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Mass Customization versus Mass Production:

Variety and Price Competition

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Mass Customization versus Mass Production: Variety and Price Competition

Abstract: We study competition between two multi-product firms with distinct production technologies in a market where customers have heterogeneous preferences on a single taste attribute. The *mass customizer* (MC) has a perfectly flexible production technology, thus can offer any variety within a product space, represented by Hotelling's (1929) linear city. The *mass producer* (MP) has a more focused production technology, and therefore, it offers a finite set of products in the same space. MP can invest in more flexible technology, which reduces its cost of variety and hence allows it to offer a larger set of products; in the extreme, MP can emulate MC's technology and offer infinite variety. The firms simultaneously decide whether to enter the market, and MP chooses its degree of product-mix flexibility upon entry. Next, MP designs its product line, i.e., the number and position of its products; MC's perfectly flexible technology makes this unnecessary. Finally, both firms simultaneously set prices. We analyze the sub-game perfect Nash equilibrium in this three-stage game, allowing firm-specific fixed and variable costs that together characterize their production technology. We find that an MP facing competition from an MC offers *lower* product variety compared to an MP monopolist, in order to reduce the intensity of price competition. We also find that MP *can* survive this competition even if it has higher fixed cost of production technology or higher marginal cost of production or both.

1 Introduction

A quintessential feature of modern commerce is the proliferation of product variety in virtually every industry. At the dawn of mass production, Ford offered the *Model T* in any color as long as it was black; the *Mazda 323* recently came in four shades of black from a palette of 180 different colors (Fisher et al. 1995). As a logical outcome of this ever-increasing product variety, many companies have begun formulating their marketing and operations strategies around customization (Pine 1993, Kotha 1995, Lampel and Mintzberg 1996, Feitzinger and Lee 1997, Zipkin 2001), while others still use mass production technologies that limit them to a predetermined set of products. Hence, we witness customizing firms competing against non-customizing firms in many industries ranging from health care to cosmetics.

The business press provides many examples of customization: vitamins (Acumin); sports shoes (Adidas); hockey sticks (Branches Hockey); industrial detergents (ChemStation); notebook and desktop computers (Dell); industrial plastics (GE Plastics); pants and shirts (Lands' End); lighting systems (Lutron); industrial electronic components (Marshall Industries); custom-color coated M&M's (Masterfoods USA); bicycles (National Bicycle); sneakers (Nike); beauty-care products (Procter & Gamble); golf clubs (TaylorMade); messenger bags (Timbuk2); plastic food containers (Ultra Pac); and candles (Yankee Candle). This list is by no means exhaustive, but reflects the diversity of industries in which customization is gaining ground. Its main appeal to customers is the promise of products that fit better with their needs or desires. In some cases, as in hip-joint replacements or house windows, perfect fit is essential. In others, as in fragrance or birthday cards, the quest for better fit is more driven by fashion or desire for uniqueness than by need.

Many scholars and practitioners believe that, due to advances in manufacturing and information technologies, very high levels of product variety – high enough to be termed "mass customization" – can now be achieved without a prohibitive cost penalty from reduced efficiency. This not only requires overcoming the traditional trade-off between product variety and manufacturing efficiency, but also an ability to market (almost) infinite product variety. National Bicycle, a Japanese company that custom-makes eight million variations of a high-end bicycle (based on different model types, colors and frame sizes), is a successful example of such operations-marketing coordination (Kotha 1995).

Mass customization capabilities have several other benefits, in addition to matching customer needs more closely. For instance, customization typically eliminates finished goods inventories and all the associated costs. It can also be an accurate tool for learning customer preferences, as shown by National Bicycle.

The prevailing view is that the competitive benefits of higher product variety lead to higher profits (Kekre and Srinivasan 1990). Empirical and analytical studies have shown however that incongruence between product plans and supply process leads to poor performance (Randall and Ulrich 2001, Berry and Cooper 1999, Safizadeh et al. 1996, de Groote 1994). Hence, adopting customization requires shifts in both marketing and operations strategy. Often this means radical changes in an entire supply chain, which might explain why mass customization has eluded the auto industry for so long. Therefore, customization tends to be employed by new entrants (e.g. Dell) rather than established mass production firms (e.g. Compaq). Besides, firms differ in their ability to adopt new technologies. We can thus expect mass customizers and mass producers to compete in many industries, at least in the short run, as new entrants adopt customization before established firms respond.

In this paper we study competition between two multi-product firms with distinct production technologies, characterized by different fixed and variable costs. The *mass customizer* (MC) has a perfectly flexible production technology, thus can offer any variety within a horizontally differentiated product space, represented by Hotelling's (1929) linear city. The *mass producer* (MP) has a continuum of choices between perfectly flexible and more focused production technologies; in the latter case, it can only offer a finite, predefined set of products. Customers are heterogenous on a single taste attribute, so firms compete on price and proximity to each customer's "ideal" product. The firms first simultaneously decide whether to enter the market, and MP chooses its degree of product-mix flexibility upon entry. Then MP designs its product line, deciding the number and location of its products. Finally, both firms simultaneously set prices; in line with our focus on horizontal product differentiation, neither firm applies price discrimination based on taste. This is consistent with existing examples of mass customization in a horizontal differentiation setting, such as Masterfoods, Timbuk2, and others mentioned above. We characterize the sub-game perfect Nash equilibrium in this three-stage game.

We address the following research questions: How would MC and MP set prices when competing against each other? How would MP design its product line? Will MP increase or decrease its product variety in response to competition from MC? Under what conditions can both firms coexist? When will one technology dominate the other? The contribution of this paper lies in its assessment of the competitive value of perfect product-mix flexibility and in its incorporation of a supply-side element (production technology) in modeling product competition, which has predominantly been studied from the demand side. Also, the comprehensive treatment we offer here, in contrast to most existing work which tends

to focus on selected special cases, reveals interactions that would otherwise remain hidden. We return to this in the discussion section.

One could study many other types of competition, such as competition between two mass producers of multiple products. However, the problem of endogenous product line design followed by price competition is recognized as unsolvable (Klemperer 1992, Teitz 1968). The current work is a first step toward understanding the competitive effects of product variety under different production technologies, and we believe that competition between MP and MC is the most interesting place to start.

The rest of the paper is organized as follows. We first review literature from economics, marketing and operations. In §3 and §4 we present the model and analysis. In §5 and §6 we discuss the results and limitations of our study. All the proofs are provided in a separate appendix.

2 Literature Review

We draw from the economics, marketing and operations management literatures on horizontal product differentiation, emphasizing research inspired by Hotelling's (1929) model.

The economics literature on product variety generally focuses on market equilibrium and social optimum. For an in-depth review, see Lancaster (1990). Here we review three papers on competition between multi-product firms, an under-studied subject in economics (Klemperer 1992, Lancaster 1990).

Heal (1980) explores spatial distribution of retail outlets in a circular city with a producer located at the center, focusing on social optimality, profit-maximizing behavior of a

single retail chain, and competition between single-outlet retailers. Heal's producer is analogous to our MC, and his collection of retail outlets to our MP. However, his central producer is passive (does not set its own price) in contrast to MC in our model.

Eaton and Schmitt (1994) model flexibility in a Hotelling framework by introducing a cost of producing variations on a basic product. They then study competition between many flexible firms for examining the conditions under which flexible manufacturing leads to market concentration.

Ulph and Vulkan (2001) study variety-price competition between two firms that are based at the endpoints of a 'linear city' and can either standardize (offer a single product) or customize (offer non-overlapping ranges of custom products). Also, they can price discriminate on customer location (first degree) or on product location (second degree). Although firms are better off adopting neither customization nor first-degree price discrimination, in equilibrium they adopt both and make lower profits. The authors do not consider a finite-variety multi-product firm (our MP), and restrict firms' location choices more than we do.

The marketing literature emphasizes decision aid for real business problems (e.g. Hauser and Shugan 1983) and testing empirical implications of theoretical models (e.g. Bayus and Putsis 1999). Analytical models tend to focus on the marketing mix variables, but do not usually consider production technologies as we do here (Ratchford 1990). For instance, Balachander and Farquhar (1994) ask how product availability affects Hotelling-type price competition, and Moorthy (1988) examines two single-product firms' choice of quality and price in a market with heterogenous valuations.

Balasubramanian (1998) studies price competition between a direct marketer and multiple retailers, evenly spaced on a circular market. He characterizes the price equilibrium,

predicts the impact of a direct marketer's entry on the retail market, and examines the strategic implications of informing only a fraction of customers about the direct channel. This can also represent competition between multiple specialized brands and a mass customizer. Our model differs in several ways: His retailers are analogous to independent single-product firms, while our "retailers" are managed by a single firm, MP. His retailers are evenly spaced, while we place no ex ante restrictions on positioning. His reservation price is sufficiently high that no consumer is priced out, an assumption we do not make.

Product variety has been handled in the operations literature almost exclusively from a supply-side perspective (Ramdas 2003, Ho and Tang 1998), often focusing on how increased product variety affects manufacturing or supply chain performance (e.g. Thonemann and Bradley 2002, Randall and Ulrich 2001, Fisher and Ittner 1999, Gupta and Srinivasan 1998, MacDuffie et al. 1996). A few exceptions recognize both demand and supply sides in exploring optimal assortment to balance market gains and inventory costs due to variety (Gaur and Honhon 2004, van Ryzin and Mahajan 1999); interaction of product line, pricing and make-to-order / make-to-stock decisions (Dobson and Yano 2002); link between conformance / performance quality and product strategy (Karmarkar and Pitbladdo 1997); marketing - operations impact of higher variety and the resulting coordination issues (Netessine and Taylor 2005; Yunes et al. 2004; Cattani, Dahan and Schmidt 2003; De Groote 1994; Kekre and Srinivasan 1990).

Dewan et al. (2000) study product strategy - location - price competition of two firms that can offer a single standard product or a range of custom products on a circular space. The equilibrium involves both firms choosing to customize, because the first stage poses a Prisoner's Dilemma. The assumption of zero marginal cost is critical, as the customizing

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firm can always replicate (hence dominate) a standard-product firm. Our model allows more general cost parameters, giving either MC or MP a cost advantage; besides, our MP can offer multiple products. Mendelson and Parlakturk (2004) offers an alternative to Dewan et al. (2000), emphasizing the practical reality that customization is rarely perfect in matching every customer's wish.

In a study independent of but parallel to ours, Xia and Rajagopalan (2004) find that the longer leadtime associated with custom products can give the standard product firm an advantage. While they have a more general functional form for the cost of variety than we do, they restrict analysis to the case with a sufficiently high reservation price, and do not model market entry. Moreover, lead time is determined only by the MC's investment and is not affected by congestion. As a result, the effect of lead time in their model is primarily to modify the consumers' reservation price. For this reason, the analysis of time-variety competition is not included in the current paper, despite being studied in the first author's dissertation (Alptekinoğlu 2004). An extended abstract of an earlier version of the current paper appeared as Alptekinoğlu and Corbett (2004).

We contribute to this literature in several ways. First, our product differentiation model incorporates a key supply-side factor: production technology with varying product-mix flexibility. Hence, our model can help assess the strategic value of perfect flexibility. Second, within the Hotelling model, we introduce a mass customizing firm that can produce and market any variety with no penalty, in line with recent advances in manufacturing and information technologies. Third, we find that some common assumptions in the operations literature about the effect of variety are not always true.

3 Model

In this section we present a model of competition between two multi-product firms with fundamentally different production technologies. The *mass customizer* (MC) is able to offer an infinite variety of custom products (within the bounds of the product space) by using a perfectly flexible production technology. The *mass producer* (MP) offers a finite set of standard products by using a focused production technology. The degree of product-mix flexibility associated with this technology is a decision made by MP by investing in setup cost reduction as in Porteus (1985). At the extreme, MP has the option to invest enough to make its technology perfectly flexible. If MP invests in perfectly flexible technology, it becomes indistinguishable from MC. The competition between MP and MC revolves around price and the ability to match each customer's "ideal" product. We assume the standard and custom products are available equally soon. We do this in order to study competitive effects of product variety in isolation, rather than commingling them with time-based competition issues that would arise if customers had to wait (longer) for custom products.

We consider a set of horizontally differentiated products, each fully characterized by a single taste attribute, quantified by a real number between 0 and 1. We refer to this as the product's *location* on the *product space* [0, 1].

The Mass Producer (MP) makes four decisions: *entry* and *flexibility*, *variety*, and *price*. MP first decides whether to enter the market. If it enters, it pays a fixed cost $F_p(f)$ for a mass production technology with a degree of product-mix flexibility that results in a fixed cost f per distinct product offered. There is some evidence from the operations literature that the cost of variety for a multi-product firm is indeed approximately linear in the number

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of products, assuming a cyclical base stock policy or something similar (Federgruen, Gallego and Katalan 2000; DeGroote, Yucesan and Kavadias 2002; Thonemann and Bradley 2002; Benjaafar et al. 2004). We follow Porteus (1985) and assume that the fixed cost $F_p(f)$ is convex decreasing in f through $F_p(f) = \kappa - \theta \ln(\gamma + f)$ (where κ, θ, $\gamma > 0$ are scalars such that $F_p(\cdot)$ is strictly positive for a domain of f relevant to our model). Note that MP can achieve perfect product-mix flexibility, i.e., choose $f = 0$ and offer infinite variety in an effort to emulate MC, at a fixed cost $F_p(0) = \kappa - \theta \ln(\gamma)$. By letting γ drop to 0, one can make it arbitrarily costly for MP to emulate MC. Second, MP decides on variety, i.e., the number and location of standard products. For each member of its product line, MP pays the fixed cost f. Let $n \in \{1, 2, 3, ...\}$ denote the number of products and $x_i \in [0, 1]$ the location of product $i \in \{1, 2, ..., n\}$. The variety decision thus involves setting the dimension n and the elements of the vector $\mathbf{x} \equiv (x_1, ..., x_n)$, and committing to a cost of fn. Third, MP sets the prices $\mathbf{p} \equiv (p_1, ..., p_n)$ for products $1, ..., n$ respectively. The variable unit cost of production c_p is the same for all products. We will see later that MP does always choose to charge a uniform price for all of its products, so restricting MP upfront to uniform pricing (i.e., no price discrimination, as mentioned in the introduction) yields the same results.

The Mass Customizer (MC) makes two decisions: *entry*, and *price*. MC, if it enters, pays a fixed cost F_c for a perfectly flexible technology capable of custom-producing any product within the product space. Typically one would expect to find $F_c > F_p(f)$, i.e., the flexibility of MC's technology comes at a price, but we do not restrict our analysis to that case. Because of its infinite flexibility within the product space, MC has no distinct "products" and incurs no "per-product" cost for variety. For instance, in the case of custom-blended paint or custom-made clothing, once MC has the necessary technology there is no

incremental cost for any particular color or garment. Second, MC sets a uniform price p_c for its products. The variable unit cost of production c_c is again independent of location. The apparent asymmetry between MC and MP in ability to price-discriminate has no effect, as MP will never use its ability to price-discriminate given that the MC does not do so. We present the model allowing MP to price-discriminate as it is slightly more general, and yields the exact same results as a model where neither firm can price-discriminate. The setting studied here is consistent with horizontal differentiation, where price discrimination based on taste is not common. Allowing both firms to price-discriminate does change the analysis; that setting would be more consistent with vertical differentiation and is left for future research.

Each customer buys one unit (if any) from either MP or MC. Customers are heterogenous in their tastes: they each have a *location* on $[0, 1]$, representing their ideal product. They pay a *transportation* cost (or disutility) d per unit distance between their location and the purchased product; there is no transportation cost for custom products. Each customer buys the product with the smallest *delivered price*, the list price plus the transportation cost, as long as this does not exceed a reservation price $\bar{p}$. A customer located at $z \in [0, 1]$ receives a utility of $U(z, x, p) = \bar{p} - p - d \ |z - x|$ by purchasing a product at x for price p. This customer will therefore purchase the custom product located at z if $p_c < p_i + d \ |z - x_i|$ for all $i \in \{1, 2, ..., n\}$, and $p_c \leq \bar{p}$; the standard product $i \in \{1, 2, ..., n\}$ located at x_i if $i = \underset{j \in \{1,2,...,n\}}{\arg \min} \ [p_j + d \ |z - x_j|]$ and $p_i + d \ |z - x_i| \leq \min (p_c, \bar{p})$; and will make no purchases if $p_c > \bar{p}$, and $p_i + d \ |z - x_i| > \bar{p}$ for all $i \in \{1, 2, ..., n\}$. Ties can be broken arbitrarily. We assume that an indifferent customer will prefer the standard product over the custom product, and a purchase over no purchase. The total number of customers is λ, their locations

uniformly spread between 0 and 1.

The interaction between MP and MC unfolds in three stages. In the first stage, the *entry game*, firms simultaneously decide whether to enter the market, and MP decides how much to invest in product-mix flexibility. The entry game has four possible outcomes. If both firms decide to enter, then a *duopoly* competition ensues. If one firm decides not to enter, the other acts as a *monopolist*. Finally, if neither firm enters, *market breakdown* occurs. Throughout, we assume that $c_c < \bar{p}$ and $c_p < \bar{p}$; otherwise, one or both firms would never enter. In the second stage, MP - if it enters - determines its product variety, i.e., sets n and designs its product line $\mathbf{x}$. No decision by MC is needed in the second stage. Finally, in the third stage, firms in the market simultaneously set their prices, p_c and $\mathbf{p}$. This sequence is consistent with the hierarchy of decisions in marketing and economics models of product strategy. Note that we only let one firm, MP, choose its production technology upon entry. This is because endogenous technology choice by both firms would require a treatment of competition between two MPs, which appears to have no equilibrium (Klemperer 1992). Moreover, we believe the MP - MC competition is the most interesting focus of analysis.

MC's profit is given by $\pi_c (p_c, f, n, \mathbf{x}, \mathbf{p}) = (p_c - c_c) \, \lambda \, y_c (p_c, f, n, \mathbf{x}, \mathbf{p}) - F_c$, where $y_c(\cdot)$ is the total market share captured by the custom products. MP's profit is given by $\pi_p (p_c, f, n, \mathbf{x}, \mathbf{p}) = \sum_{i=1}^{n} [(p_i - c_p) \, \lambda \, y_i (p_c, f, n, \mathbf{x}, \mathbf{p}) - f] - F_p(f)$, where $y_i(\cdot)$ is the length of the market segment captured by the standard product $i \in \{1, 2, ..., n\}$. y_c and $\sum_{i=1}^{n} y_i$ are MC's and MP's total market share. Furthermore, $y_c + \sum_{i=1}^{n} y_i \leq 1$, with strict inequality only when MC sets a non-competitive price ($p_c > \bar{p}$, which implies $y_c = 0$) and MP chooses not to cover the entire market. Our analysis in §4 shows that the entire market will be covered whenever any firm chooses to enter the market.

We develop a full analysis of this three-stage game in the next section to address the following research questions: What conditions result in duopoly, or in a monopoly of either firm? How would the firms set prices as monopolists or duopolists? How would MP design its product line when competing with MC, as opposed to when acting as a monopolist? Does duopoly lead to higher or lower variety of standard products than monopoly? Under what conditions, if ever, can MP profitably coexist with MC?

4   Analysis

In this section we determine the sub-game perfect Nash equilibrium of the three-stage game defined in §3. As usual, the analysis proceeds by backward induction, starting with the last stage, where firms set prices in the monopoly or duopoly outcome of the entry game. The first part of the analysis is largely standard, but necessary for the competitive analysis that follows, which is where our main contribution lies.

4.1   Monopoly of the Mass Producer (MP)

We first consider the case in which only MP enters and chooses a technology with $f > 0$. We establish some structural properties of MP's positioning and pricing decisions, slightly more general than those in de Groote (1994), that considerably reduce the solution space.

Proposition 1 *Given $f > 0$ and $n < \infty$, MP's optimal positioning and pricing decisions when acting as a monopolist obey two structural properties: the products must be located such that the market segment captured by each is contiguous and of equal length, i.e. $y_1 = y_2 = \cdots = y_n$; and priced uniformly, i.e. $p_1 = p_2 = \cdots = p_n$.*

Optimality of a symmetric product line design is not surprising under the assumption of uniformly spread customer locations. In effect, Proposition 1 reduces MP's second-stage decision to choosing n, and its third-stage decision to setting a uniform price p_p. MP's pricing problem for a given n is then:

$$\max_{p_p} \quad \pi_p(f, n, p_p) = (p_p - c_p)\lambda n y_p - nf - F_p(f)$$

$$st. \quad y_p = \begin{cases} 1/n & \text{, if } p_p < \bar{p} - d/2n \\ 2(\bar{p} - p_p)/d & \text{, if } \bar{p} - d/2n \leq p_p \leq \bar{p} \\ 0 & \text{, if } p_p > \bar{p} \end{cases}$$

where y_p denotes the same market segment length captured by each one of MP's products.

Proposition 2 *For fixed values of $f > 0$ and $n < \infty$, MP's optimal monopoly price is:*

$$p_p^* = \begin{cases} \bar{p} - d/2n & \text{, if } \bar{p} > c_p + d/n \\ (\bar{p} + c_p)/2 & \text{, if } \bar{p} \leq c_p + d/n \end{cases} \tag{1}$$

As expected, a monopolist MP will charge a higher price if it offers more products (hence closer to each customer's ideal point). Also, p_p^* is increasing in c_p and $\bar{p}$, and decreasing in d. When the customers' reservation price is not high enough, i.e. $\bar{p} < c_p + d/n$, MP chooses not to cover the entire market. This contrasts with most of the existing literature, which explicitly or implicitly assumes full market coverage. Analysis of the entry game in §4.4 shows that if MP enters, it will indeed always cover the entire market. Nevertheless, Proposition 2 (in concert with Proposition 3 below) establishes that the common assumption of full market coverage is not always satisfied when firm entry and/or number of products are given.

We now turn to the second stage: the variety decision. MP sets the number of standard products n knowing that the optimal price in (1) will follow in the third stage. MP's product

variety problem can thus be stated as follows:

$$
\max_{n \geq 1} \quad \pi_p(f, n, p_p^*) = \begin{cases} (\bar{p} - c_p - \frac{d}{2n})\lambda - nf - F_p(f) & \text{, if } \bar{p} > c_p + d/n \\[2ex] \frac{\lambda n}{2d}(\bar{p} - c_p)^2 - nf - F_p(f) & \text{, if } \bar{p} \leq c_p + d/n \end{cases}
$$

For ease of exposition we relax the integrality of n, a standard practice in the literature (e.g. Balasubramanian 1998). Define parameter regions ($\mu1$) through ($\mu4$) as follows:

$$(\mu1) \equiv \{ \bar{p} > c_p + d \text{ and } f < \frac{\lambda d}{2} \} \text{ or } \{ \bar{p} \leq c_p + d \text{ and } f < \frac{\lambda(\bar{p} - c_p)^2}{2d} \}$$

$$(\mu2) \equiv \{ \bar{p} > c_p + d \text{ and } f \geq \frac{\lambda d}{2} \}$$

$$(\mu3) \equiv \{ \bar{p} \leq c_p + d \text{ and } f > \frac{\lambda(\bar{p} - c_p)^2}{2d} \}$$

$$(\mu4) \equiv \{ \bar{p} \leq c_p + d \text{ and } f = \frac{\lambda(\bar{p} - c_p)^2}{2d} \}$$

The first portion of ($\mu1$) is the usual case implicitly assumed in the literature; the other regions have either a low reservation price and/or a high fixed cost per product. In §5 we will see that considering all possible cases rather than just those usually studied in the literature reveals some interactions that would otherwise not be apparent.

Proposition 3 *For MP, when acting as a monopolist with $f > 0$, the optimal product variety (number of products), the optimal price, and the resulting profit are given by:*

Region	Variety, n^*	Price, p_p^*	Profit, $\pi_p^{monop}(f) \equiv \pi_p(f, n^*, p_p^*)$
($\mu1$)	$\sqrt{\frac{\lambda d}{2f}}$	$\bar{p} - \sqrt{\frac{df}{2\lambda}}$	$\lambda(\bar{p} - c_p) - \sqrt{2\lambda df} - F_p(f)$
($\mu2$)	1	$\bar{p} - \frac{d}{2}$	$\lambda\left(\bar{p} - c_p - \frac{d}{2}\right) - f - F_p(f)$
($\mu3$)	1	$\frac{\bar{p} + c_p}{2}$	$\frac{\lambda}{2d}(\bar{p} - c_p)^2 - f - F_p(f)$
($\mu4$)	$n^* \in \left[1, \frac{d}{\bar{p} - c_p}\right]$	$\frac{\bar{p} + c_p}{2}$	$-F_p(f)$

Larger markets (λ) and markets with higher willingness-to-pay ($\bar{p}$) can support higher product variety, while variety is (weakly) decreasing in f and c_p. In regions ($\mu2$) and ($\mu3$),

the fixed cost f per product is so high that the firm offers only a single product. In regions $(\mu3)$ and $(\mu4)$, MP chooses not to cover the entire market but is also sure to make a loss. MP will of course decide not to enter in these cases.

4.2 Monopoly of the Mass Customizer (MC)

Now suppose that only MC enters, or that only MP enters and chooses an infinitely flexible technology so that $f = 0$ (i.e., MP essentially becomes a monopolist MC). This is a trivial case, where pricing is the only post-entry decision. Since customers buy as long as the price p_c is at or below their reservation price, MC will set the highest price possible, i.e., $p_c^* = \bar{p}$, while still capturing the entire market $(y_c = 1)$. An MC monopolist obtains a profit of $\pi_c^{monop} \equiv \pi_c(p_c^*) = (\bar{p} - c_c)\lambda - F_c$. Of course, if the fixed cost is too high, MC will not enter.

4.3 Duopoly Competition between MP and MC

If both firms enter and MP invests in perfect flexibility so that $f = 0$, pure price competition ensues, in which case it is not possible for both firms to recoup their fixed costs. Therefore, when both firms enter, MP must have chosen a technology with $f > 0$. MP's optimal positioning and pricing shares structural properties with the monopoly case shown in Proposition 1: MP evenly spaces out and uniformly prices its products[1]. This reduces MP's second-stage decision to choosing n, and its third-stage decision to setting a uniform price p_p.

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[1]We state this without proof. The idea is that, going from monopoly to duopoly, MC's price replaces the reservation price in forming an upper limit to MP's delivered price. Including an endogenous but not congestion-dependent lead time for MC similarly has the effect of shifting this upper limit.

4.3.1 Third Stage: Price Competition

To state the Nash equilibrium in prices for a given n, which is known to both parties, we first need to define four parameter regions $(\eta 1)$ through $(\eta 4)$ as follows:

$$(\eta 1) \equiv \ \{ \ c_c - c_p \geq \tfrac{d}{n} \ \}$$

$$(\eta 2) \equiv \ \{ \ \bar{p} \leq \tfrac{d}{3n} + \tfrac{2c_c + c_p}{3} \ \}$$

$$(\eta 3) \equiv \ \{ \ -\tfrac{d}{2n} < c_c - c_p < \tfrac{d}{n} \ \text{and} \ \ \bar{p} > \tfrac{d}{3n} + \tfrac{2c_c + c_p}{3} \ \}$$

$$(\eta 4) \equiv \ \{ \ c_c - c_p \leq -\tfrac{d}{2n} \ \}$$

Proposition 4 *For fixed values of $f > 0$ and $n < \infty$, the unique Nash equilibrium in prices is given by:*

Region	Price Equilibrium, $(\hat{p}_c, \hat{p}_p)$
$(\eta 1)$	$\left(c_c \quad , \quad c_c - \tfrac{d}{2n} \right)$
$(\eta 2)$	$\left(\bar{p} \quad , \quad \tfrac{\bar{p} + c_p}{2} \right)$
$(\eta 3)$	$\left(\tfrac{d}{3n} + \tfrac{2c_c + c_p}{3} \quad , \quad \tfrac{d}{6n} + \tfrac{c_c + 2c_p}{3} \right)$
$(\eta 4)$	$\left(c_p \quad , \quad c_p \right)$

Proposition 4 shows that the equilibrium prices $(\hat{p}_c, \hat{p}_p)$ are increasing in c_c, c_p and $\bar{p}$. In regions $(\eta 1)$ and $(\eta 4)$, MP and MC respectively completely dominate. These two extremes become more likely as n increases. In regions $(\eta 2)$ and $(\eta 3)$, the cost differential $|c_c - c_p|$ is small enough that both firms capture some market share, but the bounds on $|c_c - c_p|$ become tighter as n increases. Note that $(\eta 2)$, together with the assumptions $c_c < \bar{p}$ and $c_p < \bar{p}$, implies $-\tfrac{d}{2n} < c_c - c_p < \tfrac{d}{n}$, so the four regions are mutually exclusive.

Naively, one might expect that a higher variety would allow MP to charge a higher price. This is true in $(\eta 1)$, where MC's costs are so high that MP acts as a monopolist.

But in $(\eta 3)$, MP's equilibrium price $\hat{p}_p$ is strictly decreasing in n. Therefore, under duopoly competition, *price decrease* and *variety increase* can coexist for MP. This is because MC's reaction to an increase in n by MP would be to cut price. Hence, MP may prefer *lower* variety, in order to soften the price competition; we discuss this further in §5.

4.3.2 Second Stage: Product Variety Decision of the Mass Producer (MP)

Having established the duopoly price equilibrium for any given n, we can now analyze MP's variety decision for a fixed value of $f > 0$. MP maximizes profit $\pi_p(\hat{p}_c, f, n, \hat{p}_p)$ subject to $n \geq 1$. We again relax the integrality of n. Define parameter regions $(\delta 1)$ through $(\delta 4)$ as follows:

$$(\delta 1) \equiv \ \{\ c_c - c_p > d \ \text{ and } \ f < \tfrac{\lambda d}{2} \ \} \text{ or } \{\ 0 < c_c - c_p \leq d \ \text{ and } \ f < \bar{f} \ \}$$

$$(\delta 2) \equiv \ \{\ c_c - c_p > d \ \text{ and } \ f \geq \tfrac{\lambda d}{2} \ \}$$

$$(\delta 3) \equiv \ \{\ 0 < c_c - c_p \leq d \ \text{ and } \ f \geq \bar{f} \ \} \text{ or } \{\ -\tfrac{d}{2} \leq c_c - c_p \leq 0 \ \}$$

$$(\delta 4) \equiv \ \{\ c_c - c_p < -\tfrac{d}{2} \ \}$$

where $\bar{f}$ uniquely satisfies $\frac{2\lambda(c_c - c_p)}{9d}(c_c - c_p - \tfrac{d}{4}) \leq \bar{f} < \frac{\lambda(c_c - c_p)^2}{2d}$ and $\sqrt{2\lambda d\,\bar{f}} - \bar{f} = \frac{2\lambda}{9d}\left[-(c_c - c_p)^2 + \tfrac{7d}{2}(c_c - c_p) - \tfrac{d^2}{4}\right]$. Also define regions $(\varepsilon 1)$ through $(\varepsilon 4)$ as follows:

$$(\varepsilon 1) \equiv \ \{\ 0 < c_c - c_p < d \ , \ f < \breve{f} \ \}$$

$$(\varepsilon 2) \equiv \ \{\ 0 < c_c - c_p < d \ , \ \breve{f} \leq f < \tfrac{\lambda(\bar{p} - c_p)^2}{2d} \ \} \text{ or } \{\ -\tfrac{d}{2} < c_c - c_p \leq 0 \ , \ f < \tfrac{\lambda(\bar{p} - c_p)^2}{2d} \ \}$$

$$(\varepsilon 3) \equiv \ \{\ -\tfrac{d}{2} < c_c - c_p < d \ , \ f = \tfrac{\lambda(\bar{p} - c_p)^2}{2d} \ \}$$

$$(\varepsilon 4) \equiv \ \{\ -\tfrac{d}{2} < c_c - c_p < d \ , \ f > \tfrac{\lambda(\bar{p} - c_p)^2}{2d} \ \}$$

where $\breve{f}$ uniquely satisfies $\frac{\lambda(c_c - c_p)}{6d}(2c_c - c_p - \bar{p}) \leq \breve{f} < \frac{\lambda(c_c - c_p)^2}{2d}$ and $(3\bar{p} - 2c_c - c_p) \cdot \sqrt{2\lambda d\breve{f}} - d\breve{f} = 2\lambda(\bar{p} - c_c)(c_c - c_p) + \tfrac{\lambda}{2}(\bar{p} - c_p)(2c_c - c_p - \bar{p})$.

Proposition 5 *For MP competing with MC, having chosen $f > 0$ in the first stage, the optimal product variety and the resulting price equilibrium and market shares are:*

Case 1: *High reservation price:* $\bar{p} > \frac{d}{3} + \frac{2c_c + c_p}{3}$

Region	Variety, $\hat{n}$	Price equilibrium, $(\hat{p}_c, \hat{p}_p)$	Market shares, $(y_c, \hat{n}y_p)$
(δ1)	$\sqrt{\frac{\lambda d}{2f}}$	$\left(c_c, c_c - \sqrt{\frac{df}{2\lambda}}\right)$	$(0, 1)$
(δ2)	1	$\left(c_c, c_c - \frac{d}{2}\right)$	$(0, 1)$
(δ3)	1	$\left(\frac{d}{3} + \frac{2c_c + c_p}{3}, \frac{d}{6} + \frac{c_c + 2c_p}{3}\right)$	$\left(\frac{2}{3} - \frac{2(c_c - c_p)}{3d}, \frac{1}{3} + \frac{2(c_c - c_p)}{3d}\right)$
(δ4)	1	(c_p, c_p)	$(1, 0)$

Case 2: *Low reservation price:* $\bar{p} \leq \frac{d}{3} + \frac{2c_c + c_p}{3}$

Region	Variety, $\hat{n}$	Price equilibrium, $(\hat{p}_c, \hat{p}_p)$	Market shares, $(y_c, \hat{n}y_p)$
(ε1)	$\sqrt{\frac{\lambda d}{2f}}$	$\left(c_c, c_c - \sqrt{\frac{df}{2\lambda}}\right)$	$(0, 1)$
(ε2)	$\frac{d}{3\bar{p} - 2c_c - c_p}$	$\left(\bar{p}, \frac{\bar{p} + c_p}{2}\right)$	$\left(\frac{2(\bar{p} - c_c)}{3\bar{p} - 2c_c - c_p}, \frac{\bar{p} - c_p}{3\bar{p} - 2c_c - c_p}\right)$
(ε3)	$\hat{n} \in \left[1, \frac{d}{3\bar{p} - 2c_c - c_p}\right]$	$\left(\bar{p}, \frac{\bar{p} + c_p}{2}\right)$	$\left(1 - \frac{\hat{n}(\bar{p} - c_p)}{d}, \frac{\hat{n}(\bar{p} - c_p)}{d}\right)$
(ε4)	1	$\left(\bar{p}, \frac{\bar{p} + c_p}{2}\right)$	$\left(1 - \frac{\bar{p} - c_p}{d}, \frac{\bar{p} - c_p}{d}\right)$

where region (ε3) yields multiple optima. (Profit expressions, $\pi_c^{duop}(f) \equiv \pi_c(\hat{p}_c, f, \hat{n}, \hat{p}_p)$ and $\pi_p^{duop}(f) \equiv \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$, are provided in a separate appendix.)

Regardless of the reservation price $\bar{p}$, the variable cost differential between MC and MP must be small enough for both firms to attract positive market share. MP's optimal variety $\hat{n}$ is decreasing in f, and increasing in λ and d.

4.4 Entry Game

Having determined variety and price(s) under monopoly and duopoly, we now analyze the entry game, our ultimate objective. In the entry game, both firms decide whether to enter the market, and MP sets f if it does enter. The entry game can be summarized as follows:

$MC \setminus MP$	enter with $f \geq 0$	do not enter
enter	$\pi_c^{duop}(f)$, $\pi_p^{duop}(f)$	π_c^{monop} , 0
do not enter	0 , $\pi_p^{monop}(f)$	0 , 0

Therefore, each firm needs to determine a best response to no entry or entry (with a choice of f in the case of MP) by the other firm. Depending on how monopoly and duopoly profits compare to not entering the market (i.e., zero profit), there may be unique or multiple equilibria in this game. We adopt the convention that the firms enter the market when they are indifferent between entering and not entering.

In the absence of entry by MC, MP sets its marginal cost of variety f so as to maximize $\pi_p^{monop}(f) = \pi_p(f, n^*, p_p^*)$ given in Proposition 3. If MC enters, on the other hand, MP maximizes $\pi_p^{duop}(f) = \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$, which is a result of Proposition 5 and is given in a separate appendix. The entry game equilibrium can be fully characterized by solving these two problems. In Proposition 6 below, we only present the results for the duopoly competition outcome, in which MP and MC coexist in the market at equilibrium. Details of how to characterize the remaining outcomes of the entry game are provided in the proof of Proposition 6 given in a separate appendix.

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Proposition 6 *MP and MC coexist in the market* iff *the parameter values satisfy one of two mutually exclusive regions:*

<div align="center">Equilibrium Outcome: Duopoly Competition</div>

Parameter Regions	Cost of Variety, $\hat{f}$	Variety, $\hat{n}$	Price Equilibrium, $(\hat{p}_c, \hat{p}_p)$
$(v3) \quad \pi_p^{duop}(\hat{f}), \pi_c^{duop}(\hat{f}) \geq 0$	$\max\left\{\breve{f}, f_3\right\}$	$\frac{d}{3\bar{p}-2c_c-c_p}$	$\left(\bar{p}, \frac{\bar{p}+c_p}{2}\right)$
$(v4) \quad \pi_p^{duop}(\hat{f}), \pi_c^{duop}(\hat{f}) \geq 0$	$\max\left\{\theta - \gamma, \bar{f}\right\}$	1	$\left(\frac{d}{3} + \frac{2c_c+c_p}{3}, \frac{d}{6} + \frac{c_c+2c_p}{3}\right)$

where $f_3 \equiv \frac{\theta}{d}\left(3\bar{p} - 2c_c - c_p\right) - \gamma$, *and the parameter regions are defined as:*

$$(v3) \equiv \quad \bar{p} \leq \frac{d}{3} + \frac{2c_c+c_p}{3} \ , \ \{ \ [\ 0 < c_c - c_p \leq d \ , \ \breve{f} > 0 \ ]$$

$$or \ [\ -\frac{d}{2} \leq c_c - c_p \leq d \ , \ 0 < f_3 < \frac{\lambda(\bar{p}-c_p)^2}{2d} \ ] \ \}$$

$$(v4) \equiv \quad \bar{p} > \frac{d}{3} + \frac{2c_c+c_p}{3} \ , \ \{ \ [\ 0 < c_c - c_p \leq d \ , \ \bar{f} > 0 \ ]$$

$$or \ [\ -\frac{d}{2} \leq c_c - c_p \leq d \ , \ \theta > \gamma \ ] \ \}$$

(Profit expressions, $\pi_c^{duop}(\hat{f}) = \pi_c(\hat{p}_c, \hat{f}, \hat{n}, \hat{p}_p)$ and $\pi_p^{duop}(\hat{f}) = \pi_p(\hat{p}_c, \hat{f}, \hat{n}, \hat{p}_p)$, are provided in a separate appendix.)

Observe that MP only offers multiple products in duopoly when the reservation price is relatively low; in other words, smaller profit margins lead to higher variety. Also, MP never invests in perfect product-mix flexibility ($f = 0$) in a duopoly outcome. This is because, if MP emulated MC by choosing $f = 0$ and offering infinite variety, MP and MC would lock themselves in a pure-price competition: the firm with higher unit cost is unable to capture any of the market, earns no revenue, hence makes a negative profit.

5 Discussion

5.1 Competition, Price and Variety

Consider a world in which F_c is sufficiently high (and F_p sufficiently low) that MP monopolizes the market. Suppose, over time, production technology improves so that F_c drops significantly; and suppose this moves the market to a duopoly. Comparing the scenario with high F_c (monopoly of MP) to the scenario with low F_c (duopoly), we observe the following. First, prices are lower under duopoly than under monopoly, i.e. $\hat{p}_c \leq p_c^*$ and $\hat{p}_p \leq p_p^*$. Note that this is not an obvious result in light of the fact that firms seek to soften the price competition in duopoly equilibrium, our next observation.

Second, and more importantly, MP chooses to be *less* flexible and offers *lower* product variety under duopoly competition than under monopoly, i.e. $\hat{f} \geq f^*$ and $\hat{n} \leq n^*$. Naively, one might expect MP to increase its product variety to better compete against MC. However, the price equilibrium result of Proposition 4 indicates that, in the regions $(\eta 2)$ and $(\eta 3)$ where both firms may enter, higher variety leads to increased price competition, hence lower prices. Therefore, MP prefers to cut product variety in order to soften the price competition at the third stage. Note that in significant portions of the parameter space, the inequality $\hat{n} < n^*$ is strict, e.g. when region $(\upsilon 4)$ overlaps with $(\tau 1)$ or $(\tau 2)$, regions that are relevant for the MP monopoly outcome (given in a separate appendix).

This result extends a recurring theme from the economics and marketing literatures to the OM literature: the incentive to soften price competition often trumps other considerations. For instance, in a model of horizontal product differentiation under shopping costs, Klemperer (1992) shows how exactly-matching ("head-to-head") product lines may

help soften price competition. In a model of vertical product differentiation with two firms, each offering an interval of quality on $[0,\infty)$, Champsaur and Rochet (1989) show that the firms leave a gap between their product lines to relax price competition. Balachander and Farquhar (1994) study fixed-location price competition in a Hotelling framework with two firms setting their product availability, modeled as the probability of serving all customers or none. If shopping costs are so low that customers choose to visit the other store when faced with a stockout, the firms allow occasional stockouts, again to soften price competition.

5.2   Coexistence or Dominance?

For the duopoly outcome to occur, both firms must earn a non-negative profit when they both enter the market. From Proposition 6, this occurs only in two cases:

	MP's optimal variety, $\hat{n}$	**Price equilibrium**, $(\hat{p}_c, \hat{p}_p)$
Case 1: region $(v3)$	$\frac{d}{3\bar{p}-2c_c-c_p}$	$\left(\bar{p}, \frac{\bar{p}+c_p}{2}\right)$
Case 2: region $(v4)$	1	$\left(\frac{d}{3}+\frac{2c_c+c_p}{3}, \frac{d}{6}+\frac{c_c+2c_p}{3}\right)$

Note that MP offers multiple products only when the reservation price is relatively low (case 1), and offers a single product when the reservation price is high (case 2). In both cases of the duopoly outcome, the difference between c_c and c_p must be small relative to d, i.e., $-\frac{d}{2} \leq c_c - c_p \leq d$. These bounds are skewed in favor of MC: MP drops out with a unit cost disadvantage of $d/2$, whereas MC can withstand a unit cost disadvantage of up to d. Also, MC charges a higher price and captures a larger market share than MP, i.e. $\hat{p}_c \geq \hat{p}_p$ and $y_c \geq \hat{n}\, y_p$.

A key question is whether a finite-variety MP can survive competition from an infinite-variety MC. Depending on the relative magnitude of the firms' fixed and variable costs, MP

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can often profitably compete. This may not be a surprise when MP has lower fixed and variable costs than MC. However, contrary to basic intuition, MP *can* profitably compete even if MC has a variable production cost advantage, as long as that advantage is not too large, i.e. $c_p - \frac{d}{2} < c_c < c_p$. This can occur in either region, $(v3)$ or $(v4)$. Also, MP *can* still profitably compete even if MC also has a lower fixed cost, i.e. $F_p(f) > F_c$ for all f.

It is a surprising conclusion that MP can coexist with MC despite simultaneous fixed and variable cost disadvantages. The underlying reason is that even if MC has a lower variable cost, it may choose not to lower its uniform price enough to deter entry by MP. Cutting price would entice a few MP customers to switch to MC, but would reduce MC's revenue in the rest of the market. This hinges on our focus on horizontal differentiation and hence MC's inability to price-discriminate, similar to the situation in De Groote (1994) and Balasubramanian (1998). If we do let an MC with a variable cost advantage charge location-specific prices, it could always undercut MP's delivered price by a small margin, hence rendering the competitive analysis entirely uninteresting. Assuming uniform prices would be questionable in the case of vertical differentiation, but is consistent with the horizontal differentiation setting studied here.

MC having fixed and variable cost advantages may not be a common occurrence, but does happen, as illustrated by Dell. Our finding that an MP with a variable cost disadvantage can coexist with an MC is consistent with the literature on contestable markets, although the context is quite different. Gelman and Salop (1983) show that an incumbent may let an entrant with no cost advantage survive if the latter can commit to limiting capacity. In that case, it is better for the incumbent to surrender a small part of the market to the entrant, rather than driving him out by offering a lower price to the entire market. Drawing the

analogy with our work would require treating the MC as the incumbent and viewing MP's choice of variety as a form of capacity commitment.

5.3 When Does Higher Variety Imply Higher Prices?

The prevailing view in the OM community seems to be that higher product variety always allows charging higher prices, because perceived value to customers increases as product variety grows. In the monopoly analysis for MP, higher variety does indeed imply (weakly) higher prices (Proposition 2). However, under duopoly, a higher variety offered by MP may result in a lower, equal, or higher equilibrium price (Proposition 4). Therefore, our analysis points out that whether higher product variety implies higher prices depends on the prevailing competitive structure (monopoly versus duopoly) and relative costs of different production technologies. This is in agreement with the finding by Norman and Thisse (1999) that introduction of FMS technology, which makes higher product variety economical, can lead to more aggressive pricing between competitors.

In a widely-cited study based on the PIMS (Profit Impact of Marketing Strategies) database, Kekre and Srinivasan (1990) hypothesized that "a broader product line may lead to higher relative prices," but found only marginal support in both industrial and consumer markets. In contrast, our study offers a theoretical explanation for observing higher variety and lower prices concurrently (under competition), though further study would be needed to verify whether this effect does indeed contribute to Kekre and Srinivasan's finding.

6 Concluding Remarks

We model competition between two multi-product firms on the basis of product variety and price, where one firm is a mass customizer with infinite variety, while the other is a mass producer with finite variety. We construct a three-stage game using Hotelling's (1929) model of horizontal differentiation. First, both firms decide whether to enter the market, and in MP's case, how much to invest in product-mix flexibility; next, MP designs its product line; and then both firms set prices. We characterize the sub-game perfect Nash equilibria of this game, including a full analysis of when either or both firms decide to enter. We find that MP will *decrease* flexibility and variety upon entry by MC. This reflects the firms' desire to soften the price competition under duopoly. Also, the finite-variety MP *can* profitably compete even if it has a disadvantage against the infinite-variety MC in fixed cost of production technology, in variable cost of production, or both.

We contribute to the product variety literature in several ways. First, we introduce a supply side element (production technology) to the study of product differentiation, conducted so far mostly from a demand perspective. Our second contribution is to model an MC's horizontal product differentiation in a Hotelling framework. Third, we partly contradict a prevailing view in OM, by finding that MP in a duopoly may wish to *reduce* product variety (compared to an MP monopoly) in order to soften price competition with MC.

A more streamlined analysis, focusing only on one or a few of the cases identified in the analysis, would not permit a full comparison between the outcomes under monopoly and duopoly, and would not reveal the full range of possible interactions between variety and price. This illustrates the value of providing a complete treatment as we do here, despite

the resulting tediousness of the analysis.

Our model has several limitations. First, we do not explicitly model the entry deterrence effect of product variety (Schmalensee 1978, Salop 1979, Heal 1980). This would require a model of sequential entry, where MP (the incumbent) may offer excessively high product variety as a deterrent against the threat of entry by MC. Second, production technology is exogenous for MC. If both firms were allowed to choose their production technology, we would need to know the equilibrium product line design and pricing policies of two firms with finite multiple products. This is an open problem in the economics literature, but the prevailing conjecture is that location-price equilibrium does not exist even for the case of fixed numbers of products (Klemperer 1992, Teitz 1968). An alternative approach would be to simply postulate a specific variety and pricing strategy for both firms, but this would no longer be a true equilibrium analysis.

This paper offers a step towards understanding supply- and demand-side forces that shape product strategy decisions of multi-product firms. A useful direction for future research is to explore the robustness of our findings under other models of product differentiation, such as the representative consumer or random utility models (Anderson et al. 1992).

7   Appendix

All proofs are provided in a separate document that can be accessed online at http://~/.

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Online Appendix for
"Mass Customization versus Mass Production: Variety and Price Competition"

Proof of Proposition 1. First, the market segment captured by a standard product is contiguous: if we take any two locations served by a product, all locations in between must also be served by the same product. Second, the delivered price at boundary points between any two adjacent market segments and at end points (0 and 1) must be at least $\bar{p}$. Suppose not. In both cases (interior point or end point) the associated product(s) can be replaced by products that are priced higher and that cover exactly the same market area(s). Third, market segments captured by standard products must be of equal length and all products must be priced the same. Suppose not; i.e., in MP's optimal product line design, there exist two neighboring products that cover market segments of unequal length a and $l-a$ ($a \neq l/2$), and that are priced differently at $p_1 = \bar{p} - da/2$ and $p_2 = \bar{p} - d(l-a)/2$, respectively. (Higher prices would not allow the given segment lengths; lower prices are not optimal.) Without loss of generality, assume $0 < a < l - a < l < 1$. MP's profit from these two products is $\triangle \pi_p(a) = (\bar{p} - c_p)\lambda l - \frac{\lambda d}{2}\left[a^2 + (l-a)^2\right]$. We can rearrange these two segments without affecting any other segment by varying a. Solving $\max_{0 \leq a \leq l/2} \triangle \pi_p(a)$ gives $a^* = l/2$. That is, MP is better off by replacing the two unequal segments with two equal segments. This is accomplished by equalizing the prices at $p_1 = p_2 = \bar{p} - dl/4$ while keeping the total coverage of the market segments the same. This improvement affects no other segments. Therefore the optimal product line design cannot have unequal segments. There may be alternative

optima for the product locations. The vector of locations symmetric around the center,
$\mathbf{x} = \left(\frac{1}{2n}, \frac{3}{2n}, \cdots, \frac{2n-1}{2n}\right)$, is optimal; but in some regions of parameter values, $\mathbf{x}$ (or subsets
of $\mathbf{x}$) can be shifted within the bounds of the product space (at most by $\pm \left|\frac{1}{2n} - \left(\frac{\bar{p}-p_1}{d}\right)\right|$)
without reducing profits. ∎

Proof of Proposition 2. MP's pricing problem can be restated as: $\max\limits_{p_p} \pi_p(f, n, p_p) =$
$(p_p - c_p)\lambda n y_p - nf - F_p(f)$ subject to $\bar{p} - d/2n \leq p_p \leq \bar{p}$ and $y_p = 2(\bar{p} - p_p)/d$. The expression
for y_p in the second constraint is valid only for the range of prices in the first constraint.
This restriction is justified by two simple observations. First, MP would never reduce p_p
below $\bar{p} - d/2n$ because it already captures the entire market at that price. Second, with
$p_p > \bar{p}$, MP attracts no demand. Substituting y_p, as defined by the second constraint above,
into π_p, one can verify that π_p is concave in p_p. The result then follows from the Lagrangean
method. ∎

Proof of Proposition 3. Our solution strategy is to decompose the problem into two
sub-problems based on the two regions of the objective function, solve them separately, and
combine the solutions. The two sub-problems are:

$(M1)$: $\max \pi_p(f, n, p_p^*) = (\bar{p} - c_p - \frac{d}{2n})\lambda - nf - F_p(f)$ subject to $n \geq \frac{d}{\bar{p}-c_p}$ and $n \geq 1$.

$(M2)$: $\max \pi_p(f, n, p_p^*) = \frac{\lambda n}{2d}(\bar{p} - c_p)^2 - nf - F_p(f)$ subject to $n \leq \frac{d}{\bar{p}-c_p}$ and $n \geq 1$.

Let n_1^* and n_2^* be the optimal solutions to $(M1)$ and $(M2)$. The objective function of $(M1)$
is strictly concave in n, while that of $(M2)$ is linear in n. The Lagrangean method gives:

$$
n_1^* = \begin{cases} \sqrt{\frac{\lambda d}{2f}} & \text{, if } f < \min\left\{\frac{\lambda d}{2}, \frac{\lambda(\bar{p}-c_p)^2}{2d}\right\} \\ \max\left(1, \frac{d}{\bar{p}-c_p}\right) & \text{, otherwise} \end{cases}
$$

$$n_2^* = \begin{cases} \frac{d}{\bar{p}-c_p} & \text{, if } f < \frac{\lambda(\bar{p}-c_p)^2}{2d} \text{ and } \bar{p} \le c_p + d \\[3mm] \in \left[1, \frac{d}{\bar{p}-c_p}\right] & \text{, if } f = \frac{\lambda(\bar{p}-c_p)^2}{2d} \text{ and } \bar{p} \le c_p + d \\[3mm] 1 & \text{, if } f > \frac{\lambda(\bar{p}-c_p)^2}{2d} \text{ and } \bar{p} \le c_p + d \end{cases}$$

The result follows from combining these, choosing the better solution in case of overlap. ∎

Proof of Proposition 4. Suppose that MP knows MC's price, p_c ($c_c \le p_c \le \bar{p}$), hence solves the following problem to set its own price:

$$\max_{c_p \le p_p \le \bar{p}} \quad \pi_p(p_c, f, n, p_p) = (p_p - c_p)\lambda n y_p - nf - F_p(f)$$

$$st. \quad y_p = \begin{cases} 1/n & \text{, if } p_p \le p_c - d/2n \\[2mm] 2(p_c - p_p)/d & \text{, if } p_c - d/2n \le p_p \le p_c \\[2mm] 0 & \text{, if } p_p \ge p_c \end{cases}$$

Employing the same arguments as in Proposition 2, this problem can be further restricted to the mid-region where $p_c - d/2n \le p_p \le p_c$. Using the Lagrangean method, we obtain MP's best response function:

$$p_p^*(n, p_c) = \begin{cases} p_c - d/2n & \text{, if } p_c \ge c_p + d/n \\[2mm] (p_c + c_p)/2 & \text{, if } c_p < p_c < c_p + d/n \\[2mm] \in [c_p, \bar{p}] & \text{, if } p_c \le c_p \end{cases}$$

Suppose now that MC knows MP's price, p_p ($c_p \le p_p \le \bar{p}$), hence solves:

$$\max_{c_c \le p_c \le \bar{p}} \quad \pi_c(p_c, f, n, p_p) = (p_c - c_c)\lambda y_c - F_c$$

$$st. \quad y_c = \begin{cases} 1 & \text{, if } p_c \le p_p \\[2mm] 1 - 2n(p_c - p_p)/d & \text{, if } p_p \le p_c \le p_p + d/2n \text{ and } p_c \le \bar{p} \\[2mm] 0 & \text{, if } p_p + d/2n \le p_c \le \bar{p} \end{cases}$$

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Using exactly the same approach, we obtain MC's best response function:

$$
p_c^*(n, p_p) = \begin{cases}
p_p & , \text{ if } p_p \geq c_c + d/2n \\[2mm]
\frac{d}{4n} + \frac{p_p + c_c}{2} & , \text{ if } c_c - d/2n < p_p < c_c + d/2n \ , \ p_p < 2\bar{p} - c_c - d/2n \\[2mm]
\bar{p} & , \text{ if } p_p \geq 2\bar{p} - c_c - d/2n \\[2mm]
\in [c_c, \bar{p}] & , \text{ if } p_p \leq c_c - d/2n
\end{cases}
$$

Given the two best response functions, we find the equilibrium pairs $(\hat{p}_c, \hat{p}_p)$ by solving

$\hat{p}_p = p_p^*(n, \hat{p}_c)$ and $\hat{p}_c = p_c^*(n, \hat{p}_p)$ simultaneously. ∎

Proof of Proposition 5. We decompose the problem into four sub-problems based on

the regions $(\eta 1)$ - $(\eta 4)$, solve these separately, and combine their solutions. The sub-problem

that corresponds to region (ηk), labeled (Ck) for $k = 1, 2, 3, 4$, can be stated as follows.

$(C1)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = \lambda(c_c - c_p - \frac{d}{2n}) - nf - F_p(f)$ st. $n \geq \frac{d}{c_c - c_p}$ and $n \geq 1$.

$(C2)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = \frac{\lambda n}{2d}(\bar{p} - c_p)^2 - nf - F_p(f)$ st. $1 \leq n \leq \frac{d}{3\bar{p} - 2c_c - c_p}$.

$(C3)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = \frac{2\lambda n}{9d}(\frac{d}{2n} + c_c - c_p)^2 - nf - F_p(f)$ st. $1 \leq n \leq \max\left[\frac{d}{c_c - c_p}, \frac{-d}{2(c_c - c_p)}\right]$

and $n \geq \frac{d}{3\bar{p} - 2c_c - c_p}$.

$(C4)$: $\max \pi_p(\hat{p}_c, f, n, \hat{p}_p) = -nf - F_p(f)$ st. $n \geq \frac{-d}{2(c_c - c_p)}$ and $n \geq 1$.

It is easy to see that the objective functions of $(C1)$, $(C2)$, $(C3)$ and $(C4)$ are strictly

concave, linear, strictly convex and linear in n, respectively. The Lagrangean method yields

the following, where $\hat{n}_k$ represents the optimal solution for (Ck):

$$
\hat{n}_1 = \begin{cases}
\sqrt{\frac{\lambda d}{2f}} & , \text{ if } c_c - c_p > 0 \text{ and } f < \frac{\lambda d}{2} \text{ and } f < \frac{\lambda(c_c - c_p)^2}{2d} \\[2mm]
\frac{d}{c_c - c_p} & , \text{ if } 0 < c_c - c_p \leq d \text{ and } f \geq \frac{\lambda(c_c - c_p)^2}{2d} \\[2mm]
1 & , \text{ if } c_c - c_p > d \text{ and } f \geq \frac{\lambda d}{2}
\end{cases}
$$

$$\hat{n}_2 = \begin{cases} \frac{d}{3\bar{p}-2c_c-c_p} & \text{, if } 3\bar{p}-2c_c-c_p \le d \text{ and } f < \frac{\lambda(\bar{p}-c_p)^2}{2d} \\[2ex] \in \left[1, \frac{d}{3\bar{p}-2c_c-c_p}\right] & \text{, if } 3\bar{p}-2c_c-c_p \le d \text{ and } f = \frac{\lambda(\bar{p}-c_p)^2}{2d} \\[2ex] 1 & \text{, if } 3\bar{p}-2c_c-c_p \le d \text{ and } f > \frac{\lambda(\bar{p}-c_p)^2}{2d} \end{cases}$$

$$\hat{n}_3 = \begin{cases} \frac{d}{c_c-c_p} & \text{, if } \{\, 0 < c_c - c_p \le d \,,\, 3\bar{p}-2c_c-c_p \le d \,,\, f < \frac{\lambda(c_c-c_p)^2}{2d} \,\} \\[1ex] & \quad \text{or } \{\, 0 < c_c - c_p \le d \,,\, 3\bar{p}-2c_c-c_p > d \,, \\[1ex] & \qquad \text{and } f < \frac{2\lambda(c_c-c_p)}{9d}(c_c - c_p - \frac{d}{4}) \,\} \\[2ex] \frac{d}{3\bar{p}-2c_c-c_p} & \text{, if } \{\, 0 < c_c - c_p \le d \,,\, 3\bar{p}-2c_c-c_p \le d \,,\, f \ge \frac{\lambda(c_c-c_p)^2}{2d} \,\} \\[1ex] & \quad \text{or } \{\, -\frac{d}{2} \le c_c - c_p \le 0 \,,\, 3\bar{p}-2c_c-c_p \le d \,\} \\[2ex] 1 & \text{, if } \{\, 0 < c_c - c_p \le d \,,\, 3\bar{p}-2c_c-c_p > d \,, \\[1ex] & \qquad \text{and } f \ge \frac{2\lambda(c_c-c_p)}{9d}(c_c - c_p - \frac{d}{4}) \,\} \\[1ex] & \quad \text{or } \{\, -\frac{d}{2} \le c_c - c_p \le 0 \,,\, 3\bar{p}-2c_c-c_p > d \,\} \end{cases}$$

$$\hat{n}_4 = \begin{cases} \frac{d}{2(c_p-c_c)} & \text{, if } -\frac{d}{2} < c_c - c_p < 0 \\[2ex] 1 & \text{, if } c_c - c_p \le -\frac{d}{2} \end{cases}$$

Combining these four solutions by choosing the best one wherever they overlap, we obtain the optimal number of standard products under duopoly competition. The resulting optimal variety by MP is given in the proposition under four parameter regions; associated profit expressions are as follows:

Case 1: High reservation price: $\bar{p} > \frac{d}{3} + \frac{2c_c + c_p}{3}$

Region	$\pi_c^{duop}(f) \equiv \pi_c(\hat{p}_c, f, \hat{n}, \hat{p}_p)$	$\pi_p^{duop}(f) \equiv \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$
$(\delta 1)$	$-F_c$	$\lambda\left(c_c - c_p\right) - \sqrt{2\lambda df} - F_p(f)$
$(\delta 2)$	$-F_c$	$\lambda\left(c_c - c_p - \frac{d}{2}\right) - f - F_p(f)$
$(\delta 3)$	$\frac{2\lambda}{9d}\left[d - (c_c - c_p)\right]^2 - F_c$	$\frac{2\lambda}{9d}\left(c_c - c_p + \frac{d}{2}\right)^2 - f - F_p(f)$
$(\delta 4)$	$\lambda\left(c_p - c_c\right) - F_c$	$-f - F_p(f)$

Case 2: Low reservation price: $\bar{p} \leq \frac{d}{3} + \frac{2c_c + c_p}{3}$

Region	$\pi_c^{duop}(f) \equiv \pi_c(\hat{p}_c, f, \hat{n}, \hat{p}_p)$	$\pi_p^{duop}(f) \equiv \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$
$(\varepsilon 1)$	$-F_c$	$\lambda\left(c_c - c_p\right) - \sqrt{2\lambda df} - F_p(f)$
$(\varepsilon 2)$	$\frac{2\lambda(\bar{p} - c_c)^2}{3\bar{p} - 2c_c - c_p} - F_c$	$\frac{\lambda(\bar{p} - c_p)^2}{2(3\bar{p} - 2c_c - c_p)} - \frac{df}{(3\bar{p} - 2c_c - c_p)} - F_p(f)$
$(\varepsilon 3)$	$\lambda(\bar{p} - c_c)\left(1 - \frac{\hat{n}(\bar{p} - c_p)}{d}\right) - F_c$	$-F_p(f)$
$(\varepsilon 4)$	$\lambda(\bar{p} - c_c)\left(1 - \frac{\bar{p} - c_p}{d}\right) - F_c$	$\frac{\lambda(\bar{p} - c_p)^2}{2d} - f - F_p(f)$

∎

Proof of Proposition 6. We first characterize MP's best response to no entry by MC. In this case, MP sets its marginal cost of variety f so as to maximize $\pi_p^{monop}(f) = \pi_p(f, n^*, p_p^*)$ given in Proposition 3. We decompose the problem into two sub-problems based on the regions $(\mu 1)$ and $(\mu 2)$, the only regions in Proposition 3 that can lead to a non-negative profit for MP. The two sub-problems - labeled $(D1)$ and $(D2)$, respectively - can be stated as follows.

$(D1)$: $\pi_p(f, n^*, p_p^*) = \lambda\left(\bar{p} - c_p\right) - \sqrt{2\lambda df} - F_p(f)$ st. $(\mu 1)$.

$(D2)$: $\pi_p(f, n^*, p_p^*) = \lambda\left(\bar{p} - c_p - \frac{d}{2}\right) - f - F_p(f)$ st. $(\mu 2)$.

The objective functions of $(D1)$ and $(D2)$ are decreasing-increasing-decreasing and strictly

concave in f, respectively. Therefore, for $(D1)$ the larger of two roots (if they exist) and for

$(D2)$ the unique root (if it exists) of the first order condition gives the respective solutions,

$f^*_{(\mu 1)}$ and $f^*_{(\mu 2)}$ below. The result obtains when these solutions are combined by choosing the

better one wherever they overlap.

$$
f^*_{(\mu 1)} = \begin{cases} 0 & , \text{ if } \frac{\theta^2}{\lambda d} \leq 2\gamma \text{ or } \{ \frac{\theta^2}{\lambda d} > 2\gamma \text{ and } \ln(\gamma + f_0) - \ln(\gamma) < \frac{2f_0}{\gamma + f_0} \} \\[2mm] f_1 & , \text{ if } \frac{\theta^2}{\lambda d} > 2\gamma \text{ and } \ln(\gamma + f_0) - \ln(\gamma) \geq \frac{2f_0}{\gamma + f_0} \end{cases}
$$

$$
f^*_{(\mu 2)} = f_2 \equiv \max \left\{ \frac{\lambda d}{2}, \theta - \gamma \right\}
$$

where $f_1 \equiv \min \left\{ \frac{\lambda d}{2}, \frac{\lambda(\bar{p} - c_p)^2}{2d}, \frac{\theta^2}{\lambda d} - \gamma + \sqrt{\left(\frac{\theta^2}{\lambda d}\right)^2 - 2\gamma \left(\frac{\theta^2}{\lambda d}\right)} \right\}$.

As a result, MP's best response to no entry by MC is to enter the market and choose cost

of variety f^* if $\pi_p^{monop}(f^*) \geq 0$; MP's optimal decisions and profits are:

Region	Cost of Variety, f^*	Variety, n^*	Price, p_p^*
$(\tau 1)$	0	∞	$\bar{p}$
$(\tau 2)$	f_1	$\sqrt{\frac{\lambda d}{2 f_1}}$	$\bar{p} - \sqrt{\frac{d f_1}{2\lambda}}$
$(\tau 3)$	f_2	1	$\bar{p} - \frac{d}{2}$

Region	MP's Monopoly Profit, $\pi_p^{monop}(f^*) = \pi_p(f^*, n^*, p_p^*)$
$(\tau 1)$	$\lambda(\bar{p} - c_p) - \kappa + \theta \ln(\gamma)$
$(\tau 2)$	$\lambda(\bar{p} - c_p) - \kappa - \theta \left[\frac{2 f_1}{\gamma + f_1} - \ln(\gamma + f_1) \right]$
$(\tau 3)$	$\lambda \left(\bar{p} - c_p - \frac{d}{2}\right) - f_2 - [\kappa - \theta \ln(\gamma + f_2)]$

where $f_1 \equiv \min \left\{ \frac{\lambda d}{2}, \frac{\lambda(\bar{p} - c_p)^2}{2d}, \frac{\theta^2}{\lambda d} - \gamma + \sqrt{\left(\frac{\theta^2}{\lambda d}\right)^2 - 2\gamma \left(\frac{\theta^2}{\lambda d}\right)} \right\}$, $f_2 \equiv \max \left\{ \frac{\lambda d}{2}, \theta - \gamma \right\}$, $0 \leq f_1 \leq$

f_2, and the parameter regions are defined as:

$$(\tau 1) \equiv \ \{\ \theta \leq \sqrt{2\lambda d\gamma} \text{ or } [\ \theta > \sqrt{2\lambda d\gamma} \ , \ \ln(\gamma + f_1) - \ln(\gamma) < \tfrac{2f_1}{\gamma + f_1}\ ]\ \}$$

$$\text{and } \{\ \bar{p} \leq c_p + d \text{ or } [\ \bar{p} > c_p + d \ , \ \ln(\gamma + f_2) - \ln(\gamma) < \tfrac{f_2 + \lambda d/2}{\theta}\ ]\ \}$$

$$(\tau 2) \equiv \ \{\ \theta > \sqrt{2\lambda d\gamma} \ , \ \ln(\gamma + f_1) - \ln(\gamma) \geq \tfrac{2f_1}{\gamma + f_1}\ \} \text{ and } \{\ \bar{p} \leq c_p + d \text{ or }$$

$$[\ \bar{p} > c_p + d \ , \ \ln(\gamma + f_2) - \ln(\gamma) < \tfrac{f_2 + \lambda d/2}{\theta} - \tfrac{2f_1}{\gamma + f_1}\ ]\ \}$$

$$(\tau 3) \equiv \ \bar{p} > c_p + d \ , \text{ and}$$

$$\{\ \{\ \{\ \theta \leq \sqrt{2\lambda d\gamma} \text{ or } [\ \theta > \sqrt{2\lambda d\gamma} \ , \ \ln(\gamma + f_1) - \ln(\gamma) < \tfrac{2f_1}{\gamma + f_1}\ ]\ \}$$

$$\text{and } \ln(\gamma + f_2) - \ln(\gamma) \geq \tfrac{f_2 + \lambda d/2}{\theta}\ \}$$

$$\text{or } \{\ \theta > \sqrt{2\lambda d\gamma} \ , \ \ln(\gamma + f_1) - \ln(\gamma) \geq \tfrac{2f_1}{\gamma + f_1}\ ,$$

$$\text{and } \ln(\gamma + f_2) - \ln(\gamma) \geq \tfrac{f_2 + \lambda d/2}{\theta} - \tfrac{2f_1}{\gamma + f_1}\ \}\ \}$$

Next, we characterize MP's best response to entry by MC. In this case, MP maximizes $\pi_p^{duop}(f) = \pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p)$, which is a result of Proposition 5 and is given in its proof. We decompose MP's problem into two sub-problems based on the regions $(\delta 3)$ and $(\varepsilon 2)$, the only regions in Proposition 5 that can lead to non-negative profits for both firms. The two sub-problems - labeled $(E1)$ and $(E2)$, respectively - can be stated as follows.

$(E1)$: $\pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p) = \frac{2\lambda}{9d}\left(c_c - c_p + \frac{d}{2}\right)^2 - f - F_p(f)$ st. $(\delta 3)$.

$(E2)$: $\pi_p(\hat{p}_c, f, \hat{n}, \hat{p}_p) = \frac{\lambda(\bar{p} - c_p)^2}{2(3\bar{p} - 2c_c - c_p)} - \frac{df}{(3\bar{p} - 2c_c - c_p)} - F_p(f)$ st. $(\varepsilon 2)$.

The objective functions of $(E1)$ and $(E2)$ are both concave in f. Therefore, the first order condition is necessary and sufficient for global optimality of each sub-problem; and gives the respective solutions, $\hat{f}_{(\delta 3)}$ and $\hat{f}_{(\varepsilon 2)}$ below, which yield the result when combined (there are

no overlaps in this case because ($\delta 3$) and ($\varepsilon 2$) are mutually exclusive).

$$\hat{f}_{(\delta 3)} = \begin{cases} \max\left\{0, \bar{f}, \theta - \gamma\right\} & , \text{if } 0 < c_c - c_p \leq d \\[2mm] \max\left\{0, \theta - \gamma\right\} & , \text{if } -\frac{d}{2} \leq c_c - c_p \leq 0 \end{cases}$$

$$\hat{f}_{(\varepsilon 2)} = \begin{cases} \max\left\{0, \breve{f}\right\} & , \text{if } 0 < c_c - c_p < d \text{ and } f_2 < \breve{f} \\[2mm] \max\left\{0, f_3\right\} & , \text{if } \left\{ 0 < c_c - c_p < d \text{ and } \breve{f} \leq f_3 < \frac{\lambda(\bar{p}-c_p)^2}{2d} \right\} \\[2mm] & \text{or } \left\{ -\frac{d}{2} < c_c - c_p \leq 0 \text{ and } f_3 < \frac{\lambda(\bar{p}-c_p)^2}{2d} \right\} \\[2mm] \frac{\lambda(\bar{p}-c_p)^2}{2d} & , \text{if } \left\{ 0 < c_c - c_p < d \text{ and } f_3 \geq \frac{\lambda(\bar{p}-c_p)^2}{2d} \right\} \\[2mm] & \text{or } \left\{ -\frac{d}{2} < c_c - c_p \leq 0 \text{ and } f_3 \geq \frac{\lambda(\bar{p}-c_p)^2}{2d} \right\} \end{cases}$$

where $f_3 \equiv \frac{\theta}{d}\left(3\bar{p} - 2c_c - c_p\right) - \gamma$.

As a result, MP's best response to entry by MC is to enter the market and choose cost of variety $\hat{f}$ if $\pi_p^{duop}(\hat{f}) \geq 0$; the firms' equilibrium decisions and profits are:

Region	Cost of Variety, $\hat{f}$	Variety, $\hat{n}$	Price Equilibrium, $(\hat{p}_c, \hat{p}_p)$
($v1$)	0	∞	$(\min(c_c, c_p), \min(c_c, c_p))$
($v2$)	$\frac{\lambda(\bar{p}-c_p)^2}{2d}$	$\frac{d}{3\bar{p}-2c_c-c_p}$	$(\bar{p}, \frac{\bar{p}+c_p}{2})$
($v3$)	$\max\left\{\breve{f}, f_3\right\}$	$\frac{d}{3\bar{p}-2c_c-c_p}$	$(\bar{p}, \frac{\bar{p}+c_p}{2})$
($v4$)	$\max\left\{\theta - \gamma, \bar{f}\right\}$	1	$\left(\frac{d}{3} + \frac{2c_c+c_p}{3}, \frac{d}{6} + \frac{c_c+2c_p}{3}\right)$

Region	MC's Duopoly Profit, $\pi_c^{duop}(\hat{f}) = \pi_c(\hat{p}_c, \hat{f}, \hat{n}, \hat{p}_p)$
($v1$)	$\begin{array}{ll} -F_c & , \text{if } c_p \leq c_c \\[2mm] \lambda(\bar{p} - c_c) - F_c & , \text{if } c_c < c_p \end{array}$
($v2$)	$\frac{2\lambda(\bar{p}-c_c)^2}{3\bar{p}-2c_c-c_p} - F_c$
($v3$)	$\frac{2\lambda(\bar{p}-c_c)^2}{3\bar{p}-2c_c-c_p} - F_c$
($v4$)	$\frac{2\lambda}{9d}\left[d - (c_c - c_p)\right]^2 - F_c$

Region	MP's Duopoly Profit, $\pi_p^{duop}(\hat{f}) = \pi_p(\hat{p}_c, \hat{f}, \hat{n}, \hat{p}_p)$
$(\upsilon1)$	$\lambda\left(\bar{p} - c_p\right) - \left[\kappa - \theta\ln(\gamma)\right]$, if $c_p \leq c_c$ $-\left[\kappa - \theta\ln(\gamma)\right]$, if $c_c < c_p$
$(\upsilon2)$	$-\left[\kappa - \theta\ln(\gamma + \frac{\lambda(\bar{p}-c_p)^2}{2d})\right]$
$(\upsilon3)$	$\frac{\lambda(\bar{p}-c_p)^2}{2(3\bar{p}-2c_c-c_p)} - \frac{d\max\left(\breve{f},f_3\right)}{3\bar{p}-2c_c-c_p} - \kappa + \theta\ln\left[\gamma + \max\left(\breve{f},f_3\right)\right]$
$(\upsilon4)$	$\frac{2\lambda}{9d}\left(c_c - c_p + \frac{d}{2}\right)^2 - \max\left(\theta - \gamma, \bar{f}\right) - \kappa + \theta\ln\left[\gamma + \max\left(\theta - \gamma, \bar{f}\right)\right]$

where $f_3 \equiv \frac{\theta}{d}\left(3\bar{p} - 2c_c - c_p\right) - \gamma$, and the parameter regions are defined as:

$(\upsilon1) \equiv \{\ \bar{p} > \frac{d}{3} + \frac{2c_c+c_p}{3}\ ,\ \theta \leq \gamma\ ,\ \{\ [\ 0 < c_c - c_p \leq d\ ,\ \bar{f} \leq 0\ ]\ \text{or}\ [\ -\frac{d}{2} \leq c_c - c_p \leq 0\ ]\ \}\ \}$

$\qquad \text{or}\ \{\ \bar{p} \leq \frac{d}{3} + \frac{2c_c+c_p}{3}\ ,\ \{\ [\ 0 < c_c - c_p \leq d\ ,\ f_3 \leq \breve{f} \leq 0\ ]$

$\qquad\qquad\qquad\qquad \text{or}\ [\ -\frac{d}{2} \leq c_c - c_p \leq 0\ ,\ \breve{f} \leq\ f_3 \leq 0\ ]\ \}\ \}$

$(\upsilon2) \equiv\ \bar{p} \leq \frac{d}{3} + \frac{2c_c+c_p}{3}\ ,\ -\frac{d}{2} \leq c_c - c_p \leq d\ ,\ f_3 \geq \frac{\lambda(\bar{p}-c_p)^2}{2d}$

$(\upsilon3) \equiv\ \bar{p} \leq \frac{d}{3} + \frac{2c_c+c_p}{3}\ ,\ \{\ [\ 0 < c_c - c_p \leq d\ ,\ \breve{f} > 0\ ]$

$\qquad\qquad\qquad \text{or}\ [\ -\frac{d}{2} \leq c_c - c_p \leq d\ ,\ 0 < f_3 < \frac{\lambda(\bar{p}-c_p)^2}{2d}\ ]\ \}$

$(\upsilon4) \equiv\ \bar{p} > \frac{d}{3} + \frac{2c_c+c_p}{3}\ ,\ \{\ [\ 0 < c_c - c_p \leq d\ ,\ \bar{f} > 0\ ]\ \text{or}\ [\ -\frac{d}{2} \leq c_c - c_p \leq d\ ,\ \theta > \gamma\ ]\ \}$

Finally, the entry game equilibrium can be explicitly but tediously characterized by using the following logical statements and the profit expressions given in the proofs of Propositions 5 and 6 (we assume that the firms enter when they are indifferent between entering and not

entering):

Condition		Outcome
$\pi_p^{duop}(\hat{f}) \geq 0$, $\pi_c^{duop}(\hat{f}) \geq 0$	$\implies$	Duopoly
$\pi_p^{duop}(\hat{f}) \geq 0$, $\pi_c^{duop}(\hat{f}) < 0$	$\implies$	MP Monopoly
$\pi_p^{monop}(f^*) \geq 0$, $\pi_c^{monop} < 0$	$\implies$	MP Monopoly
$\pi_p^{duop}(\hat{f}) < 0$, $\pi_c^{duop}(\hat{f}) \geq 0$	$\implies$	MC Monopoly
$\pi_p^{monop}(f^*) < 0$, $\pi_c^{monop} \geq 0$	$\implies$	MC Monopoly
$\pi_j^{duop}(\hat{f}) < 0 \leq \pi_j^{monop}$ for $j = p, c$	$\implies$	MP or MC Monopoly
$\pi_p^{monop}(f^*) < 0$, $\pi_c^{monop} < 0$	$\implies$	Market Breakdown

In the only remaining case, if both firms earn negative profit under competition but non-negative profit under monopoly, then both MC and MP monopoly are Nash equilibria. ∎