of consumption goods, a broader set of preferences will imply Sandmo's result. For instance, if they are independent,[11] it is required only that the producer is averse to risk in the Arrow-Pratt sense.

It is important to emphasize that our discussion in the two preceding sections of the paper also remains valid. An empirical analysis that does not explicitly consider the producer's ordinal preferences for goods will yield biased results, if interest is in estimating risk attitudes. Also, aversion to a specific income risk, as defined by $S > 0$, would imply positive expected profits in the long-run and a more risk averse producer will demand a higher expected price to stay in business.

Conclusions

This paper has shown that many of the familiar results from models of output price uncertainty may no longer hold when producers consume a significant share of their own output. In the short-run, a peasant producer who is averse to risk may find it optimal to produce either more or less output than the level that maximizes expected profits. The necessary and sufficient conditions for each of these alternatives were derived. The univariate (Arrow-Pratt) measure of aversion to risk is not sufficient to determine which will occur. However, the alternative risk premium developed in the paper, which measures aversion to risk in income when other random variables affect utility, does provide a characterization of risk-aversion and of the level of output.

Implications were drawn for empirical studies that attempt to determine risk attitudes from observed behavior. Estimates or predictions that use univariate models will, in general, be biased for the case of peasants. Finally, in the long-run, the average price the peasant producer requires to remain in business may be larger, equal to, or smaller than average cost.

NOTES

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1. Another example of multivariate risk was examined by Herath, Hardaker, and Anderson, in the context of rice production in Sri Lanka in the presence of yield risk. They considered the varietal choice problem when utility depends on both income and subsistence rice consumption.

2. The fact that such households have limited access to insurance or capital markets makes it especially important to examine the effects of the entire set of risks they face. Of course, the same comments apply to any agricultural household, but the problem seems most acute for low-income, marketed-surplus producers. Higher income farmers are less likely to be consuming part of their output, and the other risks they face are less likely to be correlated with the price of their output.

3. This assumption is not essential, but simplifies the analysis and allows comparison with Sandmo's model. The main results hold under more general forms of stochastic profits, provided, of course, that p remains random.

4. At first glance, such a risk premium seems to attach a value to an impossible stabilization scheme. The price of the output x could be stabilized independently of the consumption

price, however. For instance, the government could operate a consumer subsidy scheme where the price to consumers was uncertain and there was a pre-announced price at which all output would be purchased from producers. A similar outcome could occur if the producer forward contracts all of his output at an unbiased futures price and then decides consumption based on the realization of p. In any event, the concept of the risk premium does not require that such a partial insurance scheme actually be offered.

5. Independence is sufficient for that to occur only for small risks. For large risks, independence would not insure equality, but would guarantee that S and the univariate risk premium would have the same sign.

6. A sufficient condition that holds for large risks as well as small risks is $\eta > r > 0$.

7. The required purchasing power comes, presumably, from other outputs or from off-farm sources of income.

8. Typically, researchers have assumed small risks, so only the second moment, variance, matters.

9. That this might occur seems rather surprising, but the data mentioned earlier from Bangladesh, cited by Ahmed and Bernard, show that it can.

10. The modified risk premium is also useful for interpersonal comparisons. For example, it can be shown that a peasant who is more averse to income risk would require a higher expected price to remain in farming.

11. For instance, if the producer receives a certain price for his output but faces yield risk that is independent of the risk in the prices of consumption goods.

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Figure 1: Effects of Risk and Risk Aversion on Output Levels