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Massless higher spin cubic vertices in flat four dimensional space

M.V. Khabarov[a] and Yu.M. Zinoviev[a,b]

[a] *Institute for High Energy Physics of National Research Center "Kurchatov Institute", Protvino, Moscow Region, 142281, Russia*

[b] *Moscow Institute of Physics and Technology (State University), Dolgoprudny, Moscow Region, 141701, Russia*

E-mail: maksim.khabarov@ihep.ru, yurii.zinoviev@ihep.ru

Abstract: In this paper we construct a number of cubic interaction vertices for massless bosonic and fermionic higher spin fields in flat four dimensional space. First of all, we construct these cubic vertices in AdS_4 space using a so-called Fradkin-Vasiliev approach, which works only for the non-zero cosmological constant. Then we consider a flat limit taking care on all the higher derivative terms which FV-approach generates. We restrict ourselves with the four dimensions because this allows us to use the frame-like multispinor formalism which greatly simplifies all calculations and provides a description for bosons and fermions on equal footing.

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Contents

1 Introduction

The construction of the cubic interaction vertices for the higher spin fields is the very first but important step in the investigation of their consistent interactions. The complete classification of all cubic vertices for massless and massive bosonic and fermionic fields were obtained in the light-cone formalism for $d \geq 4$ dimensions by Metsaev [1–3], while the classification for the massless fields in $d = 3$ appeared only quite recently [4, 5]. As for the Lorentz covariant realisation for these vertices, till now most results deal with the massless fields, where the main guiding principle is the gauge invariance, which severely restricts a possible form of the interactions. A lot of interesting results were developed in the so-called metric-like formalism (see e.g. [6–28] for the bosons and [29, 30] for the fermions). As for the frame-like formalism (which usually leads to the much more compact and elegant expressions, especially when one uses the differential form language) the most general results were obtained in [31] (see also [32]) where the generic cubic vertices for the massless bosonic fields with spins $s_1 \geq s_2 \geq s_3$ satisfying a triangular relation $s_1 < s_2 + s_3$ for AdS_d space with $d \geq 4$ have been constructed. The construction was based on the so-called Fradkin-Vasiliev approach [33, 34] where the non-zero cosmological constant plays a crucial role so that taking a flat limit appears to be a non-trivial task.

Let us briefly describe the Fradkin-Vasiliev approach to the construction of cubic vertices. First of all, recall that in the frame-like formalism a massless higher spin field is described by the set of one-forms Φ, each one having its own gauge transformations (schematically)

$$\delta_0 \Phi \sim D\eta + e\eta$$

where e is the background frame. For each one-form a corresponding gauge invariant two-form (curvature) can be constructed

$$\mathcal{R} \sim D\Phi + e\Phi$$

Moreover, for the non-zero cosmological constant the free Lagrangian can be rewritten in the explicitly gauge invariant form

$$\mathcal{L}_0 \sim \sum a_k \mathcal{R}_k \mathcal{R}_k$$

where coefficients a_k are determined by the so-called extra field decoupling conditions.

The construction of the interactions begins with the most general quadratic deformations for the initial curvatures

$$\mathcal{R} \Rightarrow \hat{\mathcal{R}} = \mathcal{R} + \Delta\mathcal{R}, \qquad \Delta\mathcal{R} \sim \Phi\Phi$$

One of the nice features of such approach is that these deformations simultaneously determine the corresponding form for the corrections to the gauge transformations that can be directly read from that of the curvatures

$$\delta_1 \Phi \sim \Phi\eta$$

At this step the main requirement is that these deformed curvatures must transform covariantly

$$\delta\hat{\mathcal{R}} \sim \mathcal{R}\eta$$

Note that the deformation procedure is independent for each of the three fields. Then one has to take the sum of the three Lagrangians, replace initial curvatures by the deformed ones and require that the resulting Lagrangian be gauge invariant. This leads to the relations on the previously independent constants and results in the cubic vertex that is (on-shell) gauge invariant.

Recall that all cubic vertices can be subdivided into three different types. The first one we call "trivially gauge invariant" because they can be written in terms of gauge invariant objects and deform neither gauge transformations nor gauge algebra. The second type — so-called abelian or Chern-Simons like vertices which do have non-trivial corrections to the gauge transformations, but the algebra remains to be abelian. At last, the third type — non-abelian or Yang-Mills type vertices which deform both the gauge transformations and the algebra. In [31] Vasiliev has constructed the most general cubic vertices for the three massless higher spin bosonic fields in $d \geq 4$ dimensions and shown that they appear to be the combinations of the non-abelian and abelian vertices, so that all such vertices from the Metsaev classification [1] satisfying the triangular relation $s_1 < s_2 + s_3$ (assuming $s_1 \geq s_2 \geq s_3$) are reproduced. Since these vertices have different number of derivatives, it is not a trivial task to extract a particular vertex and/or take a flat limit.

The situation is drastically simplified in four dimensions. Indeed, as has been shown by Metsaev [1, 2] (see also [6, 24] for bosonic cubic vertices in four dimensions), all abelian vertices are absent leaving us only the non-abelian ones. In the frame-like formalism this result is easy to understand because the abelian vertices look like $\mathcal{RR}\Phi$ and so must be five-forms. But even in this case to take the flat limit is not so simple because the general procedure described above still generate a lot of terms with a number of derivatives greater than the correct one ($s_1 + s_2 - s_3$ for bosons and $s_1 + s_2 - s_3 - 1$ for fermions). In this paper we restrict ourselves to the four dimensions and use the multispinor frame-like formalism (which greatly simplifies all calculations and allows us to treat bosons and fermions on equal footing) to reconstruct all non-abelian bosonic and fermionic vertices. We have managed to show that all these higher derivative terms combine into total derivatives or cancel on-shell so that we can safely take a flat limit and obtain (surprisingly) simple form for the flat vertices. Note that the procedure for the construction of cubic vertices we use produces only parity even ones, while the results of [24] show that there exist parity odd vertices with the same number of derivatives, How these vertices can be reproduced is still an open question.

The paper is organised as follows. In section 2 we provide all necessary information on the multispinor frame-like description for the massless higher spin bosons and fermions. Sections 3 and 4 contain a number of simple but instructive examples of the vertices with spin-2 and spin-3/2 correspondingly (and, to our opinion, they are of some interest by themselves). Section 5 contains our results for the cubic vertices with arbitrary spin bosons and fermions, while most technical details were moved into two appendices.

Notations and conventions. We use a formalism where all objects are multispinors $\Phi^{\alpha(k)\dot{\alpha}(l)}$, $\alpha, \dot{\alpha} = 1, 2$ which have k completely symmetric undotted and l completely symmetric dotted indices. In all expressions where indices are denoted with the same letter and are placed on the same level, e.g.

$$\Phi^{\alpha(k)}\Psi^{\alpha(l)}$$

they are assumed to be symmetrized and symmetrization is defined as the sum of the minimal number of necessary terms. Besides, all the fields we consider are the one-forms (and the gauge parameters are zero-forms), while all the terms in the Lagrangians are the four-forms. In this, all the wedge product signs $\wedge$ will be systematically omitted.

We work in AdS_4 space (and its flat limit) described by the background frame $e^{\alpha\dot{\alpha}}$ and the background Lorentz covariant derivative D satisfying

$$De^{\alpha\dot{\alpha}} = 0, \qquad DD\Phi^{\alpha(k)\dot{\alpha}(l)} = -\lambda^2[E^{\alpha}{}_{\beta}\Phi^{\alpha(k-1)\beta\dot{\alpha}(l)} + E^{\dot{\alpha}}{}_{\dot{\beta}}\Phi^{\alpha(k)\dot{\alpha}(l-1)\dot{\beta}}] \tag{1.1}$$

where two-forms $E^{\alpha(2)}$ and $E^{\dot{\alpha}(2)}$ are defined as follows

$$e^{\alpha\dot{\alpha}}e^{\beta\dot{\beta}} = \epsilon^{\alpha\beta}E^{\dot{\alpha}\dot{\beta}} + \epsilon^{\dot{\alpha}\dot{\beta}}E^{\alpha\beta} \tag{1.2}$$

2 Kinematics

In this section we provide all necessary information on the frame-like multispinor formalism for the massless higher spin bosonic and fermionic fields.

A massless integer spin-s $s > 2$ boson is described by the set of multispinor one-forms $\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)}$, $0 \le |m| \le s-1$, where $m = 0$ corresponds to the physical field, $m = \pm 1$ — auxiliary ones, while others are the so-called extra fields. All fields have their own gauge transformations:

$$\begin{aligned}
\delta\Omega^{\alpha(2s-2)} &= D\eta^{\alpha(2s-2)} + \lambda^2 e^{\alpha}{}_{\dot{\alpha}}\eta^{\alpha(2s-3)\dot{\alpha}} \\
\delta\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= D\eta^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} + e_{\beta}{}^{\dot{\alpha}}\eta^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)} \\
&\quad + \lambda^2 e^{\alpha}{}_{\dot{\beta}}\eta^{\alpha(s-2+m)\dot{\alpha}(s-1-m)\dot{\beta}} \\
\delta H^{\alpha(s-1)\dot{\alpha}(s-1)} &= D\eta^{\alpha(s-1)\dot{\alpha}(s-1)} + e_{\beta}{}^{\dot{\alpha}}\eta^{\alpha(s-1)\beta\dot{\alpha}(s-2)} + e^{\alpha}{}_{\dot{\beta}}\eta^{\alpha(s-2)\dot{\alpha}(s-1)\dot{\beta}}
\end{aligned} \tag{2.1}$$

Moreover, for each field a gauge invariant two-form can be constructed:

$$\begin{aligned}
\mathcal{R}^{\alpha(2s-2)} &= D\Omega^{\alpha(2s-2)} + \lambda^2 e^{\alpha}{}_{\dot{\alpha}}\Omega^{\alpha(2s-3)\dot{\alpha}} \\
\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= D\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} + e_{\beta}{}^{\dot{\alpha}}\Omega^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)} \\
&\quad + \lambda^2 e_{\alpha}{}^{\dot{\beta}}\Omega^{\alpha(s-2+m)\dot{\alpha}(s-1-m)\dot{\beta}} \\
\mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)} &= DH^{\alpha(s-1)\dot{\alpha}(s-1)} + e_{\beta}{}^{\dot{\alpha}}\Omega^{\alpha(s-1)\beta\dot{\alpha}(s-2)} + e^{\alpha}{}_{\dot{\beta}}\Omega^{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}}
\end{aligned} \tag{2.2}$$

We refer to such two-forms as curvatures. These curvatures satisfy the following differential identities:

$$\begin{aligned}
D\mathcal{R}^{\alpha(2s-2)} &= -\lambda^2 e^{\alpha}{}_{\dot{\alpha}}\mathcal{R}^{\alpha(2s-3)\dot{\alpha}} \\
D\mathcal{R}^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= -e_{\beta}{}^{\dot{\alpha}}\mathcal{R}^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)} - \lambda^2 e_{\alpha}{}^{\dot{\beta}}\mathcal{R}^{\alpha(s-2+m)\dot{\alpha}(s-1-m)\dot{\beta}} \\
D\mathcal{T}^{\alpha(s-1)\dot{\alpha}(s-1)} &= -e_{\beta}{}^{\dot{\alpha}}\mathcal{R}^{\alpha(s-1)\beta\dot{\alpha}(s-2)} - e^{\alpha}{}_{\dot{\beta}}\mathcal{R}^{\alpha(s-1)\dot{\alpha}(s-1)\dot{\beta}}
\end{aligned} \tag{2.3}$$

On-shell all the curvatures, except the highest ones, are zero, while the highest one satisfy

$$D\mathcal{R}^{\alpha(2s-2)} \approx 0, \qquad e_{\beta}{}^{\dot{\alpha}}\mathcal{R}^{\alpha(2s-3)\beta} \approx 0 \tag{2.4}$$

Note that zero-curvature conditions imply that on-shell

$$\begin{aligned}
DH^{\alpha(s-1)\dot{\alpha}(s-1)} &= -e_{\beta}{}^{\dot{\alpha}}\Omega^{\alpha(s-1)\beta\dot{\alpha}(s-2)} - h.c. \\
D\Omega^{\alpha(s-1+m)\dot{\alpha}(s-1-m)} &= -e_{\beta}{}^{\dot{\alpha}}\Omega^{\alpha(s-1+m)\beta\dot{\alpha}(s-2-m)} + O(\lambda^2)
\end{aligned} \tag{2.5}$$

Hence, on-shell the auxiliary field expresses the non-zero derivatives of the physical field, the extra field $\Omega^{\alpha(s+1)\dot\alpha(s-3)}$ expresses the non-zero derivatives of the auxiliary field etc. The field $\Omega^{\alpha(s-1+m)\dot\alpha(s-1-m)}$ thus expresses the mth derivatives of the physical field which do not vanish on-shell. Whenever we talk about the number of derivatives, we imply the number of derivatives of the physical field and count the mth extra field as an mth derivative.

At last, the free Lagrangian can be written in the explicitly gauge invariant form

$$\begin{aligned}\mathcal{L}_0 = i(-1)^s \sum_{m=1}^{s-1} \frac{(2s-2)!}{(s-1+m)!(s-1-m)!\lambda^{2m}} \Big[&\mathcal{R}_{\alpha(s-1+m)\dot\alpha(s-1-m)}\mathcal{R}^{\alpha(s-1+m)\dot\alpha(s-1-m)} \\ &- \mathcal{R}_{\alpha(s-1-m)\dot\alpha(s-1+m)}\mathcal{R}^{\alpha(s-1-m)\dot\alpha(s-1+m)}\Big]\end{aligned} \tag{2.6}$$

Note that the torsion $\mathcal{T}^{\alpha(s-1)\dot\alpha(s-1)}$ is absent in this expression. Formally, this Lagrangian contains a lot of higher derivative terms. However, due to the smart choice of the coefficients (coming from the so-called extra fields decoupling conditions) all these terms vanish (up to the total derivatives). So written in components the Lagrangian reduces to the usual form in terms of the physical and auxiliary fields only. In particular, it does not contain any terms singular in the flat limit $\lambda \to 0$. Recall also that in the multispinor formalism we use parity operation simply interchanges the dotted and undotted indices and so it correlates with the conjugation. The choose made (with the imaginary unit i and minus sign) takes into account that the Lagrangian being four-form implicitly contains a Levi-Civita symbol.

A massless half-integer spin-s $s > 3/2$ fermion is described by a set of multispinor one-forms $\Phi^{\alpha(s-1+m)\dot\alpha(s-1-m)}$, $1/2 \le |m| \le s-1$, where $m = \pm 1/2$ correspond to the physical fields, all others being the extra ones. The gauge transformations look very similar to the bosonic case the main difference is the transformation for the physical fields:

$$\begin{aligned}\delta\Phi^{\alpha(2s-2)} &= D\zeta^{\alpha(2s-2)} + \lambda^2 e^\alpha{}_{\dot\alpha}\zeta^{\alpha(2s-3)\dot\alpha} \\ \delta\Phi^{\alpha(s-1+m)\dot\alpha(s-1-m)} &= D\zeta^{\alpha(s-1+m)\dot\alpha(s-1-m)} + e_\beta{}^{\dot\alpha}\zeta^{\alpha(s-1+m)\beta\dot\alpha(s-2-m)} \\ &\quad + \lambda^2 e^\alpha{}_{\dot\beta}\zeta^{\alpha(s-2+m)\dot\alpha(s-1-m)\dot\beta} \\ \delta\Phi^{\alpha(s-1/2)\dot\alpha(s-3/2)} &= D\zeta^{\alpha(s-1/2)\dot\alpha(s-3/2)} + e_\beta{}^{\dot\alpha}\zeta^{\alpha(s-1/2)\beta\dot\alpha(s-5/2)} + \lambda e^\alpha{}_{\dot\beta}\zeta^{\alpha(s-3/2)\dot\alpha(s-3/2)\dot\beta}\end{aligned} \tag{2.7}$$

Similarly, a set of the gauge invariant two-forms can be constructed:

$$\begin{aligned}\mathcal{F}^{\alpha(2s-1)} &= D\Phi^{\alpha(2s-1)} + \lambda^2 e^\alpha{}_{\dot\alpha}\Phi^{\alpha(2s-2)\dot\alpha} \\ \mathcal{F}^{\alpha(s-1+m)\dot\alpha(s-1-m)} &= D\Phi^{\alpha(s-1+m)\dot\alpha(s-1-m)} + e_\beta{}^{\dot\alpha}\Phi^{\alpha(s-1+m)\beta\dot\alpha(s-2-m)} \\ &\quad + \lambda^2 e^\alpha{}_{\dot\beta}\Phi^{\alpha(s-2+m)\dot\alpha(s-1-m)\dot\beta} \\ \mathcal{F}^{\alpha(s-1/2)\dot\alpha(s-3/2)} &= D\Phi^{\alpha(s-1/2)\dot\alpha(s-3/2)} + e_\beta{}^{\dot\alpha}\Phi^{\alpha(s-1/2)\beta\dot\alpha(s-5/2)} + \lambda e^\alpha{}_{\dot\beta}\Phi^{\alpha(s-3/2)\dot\alpha(s-3/2)\dot\beta}\end{aligned} \tag{2.8}$$

The differential identities for them have the form:

$$\begin{aligned}D\mathcal{F}^{\alpha(2s-2)} &= -\lambda^2 e^\alpha{}_{\dot\alpha}\mathcal{F}^{\alpha(2s-3)\dot\alpha} \\ D\mathcal{F}^{\alpha(s-1+m)\dot\alpha(s-1-m)} &= -e_\beta{}^{\dot\alpha}\mathcal{F}^{\alpha(s-1+m)\beta\dot\alpha(s-2-m)} - \lambda^2 e^\alpha{}_{\dot\beta}\mathcal{F}^{\alpha(s-2+m)\dot\alpha(s-1-m)\dot\beta} \\ D\mathcal{F}^{\alpha(s-1/2)\dot\alpha(s-3/2)} &= -e_\beta{}^{\dot\alpha}\mathcal{F}^{\alpha(s-1/2)\beta\dot\alpha(s-5/2)} - \lambda e^\alpha{}_{\dot\beta}\mathcal{F}^{\alpha(s-3/2)\dot\alpha(s-3/2)\dot\beta}\end{aligned} \tag{2.9}$$

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On-shell all these curvatures, except the highest ones, are zero, while the highest ones satisfy

$$D\mathcal{F}^{\alpha(2s-2)} \approx 0, \qquad e_\beta{}^{\dot\alpha} \mathcal{F}^{\alpha(2s-3)\beta} \approx 0 \tag{2.10}$$

Again, the zero-curvature conditions imply that the field $\Phi^{\alpha(s-1/2+m)\dot\alpha(s-3/2-m)}$ expresses the mth derivatives of the physical field $\Phi^{\alpha(s-1/2)\dot\alpha(s-3/2)}$ which do not vanish on-shell.

At last, the free Lagrangian can be written as

$$\begin{aligned} \mathcal{L}_0 = (-1)^{s+1/2} \sum_{m=1/2}^{s-1} & \frac{(2s-2)!}{(s-1+m)!(s-1-m)!\lambda^{2m}} \\ & \mathcal{F}_{\alpha(s-1+m)\dot\alpha(s-1-m)} \mathcal{F}^{\alpha(s-1+m)\dot\alpha(s-1-m)} + h.c. \end{aligned} \tag{2.11}$$

The same comments on the higher derivative terms, flat limit and parity as above are applicable here, note however that the absence of imaginery unit is related with anticommutativity of fermions.

3 Graviton

In this section we consider all possible vertices with spin-2 field. They will serve as the simple illustration for both the general method and all four possible types of vertices. Besides, interaction with gravity is always of some interest by itself.

We describe a free massless spin-2 field with the one-forms $h^{\alpha\dot\alpha}$, $\omega^{\alpha(2)} + h.c.$ with the initial gauge transformations

$$\begin{aligned} \delta\omega^{\alpha(2)} &= D\eta^{\alpha(2)} - \lambda^2 e^\alpha{}_{\dot\alpha} \xi^{\alpha\dot\alpha} \\ \delta h^{\alpha\dot\alpha} &= D\xi^{\alpha\dot\alpha} + e_\beta{}^{\dot\alpha} \eta^{\alpha\beta} + e^\alpha{}_{\dot\beta} \eta^{\dot\alpha\dot\beta} \end{aligned} \tag{3.1}$$

The corresponding linearized gauge invariant curvature and torsion have the form:

$$\begin{aligned} R^{\alpha(2)} &= D\omega^{\alpha(2)} + \lambda^2 e^\alpha{}_{\dot\alpha} h^{\alpha\dot\alpha} \\ T^{\alpha\dot\alpha} &= Dh^{\alpha\dot\alpha} + e_\beta{}^{\dot\alpha} \omega^{\alpha\beta} + e^\alpha{}_{\dot\beta} \omega^{\dot\alpha\dot\beta} \end{aligned} \tag{3.2}$$

On-shell we have (note the difference with (2.4))

$$T^{\alpha\dot\alpha} \approx 0, \qquad DR^{\alpha(2)} \approx 0, \qquad e_\alpha{}^{\dot\alpha} R^{\alpha\beta} + e^\alpha{}_{\dot\beta} R^{\dot\alpha\dot\beta} \approx 0 \tag{3.3}$$

At last, the free Lagrangian can be written as

$$\mathcal{L}_0 = \frac{i}{\lambda^2} R_{\alpha(2)} R^{\alpha(2)} + h.c. \tag{3.4}$$

There are only two possible types of vertices satisfying the triangular relation, namely $(s+1, s, 2)$ and $(s, s, 2)$. For both of them, the cases with $s = 2$ turn out to be special, resulting in four different cases in total. We consider them in turn.

3.1 Vertex $(s+1, s, 2)$, $s > 2$

We use Σ and $\mathcal{F}$ for the field with spin $s+1$ and its curvatures and Ω and $\mathcal{R}$ — for spin s. Using the general formulas given in appendix A, it easy to construct deformations for the curvatures of all three fields.[1] For the spin $s+1$ components we obtain:

$$\begin{aligned} \Delta\mathcal{F}^{\alpha(2s)} &= a_0\lambda^2\Omega^{\alpha(2s-2)}\omega^{\alpha(2)} \\ \Delta\mathcal{F}^{\alpha(2s-1)\dot\alpha} &= a_0\lambda^2\Omega^{\alpha(2s-2)}h^{\alpha\dot\alpha} + a_0\lambda^2\Omega^{\alpha(2s-3)\dot\alpha}\omega^{\alpha(2)} \\ \Delta\mathcal{F}^{\alpha(2s-2)\dot\alpha(2)} &= a_0\Omega^{\alpha(2s-2)}\omega^{\dot\alpha(2)} + O(\lambda^2) \end{aligned} \tag{3.5}$$

where a_0 is a coupling constant and we always choose normalization so that all coefficients in the deformations are proportional to the positive degree of λ. The only variations of the deformed curvatures that do not vanish on-shell are

$$\delta\hat{\mathcal{F}}^{\alpha(2s)} = a_0\lambda^2\left[\mathcal{R}^{\alpha(2s-2)}\eta^{\alpha(2)} - \eta^{\alpha(2s-2)}R^{\alpha(2)}\right] \tag{3.6}$$

Now we turn to the spin-s components and obtain:

$$\begin{aligned} \Delta\mathcal{R}^{\alpha(2s-2)} &= b_0\Sigma^{\alpha(2s-2)\beta(2)}\omega_{\beta(2)} + 2b_0\lambda^2\Sigma^{\alpha(2s-2)\beta\dot\beta}h_{\beta\dot\beta} + b_0\lambda^2\Sigma^{\alpha(2s-2)\dot\beta(2)}\omega_{\dot\beta(2)} \\ \Delta\mathcal{R}^{\alpha(2s-3)\dot\alpha} &= b_0\Sigma^{\alpha(2s-3)\beta(2)\dot\alpha}\omega_{\beta(2)} + O(\lambda^2) \end{aligned} \tag{3.7}$$

In this case, the variations of the deformed curvatures that do not vanish on-shell are

$$\delta\hat{\mathcal{R}}^{\alpha(2s-2)} = b_0\left[\mathcal{F}^{\alpha(2s-2)\beta(2)}\eta_{\beta(2)} - \zeta^{\alpha(2s-2)\beta(2)}R_{\beta(2)}\right] \tag{3.8}$$

At last, for the spin-2 we get

$$\begin{aligned} \Delta R^{\alpha(2)} &= c_0\Sigma^{\alpha(2)\beta(2s-2)}\Omega_{\beta(2s-2)} + (2s-2)c_0\lambda^2\Sigma^{\alpha(2)\beta(2s-3)\dot\beta}\Omega_{\beta(2s-3)\dot\beta} \\ &\quad + c_0\lambda^2\Sigma^{\alpha(2)\dot\beta(2s-2)}\Omega_{\dot\beta(2s-2)} + O(\lambda^4) \\ \Delta T^{\alpha\dot\alpha} &= c_0\Sigma^{\alpha\beta(2s-2)\dot\alpha}\Omega_{\beta(2s-2)} + c_0\Sigma^{\alpha\dot\alpha\dot\beta(2s-2)}\Omega_{\dot\beta(2s-2)} + O(\lambda^2) \end{aligned} \tag{3.9}$$

with the non-vanishing variations being:

$$\delta\hat{R}^{\alpha(2)} = c_0\left[\mathcal{F}^{\alpha(2)\beta(2s-2)}\eta_{\beta(2s-2)} - \zeta^{\alpha(2)\beta(2s-2)}\mathcal{R}_{\beta(2s-2)}\right] \tag{3.10}$$

Now we take the sum of the free Lagrangians and replace the free curvatures by the deformed ones. The gauge variation of the resulting Lagrangian produces:

$$\begin{aligned} \delta\hat{\mathcal{L}} &= \left[\frac{(-1)^{s+1}s(2s-1)a_0}{\lambda^{2s-2}} + \frac{(-1)^s b_0}{\lambda^{2s-2}}\right]\mathcal{F}_{\alpha(2s-2)\beta(2)}\mathcal{R}^{\alpha(2s-2)}\eta^{\beta(2)} \\ &\quad + \left[\frac{c_0}{\lambda^2} - \frac{(-1)^{s+1}s(2s-1)a_0}{\lambda^{2s-2}}\right]\mathcal{F}_{\alpha(2s-2)\beta(2)}\eta^{\alpha(2s-2)}R^{\beta(2)} \\ &\quad - \left[\frac{c_0}{\lambda^2} + \frac{(-1)^s b_0}{\lambda^{2s-2}}\right]\mathcal{R}_{\alpha(2s-2)}\zeta^{\alpha(2s-2)\beta(2)}R_{\beta(2)} \end{aligned} \tag{3.11}$$

[1] Here and in what follows we provide only the terms which give non-zero contribution to the flat vertices.

Thus the invariance of the deformed Lagrangian requires

$$(-1)^{s+1}s(2s-1)a_0 = \lambda^{2s-4}c_0, \qquad (-1)^s b_0 = -\lambda^{2s-4}c_0 \tag{3.12}$$

Now we consider a cubic vertex that follows from the deformed Lagrangian. Due to the relations on the coupling constants given above we find that the terms with the highest number of derivatives (and singular in the flat limit) combine into the total derivative and can be dropped out. At the next level we obtain terms with the correct number $N = 2s-1$ of derivatives, so we can safely take a flat limit and, after a number of cancellations, obtain a very simple result:

$$\mathcal{L}_1 = c_0 D\omega_{\alpha(2)}\Sigma^{\alpha(2)\dot\alpha(2s-2)}\Omega_{\dot\alpha(2s-2)} + h.c. \tag{3.13}$$

We see that the spin-2 field enters through the gauge invariant curvature, while the invariance of the vertex under the other gauge transformations can be checked using the on-shell identities (3.3) and the corrections to the physical graviton transformations:

$$\delta h^{\alpha\dot\alpha} = c_0\Sigma^{\alpha\dot\alpha\dot\beta(2s-2)}\eta_{\dot\beta(2s-2)} - c_0\zeta^{\alpha\dot\alpha\dot\beta(2s-2)}\Omega_{\dot\beta(2s-2)} + h.c. \tag{3.14}$$

Let us stress that this result holds also for the case when s is half-integer, i.e. both higher spin fields are fermions.

3.2 Vertex $(s, s, 2)$, $s > 2$

In this case the vertex is symmetric on the two spin-s fields, so for simplicity we assume that we have just one such field. The part of the deformation for the spin-s components we need have the form:

$$\begin{aligned}
\Delta\mathcal{R}^{\alpha(2s-2)} &= a_0\Omega^{\alpha(2s-3)\beta}\omega^\alpha{}_\beta + a_0\lambda^2\Omega^{\alpha(2s-3)\dot\beta}h^\alpha{}_{\dot\beta} \\
\Delta\mathcal{R}^{\alpha(2s-3)\dot\alpha} &= a_0\Omega^{\alpha(2s-3)\beta}h^{\dot\alpha}{}_\beta + a_0\Omega^{\alpha(2s-4)\beta\dot\alpha}\omega^\alpha{}_\beta \\
&\quad + a_0\Omega^{\alpha(2s-3)\dot\beta}\omega^{\dot\alpha}{}_{\dot\beta} + O(\lambda^2)
\end{aligned} \tag{3.15}$$

while the non-vanishing variations of the deformed curvatures look like:

$$\delta\hat{\mathcal{R}}^{\alpha(2s-2)} = a_0[\mathcal{R}^{\alpha(2s-3)\beta}\eta^\alpha{}_\beta - \eta^{\alpha(2s-3)\beta}R^\alpha{}_\beta] \tag{3.16}$$

The corresponding expressions for the deformations of spin-2 curvature and torsion are:

$$\begin{aligned}
\Delta R^{\alpha(2)} &= c_0\Omega^{\alpha\beta(2s-3)}\Omega^\alpha{}_{\beta(2s-3)} + c_0\lambda^2\Omega^{\alpha\beta(2s-4)\dot\beta}\Omega^\alpha{}_{\beta(2s-4)\dot\beta} \\
&\quad + c_0\lambda^2\Omega^{\alpha\dot\beta(2s-3)}\Omega^\alpha{}_{\dot\beta(2s-3)} + O(\lambda^4) \\
\Delta T^{\alpha\dot\alpha} &= c_0\Omega^{\alpha\beta(2s-3)}\Omega^{\dot\alpha}{}_{\beta(2s-3)} + c_0\Omega^{\alpha\dot\beta(2s-3)}\Omega^{\dot\alpha}{}_{\dot\beta(2s-3)} + O(\lambda^2)
\end{aligned} \tag{3.17}$$

and for the non-vanishing variations

$$\delta\hat R^{\alpha(2)} \sim 2c_0\mathcal{R}^{\alpha\beta(2s-3)}\eta^\alpha{}_{\beta(2s-3)} \tag{3.18}$$

The invariance of the deformed Lagrangian requires

$$(-1)^s(2s-2)a_0 = 4\lambda^{2s-4}c_0 \tag{3.19}$$

As in the previous case, due to this relation the terms in the cubic vertex with $2s$ derivatives combine into the total derivative and can be dropped out so that we can safely take a flat limit and obtain one more simple result:

$$\mathcal{L}_1 = 2c_0 D\omega_{\alpha\beta}\Omega^{\alpha\dot\alpha(2s-3)}\Omega^\beta{}_{\dot\alpha(2s-3)} + h.c. \tag{3.20}$$

Here the spin-2 also enters only through the gauge invariant curvature, while the invariance under remaining gauge transformations holds due to the on-shell identities (3.3) and the corresponding corrections to the physical graviton transformations:

$$\delta h^{\alpha\dot\alpha} = c_0\Omega^{\alpha\dot\beta(2s-3)}\eta^{\dot\alpha}{}_{\dot\beta(2s-3)} - c_0\eta^{\alpha\dot\beta(2s-3)}\Omega^{\dot\alpha}{}_{\dot\beta(2s-3)} + h.c. \tag{3.21}$$

Note that these results are in agreement with the particular case of the $(3,3,2)$ vertex which has been considered in [35] (see also [7, 10, 11] for the metric-like formulation). Note also that in this case this results works for the fermionic case where s is half-integer as well.

3.3 Vertex $(3,2,2)$

This case is special and provides a simple example of the whole class of vertices where two lower spins are equal. As far as we know, in the metric-like formulation this vertex was considered for the first time in [7], while in the frame-like formalism — in [35]. Note that this vertex is antisymmetric on the spin-2 fields so that we must have two different spin-2 particles.

The deformations for all curvatures have the form now:

$$\begin{aligned}
\Delta\mathcal{F}^{\alpha(4)} &= a_0\Omega^{\alpha(2)}\omega^{\alpha(2)} \\
\Delta\mathcal{F}^{\alpha(3)\dot\alpha} &= a_0\Omega^{\alpha(2)}h^{\alpha\dot\alpha} + a_0 H^{\alpha\dot\alpha}\omega^{\alpha(2)} \\
\Delta\mathcal{R}^{\alpha(2)} &= b_0\Sigma^{\alpha(2)\beta(2)}\omega_{\beta(2)} + 2b_0\lambda^2\Sigma^{\alpha(2)\beta\dot\beta}h_{\beta\dot\beta} + b_0\lambda^2 H^{\alpha(2)\dot\beta(2)}\omega_{\dot\beta(2)} \\
\Delta R^{\alpha(2)} &= c_0\Sigma^{\alpha(2)\beta(2)}\Omega_{\beta(2)} + 2c_0\lambda^2\Sigma^{\alpha(2)\beta\dot\beta}H_{\beta\dot\beta} + c_0\lambda^2 H^{\alpha(2)\dot\beta(2)}\Omega_{\dot\beta(2)}
\end{aligned} \tag{3.22}$$

while non-vanishing variations are:

$$\begin{aligned}
\delta\hat{\mathcal{F}}^{\alpha(4)} &= a_0[\mathcal{R}^{\alpha(2)}\eta^{\alpha(2)} - \zeta^{\alpha(2)}R^{\alpha(2)}] \\
\delta\hat{\mathcal{R}}^{\alpha(2)} &= b_0[\mathcal{F}^{\alpha(2)\beta(2)}\eta_{\beta(2)} - \zeta^{\alpha(2)\beta(2)}R_{\beta(2)}] \\
\delta\hat{R}^{\alpha(2)} &= c_0[\mathcal{F}^{\alpha(2)\beta(2)}\zeta_{\beta(2)} - \zeta^{\alpha(2)\beta(2)}\mathcal{R}_{\beta(2)}]
\end{aligned} \tag{3.23}$$

The invariance of the deformed Lagrangian requires

$$6a_0 = \lambda^2 b_0, \qquad c_0 = -b_0 \tag{3.24}$$

As usual, the terms with 5 derivatives combine into total derivative, while 3-derivative terms give the following flat vertex:

$$\mathcal{L}_1 = b_0 D\Omega_{\alpha(2)}H^{\alpha(2)\dot\alpha(2)}\omega_{\dot\alpha(2)} + 2b_0\Omega_{\alpha(2)}\Sigma^{\alpha(2)\beta\dot\alpha}e_\beta{}^{\dot\beta}\omega_{\dot\alpha\dot\beta} - (\Omega \leftrightarrow \omega) + h.c. \tag{3.25}$$

3.4 Vertex $(2, 2, 2)$

This very well known vertex provides the simplest example of self-interaction, so for completeness we briefly give it here. The curvature deformation looks like:

$$\Delta R^{\alpha(2)} = a_0 \omega^{\alpha\beta} \omega^{\alpha}{}_{\beta} + a_0 \lambda^2 h^{\alpha\dot{\beta}} h^{\alpha}{}_{\dot{\beta}} \tag{3.26}$$

The deformed Lagrangian is automatically gauge invariant. The terms in the cubic vertex with four derivatives combine into the total derivative leaving us with:

$$\mathcal{L}_1 = a_0 D\omega_{\alpha\beta} h^{\alpha\dot{\alpha}} h^{\beta}{}_{\dot{\alpha}} - a_0 e_{\alpha}{}^{\dot{\alpha}} h_{\beta\dot{\alpha}} \omega^{\alpha\gamma} \omega^{\beta}{}_{\gamma} + h.c. \tag{3.27}$$

4 Gravitino

In this section we present two more simple examples — vertices with the spin-3/2 field. Taking into account the even in the higher spin theory the supersymmetry plays a distinguished role, we think they worth to be considered. The spin-3/2 itself is described by the one-forms ψ^{α}, $\psi^{\dot{\alpha}}$ with the gauge invariant two-forms:

$$\begin{aligned} F^{\alpha} &= D\psi^{\alpha} + \lambda e^{\alpha}{}_{\dot{\alpha}} \psi^{\dot{\alpha}} \\ F^{\dot{\alpha}} &= D\psi^{\dot{\alpha}} + \lambda e_{\alpha}{}^{\dot{\alpha}} \psi^{\alpha} \end{aligned} \tag{4.1}$$

and the free Lagrangian

$$\mathcal{L}_0 = \frac{1}{\lambda} F_{\alpha} F^{\alpha} + h.c. \tag{4.2}$$

There are two types of vertices satisfying the strict triangle inequality and corresponding to the two types of the massless supermultiplets — $(s + 1/2, s, 3/2)$ and $(s + 1, s + 1/2, 3/2)$.

4.1 Vertex $(s + 1/2, s, 3/2)$, $s \geq 2$

We begin with the deformations for all curvatures (keeping only necessary terms):

$$\begin{aligned} \Delta\mathcal{F}^{\alpha(2s-1)} &= a_0 \lambda \Omega^{\alpha(2s-2)} \psi^{\alpha} \\ \Delta\mathcal{F}^{\alpha(2s-2)\dot{\alpha}} &= a_0 \Omega^{\alpha(2s-2)} \psi^{\dot{\alpha}} + O(\lambda) \\ \Delta\mathcal{R}^{\alpha(2s-2)} &= b_0 \Phi^{\alpha(2s-2)\beta} \psi_{\beta} + b_0 \lambda \Phi^{\alpha(2s-2)\dot{\beta}} \psi_{\dot{\beta}} \\ \Delta F^{\alpha} &= c_0 \Phi^{\alpha\beta(2s-2)} \Omega_{\beta(2s-2)} + c_0 \lambda \Phi^{\alpha\dot{\beta}(2s-2)} \Omega_{\dot{\beta}(2s-2)} + O(\lambda^2) \end{aligned} \tag{4.3}$$

Non-vanishing variations have the form:

$$\begin{aligned} \delta\hat{\mathcal{F}}^{\alpha(2s-1)} &= a_0 \lambda [\mathcal{R}^{\alpha(2s-2)} \zeta^{\alpha} - \eta^{\alpha(2s-2)} F^{\alpha}] \\ \delta\hat{\mathcal{R}}^{\alpha(2s-2)} &= b_0 [\mathcal{F}^{\alpha(2s-2)\beta} \zeta_{\beta} - \zeta^{\alpha(2s-2)\beta} F_{\beta}] \\ \delta\hat{F}^{\alpha} &= c_0 [\mathcal{F}^{\alpha\beta(2s-2)} \eta_{\beta(2s-2)} - \zeta^{\alpha\beta(2s-2)} \mathcal{R}_{\beta(2s-2)}] \end{aligned} \tag{4.4}$$

The invariance of the deformed Lagrangian requires

$$(-1)^{s+1}(2s-1) a_0 = \lambda^{2s-3} c_0, \qquad (-1)^s b_0 = \lambda^{2s-3} c_0 \tag{4.5}$$

The resulting flat vertex with the correct number of derivatives $N = 2s-2$ (after the higher derivative terms combine into total derivative and were dropped out) takes the form:

$$\mathcal{L}_1 = c_0 D\psi_\alpha \Phi^{\alpha\dot{\alpha}(2s-2)}\Omega_{\dot{\alpha}(2s-2)} + h.c. \tag{4.6}$$

Once again we find that the lowest spin field enters through its gauge invariant curvature only, while to check the invariance under the remaining gauge transformations one has to take into account the corrections to the gravitino gauge transformations:

$$\delta\psi^{\dot{\alpha}} = c_0 \Phi^{\dot{\alpha}\dot{\beta}(2s-2)}\eta_{\dot{\beta}(2s-2)} - c_0 \zeta^{\dot{\alpha}\dot{\beta}(2s-2)}\Omega_{\dot{\beta}(2s-2)} + h.c. \tag{4.7}$$

4.2 Vertex $(s+1, s+1/2, 3/2)$, $s \geq 2$

This case appears to be very similar, so we will be brief. The appropriate deformations look like:

$$\begin{aligned}
\Delta\mathcal{R}^{\alpha(2s)} &= a_0 \lambda \Phi^{\alpha(2s-1)}\psi^\alpha \\
\Delta\mathcal{R}^{\alpha(2s-1)\dot{\alpha}} &= a_0 \Phi^{\alpha(2s-1)}\psi^{\dot{\alpha}} + O(\lambda) \\
\Delta\mathcal{F}^{\alpha(2s-1)} &= b_0 \Omega^{\alpha(2s-1)\beta}\psi_\beta + b_0 \lambda \Omega^{\alpha(2s-1)\dot{\beta}}\psi_{\dot{\beta}} \\
\Delta F^\alpha &= c_0 \Omega^{\alpha\beta(2s-1)}\Phi_{\beta(2s-1)} + c_0 \lambda \Omega^{\alpha\dot{\beta}(2s-1)}\Phi_{\dot{\beta}(2s-1)}
\end{aligned} \tag{4.8}$$

while the relations on the coupling constants are:

$$(-1)^{s+1} 2s a_0 = -\lambda^{2s-2} c_0, \qquad (-1)^{s+1} b_0 = \lambda^{2s-2} c_0 \tag{4.9}$$

The resulting flat cubic vertex with $N = 2s-1$ derivatives appears to be

$$\mathcal{L}_1 = c_0 D\psi_\alpha \Omega^{\alpha\dot{\alpha}(2s-1)}\Phi_{\dot{\alpha}(2s-1)} + h.c. \tag{4.10}$$

The results given above hold only for $s \geq 2$, while the case $s = 1$ turns out to be special (as all cases where two lowest spins are equal). This vertex $(2, 3/2, 3/2)$ is very well known being a part of the $N = 1$ supergravity, but for completeness we briefly provide this vertex in our current formalism.

The deformations now are very simple

$$\begin{aligned}
\Delta R^{\alpha(2)} &= \frac{i}{4} c_0 \lambda \psi^\alpha \psi^\alpha \\
\Delta T^{\alpha\dot{\alpha}} &= \frac{i}{2} c_0 \psi^\alpha \psi^{\dot{\alpha}} \\
\Delta F^\alpha &= c_0 \omega^{\alpha\beta}\psi_\beta + c_0 \lambda h^{\alpha\dot{\alpha}}\psi_{\dot{\alpha}}
\end{aligned} \tag{4.11}$$

and the flat vertex has the form:

$$\mathcal{L}_1 = c_0 D\psi_\alpha h^{\alpha\dot{\alpha}}\psi_{\dot{\alpha}} - c_0 e_\alpha{}^{\dot{\alpha}}\psi_{\dot{\alpha}}\omega^{\alpha\beta}\psi_\beta + h.c. \tag{4.12}$$

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5 Arbitrary spins

In this section we consider general case of three arbitrary spins $s_1 \geq s_2 \geq s_3$. We introduce their convenient combinations:

$$\hat{s}_1 = s_2 + s_3 - s_1 - 1, \quad \hat{s}_2 = s_1 + s_3 - s_2 - 1, \quad \hat{s}_3 = s_1 + s_2 - s_3 - 1 \tag{5.1}$$

Note that if spins $s_{1,2,3}$ satisfy the triangular relations these combinations are always non-negative: $\hat{s}_{1,2,3} \geq 0$. Moreover, even if two of the three fields are fermions and two of the three $s_{1,2,3}$ are half-integer, the corresponding $\hat{s}_{1,2,3}$ are always integer. Let us give here some useful relations on them:

$$\hat{s}_1 + \hat{s}_2 = 2(s_3 - 1), \qquad \hat{s}_1 + \hat{s}_3 = 2(s_2 - 1), \qquad \hat{s}_2 + \hat{s}_3 = 2(s_1 - 1) \tag{5.2}$$

We begin with the bosonic case and then make necessary adjustment for the fermionic one. We use notations Σ, $\mathcal{F}$ for the fields component and curvatures for the highest spin s_1, Ω, $\mathcal{R}$ for spin s_2 and ω, R for the lowest spin s_3 correspondingly.

The deformations for all curvatures of the highest spin s_1 have the form:

$$\Delta\mathcal{F}^{\alpha(2s_1-2-m)\dot{\alpha}(m)} = \sum_{k=0}^{\hat{s}_1} \sum_{l=0}^{\min(m,\hat{s}_2)} a_k \Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)} \omega^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \tag{5.3}$$

where coefficients a_k (see appendix A for details) look like

$$a_k = \frac{(\hat{s}_1)!}{(\hat{s}_1 - k)!k!} a_0 \tag{5.4}$$

Strictly speaking, these coefficients must be multiplied by λ raised to some positive power, but to simplify formulas we temporarily set $\lambda = 1$. We restore them by dimensionality of terms whenever it is necessary.

Similarly, for the two other spins $s_{2,3}$ we consider

$$\Delta\mathcal{R}^{\alpha(2s_2-2-m)\dot{\alpha}(m)} = \sum_{k=0}^{\hat{s}_2} \sum_{l=0}^{min(m,\hat{s}_1)} b_k \Sigma^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_2-k)\dot{\alpha}(m-l)\dot{\beta}(k)} \omega^{\alpha(\hat{s}_1-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_2-k)\dot{\beta}(k)} \tag{5.5}$$

$$\Delta R^{\alpha(2s_3-2-m)\dot{\alpha}(m)} = \sum_{k=0}^{\hat{s}_3} \sum_{l=0}^{\min(m,\hat{s}_1)} c_k \Sigma^{\alpha(\hat{s}_2-m+l)\beta(\hat{s}_3-k)\dot{\alpha}(m-l)\dot{\beta}(k)} \Omega^{\alpha(\hat{s}_1-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_3-k)\dot{\beta}(k)} \tag{5.6}$$

with the corresponding coefficients

$$b_k = \frac{(\hat{s}_2)!}{(\hat{s}_2 - k)!k!} b_0, \qquad c_k = \frac{(\hat{s}_3)!}{(\hat{s}_3 - k)!k!} c_0 \tag{5.7}$$

Now we take a sum of the three Lagrangians, replace the initial curvatures by the deformed ones and require the resulting deformed Lagrangian to be invariant. The non-vanishing on-shell variations have the form:

$$
\begin{aligned}
\delta\hat{\mathcal{F}}^{\alpha(2s_1-2)} &= a_0[\mathcal{R}^{\alpha(\hat{s}_3)\beta(\hat{s}_1)}\eta^{\alpha(\hat{s}_2)}{}_{\beta(\hat{s}_1)} - \eta^{\alpha(\hat{s}_3)\beta(\hat{s}_1)}R^{\alpha(\hat{s}_2)}{}_{\beta(\hat{s}_1)}] \\
\delta\hat{\mathcal{R}}^{\alpha(2s_2-2)} &= b_0[\mathcal{F}^{\alpha(\hat{s}_3)\beta(\hat{s}_2)}\eta^{\alpha(\hat{s}_1)}{}_{\beta(\hat{s}_2)} - \eta^{\alpha(\hat{s}_3)\beta(\hat{s}_2)}R^{\alpha(\hat{s}_1)}{}_{\beta(\hat{s}_2)}] \\
\delta\hat{R}^{\alpha(2s_3-2)} &= c_0[\mathcal{R}^{\alpha(\hat{s}_2)\beta(\hat{s}_3)}\eta^{\alpha(\hat{s}_1)}{}_{\beta(\hat{s}_3)} - \eta^{\alpha(\hat{s}_2)\beta(\hat{s}_3)}R^{\alpha(\hat{s}_1)}{}_{\beta(\hat{s}_3)}]
\end{aligned}
\tag{5.8}
$$

Then the invariance of the Lagrangian requires (for what follows it is important to restore the λ dependence here):

$$
(-1)^{s_1}\frac{(\hat{s}_2+\hat{s}_3)!}{(\hat{s}_2)!(\hat{s}_3)!}\frac{a_0}{\lambda^{2s_1-2}} = -(-1)^{s_2}\frac{(\hat{s}_1+\hat{s}_3)!}{(\hat{s}_1)!(\hat{s}_3)!}\frac{b_0}{\lambda^{2s_2-2}} = (-1)^{s_3}\frac{(\hat{s}_1+\hat{s}_2)!}{(\hat{s}_1)!(\hat{s}_2)!}\frac{c_0}{\lambda^{2s_3-2}} \tag{5.9}
$$

Now let us turn to the cubic vertex. Recall, that all the curvatures except the highest ones, i.e. $\mathcal{F}^{\alpha(2s_1-2)}$, $\mathcal{R}^{\alpha(2s_2-2)}$ and $R^{\alpha(2s_3-2)}$ (and their conjugates), vanish on-shell. So it seems that the simplest way to obtain the cubic vertex is to take into account their deformations only. But this produce a lot of terms with the number of derivatives greater than $N = s_1 + s_2 - s_3$, moreover, their coefficients will be proportional to the negative degrees of λ and so will be singular in the flat limit. Note that due to relation on the constants given above the terms with the highest number of derivatives, namely $s_1 + s_2 + s_3 - 2$ combine into total derivative and can be dropped out, but it still leaves a lot of other dangerous terms (exceptions are the vertices with lowest spin-2 and spin-$3/2$). So before taking a flat limit we must show that all these terms somehow vanish on-shell. It turns out that the best strategy is to keep all the curvatures and all their deformations. In this way we managed to show (see appendix B for details) that all such terms combine into total derivatives or cancel each other so we safely can take a flat limit. The procedure we followed produce also a lot of terms which have the correct number $s_1 + s_2 - s_3$ of derivatives and contribute to the flat vertex. By rather long but straightforward calculations (ones again see appendix B) we reduced the final results to (we dare say) the simplest form possible.

Among all cubic vertices there are four possible types, namely $s_1 > s_2 > s_3$, $s_1 = s_2 > s_3$, $s_1 > s_2 = s_3$ and $s_1 = s_2 = s_3$, and, as we have seen on the simple examples above, have to be considered separately.

5.1 Vertex $s_1 > s_2 > s_3$

First of all note that the relation (5.9) implies that

$$
a_0 \sim \lambda^{2(s_1-s_3)}c_0, \qquad b_0 \sim \lambda^{2(s_2-s_3)}c_0
$$

It means that in the flat limit all deformations for the two higher spins vanish and as a result the flat vertex must be trivially invariant under the lowest spin field gauge transformations. And indeed, we managed to reduce this vertex to very simple form

$$
\mathcal{L}_1 = 2c_0 D\omega_{\alpha(\hat{s}_2)\beta(\hat{s}_1)}\Sigma^{\alpha(\hat{s}_2)\dot{\alpha}(\hat{s}_3)}\Omega^{\beta(\hat{s}_1)}{}_{\dot{\alpha}(\hat{s}_3)} + h.c. \tag{5.10}
$$

where the lowest spin field enters through the gauge invariant curvature. As for the invariance under the other gauge transformations, it can be easily checked with the help of on-shell identities (2.4) or (2.10). Recall that even if the two of the three fields are fermions so that two of the three $s_{1,2,3}$ are half-integer, the combinations $\hat{s}_{1,2,3}$ are always integer and so the formula above works for the fermionic vertices as well.

5.2 Vertex $s_1 = s_2 > s_3$

First of all note that these vertices are symmetric on the two higher spin fields if s_3 is even (so it may be one and the same field) and antisymmetric if s_3 is odd. In all other respects, including considerations on the gauge invariance, they are very similar to the previous case. The flat vertex turns out to be

$$\mathcal{L}_1 = 2c_0 D\omega_{\alpha(s_3-1)\beta(s_3-1)}[\Sigma^{\alpha(s_3-1)\dot\alpha(\hat s_3)}\Omega^{\beta(s_3-1)}{}_{\dot\alpha(\hat s_3)} + (-1)^{s_3}(\Sigma \leftrightarrow \Omega)] + h.c. \tag{5.11}$$

so the lowest spin field also enters only through the gauge invariant curvature. Note also, that in this case the two higher spin fields can be fermions, but lower spin field is always boson.

5.3 Vertex $s_1 > s_2 = s_3$

For the even highest spin s_1 such vertex must be symmetric on the two lower spin ones, so it may be one and the same field, while for the odd s_1 it must be antisymmetric and we must have two different fields with the same spin. We have seen on the simple examples above that this case is indeed special and the vertex has a more complicated form. Indeed, the relations on the coupling constants

$$a_0 \sim \lambda^{2(s_1-s_3)}c_0, \qquad b_0 \sim c_0$$

show that only corrections to the higher spin transformations vanish in the flat limit and so the vertex cannot be trivially gauge invariant under the gauge transformations of the lower spin fields. The most simple result we have managed to obtain looks like:

$$\begin{aligned}\mathcal{L}_1 &= c_0 D\omega_{\alpha(s_1-1)\beta(\hat s_1)}H^{\alpha(s_1-1)\dot\beta(s_1-1)}\Omega^{\alpha(\hat s_1)}{}_{\dot\beta(s_1-1)} \\ &\quad + c_0\sum_{k=0}^{\hat s_1}\frac{(s_1-1)(\hat s_1)!}{(\hat s_1-k)!k!}e^{\gamma}{}_{\dot\gamma}\Sigma_{\alpha(s_1-1)\gamma\dot\alpha(s_1-2)}\Omega^{\alpha(s_1-1)\beta(\hat s_1-k)\dot\beta(k)}\omega^{\alpha(s_1-2)\dot\gamma}{}_{\beta(\hat s_1-k)\dot\beta(k)} \\ &\quad + (-1)^{s_1}(\Omega\leftrightarrow\omega) + h.c\end{aligned} \tag{5.12}$$

The first term has the same structure as in the general case the main difference is that the highest spin enters through its physical component that has different on-shell relations. As a result, the first term is not gauge invariant by itself and the gauge invariance requires that the number of algebraic terms to be added.

Note that in this case the two lower spin fields can be fermions, while the highest spin one is always boson.

5.4 Vertex $s_1 = s_2 = s_3 = s$

Similarly to the previous case, for the even spin s this vertex must be completely symmetric on all three fields so that it may be just one and the same field and the vertex describes its self interaction; for the odd spin s the vertex must be completely antisymmetric so we must have three different fields with the same spin. In this case

$$a_0 \sim b_0 \sim c_0$$

so that the corrections to the gauge transformations for all three fields survive in the flat limit and the resulting vertex looks very similar to the previous one:

$$
\begin{aligned}
\mathcal{L}_1 &= c_0 D \Sigma_{\alpha(s-1)\beta(s-1)} \Phi^{\alpha(s-1)\dot\alpha(s-1)} \phi^{\beta(s-1)}{}_{\dot\alpha(s-1)} \\
&\quad + c_0 \sum_{k=1}^{s-1} \frac{(s-1)(s-1)!}{(s-1-k)!k!} e^{\gamma}{}_{\dot\gamma} \Sigma_{\alpha(s-1)\gamma\dot\alpha(s-2)} \Omega^{\alpha(s-1)\beta(s-1-k)\dot\beta(k)} \omega^{\alpha(s-2)\dot\gamma}{}_{\beta(s-1-k)\dot\beta(k)} \\
&\quad + min.\ perm.(\Sigma, \Omega, \omega) + h.c.
\end{aligned}
\tag{5.13}
$$

Here *min. perm.* stands for the two cyclic permutations of Σ, Ω, ω in the first term and five permutations in the second one. It is clear that such vertices exist only for bosons.

6 Conclusion

In this paper we have constructed a number of non-trivial cubic vertices for the massless higher spin bosonic and fermionic fields in flat four dimensional space. We begin with Fradkin-Vasiliev approach in AdS_4 space and then consider the flat limit. The procedure appears to be not so simple, because we have to take care on all the higher derivative terms, which such approach generates, but the final results happen to be very simple. So we hope that they could be useful for the future investigations. Let us stress once more that the procedure we use produce only parity even vertices, while the construction of the corresponding parity odd ones [24] is an open question.

As one of the future directions we see a construction of the cubic vertices for massive and partially massless fields. The frame-like formalism for such fields is known [36–38], but there are just a few examples of interactions till now [39–44].

One more interesting direction is the cubic vertices for the higher spin massless supermultiplets. Their classification was elaborated quite recently in the light-cone formalism [45, 46], but for the Lorentz covariant realization there are also just a few non-trivial results [47–51].

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A Deformations

In this appendix we calculate the combinatoric coefficients for the deformations of the gauge invariant curvatures. Let us take as an example the curvatures of the highest spin components. Their most general quadratic deformations are given by ansatz (5.3), which we repeat here for the reader convenience:

$$
\Delta \mathcal{F}^{\alpha(2s_1-2-m)\dot\alpha(m)} = \sum_{k=0}^{\hat s_1} \sum_{l=0}^{\min(m,\hat s_2)} a_k \Omega^{\alpha(\hat s_3-m+l)\beta(\hat s_1-k)\dot\alpha(m-l)\dot\beta(k)} \omega^{\alpha(\hat s_2-l)\dot\alpha(l)}{}_{\beta(\hat s_1-k)\dot\beta(k)}
$$

Recall also that

$$\hat{s}_1 = s_2 + s_3 - s_1 - 1, \quad \hat{s}_2 = s_1 + s_3 - s_2 - 1, \quad \hat{s}_3 = s_1 + s_2 - s_3 - 1$$

Now let us consider variations of the deformed curvatures $\hat{\mathcal{F}} = \mathcal{F} + \Delta\mathcal{F}$ under the lowest spin ω gauge transformations. From the ansatz given above we can immediately read the corrections to the gauge transformations:

$$\delta\Sigma^{\alpha(2s_1-2-m)\dot{\alpha}(m)} = a_{k,l,m}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \tag{A.1}$$

Taking into account these corrections, the variations of the deformed curvatures $\hat{\mathcal{F}} = \mathcal{F} + \Delta\mathcal{F}$ appear to be

$$\begin{aligned}
\delta\hat{\mathcal{F}}^{\alpha(2s_1-2-m)\dot{\alpha}(m)} &= a_{k,l,m}D\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + (a_{k,l,m+1} - a_{k,l-1,m})e^{\alpha}{}_{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l-1)\beta(\hat{s}_1-k)\dot{\alpha}(m-l+1)\dot{\beta}(k)} \\
&\qquad\quad \eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l-1)\dot{\gamma}}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + (k+1)a_{k+1,l,m}e^{\gamma}{}_{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k-1)\dot{\alpha}(m-l)\dot{\beta}(k)\dot{\gamma}}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k-1)\gamma\dot{\beta}(k)} \\
&\quad + a_{k,l,m+1}e^{\alpha}{}_{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l-1)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)\dot{\gamma}}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + (a_{k,l,m-1} - a_{k,l+1,m})e_{\gamma}{}^{\dot{\alpha}}\Omega^{\alpha(\hat{s}_3-m+l+1)\beta(\hat{s}_1-k)\dot{\alpha}(m-l-1)\dot{\beta}(k)} \\
&\qquad\quad \eta^{\alpha(\hat{s}_2-l-1)\gamma\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + a_{k,l,m-1}e_{\gamma}{}^{\dot{\alpha}}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\gamma\dot{\alpha}(m-l-1)\dot{\beta}(k)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + (\hat{s}_1-k+1)a_{k-1,l,m}e_{\gamma}{}^{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\gamma\dot{\alpha}(m-l)\dot{\beta}(k-1)} \\
&\qquad\quad \eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k-1)\dot{\gamma}}
\end{aligned} \tag{A.2}$$

The main requirement here is that the deformed curvatures transform covariantly, so we must have

$$\begin{aligned}
\delta\hat{\mathcal{F}}^{\alpha(2s_1-2-m)\dot{\alpha}(m)} &= a_{k,l,m}\mathcal{R}^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&= a_{k,l,m}\Big[D\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + e^{\alpha}{}_{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l-1)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)\dot{\gamma}}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + (\hat{s}_1-k)e^{\gamma}{}_{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k-1)\dot{\alpha}(m-l)\dot{\beta}(k)\dot{\gamma}}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k-1)\gamma\dot{\beta}(k)} \\
&\quad + e_{\gamma}{}^{\dot{\alpha}}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\gamma\dot{\alpha}(m-l-1)\dot{\beta}(k)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \\
&\quad + ke_{\gamma}{}^{\dot{\gamma}}\Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\gamma\dot{\alpha}(m-l)\dot{\beta}(k-1)}\eta^{\alpha(\hat{s}_2-l)\dot{\alpha}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k-1)\dot{\gamma}}\Big]
\end{aligned} \tag{A.3}$$

A comparison of these two expressions gives us a number of recurrent relations on the coefficients $a_{k,l,m}$

$$\begin{aligned}
a_{k,l,m+1} &= a_{k,l-1,m}, & a_{k,l,m-1} &= a_{k,l+1,m} \\
(\hat{s}_1-k+1)a_{k-1,l,m} &= ka_{k,l,m}, & (k+1)a_{k+1,l,m} &= (\hat{s}_1-k)a_{k,l,m} \\
a_{k,l,m+1} &= a_{k,l,m}, & a_{k,l,m-1} &= a_{k,l,m}
\end{aligned}$$

their simple solution being

$$a_{k,l,m} = \frac{(\hat{s}_1)!}{(\hat{s}_1 - k)!k!} a_0 \tag{A.4}$$

Thus the result turns out to be unique up to the one arbitrary coupling constant.

B Flat limit

The main problem with the flat limit is that the formalism we use generates a lot of terms with the number of derivatives greater than that of the flat vertex and their coefficients are singular in the limit $\lambda \to 0$. Our first task here is to show that all such terms combine into total derivatives or vanish on-shell and so they all can be dropped out allowing us to take a desired limit. Let us consider contribution to the cubic vertex from the highest spin field as an example. They have the form (schematically)

$$\Delta\mathcal{L}_1 \sim \sum_m \mathcal{F}_{\alpha(2s_1-2-m)\dot{\alpha}(m)} \Delta\mathcal{F}^{\alpha(2s_1-2-m)\dot{\alpha}(m)}$$

where $\Delta\mathcal{F}$ are given in (5.3). Recall that on-shell each auxiliary or extra field $\Sigma^{\alpha(s_1-1+m_1)\dot{\alpha}(s_1-1-m_1)}$ is equivalent to $|m_1|$ derivatives of the physical one, in this the number of derivatives for each concrete term in the cubic vertex is defined by $N = |m_1| + |m_2| + |m_3| + 1$, where

$$\begin{aligned} m_1 &= s_1 - 1 - m \\ m_2 &= s_2 - 1 - m + l - k \\ m_3 &= s_3 - 1 - k - l \end{aligned} \tag{B.1}$$

Let us consider the contributions with positive m_1, while $m_{2,3}$ can be both positive or negative. Now we consider all four possible cases, calculate the number of derivatives and focus on terms with more than $N_0 = s_1 + s_2 - s_3$ derivatives.

I) $m_2 > 0$, $m_3 > 0$

$$N = s_1 + s_2 + s_3 - 2 - 2m - 2k > s_1 + s_2 - s_3 \Rightarrow k < s_3 - 1 - m$$

II) $m_2 > 0$, $m_3 < 0$

$$N = s_1 + s_2 - s_3 - 2m + 2l > s_1 + s_2 - s_3 \Rightarrow l > m$$

III) $m_2 < 0$, $m_3 > 0$

$$N = s_1 - s_2 + s_3 - 2l > s_1 + s_2 - s_3 \Rightarrow l < s_3 - s_2 < 0$$

IV) $m_2 < 0$, $m_3 < 0$

$$N = s_1 - s_2 - s_3 + 2 + 2k > s_1 + s_2 - s_3 \Rightarrow k > s_2 - 1 > \hat{s}_1$$

So we see that only terms where all three $m_{1,2,3}$ are positive (or all three are negative) generate the higher derivatives terms. Each such contribution looks (schematically)

$$[D\Sigma - e\Sigma - \lambda^2 e\Sigma]\Omega\omega$$

so that we have terms with explicit derivative as well as the purely algebraic ones. Let us begin with terms $D\Sigma\Omega\omega$. Taking into account all combinatoric coefficients (both from the free Lagrangian as well as from the deformation parameters) we obtain

$$\Delta = C_{k,l,m} D\Sigma_{\alpha(\hat{s}_3-m+l)\delta(\hat{s}_2-l)\dot{\alpha}(m-l)\dot{\delta}(l)} \Omega^{\alpha(\hat{s}_3-m+l)\beta(\hat{s}_1-k)\dot{\alpha}(m-l)\dot{\beta}(k)} \omega^{\delta(\hat{s}_2-l)\dot{\delta}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} \tag{B.2}$$

where

$$C_{k,l,m} = \frac{(\hat{s}_2+\hat{s}_3)!(\hat{s}_1)!a_0}{(\hat{s}_3-m+l)!(\hat{s}_2-l)!(m-l)!l!(\hat{s}_1-k)!k!} \tag{B.3}$$

Calculating the inverse relations from the (B.1)

$$m = (s_1-1) - m_1, \qquad k = \frac{\hat{s}_1+\hat{m}_1}{2}, \qquad l = \frac{\hat{s}_2+\hat{m}_2}{2} \tag{B.4}$$

where we have introduced

$$\hat{m}_1 = m_1 - m_2 - m_3, \qquad \hat{m}_2 = m_2 - m_1 - m_3, \qquad \hat{m}_3 = m_3 - m_1 - m_2 \tag{B.5}$$

we can show that the denominator in the expression for $C_{k,l,m}$ can be rewritten as follows:

$$\left(\frac{\hat{s}_1+\hat{m}_1}{2}\right)!\left(\frac{\hat{s}_1-\hat{m}_1}{2}\right)!\left(\frac{\hat{s}_2+\hat{m}_2}{2}\right)!\left(\frac{\hat{s}_2-\hat{m}_2}{2}\right)!\left(\frac{\hat{s}_3+\hat{m}_3}{2}\right)!\left(\frac{\hat{s}_3-\hat{m}_3}{2}\right)!$$

Taking into account the relations on the constants a_0, b_0 and c_0, we see that such contributions are completely symmetric on the three fields. As a result, all such terms with explicit derivative combine into total derivative exactly in the same way as the terms with the highest number of derivatives do.

Now we consider purely algebraic terms of the type $\lambda^2 e\Sigma\Omega\omega$. We obtain

$$\begin{aligned}\Delta_1 = (\hat{s}_3-m+l)C_{k,l,m} e_\gamma{}^{\dot{\gamma}} \Sigma_{\alpha(\hat{s}_3-m+l-1)\delta(\hat{s}_2-l)\dot{\alpha}(m-l)\dot{\delta}(l)\dot{\gamma}} \\ \Omega^{\alpha(\hat{s}_3-m+l-1)\beta(\hat{s}_1-k)\gamma\dot{\alpha}(m-l)\dot{\beta}(k)} \omega^{\delta(\hat{s}_2-l)\dot{\delta}(l)}{}_{\beta(\hat{s}_1-k)\dot{\beta}(k)} + \dots\end{aligned} \tag{B.6}$$

where dots stand for the similar terms with index γ contracted with one of the indices of the field ω. On the other hand, if we take the contribution of the type $e\Omega\Sigma\omega$ from the deformations of the Ω field, we obtain

$$\begin{aligned}\Delta_2 = (\tilde{m}-\tilde{l})\tilde{C}_{\tilde{k},\tilde{l},\tilde{m}} e^\gamma{}_{\dot{\gamma}} \Omega_{\alpha(\hat{s}_3-\tilde{m}+\tilde{l})\delta(\hat{s}_1-\tilde{l})\dot{\alpha}(\tilde{m}-\tilde{l}-1)\dot{\delta}(\tilde{l})} \\ \Sigma^{\alpha(\hat{s}_3-\tilde{m}+\tilde{l})\beta(\hat{s}_2-\tilde{k})\dot{\alpha}(\tilde{m}-\tilde{l}-1)\dot{\beta}(\tilde{k})} \omega^{\delta(\hat{s}_1-\tilde{l})\dot{\delta}(\tilde{l})}{}_{\beta(\hat{s}_2-\tilde{k})\dot{\beta}(\tilde{k})} + \dots\end{aligned} \tag{B.7}$$

where again dots stand for the similar terms where index $\dot{\gamma}$ is contracted with one of the ω indices. We see that the structure of these two contributions is the same provided

$$\tilde{m} = m - l + k + 1, \qquad \tilde{k} = l, \qquad \tilde{l} = k \tag{B.8}$$

The resulting coefficients turn out to be equal so these two terms cancel each other. The same holds for the two other pairs of contractions, namely $(\Sigma\omega)$ and $(\Omega\omega)$.

Thus all the higher derivative terms combine into total derivatives or cancel each other and we may safely take a flat limit. We repeat our considerations but focus this time on the terms with exactly $N_0 = s_1 + s_2 - s_3$ derivatives, i.e. those which do not vanish in the flat limit. We find that there are a lot of such terms with both positive and negative $m_{2,3}$. The situation with positive $m_{2,3}$ appears to be mainly the same as before, so that they also combine into total derivatives or cancel. As for the terms with negative m_2 or/and m_3, after rather long work we have managed to show that most of them can be combined into terms proportional to the gauge invariant curvatures which vanish on-shell. All this leads to the surprisingly simple results presented in the main text.

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