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Mass Spectra of Elementary Particles and Extended σ-Model

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The renormalizability of the so-called σ-model[1] has been studied most successfully by Lee[2] and by Gervais and Lee.[3]

The purpose of this note is to extend that model to all P_{11} states of nucleon family, the scalar σ-like bosons and to the family of pions so as to derive mass relations among these particles.

For this purpose we introduce the new quantum number $n\,(=0, 1, 2, \cdots)$ which specifies the particle states from low to high mass states in succession. Let us assume a degenerate bare mass μ^2 for mesons, the common bare coupling constants g_0 and λ_0 in the starting Lagrangian as in the original σ-model. In what follows we can predict the masses ε and ε' in terms of the masses $N(939)$, $N(1470)$, $N(1780)$, $\pi(137)$ and $S^*(1000)$ consistent with experiment. Two, so-far unobserved massive pion-like states appear in this approach.

As will be discussed later, the renormalizable, chiral invariant Lagrangian, except for a term linear to $\sigma_0(n)$, for the n-th excited states may be defined by

$$\begin{aligned}L(n)=&\bar{\psi}_0(n)(i\gamma\cdot\partial-g_0(\sigma_0(n)+i\boldsymbol{\pi}_0(n)\cdot\boldsymbol{\tau}\gamma_5))\\&\times\psi_0(n)+\tfrac{1}{2}((\partial\sigma_0(n))^2+(\partial\boldsymbol{\pi}_0(n))^2)\\&-\tfrac{1}{2}\mu^2(\sigma_0(n)^2+\boldsymbol{\pi}_0(n)^2)\\&-\tfrac{1}{4}\lambda_0{}^2(\sigma_0(n)^2+\boldsymbol{\pi}_0(n)^2)^2+c_0(n)\sigma_0(n)\\&-\tfrac{1}{2}\delta\mu(n)^2(\sigma_0(n)^2+\boldsymbol{\pi}_0(n)^2),\qquad(1)\end{aligned}$$

where the space-time coordinate is suppressed for simplicity and the infinite constant of mass counter term $\delta\mu(n)^2$ is in general dependent upon n, and $c_0(0)$ corresponds to an ordinary PCAC constant.

The masses of P_{11}-nucleon family and those of associated mesons may be induced by a translation of the field variable $\sigma_0(n)$ and by a re-expression of the Lagrangian in terms of the variable

$$s_0(n)=\sigma_0(n)-v_0(n),\qquad(2)$$

where $v_0(n)$ is a non-vanishing vacuum expectation value, while

$$\langle s_0(n)\rangle=0\,.\qquad(3)$$

Substituting Eq. (2) into Eq. (1), we obtain, to within an inessential c-number,

$$L(n)=\bar{\psi}_0(n)\,(i\gamma\cdot\partial-M(n)+g_0(s_0(n)$$

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$$+i\boldsymbol{\pi}_0(n)\cdot\boldsymbol{\tau}\gamma_5))\psi_0(n)$$
$$+\tfrac{1}{2}((\partial s_0(n))^2-\mu_\sigma(n)^2 s_0(n)^2)$$
$$+\tfrac{1}{2}((\partial\boldsymbol{\pi}_0(n))^2-\mu_\pi(n)^2\boldsymbol{\pi}_0(n)^2)$$
$$-\tfrac{1}{4}\lambda_0^2(S_0(n)^2+\boldsymbol{\pi}_0(n)^2)^2$$
$$-\lambda_0(\lambda_0 v_0(n))s_0(n)(s_0(n)^2+\boldsymbol{\pi}_0(n)^2)$$
$$-\tfrac{1}{2}\delta\mu(n)^2(s_0(n)^2+\boldsymbol{\pi}_0(n)^2)$$
$$-(v_0(n)(\mu^2+\delta\mu(n)^2)-c_0(n)$$
$$+\lambda_0^2 v_0(n)^3)s_0(n), \tag{4}$$

where

$$M(n)=g_0 v_0(n),$$
$$\mu_\sigma(n)^2=\mu^2+3\left(\frac{\lambda_0}{g_0}\right)^2 M(n)^2, \tag{5}$$
$$\mu_\pi(n)^2=\mu^2+\left(\frac{\lambda_0}{g_0}\right)^2 M(n)^2.$$

By the use of the masses of $N(939)$, $N(1470)$ as N_0, N_1 member, $\pi(137)$ $(\equiv\pi_0)$ and $S^*(1000)$ $(\equiv\sigma_1)$, the masses of $\varepsilon(\equiv\sigma_0)$ and π_1, are determined to be $\mu_\sigma(0)=572$ MeV and $\mu_\pi(1)=493$ MeV, respectively. The former is consistent with the experimental value[4] ≤ 700 MeV but the latter has not yet been observed. The remaining parameters of the formula are given as $(\lambda_0/g_0)^2=0.175$, while $\mu^2=-0.136\ \mathrm{GeV}^2$ (see the Appendix of Ref. 2), which is a remark on the sign of this quantity in relation to renormalization). If we assign $N(1780)$ to N_2, we find that the masses of $\varepsilon'(=\sigma_2)$ and the associated pseudo-scalar meson are $\mu_\sigma(2)=1240$ MeV and $\mu_\pi(2)=648$ MeV, respectively. According to the recent $\pi\pi$-phase-shift analysis[5] the peak value of the argand diagram for S-wave scattering amplitude corresponds roughly to 1210 MeV. As far as the σ-like scalar meson is concerned, our model seems to be satisfactory with experimental information. It remains to confirm the predicted *ps* mesons experimentally.*)

As seen from our approach, our Lagrangian is renormalizable in the sense of the excellent work of Lee[2] and of Gervais and Lee[3] in case of parity doublet massive regulators quantized according to the anti-commutation and commutation relations. Thus, it would be sufficient to comment on the constancy of the parameters of our theory.

In our approach there arise much more φ' lines depending upon the degrees of freedom of σ-like fields. In order to remove the divergences associated with diagram $D^{(4)}$ attached to many φ' lines,[3] we renormalize fields and coupling constants according to

$$\psi_0(n)=\hat{Z}_F(n)^{1/2}\hat{\psi}(n), \tag{6}$$
$$(\boldsymbol{\pi}_0(n), s_0(n), v_0(n))$$
$$=\hat{Z}_B(n)^{1/2}(\hat{\boldsymbol{\pi}}(n), \hat{s}(n), v(n)), \tag{7}$$
$$g_0=gZ_g(n)/(\hat{Z}_F(n)\hat{Z}_B(n)^{1/2}), \tag{8}$$
$$\lambda_0^2=\lambda^2 Z_\lambda(n)/\hat{Z}_B(n)^2 \tag{9}$$

and choose the divergent parts of $\delta\mu(n)^2$, $\hat{Z}$ and Z so as to be those that make the symmetry theory finite. The conditions of renormalizability are still given by

$$g_0^2+2\sum\varepsilon_i g_i^2=0, \tag{10}$$
$$\sum\varepsilon_i g_i^2 m_i^2=0, \tag{11}$$
$$g_0^4+2\sum\varepsilon_i g_i^4=0, \tag{12}$$

where $\varepsilon_i=\pm 1$ depending on whether the i-th regulator is quantized according to the

*) It is interesting to point out that if we replace the index σ by η in Eq. (5) and assign the η meson to η_0, we find the $\eta'(\equiv\eta_1)$ mass to be $\mu_{\eta'}=957$ MeV close to the observed value, with the same inputs as those in σ-model for the rest of the inputs. Here $(\lambda_0/g_0)^2=0.160$. One can try a similar game for the vector and axial vector mesons by an appropriate assignment. We have, however, no justification for these cases but such a relation seems to work as an empirical rule.

anti-commutation or commutation relations and m_i is its mass. These conditions do not contradict the constancy of g_0. We have assumed that the other constants λ_0 and μ^2 will not strongly depend upon n through the modified renormalization constants (6)$\sim$(9) in order to apply our mass formula successfully. Perhaps the most serious point of our approach is how to interpret physically the additional PCAC-like constants $c_0(n)$ $(n\neq 0)$ introduced into the theory. We have to assume formally that they also satisfy the ordinary PCAC-like relations in order to prove the Ward-Takahashi identity for $\Gamma_\mu{}^5(k)$ consistent with the same renormalization conditions (10) and (11).

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