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MASS FLOW FROM STELLAR SYSTEMS—I

Radial Flow from Spherical Systems

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Summary

Evidence indicates that interstellar material exists within some Population II stellar systems. With a view toward understanding the behaviour of this material, a simplified model stellar system is outlined in which it is assumed that:

(i) interstellar material is injected into the system by its stellar component;
(ii) a radial, outward flow pattern is established;
(iii) the flow is in steady state.

Solutions of the differential equations governing the model give quantitative results for the velocity, temperature, and density of the interstellar matter in globular clusters and elliptical galaxies. It is seen that for the more tightly bound elliptical galaxies solutions do not exist, indicating that material does not escape in a radial, steady state flow pattern, but probably remains within the galaxy.

1. *Introduction.* It has by now become well established that to a greater or lesser extent many stars lose material. The range of mass loss runs from the relatively small, quiet outflow of the solar wind (amounting to less than 0·1 per cent of the mass of the Sun over its entire duration), to explosions expelling perhaps the majority of the stellar mass. These loss mechanisms have been well summarized by Deutsch (1960) and Weymann (1963). If the members of a compact stellar system continually emit material into interstellar space then some kind of a random or organized flow pattern must be established. Spitzer (1942) has shown that material ejected by stars, if allowed to stand until cool, will collapse toward the centre of the system. We are concerned here with the complementary problem: what distribution of material will result if mass injected into a system does not stand and cool, but rather joins a flow pattern carrying it out of the system? In order to obtain a general idea of the behaviour of the flow we will outline a model for an idealized system, necessarily much simplified compared with the actual complexities of the situation.

In the construction of the model we shall assume:

(i) *Steady state.* This assumption will be considered reasonably justified if the flow rate can completely exchange the gas in the system in a time short compared with the duration of undisturbed flow.

(ii) *Spherical symmetry.* All variables will be smooth functions only of the distance, r, from the centre of the system. A corollary of this assumption is that rotation is not considered.

(iii) The injection of mass into the system can be adequately described by a continuous function, $q(r)$, which is proportional to the mean local stellar density. Material will be injected with an enthalpy per unit mass, h. The effect of the stellar random motions on q or h will be ignored.

(iv) The gravitational field of the system is determined by the stellar component, which is much more massive than the interstellar gas. This assumption must be justified if assumption (i) is valid. The density of the stellar component will be approximated by a Gaussian function,

$$\rho \propto \exp(-r^2/b^2)$$

where b is a characteristic radius of the system.

(v) Thermal conductivity, viscosity, and radiation will be ignored. To justify the neglect of radiative cooling by the gas we must show that the flow rate is high enough to remove an element of gas from the system in less than a characteristic cooling time.

(vi) Zero pressure at infinity.

Having set up models defined by these assumptions we shall then use them to construct idealized flow patterns for globular clusters and the more spherical types of elliptical galaxies. We may obtain the equilibrium values of the velocity, density, and temperature of the interstellar gas. These values will be compared with the rather sketchy observations.

Recently evidence has accumulated indicating the presence of gas or dust within Population II stellar systems, previously thought to be devoid of interstellar material. Let us begin with a brief overview of some of this evidence as it pertains to globular clusters and elliptical galaxies.

2. *Observations of interstellar material in Population II systems*

Globular clusters. Roberts (1960) recounts many earlier observations of obscuring material in globular clusters. Fitzgerald (1955) reports seeing dust lanes in Omega Centauri. He judges them very unlikely to be foreground obscuration. Takebe & Matsunami (1957) discuss various mechanisms for insertion of gas and dust into a cluster. They estimate that densities of interstellar gas may be in the range of 10^{-25} g cm^{-3}. They have assumed that only about 1 per cent of the stellar mass is lost during the life of a cluster, and that a cluster is purged of gas by passage through the galactic plane about 50 times in this interval. Their estimate of total mass lost is certainly quite modest.

Sandage (1957) estimates that during evolution to the white dwarf stage in the cluster M3 stars may inject about $10^5 M_\odot$ (half their mass) into interstellar space. According to Roberts this corresponds to an average level of about 2000 $M_\odot$, determined by the interval between passes through the galactic plane. Sandage points out that most of this material would leave the cluster because of the low escape velocity (~ 5 km s^{-1}).

Idlis & Nikol'skii (1959) think they see dust clouds in the cluster M4. They consider the question of whether the absorption patches they detect could be just random fluctuations in the stellar density, and conclude that it is not likely. For a total of around 140 clouds, they estimate about 14 $M_\odot$ of material to be within the cluster.

Roberts has examined photographs of globular clusters and in several he finds obscured areas which he attributes to absorption within the cluster. He finds these areas to occur too frequently to be foreground clouds. His calculations of the expected density and size of such regions lead him to estimate that there may be around a thousand solar masses of material within many clusters. Presumably this level of density must as usual be established in less than the period of time it takes a cluster to orbit around the galactic centre.

On the other hand, searches have been made at 21 cm for evidence of interstellar neutral hydrogen by Roberts & Goldstein (1964). Roberts finds an upper limit to the amount of H I in M13 of 200 $M_\odot$ and in M3 of 700 $M_\odot$. Goldstein finds an upper limit in M13 of 150 $M_\odot$. But then Roberts points out that a large part of the gas may be ionized, or in small opaque clouds, thus avoiding detection at 21 cm.

Elliptical galaxies. The evidence for interstellar material in elliptical galaxies is more substantial than that for globular clusters. Some aspects of the problem are discussed by Sandage (1957). He estimates that only about 1/200 of the mass of an elliptical galaxy (for globular clusters it was 1/2) is emitted by the stars. On the other hand since escape velocities are in the hundreds of kilometers per second he expects ejected gas to remain in the system.

Minkowski & Osterbrock (1959) and Osterbrock (1960) have looked for the λ 3727 line of [O II] spectroscopically. They find that it occurs in about 14 per cent of E galaxies. In NGC 1052 this line is broadened and inclined, with a width corresponding to around 600 km s^{-1}. In NGC 4125 its width corresponds to 680 km s^{-1}. In NGC 4278 its width corresponds to 890 km s^{-1} near the centre of the nucleus. Further from the centre the velocity spread drops sharply.

The Burbidges (1965) looked for λ 6583 of [N II] and for Hα. In ellipticals they generally found the former and occasionally the latter.

Epstein (1964) observed six elliptical galaxies at 21 cm and was able to detect only continuum radiation in only two of them.

The λ 3727 observations seem to indicate that not merely is there interstellar gas present in elliptical galaxies, but that it has large velocities, possibly radially outward. In some cases rotation is indicated. There is vague evidence that collisions between elliptical galaxies in clusters may periodically purge them of interstellar material (Osterbrock 1962).

3. *Properties of flow*

Basic equations. We may write the three equations of conservation of energy, mass, and momentum in spherical symmetry as,

$$\frac{\partial}{\partial r}(r^2uP)+\frac{\partial}{\partial r}(r^2eu\rho)+\frac{\partial}{\partial r}(\tfrac{1}{2}\rho r^2u^3)+\frac{\partial}{\partial r}(r^2\varphi u\rho)-r^2q(h+\varphi) = 0, \qquad (1)$$

$$\frac{\partial}{\partial r}(r^2u\rho) = qr^2, \qquad (2)$$

$$\rho\frac{\partial}{\partial r}\left(\frac{u^2}{2}\right)+uq = -\frac{\partial P}{\partial r}-\rho\frac{\partial\varphi}{\partial r}, \qquad (3)$$

where u is velocity, P is pressure, ρ is density, e is internal energy per unit mass and φ is gravitational potential. The case of no distributed source, $q\equiv 0$, produces the conventional solar wind equations (Chamberlain 1961).

Let us put

$$e = \frac{1}{\gamma-1}\frac{kT}{m}, \qquad \rho = nm, \qquad P = nkT, \qquad (4)$$

where T is temperature, m is mean particle mass, k is Boltzmann's constant, γ is the ratio of specific heats and n is the number of particles per unit volume.

Equation (3) may be used to eliminate $\partial\varphi/\partial r$ from equation (1) giving:

$$-ukT\frac{\partial n}{\partial r}+\frac{1}{\gamma-1}unk\frac{\partial T}{\partial r}+q\left[\frac{\gamma}{\gamma-1}\frac{kT}{m}-h-\tfrac{1}{2}u^2\right] = 0, \quad (5)$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2un) = \frac{q}{m}, \quad (6)$$

$$nm\frac{\partial}{\partial r}(\tfrac{1}{2}u^2)+uq = -\frac{\partial}{\partial r}(nkT)-nm\frac{\partial\varphi}{\partial r}. \quad (7)$$

Equation (6) may be formally integrated to give

$$un = \frac{1}{mr^2}\int_0^r qr^2dr. \quad (8)$$

It will be convenient to scale the equations by the substitutions

$$\frac{\varphi}{a^2}\equiv\Phi,\quad \frac{h}{a^2}\equiv H,\quad \frac{qr_0}{ma}\equiv Q,\quad \frac{u}{a}\equiv U,\quad \frac{r}{r_0}\equiv R,\quad \frac{kT}{m}\equiv\theta\equiv a^2\Theta, \quad (9)$$

where a is a free parameter. Now equations (5), (7) and (8) become

$$Un = \frac{1}{R^2}\int_0^R QR^2dR\equiv S. \quad (10)$$

$$-\Theta\left(\frac{\partial S}{\partial R}-\frac{S}{U}\right)\frac{\partial U}{\partial R}+\frac{1}{\gamma-1}S\frac{\partial\Theta}{\partial R}+Q\left(\frac{\gamma}{\gamma-1}\Theta-H-\tfrac{1}{2}U^2\right) = 0, \quad (11)$$

$$SU\frac{\partial U}{\partial R}+QU^2 = -S\frac{\partial\Theta}{\partial R}-\Theta\frac{\partial\Theta}{\partial R}+\frac{S\Theta}{U}\frac{\partial U}{\partial R}-S\frac{\partial\Phi}{\partial R}. \quad (12)$$

If the gas is monatomic (the only case we shall consider) then $\gamma = 5/3$ and an integral of these equations may be obtained. Combining equations (11) and (12) yields

$$S\frac{\partial W}{\partial r}+QW+S\frac{\partial\Phi}{\partial R}-QH = 0, \quad (13)$$

where

$$W\equiv\frac{5}{2}\Theta+\frac{U^2}{2}. \quad (14)$$

The formal solution of equation (13) may be written in closed form

$$W = \frac{1}{SR^2}\left[\int\left(\frac{QH}{S}-\frac{\partial\Phi}{\partial R}\right)SR^2dR+\text{constant}\right]. \quad (15)$$

The condition that W remains finite at the origin requires the constant in equation (15) to be zero.

Θ may be eliminated from equation (12) by using equation (14), so that a single equation involving only $U^2\equiv y$ and the known function W may be obtained:

$$\frac{1}{5}\frac{\partial y^2}{\partial R}+\frac{y^2}{S}\left(Q-\frac{S'}{5}\right)-\frac{W}{5}\frac{\partial y}{\partial R}+y\left(\frac{S'}{S}\frac{2}{5}W+\Phi'+\frac{2}{5}W'\right) = 0. \quad (16)$$

Primes denote differentiation with respect to R. It seems very difficult to find solutions for this equation, even in the case of special or simple forms of the source function Q or the function W. We may, however, obtain an approximate solution near the origin, for a region in which Q may be taken as constant.

Expand y in a power series in R:

$$y = \sum_{i=0}^{\infty} a_i R^i$$

and substitute into equation (16). Equation coefficients and requiring $y = 0$ at the origin and $y > 0$ everywhere else, we obtain

$$y = a_2R^2 + \left(9a_2{}^2 + \frac{21}{10} g a_2\right) H^{-1} R^4 + \ldots,$$

where

$$g \equiv \frac{1}{R}\Phi'(0).$$

To order R^3

$$y = a_2 R^2. \tag{17}$$

Equation (15) gives for W in this case of constant Q,

$$W = H - g\frac{R^2}{5}, \tag{18}$$

so that

$$\Theta = \frac{2}{5}H - R^2\left(\frac{2g}{25} + \frac{a_2}{5}\right). \tag{19}$$

Equation (10) yields

$$n = \frac{QR}{3U} = \frac{Q}{3\sqrt{a_2}}. \tag{20}$$

Numerical solutions. In order to obtain complete solutions to the set of equations (10)–(12) an IBM 1620 computer was programmed to perform numerical iterations. We would *a priori* expect the topology of the solutions to be quite similar to those of the solar wind equations (Parker 1960), and in fact this is the case. Parker's discussion of asymptotic solutions, critical solutions, and effects of non-zero pressure at infinity are all relevant to the present case. Once a solution curve has passed beyond the edge of the system, so that the source function becomes zero, then the solutions correspond to analogous solar wind flow patterns.

For computational illustration we take a normalized source function to be

$$Q = Q_0 \exp\left(-R^2/R_0{}^2\right)$$

and the normalized gravitational field is then given by

$$\Phi' = \frac{C}{R^2}\int_0^R QR^2 dR.$$

Examination of equations (10)–(12) reveals that their solutions form a one parameter family. These equations and equation (9) show that that parameter, α, may be defined as

$$\alpha \equiv Q_0 C R_0{}^2 = \frac{4GM}{\pi^{1/2}a^2 b}, \tag{21}$$

where M is the total mass of the system, and b is defined in assumption (iv). H is chosen to be one, so that $a^2 = h$ is determined by the enthalpy of the injected

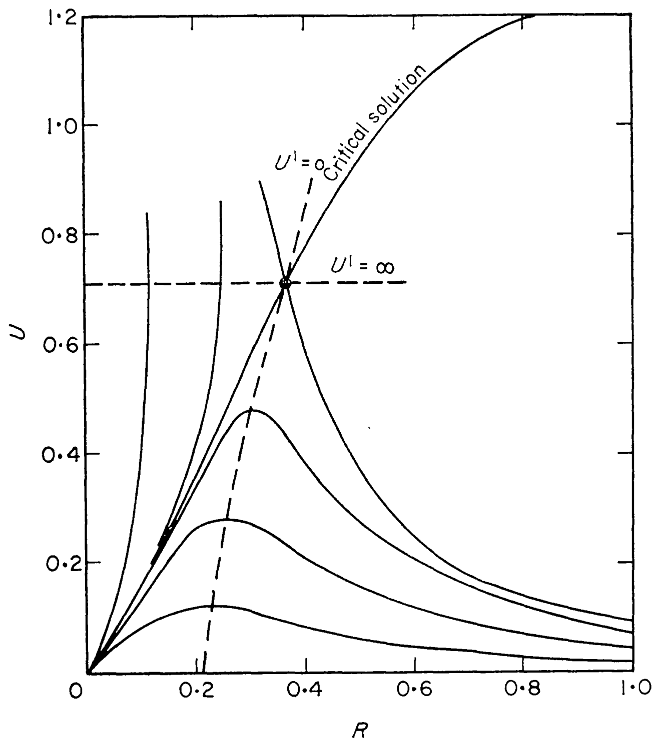

FIG. 1. *Weak field velocity solutions.* $\alpha = 0$—$0 \cdot 15$.

material. Equations (17), (19), and (20) were used to start the numerical integrations at the origin.

Solutions are obtainable only for those values of α which correspond to cases where the enthalpy of the injected material is sufficient to overcome the gravitational field of the system and the pressure gradient of the gas. This occurs for values of α

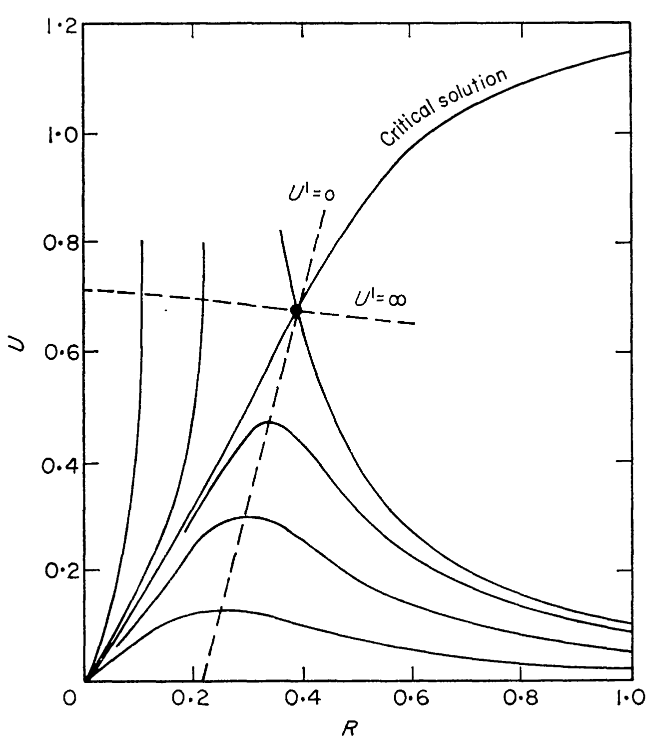

FIG. 2. *Medium field velocity solutions.* $\alpha = 0 \cdot 6$.

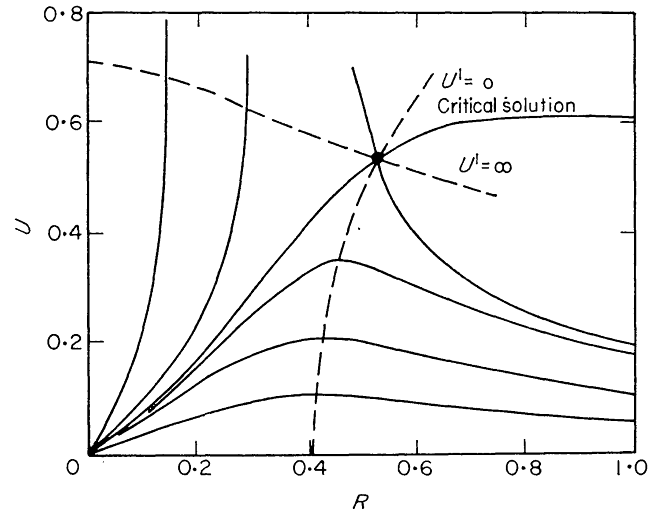

FIG 3. *Strong field velocity solutions.* $\alpha = 2 \cdot 5$.

less than about 3·5. In physical terms we expect this cut-off to occur when the characteristic gravitational potential of a particle, GM/b, is the same order of magnitude as its characteristic kinetic energy, a^2, or (by equation (21)) when $\alpha \sim 4/\pi^{1/2}$.

Cases with $\alpha > 3 \cdot 5$ correspond to the situation considered by Spitzer (1942) where no outward flow pattern is established; the material is gravitationally bound allowing it to cool and collapse to the centre of the system. If is, of course, assumed that additional energy is not added to the gas by radiation, turbulence, gravitational fluctuations, etc.

Figs 1–3 display solutions for values of α ranging from 0 to 2·5, the latter figure being near the maximum where solutions may be obtained. When $\alpha \lesssim 0 \cdot 15$ the gravitational potential is becoming so weak compared to the thermal energy of injected particles, that it has almost no effect on the flow. Thus Fig. 3 represents all solutions for $\alpha < 0 \cdot 15$. In each case the solution going through the critical point

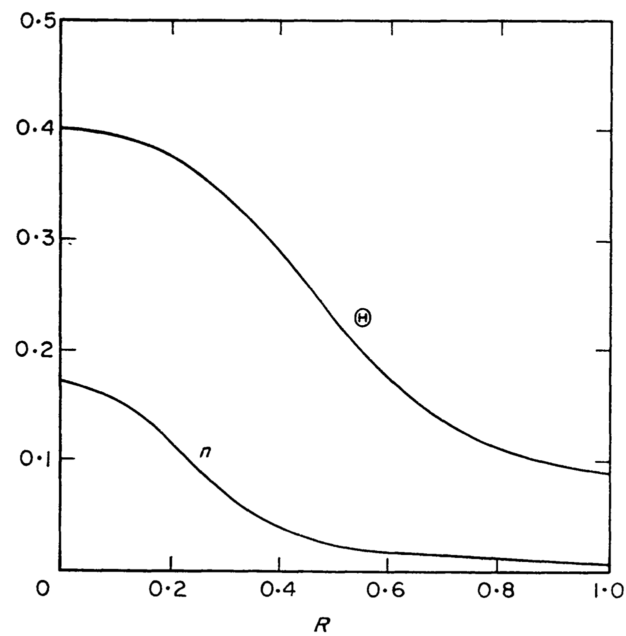

FIG. 4. *Weak field critical solution*: α = 0—0·15. *Temperature and number density.*

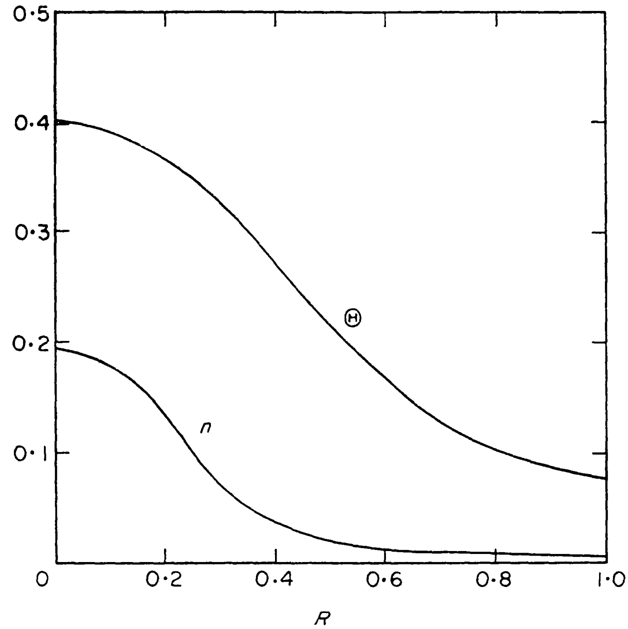

FIG. 5. *Medium field critical solution*: $\alpha = 0\cdot6$. *Temperature and number density.*

corresponds to zero pressure at infinity. Figs 4–6 show the run of temperature and density for these solutions. Values of parameters and properties of the critical solutions are listed in Table I. The scaled mass of interstellar material within radius R_0^* is given in column 4,

$$\mu(R_0^*) \equiv \int_0^{R_0^*} n^* R^{*2} dR^*.$$

Some useful auxiliary expressions, relating desired scaled quantities to quantities used in the integrations (read from the figures and denoted by asterisks), are listed below:

$$r_0 = \frac{b}{R_0}, \quad \frac{q_0 b}{ma} = Q_0 R_0, \quad \frac{n}{n^*} = \frac{Q_0 R_0}{Q_0^* R_0^*}, \quad \frac{R}{R^*} = \frac{R_0}{R_0^*}, \quad \frac{r}{b} = \frac{R^*}{R_0^*}.$$

The total actual flow rate $\mathcal{Q}$ is given by

$$\mathcal{Q} = q_0 \pi^{3/2} b^3,$$

and the source function at the centre of the system is given by

$$q_0 = \frac{Mf}{\pi^{3/2} b^3 \tau},$$

where f represents the fraction of the system lost during its lifetime, τ. Measured in units of solar masses (M), parsecs (b), 10^{10} years (τ), proton masses (m), and erg g⁻ (a^2) we may write, for convenience in selecting a solution to match an actual case

$$\alpha = Q_0 R_0^2 C = \frac{9\cdot7 \times 10^7\, M}{a^2 b} \quad (22$$

$$Q_0 R_0 = \frac{74 Mf}{b^2 \tau a m}. \quad (23$$

Equations (9) may be used to obtain c.g.s. quantities from their scaled counterparts

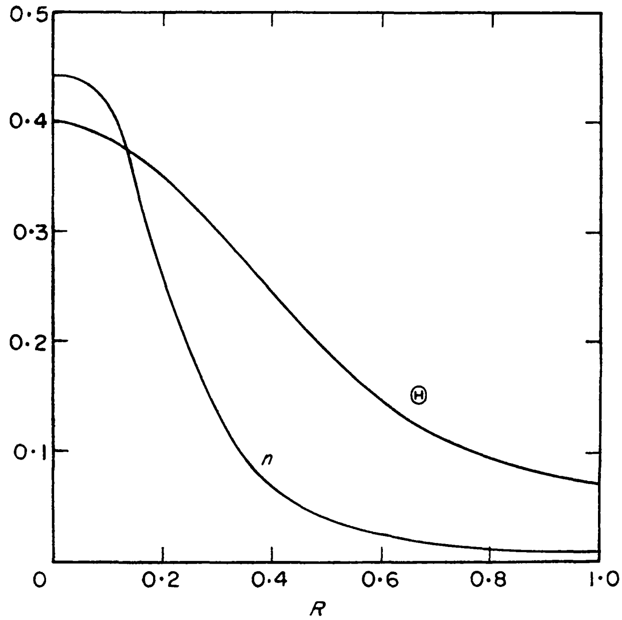

FIG. 6. *Strong field critical solution*: $\alpha = 2 \cdot 5$. *Temperature and number density.*

TABLE I

Parameters and properties of numerical solutions

Figs	α	U_{crit}^*	R_{crit}^*	$\mu(R_0^*)$
1, 4	0–0·15	0·71	0·37	$5 \cdot 2 \times 10^{-4}$
2, 5	0·6	0·67	0·39	$5 \cdot 7 \times 10^{-4}$
3, 6	2·5	0·53	0·53	$1 \cdot 2 \times 10^{-3}$

4. *Models.* We may now outline a few representative models of spherical systems similar to idealized globular clusters and elliptical galaxies. The two chief sources of indeterminacy are lack of knowledge of the total fraction of a system lost into interstellar space (f), and the average enthalpy of the injected material (h). For the former quantity we construct examples using $f = 0 \cdot 1$ (considerably more than the solar wind value). For the latter we use values bracketing the solar wind value of $h \approx 4 \times 10^{14}$ erg g^{-1}. Models with different f values will be identical, except that $n(r)$ varies proportionally to f.

Globular clusters. Table II lists the properties of two model globular clusters, one with a medium gravitational field (similar to M22), and one with a weak field (similar to M92). The total mass of interstellar gas within the cluster,

$$\mu(b) = 4\pi m \int_0^b n(r) r^2 dr$$

$$= \frac{4bMf}{\pi^{1/2} Q_0^* R_0^{*4} \tau a} \mu(R_0^*), \qquad (24)$$

is given in the last column. We note that this is a very small quantity, being less than a few solar masses for any models. The distribution of interstellar material seen by observers, if they are correct, is of course quite different from that of the

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TABLE II

Globular cluster models

$$m = \frac{m_H}{2},\ \tau = 10^{10}\ \text{years},\ f = \frac{1}{10}$$

Model	Similar system	Gravitational field	M ($M_\odot$)	b (pc)	h (10^{14} erg/g)	α	Q_0R_0	$n(0)$ (cm^{-3})	$T(0)$ (10^6 °K)	U_{crit} (km/s)	$\mu(b)$ ($M_\odot$)
1a	M22	Weak	7×10^6	4·5	0·61	2·5	0·66	1·3	0·15	41	4·2
b					2·5	0·6	0·32	0·28	0·61	110	0·97
c					10·0	0·15	0·16	0·13	2·4	230	0·44
2a	M92	Very weak	$1·4\times10^5$	5	0·68	0·04	0·01	0·0076	0·16	59	0·038
b					2·7	0·01	0·0050	0·0038	0·66	120	0·019
c					11·0	0·0025	0·0025	0·0019	2·6	230	0·0094

present models, the former being usually in clumps or small clouds. In addition we note a rather large quantitative discrepancy between amounts of material observed and allowable steady state levels according to our models. If gas is to appear and accumulate within a cluster, then it seems that it must be injected with a very small enthalpy, so that even the weak gravitational field of a cluster may prevent a more or less steady state, quiescent, outward flow pattern from being established.

A characteristic time for the establishment of steady state conditions (assuming they are attained at all) is given by

$$\tau_{\text{char}} \sim \frac{\mu(b)}{\mathcal{Q}} = \frac{\mu(b)\tau}{Mf}.$$

These times for the models of Table II amount to only tens to hundreds of thousands of years: times short compared to the interval between passages of a globular cluster through the galactic plane, when steady state conditions must be upset. Therefore, steady state conditions should persist throughout most of the orbit of a globular cluster.

Finally, we must check whether there is reasonable adherence to our assumption of no cooling of the interstellar gas. Spitzer (1942) gives times for recombination and cooling from free–free transitions. For gas under the conditions of density and temperature encountered in models of Table II, these times are in excess of 10^8 years. Since a given injected sample of gas will leave the system in a time less than 10^5 years we may safely assume that cooling by free–free transitions will not be important within the system. Even if dust were present, cutting the cooling time to about 10^6 years, this is still much longer than the time a sample of gas spends within the system.

Elliptical galaxies. The gravitational binding energy of a particle within an elliptical galaxy is generally much higher than in a globular cluster. Some models are outlined in Table III. For the more strongly bound systems, values of the enthalpy corresponding to injection velocities up to order 1000 km s^{-1} are insufficient to cause outward mass flow. These systems should retain most of their interstellar material, unless a really unusual injection enthalpy or other heating source is present. Among those models losing mass, systems range from tightly bound (NGC.3115) to loosely bound (Wolf–Lundmark system). Again in the last column we have computed the mass of interstellar material within the cluster, $\mu(b)$. We see that significantly large amounts of material may be present, although the fractional amount compared with the mass of the stellar component is small.

The characteristic velocities ($\sim u_{\text{crit}}$) derived from the models of up to several hundred kilometres per second correspond to the order of magnitude of observed velocities. Because of the high temperature a large fraction of the gas would be ionized and difficult to detect, except as a thermal source. This would account for the lack of observational detection of non-thermal 21 cm emission.

Free–free cooling times and recombination times for the conditions of the interstellar gas given in Table III are, by Spitzer's calculations, in excess of 10^9 years. Time scales for complete exchange of material vary up to around 10^6–10^7 years, so that again we may say that the no cooling approximation is reasonable. Even if dust were present, the cooling time is in excess of 10^7 years which provides enough margin to allow neglect of cooling as a first approximation.

TABLE III

Elliptical galaxy models

$$m = \frac{m_H}{2}, \tau = 10^{10} \text{ years}, f = \frac{1}{10}$$

Model	Similar system	Gravitational field	M ($M_\odot$)	b (kpc)	h (10^{14} erg/g)	α	Q_0R_0	$n(0)$ (cm^{-3})	$T(0)$ (10^6 °K)	U_{crit} (km/s)	$\mu(b)$ ($M_\odot$)
1a	M87				15·0	40·0			(no solution)		
b	(E1)	Strong	4×10^{12}	6·5	69·0	10·0			(no solution)		
c					240·0	2·5	0·0090	0·0178	58·0	820	$1·7 \times 10^8$
2a	M32				0·97	10·0			(no solution)		
b	(E2)	Medium	4×10^9	0·4	3·9	2·5	0·0189	0·037	0·94	100	$8·5 \times 10^4$
c					16·0	0·6	0·0095	0·0081	3·93	260	$2·0 \times 10^4$
3a					3·2	10·0			(no solution)		
b		Medium	10^{11}	3·0	13·0	2·5	0·46	0·90	3·1	190	$8·7 \times 10^6$
c					54·0	0·6	0·22	0·19	13·0	490	$2·0 \times 10^6$
4a	Wolf–				0·23	0·6	0·00055	0·00047	0·055	32	$7·7 \times 10^3$
b	Lundmark	Weak	10^8	0·75	0·90	0·15	0·00028	0·00021	0·22	67	$3·5 \times 10^3$
c	System				3·4	0·04	0·00014	0·00011	0·82	130	$1·8 \times 10^3$
					14·0	0·01	0·000072	0·000055	3·3	260	$0·90 \times 10^3$

5. *Summary*. We have outlined conditions under which we may set up simple models for outflow of material from spherical stellar systems. We have assumed that:

(i) interstellar material is injected into the system by its stellar component;
(ii) a radial, outward flow pattern is established;
(iii) the flow is in steady state.

Globular cluster models are all characterized by weak gravitational fields, so that if material is injected into interstellar space with an enthalpy corresponding to the order of magnitude of the solar wind enthalpy it will leave the system with velocities ranging from around 50–200 km s^{-1}. The central number density will be quite low, running from about 0·002 to 1 cm^{-3}. Accordingly, the total mass of interstellar material within the system will be less than a few solar masses, and very difficult to observe. The temperature will be of the order of hundreds of thousands of degrees Kelvin so that total ionization will be achieved. It is not surprising, therefore, that 21 cm observations show upper limits of a few hundred solar masses for this material. Clumps of material observed remaining within globular clusters, if in fact real, must behave in a totally different manner from the gas in these models.

Many ' elliptical ' (actually spherical) galaxy models have stronger gravitational fields, so that the series of models considered contains those so tightly bound that no gas is likely to escape. Rather it will probably fall to the centre, in the manner described by Spitzer. The models with weaker gravitational fields exhibit material outflows with velocities of hundreds of kilometres per second, in accordance with many observations. The total mass of gas within the system of these models varies from about $10^3 M_\odot$ (Wolf–Lundmark system) to around $10^8 M_\odot$ (M87).

Further work is now being initiated to include the effects of rotation of the system, so that more realistic models of Population II systems may be constructed. Such a modification may also permit applications to flow from galactic nuclei.

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