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Research Article

Massive MIMO Systems with Low-Resolution ADCs: Achievable Rates and Allocation of Quantization Bits

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In massive multiple-input multiple-output (MIMO) systems, the large number of high-resolution analog-to-digital converters (ADCs) lead to high hardware cost and power consumption. In this work, the uplink achievable rates of massive MIMO systems with low-resolution ADCs are studied with consideration of both "Uniform-ADC" that uses ADCs with the same number of quantization bits and "Mixed-ADC" that allows the use of ADCs with different resolutions. By leveraging an additive quantization noise model (AQNM), the asymptotic achievable rates are obtained for maximum ratio combining (MRC), zero-forcing (ZF), and linear minimum mean squared error (LMMSE) receivers in very simple forms. Taking advantages of the theoretical results, we propose two criteria for allocation of quantization bits. It is found that the optimal quantization bits allocation for LMMSE is Mixed-ADCs with number of quantization bits that are polarized, while Uniform-ADC is optimal for MRC and ZF. When there is a constraint on the total ADC power consumption, the proposed quantization-bit allocation scheme for LMMSE becomes Uniform-ADC when the transmit signal-to-noise ratio (SNR) is below a threshold, which is related to the system scale and the ADC power consumption. The theoretical results are verified by Monte-Carlo simulations.

1. Introduction

Massive multiple-input multiple-output (MIMO) systems that employ a large number of antennas at the base station (BS) are able to provide excess spatial degrees of freedom and significantly improve both the spectral and energy efficiency [1, 2], which makes it a key technology in future wireless communication systems [3]. Despite these appealing advantages, the use of a large number of antennas leads to high hardware cost and power consumption. Therefore, recent works have advocated the use of low-cost and energy-efficient devices [4–6]. Since the main source of power consumption is the power-hungry high-resolution analog-to-digital converters (ADCs) at the receiver, the use of low-resolution ADCs in massive MIMO systems has drawn extensive research interest [7].

There are different ways of exploiting the benefits of low-resolution ADCs in massive MIMO systems, i.e., "Uniform-ADC" that uses ADCs with the same resolution [8–20], and "Mixed-ADC" that uses ADCs with different resolutions [21–26]. Regarding Uniform-ADC, the extreme case of using one-bit ADCs in massive MIMO systems was studied in [8–11]. In [8], the optimal detection for receivers with one-bit ADCs was investigated and a near maximum likelihood (ML) detector was proposed. The optimal quantization threshold was designed in [9]. It is shown that using one-bit ADCs can approach the channel estimation error of unquantized systems with an increased by a factor of $\pi/2$. In [10, 11], the impact of using one-bit ADCs on the spectral efficiency of massive MIMO systems was studied. It is shown that the high spatial multiplexing gain owing to the use of large antenna array is still achievable with one-bit ADCs. To attain the same performance as massive MIMO systems with high-resolution ADCs, however, much more antennas (over 2–2.5 times) are required. Due to the extremely coarse quantization, receivers with one-bit ADCs suffer from severe quantization noise. Therefore, to relax the ADC resolution from one bit to a few bits may significantly improve the performance. The uplink achievable rates and energy efficiency of Uniform-ADC massive MIMO systems were investigated in [12, 13]. Results show that 4-bit ADCs would approach the performance of perfect quantization for MRC receivers [12], while the most energy-efficient ADC resolution is 4–8 bits and it does not decrease with more BS antennas in most cases [13]. In [14], a Bayes inference-based joint channel and data detection scheme was proposed for Uniform-ADC massive MIMO systems. A novel channel estimation scheme based on Bussgang decomposition was proposed and the approximate achievable rates in the uplink was derived in [15], which reveals that massive MIMO systems with ADCs of only a few bits can approach the performance of unquantized systems. The joint antenna and user selection in Uniform-ADC massive MIMO systems was studied in [16] by solving a cross-entropy optimization problem. In [17], performance analysis for massive MIMO systems with uniform ADCs was carried out under Nakagami-m channels. The implementation of uniform ADCs in millimeter wave (mmWave) was discussed in [18–20]. It is discovered that using ADCs with uniform low resolution can reduce the hardware cost and improve the energy efficiency.

Massive MIMO systems with all antennas connected to high-resolution ADCs cause huge power consumption, while using only low-resolution ADCs leads to spectral efficiency degradation. Therefore, Mixed-ADC architecture that uses ADCs with different resolutions was studied in [21–26]. In [21], the authors use one-bit ADCs to partially replace conventional full-precision ADCs and propose that Mixed-ADC scheme is able to achieve a large fraction of the channel capacity obtained by conventional schemes. In [22], a family of detectors were devised through Bayesian inference for uplink massive MIMO systems with mixed ADCs. In [23], the authors investigated channel estimation in Mixed-ADC architecture and analyzed the spectral efficiency. Mixed-ADC massive MIMO systems were studied under Rician fading channel in [24]. In [25], power allocation algorithms for mixed-ADC massive MIMO systems were proposed. In [26], the impact of ADCs was studied in the presence of IQ imbalance.

In the design of massive MIMO systems with Uniform-ADC or Mixed-ADC, a key topic is how to choose the optimal resolution bits for ADCs. In [13], it is found that 4–8 bits are energy-efficient. In [27, 28], efficient optimal quantization-bit allocation algorithms are designed for cell-free and mmWave MIMO systems. However, these works are mostly based on simulations and few theoretical results are provided.

In this work, we consider a general model for massive MIMO system with low-resolution ADCs that covers both Uniform-ADC and Mixed-ADC architectures. The additive quantization noise model (AQNM) is used to model the quantization process that is widely used in the literature [17, 18]. In contrast to previous work, we aim to provide more theoretical guidance for implementing low-resolution ADCs in massive MIMO systems. The asymptotic achievable rates of linear detectors (i.e., MRC, ZF, and LMMSE) are provided as functions of the ADC quantization bits. Most of the analytical results are presented in very simple forms such that more insights could be discovered. Based on the theoretical results, we formulate the optimization problem for the allocation of ADC bits with a constraint on total power consumption. By leveraging the theory of majorization, an approximate solution is derived in closed form. The main contributions of this work are summarized as follows:

(i) The asymptotic achievable rates of MRC and ZF detectors are derived in closed form. It is found that the received signal-to-interference-plus-noise ratio (SINR) is closely related to the inverse of quantization coefficients. Moreover, there is an upper bound for the SINR in the high transmit signal-to-noise ratio (SNR) region

(ii) For the LMMSE detection, approximations of the achievable rates are provided for both high and low transmit SNR region in very simple forms

(iii) The optimal ADC quantization-bit allocation is studied with or without a total power constraint. By using majorization theory, we propose simple design criteria for massive MIMO systems with low-resolution ADCs for MRC, ZF, and LMMSE receivers

(iv) Monte-Carlo simulations are carried out. The theoretical analysis and the proposed quantization-bit allocation criteria are verified by numerical results

The rest of the paper is organized as follows. The system model is described in Section 2. In Section 3, the achievable rates for MRC, ZF, and LMMSE receivers are derived. In Section 4, the optimal allocation of resolution bits is investigated. The numerical results are given in Section 5 and conclusions are drawn in Section 6.

2. System Model

As shown in Figure 1, we consider the uplink of a single-cell massive MIMO system with a BS which employs an array of M antennas and serves K single-antenna users. The received signal $y \in \mathbb{C}^M$ at the BS is described as

$$y = \sqrt{p_u} H x + n, \tag{1}$$

where $H \in \mathbb{C}^{M\times K}$ denotes the channel matrix between the BS and the users; $x = [x_1, \cdots, x_K]^T$ represents the transmitted symbol vector of all the users, the entries of which are identically independently distributed (i.i.d.) and follow the distributions of $\mathscr{CN}(0, 1)$, i.e., circularly symmetric complex Gaussian with zero mean and unit variance; p_u is the average transmit power of each user; $n \sim \mathscr{CN}(0, \sigma_n^2 I_K)$ represents the additive Gaussian white noise with zero mean and covariance given by $\sigma_n^2 I_K$.

The wireless channel H is modeled as

$$H = GD^{1/2}, \tag{2}$$

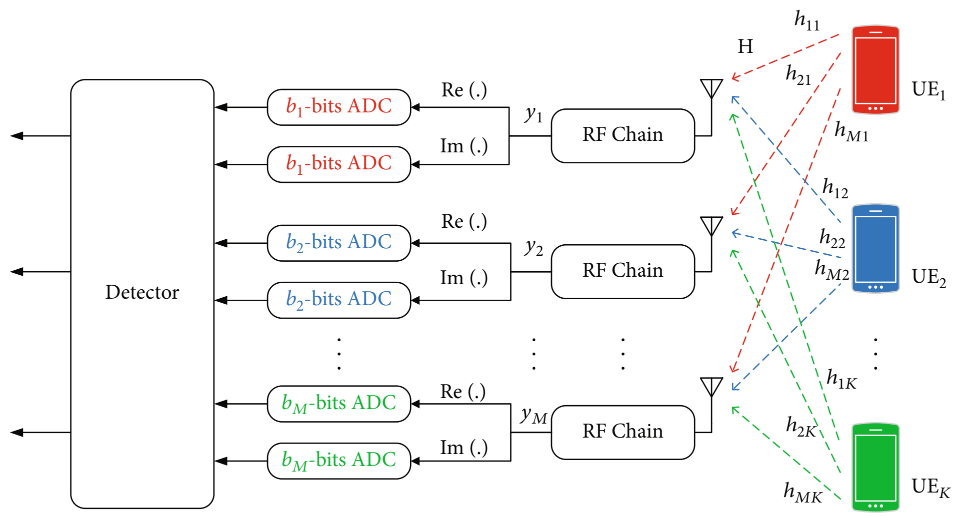

FIGURE 1: System model of the uplink of a massive MIMO system with ADCs with an arbitrary number of bits.

where $G \in \mathbb{C}^{M\times K}$ stands for small-scale fading with i.i.d entries that follow the distributions of $\mathscr{CN}(0, 1/M)$; D is a $K \times K$ diagonal matrix with diagonal entries β_n $(n = 1, \cdots, K)$, which models both geometric attenuation and shadow fading. β_n is assumed to be known at the receiver and is constant across the antenna array since the BS antennas are considered to be colocated.

In order to study the optimal allocation of bits of ADCs, we consider that each antenna at the BS is connected to an ADC with arbitrary resolutions. The numbers of ADC resolution bits of all antennas can be expressed as $b = [b_1, b_2, \cdots, b_M]^T$, where b_m can take any integer value from 1 to the maximum resolution bits $b_{\max}$.

The received signal at the BS after quantization is obtained using AQNM in [12] as

$$y_q = Ay + n_q = \sqrt{p_u} AHx + \mathrm{An} + n_q, \tag{3}$$

with diagonal matrix $A = \mathrm{diag}\,\{\alpha_1, \cdots, \alpha_M\}$ and $\alpha_m = 1 - \rho_m$, where ρ_m is the inverse of the signal-to-quantization-noise ratio. n_q is the additive Gaussian quantization noise vector that is uncorrelated with y. According to [12], the values of ρ_m are listed in Table 1 for $b_m \le 5$ and can be approximated by $\rho_m = (\pi\sqrt{3}/2) \cdot 2^{-2b_m}$ for $b_m > 5$. The covariance of the quantization noise n_q is given by

$$R_{n_q n_q} = \mathbb{E}\left\{n_q n_q^H\right\} = A(I_M - A)\mathbb{E}\left\{\mathrm{diag}\left(p_u HH^H + \sigma_n^2 I_M\right)\right\}. \tag{4}$$

Since the entries of G follow the distribution of $\mathscr{CN}(0, 1/M)$, each of the diagonal elements of $\mathbb{E}\{HH^H\}$ is obtained as L/M with L being the trace of D, i.e. $L = \mathrm{Tr}(D)$. Therefore, the covariance of the quantization noise n_q is obtained as

$$R_{n_q n_q} = A(I_M - A)\left(p_u L/M + \sigma_n^2\right). \tag{5}$$

TABLE 1: Distortion factors for different quantization bits.

b_m	1	2	3	4	5
ρ_m	0.3634	0.1175	0.03454	0.009497	0.002499

In this paper, the ADC resolution bits are considered to be arbitrary, thus to cover all cases of Uniform-ADC and Mixed-ADC schemes.

3. Achievable Rate Analysis for Linear Receivers

In this section, the uplink achievable rates of each user in massive MIMO systems with low-resolution ADCs and linear receivers (i.e., MRC, ZF, and LMMSE) are derived. An asymptotic analysis is carried out and simplified expressions for the achievable rates are also provided.

3.1. Achievable Rates Analysis for MRC Receivers. The receive filter for MRC is given by $W_{\mathrm{MRC}} = H^H A^{-1}$, where A^{-1} is introduced for ADC calibration so as to simplify signal processing, such as channel estimation. Then the estimate of the signal is obtained as

$$r_{\mathrm{MRC}} = \sqrt{p_u} H^H A^{-1} AHx + H^H A^{-1} \mathrm{An} + H^H A^{-1} n_q. \tag{6}$$

The estimated signal of user k is derived as

$$r_{\mathrm{MRC},k} = \sqrt{p_u} h_k^H h_k x_k + \sqrt{p_u} \sum_{i=1, i\ne k}^{K} h_k^H h_i x_i + h_k^H n + h_k^H A^{-1} n_q, \tag{7}$$

where h_k is the k-th column of H. According to (7), the achievable rates of MRC are derived and summarized in the following proposition.

Proposition 1. *When M and K are large, the achievable rate of user k for MRC receivers is given by $R_{MRC,k} \longrightarrow \log_2(1 + SINR_{MRC,k})$, where*

$$SINR_{MRC,k} = \frac{(M+1)p_u\beta_k}{p_u(L-\beta_k)+Tr(E)}, \tag{8}$$

where $E = \sigma_n^2 I_M + (p_u L/M + \sigma_n^2)(A^{-1} - I_M)$.

Proof. According to (7), the signal power of user k is obtained as

$$\mathbb{E}\left(p_u h_k^H h_k h_k^H h_k\right) = p_u \beta_k^2 \mathbb{E}\left(\|g_k\|_2^2\right) = \frac{(M+1)p_u\beta_k^2}{M}, \tag{9}$$

where g_k denotes the k-th column of G. The noise power is obtained as $\mathbb{E}(h_k^H nn^H h_k) = \beta_k\sigma_n^2$. Similarly, the interference power is derived as $\sum_{i=1,i\neq k}^{K}\mathbb{E}(p_u h_k^H h_i h_i^H h_k) = \beta_k p_u (L-\beta_k)/M$. The quantization noise power is given by

$$\begin{aligned}\mathbb{E}\left(h_k^H A^{-1} R_{n_q n_q} A^{-1} h_k\right) &= \mathbb{E}\left[h_k^H\left(-\sigma_n^2 I_M + E\right)h_k\right] \\ &= \frac{\beta_k}{M}\mathrm{Tr}(E) - \beta_k\sigma_n^2.\end{aligned} \tag{10}$$

Then the SINR is obtained straightforwardly as in (8). □

As can be seen from Proposition 1, the low-resolution ADCs affect $\mathrm{SINR}_{\mathrm{MRC},k}$ through E. Denoting the transmit SNR as $\mathrm{SNR}_t = p_u/\sigma_n^2$, there is an upper bound of $\mathrm{SINR}_{\mathrm{MRC},k}$ in the high SNR_t region.

Corollary 2. *The asymptotic achievable rates of user k for MRC receivers is upper bounded by*

$$SINR_{MRC,k}^{UB} = \frac{M(M+1)\beta_k}{M(L-\beta_k) + LTr\left(A^{-1} - I\right)}. \tag{11}$$

Proof. The proof is obtained by letting $p_u \longrightarrow \infty$. □

Corollary 2 indicates besides interuser interference, in the high SNR_t region the achievable rates are limited by low-resolution ADCs and different choices of A greatly affect the performance of MRC receivers.

3.2. Achievable Rates Analysis for ZF Receivers. For ZF receivers, the receive filter is given by $W_{\mathrm{ZF}} = (H^H H)^{-1} H^H A^{-1}$. Then the estimate of the signal is obtained as

$$\begin{aligned} r_{\mathrm{ZF}} &= \sqrt{p_u}\left(H^H H\right)^{-1} \mathrm{H}^{\mathrm{H}} A^{-1} A x + \left(H^H H\right)^{-1} H^H A^{-1} n \\ &\quad + \left(H^H H\right)^{-1} H^H A^{-1} n_q = \sqrt{p_u} x + \left(H^H H\right)^{-1} H^H n \\ &\quad + \left(H^H H\right)^{-1} H^H A^{-1} n_q. \end{aligned} \tag{12}$$

The expression for the achievable rates of ZF receivers are given in the following proposition.

Proposition 3. *When M and K are large, the achievable rate of user k for ZF receivers is given by* $R_{ZF,k} \longrightarrow \log_2(1 + SINR_{ZF,k})$, *where* $SINR_{ZF,k}$ *is given by*

$$SINR_{ZF,k} = \frac{(M-K)p_u\beta_k}{Tr(E)}, \tag{13}$$

where E is the same as in (8).

Proof. For ZF receivers, the estimated signal of user k is derived from (12) as

$$r_{\mathrm{ZF},k} = \sqrt{p_u} x_k + \left(H^H H\right)_k^{-1} H^H n + \left(H^H H\right)_k^{-1} H^H A^{-1} n_q, \tag{14}$$

where $\left(H^H H\right)_k^{-1}$ is the k-th row of $\left(H^H H\right)^{-1}$. Then the desired signal power is p_u and the interference power is 0. Now we only need to compute the noise and quantization noise power, which is given by $\mathbb{E}\{(H^H H)_k^{-1} H^H nn^H H [(H^H H)_k^{-1}]^{\mathrm{H}}\} = \mathbb{E}[(H^H H)_{kk}^{-1}]\sigma_n^2 =^{(a)} (M/M-K)\beta_k^{-1}\sigma_n^2$, where (a) follows the property of central Wishart's matrix.

The quantization noise power is obtained as

$$\begin{aligned}
&\mathbb{E}\left\{\left(H^H H\right)_k^{-1} H^H A^{-1} n_q n_q^H A^{-1} H\left[\left(H^H H\right)_k^{-1}\right]^H\right\} \\
&= \left(p_u L/M + \sigma_n^2\right)\frac{1}{K}\mathbb{E}\mathrm{Tr}\left[\left(H^H H\right)^{-1} H^H\left(A^{-1} - I_M\right) H\left(H^H H\right)^{-1}\right] \\
&= \left(p_u L/M + \sigma_n^2\right)\frac{1}{K}\mathbb{E}\mathrm{Tr}\left[H\left(H^H H\right)^{-2} H^H\left(A^{-1} - I_M\right)\right] \\
&= \left(p_u L/M + \sigma_n^2\right)\frac{1}{K}\mathbb{E}\left[H_{[m]}\left(H^H H\right)^{-2} H_{[m]}^H\right] Tr\left(A^{-1} - I_M\right) \\
&= \left(p_u L/M + \sigma_n^2\right)\mathrm{Tr}\left(A^{-1} - I_M\right)\frac{1}{K}\frac{1}{M}\mathbb{E}\mathrm{Tr}\left(H\left(H^H H\right)^{-2} H^H\right) \\
&= \left(p_u L/M + \sigma_n^2\right)\mathrm{Tr}\left(A^{-1} - I_M\right)\frac{\beta_k^{-1}}{M-K},
\end{aligned} \tag{15}$$

where $H_{[m]}$ is the m-th row of H.

By combining the above results, the SINR of user k is easily obtained as (13). □

Similarly as MRC, ZF receivers also have an upper bound in the high SINR_t region.

Corollary 4. *The asymptotic achievable rates of user k for ZF receiver with ADC calibration is upper bounded by*

$$SINR_{ZF,k}^{UB} = \frac{M(M-K)\beta_k}{LTr\left(A^{-1} - I\right)}. \tag{16}$$

Proof. The proof is obtained by letting $p_u \longrightarrow \infty$. □

Unlike MRC, the interuser interference diminishes for ZF. However, the achievable rates of ZF receivers are limited by low-resolution ADCs and carefully choosing A may help improve the performance.

3.3. Achievable Rates Analysis for LMMSE Receivers. We first derive the LMMSE receive filter W_{LMMSE} by solving the optimization problem $W_{\mathrm{LMMSE}} = \arg\min_W \mathbb{E}\|x - W y_q\|^2$, the solution to which is expressed as $W_{\mathrm{LMMSE}} = R_{xy_q} R_{y_q y_q}^{-1}$, where

$$R_{xy_q} = \mathbb{E}\left\{xy_q^H\right\} = \sqrt{p_u}H^H A^H,$$
$$R_{y_q y_q} = \mathbb{E}\left\{y_q y_q^H\right\} = p_u A H H^H A^H + \sigma_n^2 A A^H + R_{n_q n_q}. \tag{17}$$

Denote $Q \triangleq [p_u H H^H + \sigma_n^2 I_M + (p_u L/M + \sigma_n^2)(A^{-1} - I_M)]^{-1}$ and then we have

$$W_{\mathrm{LMMSE}} = H^H Q A^{-1} \tag{18}$$

where $\sqrt{p_u}$ is omitted since it does not affect the analysis. Note that compared with the traditional LMMSE receive filter, there are additional terms that are related to A in W_{LMMSE}. This makes the achievable rates analysis completely different, especially for Mixed-ADC schemes where the entries of A take different values. Note that the term $A^{-1} - I_M$ exists in both high and low SNR region. Therefore, LMMSE behaves differently from ZF and MRC which will become clear in the following sections.

The estimate of the signal is given by

$$\begin{aligned} r_{\mathrm{LMMSE}} &= \sqrt{p_u} W_{\mathrm{LMMSE}} A H x + W_{\mathrm{LMMSE}} \mathrm{An} + W_{\mathrm{LMMSE}} n_q \\ &= \sqrt{p_u} H^H Q H x + H^H Q n + H^H Q A^{-1} n_q, \end{aligned} \tag{19}$$

The estimated signal of user k is thus obtained by simple algebraic manipulations as

$$r_{\mathrm{LMMSE},k} = f_{k,k}\sqrt{p_u}x_k + \sum_{i=1,i\neq k}^{K} f_{k,i}\sqrt{p_u}x_i + f_{n_k} + f_{q_k}, \tag{20}$$

where $f_{k,i} = h_k^H Q h_i$ $(i = 1, \cdots, K)$ is the effective gain for the signal of user i; $f_{n_k} = h_k^H Q n$ and $f_{q_k} = h_k^H Q A^{-1} n_q$ are the effective white noise and quantization noise, respectively.

Following a similar procedure as in [29, 30], given any allocation of bits in the ADCs, i.e., A, the SINR of user k can thus be approximated as

$$\mathrm{SINR}_{\mathrm{LMMSE},k} = \frac{p_u \left|\mathbb{E}\left(f_{k,k}\right)\right|^2}{p_u \operatorname{var}\left(f_{k,k}\right) + \mathbb{E}\left(p_u \sum_{i\neq k}\left|f_{k,i}\right|^2 + \left|f_{n_k}\right|^2 + \left|f_{q_k}\right|^2\right)}. \tag{21}$$

Assuming Gaussian signaling, the expression for the achievable rates of the k-th user $R_{\mathrm{LMMSE},k}$ is given by $R_{\mathrm{LMMSE},k} = \log_2(1 + \mathrm{SINR}_{\mathrm{LMMSE},k})$.

An accurate analytical result of $\mathrm{SINR}_{\mathrm{LMMSE},k}$ is derived in our previous work and summarized as Proposition 1 in [30]. This result is derived under the assumption that $M \longrightarrow \infty$ and $K \longrightarrow \infty$, but it provides accurate approximation for limited values of M and K, though the expression is complicated. In this paper, we provide simplified expressions in Proposition 5.

Proposition 5. *For LMMSE receivers, the SINR of user k is approximated in the low SNR_t region by*

$$SINR_{LMMSE,k}^{L} \approx \frac{p_u\beta_k\left[Tr\left(E^{-1}\right)\right]^2}{p_u(L-\beta_k)Tr\left(E^{-2}\right) + MTr\left(E^{-1}\right)}, \tag{22}$$

and in the high SNR_t region by

$$SINR_{LMMSE,k}^{H} \approx \frac{p_u\beta_k Tr\left(E^{-1}\right)}{M}. \tag{23}$$

Proof. Let us rewrite $W_{\mathrm{LMMSE}} = (I_K + p_u D^{1/2} G^H E^{-1} G D^{1/2})^{-1} H^H E^{-1} A^{-1}$. Since the elements of G, i.e., g_{mk} are i.i.d and follows the distribution of $\mathscr{CN}(0, 1/M)$, according to Lemma 4 in [31], W_{LMMSE} is approximated by

$$W_{\mathrm{LMMSE}} \approx \left[I_K + p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}D\right]^{-1} H^H E^{-1} A^{-1}, \tag{24}$$

where the off-diagonal elements of $G^H E^{-1} G$ is omitted since they have little impact on the the overall matrix-inverse in W when p_u is small in the low SNR_t region.

Therefore, the received signal can be approximated by

$$r \approx \sqrt{p_u}\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}D + I_K\right]^{-1} H^H E^{-1}\left(Hx + n + A^{-1}n_q\right). \tag{25}$$

Consequently, the received signal of user k is obtained as

$$r_k = \sqrt{p_u}\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-1} h_k^H E^{-1}\left(h_k x_k + \sum_{i\neq k}^{K} h_i x_i + n + A^{-1}n_q\right). \tag{26}$$

In (26), the desired signal power is given by

$$\begin{aligned} &\mathbb{E}\left\{\left|\sqrt{p_u}\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-1} h_k^H E^{-1} h_k\right|^2\right\} \\ &\overset{(a)}{\approx} \left|\sqrt{p_u}\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-1}\frac{\beta_k\mathrm{Tr}\left(E^{-1}\right)}{M}\right|^2. \end{aligned} \tag{27}$$

where (a) follows Lemma 4 in [31]. The interference power is obtained as

$$\begin{aligned} &\mathbb{E}\left\{\sum_{i=1,i\neq k}^{K}\sqrt{p_u}\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-1} h_k^H E^{-1} h_i h_i^H E^{-1} h_k\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-1}\sqrt{p_u}\right\} \\ &= p_u\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-2}\mathbb{E}\left(h_k^H E^{-1} H_{-k} H_{-k}^H E^{-1} h_k\right) \\ &= p_u\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-2}\frac{L-\beta_k}{M}\mathbb{E}\left(h_k^H E^{-1} E^{-1} h_k\right) \\ &\overset{(a)}{\approx} p_u\left[p_u\frac{\mathrm{Tr}\left(E^{-1}\right)}{M}\beta_k + 1\right]^{-2}\frac{1}{M^2}\beta_k(L-\beta_k)\mathrm{Tr}\left(E^{-2}\right), \end{aligned} \tag{28}$$

where (a) is obtained using Lemma 4 in [31]. Similarly, the noise power is obtained as

$$\mathbb{E}\left\{\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-1}h_k^HE^{-1}nn^HE^{-1}h_k\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-1}\right\} \overset{(a)}{\approx}\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-2}\frac{1}{M}\beta_k\sigma_n^2\mathrm{Tr}(E^{-2}), \quad (29)$$

in which (a) follows Lemma 4 in [31]. The quantization noise power is given by

$$\begin{aligned}&\mathbb{E}\left\{\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-1}h_k^HE^{-1}A^{-1}n_qn_q^HA^{-1}E^{-1}h_k\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-1}\right\}\\&=\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-2}\mathbb{E}\left[h_k^HE^{-1}(I_M-A)(p_uL/M+\sigma_n^2)A^{-1}E^{-1}h_k\right]\\&\overset{(a)}{\approx}\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-2}\beta_k(p_uL/M+\sigma_n^2)\frac{1}{M}\mathrm{Tr}\left[E^{-1}(I_M-A)A^{-1}E^{-1}\right],\end{aligned} \quad (30)$$

where (a) follows Lemma 4 in [31].

According to (27), (28), (29), and (30), we obtain the approximate SINR of the user k in the low SNR_t region as

$$\mathrm{SINR}_k^L\approx\frac{p_u\mathrm{Tr}^2(E^{-1})\beta_k}{p_u(L-\beta_k)\mathrm{Tr}(E^{-2})+\sigma_n^2M\mathrm{Tr}(E^{-2})+M(p_uL/M+\sigma_n^2)\mathrm{Tr}\left[E^{-2}(A^{-1}-I_M)\right]}. \quad (31)$$

Since $E=\sigma_n^2I_M+(p_uL/M+\sigma_n^2)(A^{-1}-I_M)$, we have $A^{-1}=(1/p_uL/M+\sigma_n^2)(E+p_uL/MI_M)$. Substituting A^{-1} into (31) yields (22) in Proposition 5.

Regarding the high SNR_t region, W_{LMMSE} is approximated by

$$\begin{aligned}W_{\mathrm{LMMSE}}&=\left(I_K+p_uH^HE^{-1}H\right)^{-1}H^HE^{-1}A^{-1}\\&\approx\left(p_uH^HE^{-1}H\right)^{-1}H^HE^{-1}A^{-1},\end{aligned} \quad (32)$$

where I_K is omitted since it has little impact when p_u is significantly large.

Similarly as in the low SNR_t region, the received signal in the high SNR_t region can be expressed as

$$\begin{aligned}r&=\sqrt{p_u}W_{\mathrm{LMMSE}}AHx+W_{\mathrm{LMMSE}}\mathrm{An}+W_{\mathrm{LMMSE}}n_q\\&=\sqrt{p_u}\left(p_uH^HE^{-1}H\right)^{-1}H^HE^{-1}\left(Hx+n+A^{-1}n_q\right)\\&\approx\frac{1}{\sqrt{p_u}}x+\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}D\right]^{-1}H^HE^{-1}n\\&\quad+\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}D\right]^{-1}H^HE^{-1}A^{-1}n_q.\end{aligned} \quad (33)$$

Then the received signal of user k can be approximated by

$$\begin{aligned}r_k&\approx\frac{1}{\sqrt{p_u}}x_k+\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-1}h_k^HE^{-1}n\\&\quad+\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k+1\right]^{-1}h_k^HE^{-1}A^{-1}n_q.\end{aligned} \quad (34)$$

The desired signal power is given by

$$\frac{1}{p_u}\mathbb{E}(x_kx_k^*)=\frac{1}{p_u}. \quad (35)$$

The interference power is omitted and the noise power is given by

$$\mathbb{E}\left\{\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-1}h_k^HE^{-1}nn^HE^{-1}h_k\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-1}\right\} \overset{(a)}{\approx}\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-2}\frac{1}{M}\beta_k\sigma_n^2\mathrm{Tr}(E^{-2}), \quad (36)$$

in which (a) is obtained using Lemma 4 in [31]. The quantization noise is given by

$$\begin{aligned}&\mathbb{E}\left\{\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-1}h_k^HE^{-1}A^{-1}n_qn_q^HA^{-1}E^{-1}h_k\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-1}\right\}\\&=\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-2}h_k^HE^{-1}(I_M-A)(p_uL/M+\sigma_n^2)A^{-1}E^{-1}h_k\\&\overset{(a)}{\approx}\left[p_u\frac{\mathrm{Tr}(E^{-1})}{M}\beta_k\right]^{-2}\beta_k(p_uL/M+\sigma_n^2)\frac{1}{M}\mathrm{Tr}\left[E^{-1}(I_M-A)A^{-1}E^{-1}\right],\end{aligned} \quad (37)$$

where (a) follows Lemma 4 in [31].

Combining (35), (36), and (37), the approximated SINR of user k in high SNR_t region is obtained as

$$\mathrm{SINR}^{H}_{\mathrm{LMMSE},k} \approx \frac{1}{Mp_u\beta_k\left[p_u\mathrm{Tr}\left(E^{-1}\right)\beta_k\right]^{-2}\left\{\sigma_n^2\mathrm{Tr}\left(E^{-2}\right) + \left(p_uL/M + \sigma_n^2\right)\mathrm{Tr}\left[E^{-2}\left(A^{-1} - I_M\right)\right]\right\}}. \tag{38}$$

From (38), it easy to obtain (23). □

Due to the impact of low-resolution ADCs, the achievable rates of LMMSE receiver have an upper bound in the high SNR_t region.

Corollary 6. *The SINR of user k is upper bounded in the high SNR_t region as*

$$SINR^{H}_{LMMSE,k} \leq \frac{\beta_k}{L} Tr\left[\left(A^{-1} - I_M\right)^{-1}\right]. \tag{39}$$

Proof. The proof is obtained by letting $p_u \longrightarrow \infty$. □

From Proposition 5, it is found that ADCs with higher resolution will lead to smaller diagonal entries of E, thus resulting in better performance. When the channel gain β_k is larger, $\mathrm{SINR}_{\mathrm{LMMSE},k}$ becomes higher as expected.

Corollary 7. *Let $\bar{\mathrm{SNR}}_r \triangleq p_uL/\sigma_n^2K$ be the average received SNR of all users at the BS and denote $\gamma \triangleq K/M$. Among all the users, the performance of users that have improved channel gain will become better while that of other users will decrease as $\bar{\mathrm{SNR}}_r$ increases.*

Proof. It is ready to see that $\mathrm{SINR}_{\mathrm{LMMSE},k}$ always increases with β_k. In the high SNR region, from (23) we get

$$\begin{aligned}\mathrm{SINR}^{H}_{k} &\approx \frac{p_u\beta_k}{M}\mathrm{Tr}\left\{\left[\sigma_n^2 I_M + \left(\frac{p_uL}{M} + \sigma_n^2\right)\left(A^{-1} - I_M\right)\right]^{-1}\right\} \\ &= \frac{p_u\beta_k}{M}\mathrm{Tr}\left\{\left[\left(\frac{p_uL}{M} + \sigma_n^2\right)A^{-1} - \frac{p_uL}{MI_M}\right]^{-1}\right\},\end{aligned} \tag{40}$$

Let $\mathrm{SNR}_{r,k}$ be the received SNR of user k at the BS. By simple algebraic manipulations, we obtain

$$\begin{aligned}\mathrm{SINR}^{\mathrm{H}}_{k} &\approx \frac{p_u\beta_k}{\sigma_n^2}\frac{1}{M}\mathrm{Tr}\left\{\left[\left(\gamma\bar{\mathrm{SNR}}_r + 1\right)A^{-1} - \gamma\bar{\mathrm{SNR}}_r I_M\right]^{-1}\right\} \\ &= \mathrm{SNR}_{r,k}\frac{1}{M}\mathrm{Tr}\left\{\left[\left(\gamma\bar{\mathrm{SNR}}_r + 1\right)A^{-1} - \gamma\bar{\mathrm{SNR}}_r I_M\right]^{-1}\right\} \\ &= \mathrm{SNR}_{r,k}\frac{1}{M}\sum_{m=1}^{M}\frac{1}{\left(\gamma\bar{\mathrm{SNR}}_r + 1\right)\alpha_i^{-1} - \gamma\bar{\mathrm{SNR}}_r}.\end{aligned} \tag{41}$$

It is apparent from (41) that for fixed values of A and $\mathrm{SNR}_{r,k}$, the receive SINR will decrease with larger values of $\bar{\mathrm{SNR}}_r$.

The proof for the low SNR region is similar by using the fact that $(\partial/\partial X_{ii})[\mathrm{Tr}(X)]^2 > (\partial/\partial X_{ii})\mathrm{Tr}(X^2)$ for any positive definite matrix X with diagonal elements given by X_{ii}. □

Remarks. When there are a large number of users, $\bar{\mathrm{SNR}}_r$ is determined by their topology in the cell. Corollary 7 reveals how users interfere with each other due to the impact of low-resolution ADCs. For some specific user j, if β_j becomes larger which indicates a better channel condition, then $\mathrm{SINR}_{\mathrm{LMMSE},j}$ will improve. However, the SINR for all the other users will decrease due to the additional quantization noise caused by increased signal power of user j.

4. Optimal Allocation of Quantization Bits

In this section, the optimal allocation of quantization bits is studied. Unlike previous work in [13, 27] and [28], where the optimal resolutions for ADCs are either obtained by simulations or numerical algorithms, in this work, we aim to find theoretical criteria for the optimal quantization bits of all ADCs.

Conjecture 1. *For any two allocations of bits of ADCs A_1 and A_2 that satisfy $Tr(A_1) = Tr(A_2)$, denote α_1 and α_2 as the vectors formed by diagonal entries of A_1 and A_2, respectively. If α_1 is majorized by α_2 (i.e., $\alpha_1 \prec \alpha_2$), the achievable rates corresponding to A_1 is less than that of A_2 for LMMSE, but is greater than the latter for MRC and ZF.*

Let us start with LMMSE in low SNR_t region. After simple scaling and expansion we get

$$\frac{p_u\beta_k}{p_u\left(L - \beta_k\right) + M\mathrm{Tr}^{-1}\left(E^{-1}\right)} \leq \mathrm{SINR}^{L}_{\mathrm{LMMSE},k} \leq \frac{p_u\beta_k\mathrm{Tr}\left(E^{-1}\right)}{p_u\left(L - \beta_k\right) + M}. \tag{42}$$

Since E is a diagonal matrix, we can rewrite $\mathrm{Tr}(E^{-1})$ as $\mathrm{Tr}(E^{-1}) = \sum_{m=1}^{M}(1/\sigma_n^2 + (p_uL/M + \sigma_n^2)(\alpha_m^{-1} - 1))$, where the second order derivative of each term in the summation is given by

$$\frac{\partial^2}{\partial\alpha_m^2}E^{-1}_{mm} = \frac{2p_uL/M\left(p_uL/M + \sigma_n^2\right)}{\left[\left(p_uL/M + \sigma_n^2\right) - \alpha_m p_uL/M\right]^3}. \tag{43}$$

When $\alpha_m < 1$, (43) is larger than 0. Therefore, each term in the summation in $\mathrm{Tr}(E^{-1})$ is convex over α_m, and $\mathrm{Tr}(E^{-1})$ is a convex function of α_m's. According to the theory of majorization [32], it is obtained that when $\mathrm{Tr}(A)$ is fixed, $\mathrm{Tr}[E^{-1}(\alpha_1)] < \mathrm{Tr}[E^{-1}(\alpha_2)]$ if $\alpha_1 \prec \alpha_2$. Consequently, both the upper bound and lower bound of SINR_k^L for α_1 are higher, and thus it is reasonable to result in better performance for A_1. The analysis for MRC, ZF, and LMMSE in the high SN R_t region is similar and thus omitted.

Conjecture 1 reveals that the optimal architecture for LMMSE is Mixed-ADC with polarized quantization bits. In this case, it tends to use combination of one-bit ADCs and those with the highest resolution. For MRC and ZF, however, completely opposite conclusion is obtained, where the optimal solution is Uniform-ADC that uses ADCs with all the same resolution. Conjecture 1 indicates that Mixed-ADC does not always outperform Uniform-ADC. It depends on the specific detection method used at the receiver.

Based on Conjecture 1, we propose Majorization-based Allocation of Bits with Fixed Trace of A (MAB-FT) algorithm to find the optimal ADC quantization bits b^*. In Algorithm 1, $b_{\mathrm{value}} = [b_{\max}, b_{\max} - 1, \cdots, 1]^T$ is the group of ADC quantization bits that can be used. The quantization coefficient α_{value} is computed according to Table 1. The proposed algorithm can be considered "optimal" in the sense that it finds b^* that majorizes the other ADCs resolution profiles and thus provides the maximal achievable rates with fixed $\mathrm{Tr}(A)$.

In order to provide insights on the performance of massive MIMO systems with low-resolution ADCs, we need to take the ADC power consumption into account, i.e., $p_{\mathrm{ADC}}(b_m) = cW2^{b_m}$, where W is the sampling rate and c is the energy consumption per conversion step. Therefore, the following optimization problem is brought about as

$$\begin{aligned} \mathrm{P1}: \quad & \max_{\boldsymbol{b}} \quad R_k \\ & \text{s.t.} \quad \sum_{m=1}^{M} p_{\mathrm{ADC}}(b_m) = C, \end{aligned} \tag{44}$$

where C is the maximum power consumption of all ADCs. The objective function can be $R_{\mathrm{LMMSE},k}$, $R_{\mathrm{MRC},k}$, or $R_{\mathrm{ZF},k}$. By solving Problem P1, we aim to find the optimal quantization-bit allocation with the ADC power-consumption constraint.

Conjecture 2. *Assuming that the amount of ADC power consumption is fixed to C and $\rho_m = c_1 2^{-2b_m}$. The solution to P1 with MRC and ZF receivers is to use Uniform-ADC. However, the solution for LMMSE receivers is given by*

$$\begin{cases} \text{Mixed} - \text{ADC} & \text{if } \gamma c_1 \bar{\mathrm{SNR}}_r > 3 \cdot 2^{2b_{\max}}, \\ \text{Uniform} - \text{ADC} & \text{if } \gamma c_1 \bar{\mathrm{SNR}}_r < 12, \end{cases} \tag{45}$$

where the optimal Mixed-ADC scheme has ADC resolutions that mostly spread out.

Let us start with the proof for MRC and ZF. In order to maximize the achievable rates, we need to select A that minimizes $\mathrm{Tr}(E)$. Denote $x_m = 2^{b_m}$ and we obtain $\alpha_m = 1 - c_1 x_m^{-2}$. Therefore, P1 is reformulated as

$$\begin{aligned} & \min \quad \mathrm{Tr}(E) \\ & \text{s.t.} \quad \sum_{m=1}^{M} x_m = \frac{C}{cW}. \end{aligned} \tag{46}$$

The i-th diagonal entry of E is rewritten as $E_{ii} = \sigma_n^2 + (p_u L/M + \sigma_n^2) c_1 f(x_m)$, where $f(x) = (x^2 - c_1)^{-1}$, the second order derivative of which is given by $\mathrm{d}^2 f/\mathrm{d}x^2 = 6x^2 + 2c_1/(x^2 - c_1)^3$. According to the values of c_1, in most cases we have $x_m^2 > c_1$. Therefore, $\mathrm{d}^2 f/\mathrm{d}x^2 > 0$ and thus $f(x)$ is convex over x. Consequently, E_{ii} is convex over x_m. Then it is readily shown that $\mathrm{Tr}(E)$ is convex over x_m's. According to the theory of majorization, $\mathrm{Tr}(E)$ is minimized when x_m's take the same value, which leads to a Uniform-ADC scheme.

Regarding LMMSE, the optimization problem can be reformulated as

$$\begin{aligned} & \max \quad \mathrm{Tr}\left(E^{-1}\right) \\ & \text{s.t.} \quad \sum_{m=1}^{M} x_m = \frac{C}{cW}. \end{aligned} \tag{47}$$

The i-th diagonal entry of E^{-1} is rewritten as $E_{ii}^{-1} = (1/\sigma_n^2) + (p_u L/M + \sigma_n^2) c_1 \sigma_n^{-2} g(x_m)$, where $g(x) = -(x^2 + p_u L M^{-1} c_1 \sigma_n^{-2})^{-1}$, the second order derivative of which is given by $\mathrm{d}^2 g/\mathrm{d}x^2 = 2p_u L M^{-1} c_1 \sigma_n^{-2} - 6x^2/(x^2 + p_u L M^{-1} c_1 \sigma_n^{-2})^3$. It is now straightforward to prove when $\gamma c_1 \bar{\mathrm{SNR}}_r > 3 \cdot 2^{b_{\max}}$, $\mathrm{Tr}(E^{-1})$ is convex over x_m's. According to [32], the optimal solution to P1 is Mixed-ADC with polarized quantization bits. Similarly, when $\gamma c_1 \bar{\mathrm{SNR}}_r < 6$, $\mathrm{Tr}(E^{-1})$ is concave over x_m's and the optimal solution becomes Uniform-ADC that uses the same number of resolution bits across the antennas.

It can be seen from Conjecture 2, the optimal structure for MRC and ZF is Uniform-ADC, while that for LMMSE depends on the value of γ, c_1, and $\bar{\mathrm{SNR}}_r$. With fixed value of C, the optimal allocation of bits for Mixed-ADC can be computed similarly as Algorithm 1 by replacing the constraint. We refer to it as Majorization-based Allocation of Bits with Fixed Power consumption (MAB-FP). Details of the algorithm is omitted for conciseness.

Remark. Although it would be interesting to see the optimal bit allocation under the constraint of total number of quantization bits, this is no easy task due to the truth that there is no closed-form expression for α_m as a function of b_m. Even though we can extend the approximation $\alpha_m = 1 - \rho_m = 1 - (\sqrt{3}\pi/2) \cdot 2^{-2b_m}$ for $b_m \geq 5$ to arbitrary value of b_m, it is still very complicated to obtain useful results. Since the paper is already lengthy, we defer the detailed analysis to future work.

Input: M, $\mathbf{b}_{\text{value}}$, $\mathrm{Tr}(\mathbf{A})$;
  $i = 1$, $j = 0$, $\alpha_{\text{sum}} = 0$, $I = \text{length}(\mathbf{b}_{\text{value}})$;
  Compute α_{value} according to Table 1;
  While $i < I$ **do**
    If $\alpha_{\text{sum}} + \alpha_{\text{value}}[i] + (M-1) * \alpha_{\text{value}}[I] \le \mathrm{Tr}(\mathbf{A})$ **then**
      $j = j + 1$;
      $M = M - 1$;
      $\alpha_{\text{sum}} = \alpha_{\text{sum}} + \alpha_{\text{value}}[i]$;
      $\mathbf{b}^*[i] = \mathbf{b}_{\text{value}}[i]$;
    Else
      $i = i + 1$;
    End if
  End while
  $\mathbf{b}^*[j+1 : j+M] = \mathbf{b}_{\text{value}}[i]$;
Output: $\mathbf{b}^*$;

Algorithm 1: Majorization based Allocation of Bits for ADCs with Fixed $\mathrm{Tr}(\mathbf{A})$ (MAB-FT).

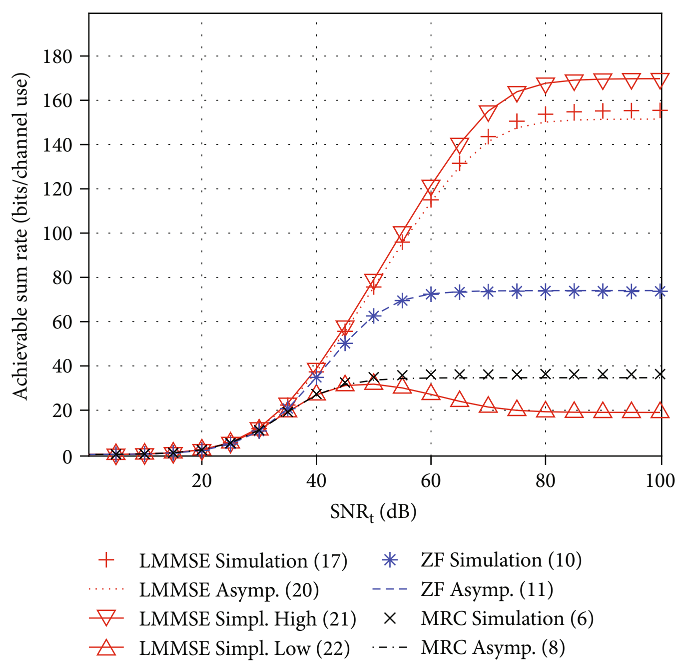

Figure 2: Achievable sum rate versus the transmit SNR, where $M = 128$ and $K = 20$.

5. Numerical Results

In this section, we carry out Monte-Carlo simulations to verify the correctness of the theoretical analysis in previous sections. All users are considered to be randomly distributed in a circular cell with a radius of 1000 meters, except for a central disk of radius $r_h = 100$ meters. The large-scale fading is modeled as $\beta_k = z_k/(r_k/r_{\mathrm{h}})^{\nu}$, where z_k is a log-normal variable with standard deviation $\sigma_{\text{shadow}} = 8$ dB, r_k is the distance between the user k and the BS, and $\nu = 4.8$ is the path loss exponent. In all the simulations of low-resolution ADCs, $b_{\max}$ is set to 6. The noise power spectrum density is set to be -174 dBm/Hz and the $c = 494$ fJ for all ADCs. The number of BS antennas and users are set to be $M = 128$ and $K = 20$, respectively, unless otherwise specified.

Figure 2 illustrates the achievable sum rates with respect to the transmit SNR. The sum rates for LMMSE, ZF, and MRC increase with respect to SNR$_t$ and then level out in the high SNR$_t$ region, due to extra quantization noise of ADCs caused by high-input signal power. The upper bounds in high SNR$_t$ region are also derived in (11), (16), and (39). It can be seen from Figure 2 that the asymptotic analysis for ZF and MRC match well with the simulation results. For LMMSE, (22) provides accurate approximation in low SNR$_t$ region and (23) is accurate for modest SNR$_t$ and is slightly higher than the simulation results when SNR$_t$ is very high. Despite an approximation error within about 10% for $M = 128$, (25) shows the same trend as the simulation results. Therefore, it is still very helpful for analysis.

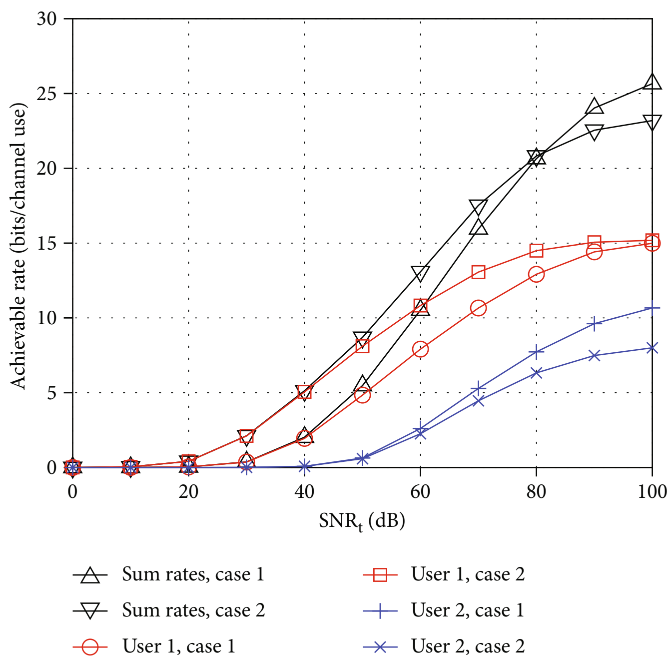

FIGURE 3: Achievable rates of two user where $M = 128$, $r_1 = 500$ m and $r_2 = 700$ m for Case 1 and $r_1 = 300$ m and $r_2 = 700$ m for Case 2.

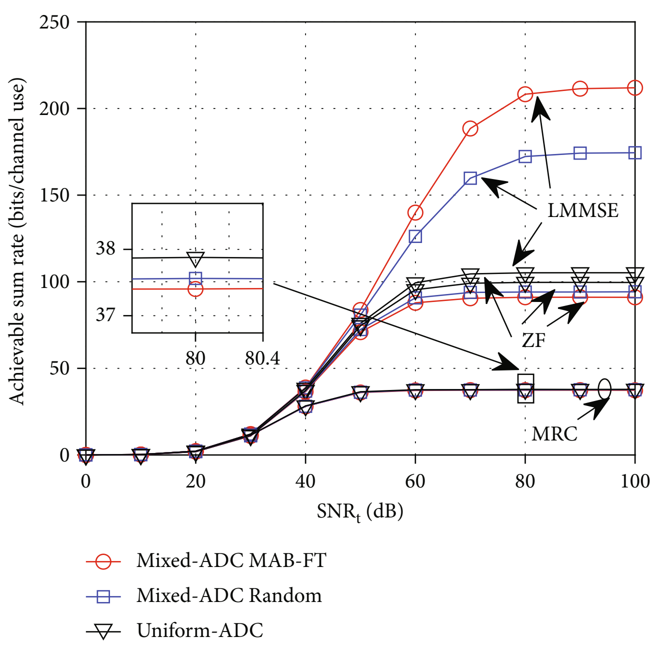

FIGURE 4: Achievable sum rates comparisons for different quantization-bit allocation with $\mathrm{Tr}(A) = 120$.

Figure 3 demonstrates how the increased channel gain of a user affects the performance of both users for LMMSE. As the distance between user 1 and the BS decreases, β_1 increases, which leads to a higher achievable rates of user 1. However, as described in Corollary 7, the performance of user 2 will decrease due to the increased quantization noise. Different from systems with perfect quantization, when low-resolution ADCs are employed, in addition to interuser interference, users can affect each other through the quantization noise.

Figure 4 shows the comparison of different allocation schemes of quantization bits for ADCs in terms of achievable sum rates, where "Mixed-ADC Random" randomly allocates a number of bits to each ADC with equal probability. In the simulations, $\mathrm{Tr}(A)$ is set to be 120. For LMMSE, the proposed MAB-FT performs the best, while uniform-ADC is the worst. In contrast, Uniform-ADC provides the highest sum rates and outperforms Mixed-ADC schemes for MRC and ZF. The simulation results show that Conjecture 1 is correct.

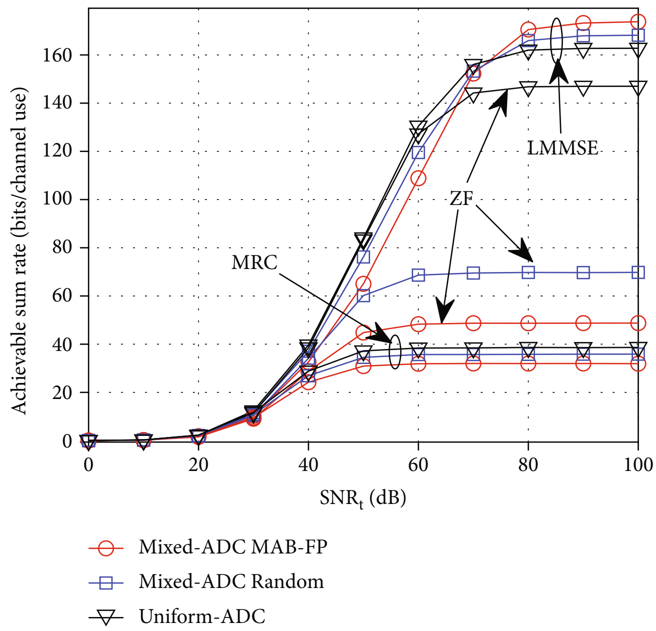

FIGURE 5: Achievable sum rates comparisons for different quantization-bit allocation with fixed total power.

Figure 5 shows the comparison for different quantization-bit allocation with fixed total power $C = 30.4$ mW. It can be seen that the "Uniform-ADC" scheme performs the best for MRC and ZF. For LMMSE, according to (45), when SN $R_t > 74$ dB, the optimal allocation scheme for LMMSE is the proposed MAB-FP, while "Uniform-ADC" is the best when $\mathrm{SNR}_t < 44$ dB. Figure 5 demonstrates that Conjecture 2 provides sufficient conditions for the optimal allocation schemes for LMMSE.

6. Conclusion

In this paper, the uplink performance of MRC, ZF, and LMMSE receivers have been investigated for massive MIMO systems with low-resolution ADCs. Leveraging AQNM and random matrix theory, the asymptotic achievable rates for MRC, ZF, znd LMMSE receivers have been derived as functions of the ADC quantization bits in closed and simple forms. Based on the theoretical analysis, we have proposed two quantization-bit allocation criteria for all the three detection schemes with or without the ADC power constraint. Results indicate that Uniform-ADC is optimal for MRC and ZF, while for LMMSE it depends on c_1 and the ratio of K to M. Numerical results verified the correctness of the theoretical analysis.

Data Availability

Data are available on request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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