Markovian and non-Markovian dynamics in quantum and classical systems

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We discuss the conceptually different definitions used for the non-Markovianity of classical and quantum processes. The well-established definition for non-Markovianity of a classical stochastic process represents a condition on the Kolmogorov hierarchy of the n-point joint probability distributions. Since this definition cannot be transferred to the quantum regime, quantum non-Markovianity has recently been defined and quantified in terms of the underlying quantum dynamical map, using either its divisibility properties or the behavior of the trace distance between pairs of initial states. Here, we investigate and compare these definitions and their relations to the classical notion of non-Markovianity by employing a large class of non-Markovian processes, known as semi-Markov processes, which admit a natural extension to the quantum case. A number of specific physical examples is constructed which allow to study the basic features of the classical and the quantum definitions and to evaluate explicitly the measures for quantum non-Markovianity. Our results clearly demonstrate several fundamental distinctions between the classical and the quantum notion of non-Markovianity, as well as between the various quantum measures for non-Markovianity.

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